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THE

PHYSICAL REVIEW.

IONIZATION AND EXCITATION OF RADIATION BY ELECTRON IMPACT IN NITROGEN.

By Bergen Davis and F. S. Goucher.

THE purpose of the present paper is to present some of the experimental results obtained in further experiments of the writers on ionization and excitation of radiation by electron impact in various gases.

The method of experimentation employed was in principle that used in connection with our experiments on mercury vapor and hydrogen and the reader is referred to our paper for a detailed description of the same.[1]

This method is simply a modification of that applied by Frank and Hertz and others for the determination of the ionizing potentials of various gases, with the end in view of differentiating between the effects due to radiation from the bombarded atoms and those due to ionization.

This modified method may be summarized by reference to the schematic diagram Fig. 1 as follows. Electrons from a source *A* are accelerated by an applied potential V_1 through a gauze *B* into a region *BCD*, *C* being a second gauze and *D* a collecting plate connected to an electrometer. V_2 is a retarding field applied between *B* and *C*, and V_2 is maintained at a constant amount greater than V_1 to prevent the original electrons from *A* reaching the plane of the gauze *C*. V_3 is small as compared with V_1 and V_2 and determines the direction of the field in the

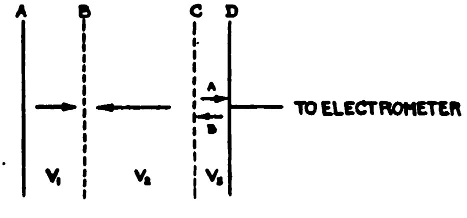

Fig. 1.

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[1] Physical Review, August, 1917.

neighborhood of C and D. The original electrons from A will be capable of making impacts with atoms or molecules in the space between A and C; and, depending on the energy imparted to them by V_1, may result in either the production of radiation from some of the impacted atoms or ionization or both. If the gauze C were not present and D in the place of C either of these results might cause a positive charging up of D due to the direction of V_2; if the cause were radiation alone it might cause a photoelectric emission of electrons from D, and if it were ionization alone the positive ions formed would travel to D. But with the small controlling field V_3 present the effects arising from these two causes can be differentiated, for if the cause be radiation alone it would produce a photo electric emission of electrons from either C or D dependent on the direction of V_3 and a consequent charging up of D—negatively if V_3 is in direction (*a*), and positively if in direction (*b*), if however the cause be ionization alone the positive ions formed in the region BC will be capable of reaching D in either case—V_3 being small as compared with V_2. As however both of these causes may occur simultaneously we may have a combination of these effects. Consequently the method consists in varying the applied potential V_1 maintaining $V_2—V_1$ at a constant small value, and obtaining two current curves as against V_1. One for V_3 being in direction (*a*); and the other for V_3 being in the direction (*b*). The shape of these curves and position of the intercept on the V_1 axis indicates the energy required to produce the different effects for any given gas.

The apparatus and experimental details were essentially the same as those used for mercury vapor and hydrogen. Certain improvements in design of the tube in which the measurements were made are worthy of note however. The same platinum thimble equipotential surface electron source was used, but the amount of metal in the tube was reduced considerably by using spiral wire grids wound on pyrex glass supports in place of the gauzes, with their heavy metal end supports. The grid B was of fine platinum wire about no. 30 and about 10 turns to the inch. The grid C was of nickel no. 20 and about 5 turns to the inch. The collecting electrode D was also of nickel and supported on a glass tube insulated by two grounding rings. All the seals were of glass, thus eliminating the use of cemented joints. This design made it possible to get rid of a much larger portion of the residual gases in the tube both on account of the reduction of the amount of metal and because the tube could be baked out as a whole and the grids glowed by means of current furnished from the outside.

The nitrogen used was prepared by the usual chemical method and was purified by washing with H_2SO_4 and NaOH and then passing over hot copper. It was collected over distilled water and before being introduced into the measuring system was dried by passing through a phosphorus pentoxide tube.

The streaming method was employed; the pumps were kept running and the gas was allowed to pass through a porous plug inserted in the bore of a stopcock. By exposing more or less of the surface of the plug and regulating the speed of the pump desirable pressures could be maintained at a constant value as measured by a McLeod gauge.

Experimental Results.—From a large number of curves obtained under varying conditions of pressure and intensity of electron emission from the source, the curves shown in Figs. 2–5 were selected as being typical.

Fig. 2 shows that by reversing V_3 which was of the order of 2 volts in all these experiments the current curve could be made to reverse, indicating that the effect is produced largely if not entirely by radiation from the bombarded atoms. It is to be noticed that the curves have a horizontal portion in common. This is due to a very slight leak to the electrometer possibly from the glass. This could not seem to be avoided within small limits and was sometimes positive and sometimes negative. But the essential thing is that the two curves break away from this natural leak line at the same point which is about 7.5 volts, a result consistent with the results obtained by one of us and also as found by other experimenters.

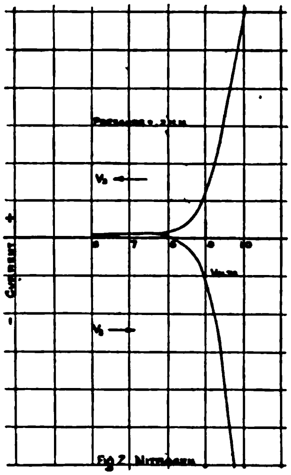

Fig. 2.

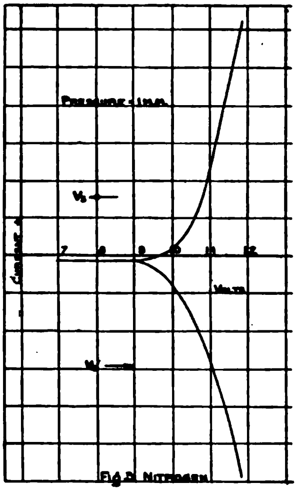

Fig. 3.

There seems to be quite a marked increase in this radiation at about

9 volts; the curves being very steep after 9 volts. This is better shown in Fig. 3 where the electron emission was much less intense, here the curves do not separate until 9 volts had been reached. The current curve, Fig. 4, was obtained with a very small emission from the source and at low pressure a condition which is most favorable for ionization effects with this type of apparatus. This shows the effect to be radiation largely up to a value of about 17.5 to 18 volts. At this value the curve rises rapidly in spite of the opposing field V_3 showing that ionization has set in.

This then shows that the ionization potential of nitrogen is about 18 volts a value considerably higher than that heretofore supposed. This result however is more in accord with the work of Johnson on the total ionization by slow electrons. His straight line relation between total ionization and volts points more nearly to 20 than to 10 volts, though this method could not be expected to give an accurate value for the minimum ionization potential.

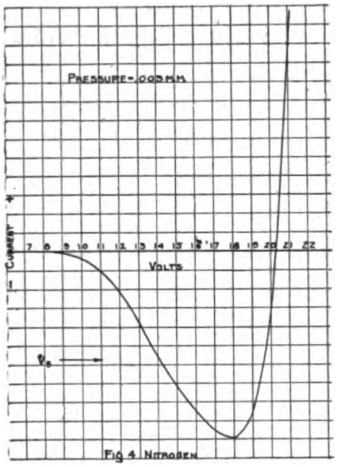

Fig. 4.

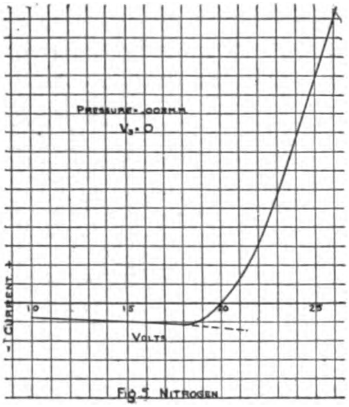

Fig. 5.

The intense radiation produced at about 9 volts is of interest in connection with the work of Meyer[1] who found that the wave-length about 1300 $A°$ stimulated the fluorescent band 3064 $A°$ in nitrogen. The value 1300 $A°$ from the quantum relation $Ve = hn$ would correspond to a voltage of the order of 9 applied to an electron.

Fig. 5 shows the current curve obtained when $V_3 = 0$. There is evidently not a complete neutralization of the radiation effect but the ionization effect is shown to set in strongly at about 18 volts.

---

[1] Physical Review, November, 1917.

SUMMARY.

These experiments show.

(*a*) Radiation can be stimulated in nitrogen molecules by electron bombardment without any appreciable ionization of the same.

(*b*) The value 7.5 volts heretofore accepted as the value of the ionizing potential really corresponds to the least energy an electron must have in order that it may excite radiation by the bombarded molecule.

(*c*) There is an indication of a more intense type of radiation setting in at about 9 volts.

(*d*) That ionization sets in at about 18 volts.

These experiments were completed more than a year ago, but stress of circumstance has delayed their publication to the present time.

Phœnix Physical Laboratory,
September, 1918.

ON A KINETIC THEORY OF MAGNETISM IN GENERAL.

By Kôtarô Honda and Junzô Ôkubo.

1. In the last fifteen years, our knowledge of the nature of magnetism has made a great advance. The electron theory was first proposed by J. J. Thomson[1] and W. Voigt[2]; but their results were not conclusive. P. Langevin[3] developed a kinetic theory of paramagnetism as well as diamagnetism, and arrived at results which agree in many points with the observed facts; but there are many other facts which cannot be explained by his theory. Prof. P. Weiss founded his well-known theory of magnetons[4] on the basis of Langevin's theory of paramagnetic gas; but subsequent investigations[5] showed that the existence of the magneton—the fundamental unit of magnetism—is very doubtful. One of the present writers[6] modified Langevin's theory of the paramagnetism of gases so as to include the magnetization of liquid and solid states, and made clear the distinction between paramagnetic and diamagnetic substances. This theory explains the fact of the dependency of diamagnetic susceptibility on the temperature, and also its change with the change of state; while according to Langevin's theory, such changes of diamagnetism are improbable. Professor Weiss, by introducing a molecular field of enormously large magnitude (some 10^7 gausses) into Langevin's theory of paramagnetic gas, tried to explain the ferromagnetism; but although he succeeded in explaining some of the thermomagnetic properties of ferromagnetic substances, it is very difficult to understand from his theory, why magnetization is so easily induced by a small external field in the presence of the large molecular field. On several occasions, one of the present writers[7] has shown that the existence of the molecular field is very improbable and that the thermomagnetic properties accounted for by means of the molecular field can be equally well explained by other theories.

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[1] Phil. Mag., 6, 1903, 673.

[2] Ann. der Phys., 9, 1902, 115.

[3] Ann. de chem. et phy., 5, 1905, 70.

[4] Arch. des Sci., 31, 1911, 401.

[5] K. Honda & T. Ishiwara, Sci. Rep., 4, 1915, 215. K. Honda & H. Takagi, Sci. Rep., 4, 1915, 261; Sci. Rep., 1, 1912, 229.

[6] K. Honda, Sci. Rep., 3, 1914, 171.

[7] K. Honda, Sci. Rep., 4, 1915, 208. K. Honda & J. Okubo, Sci. Rep., 5, 1916, 207. K. Honda, Sci. Rep., 7, 1918, 53.

Various theories of ferromagnetism have been advanced by Poisson, Weber, Maxwell and Ewing. Ewing's theory of molecular magnetism[1] explains the curve of magnetization and the phenomenon of hysteresis quite satisfactorily; but the qualitative nature of the theory is its great defect. R. Gans[2] tried to treat Ewing's model of molecular magnets mathematically; but his theory differs essentially from that of Ewing in assuming the distribution of the molecular magnets in the substance to be quite arbitrary, and in fact the conclusions from his theory are a rough approximation to the observed facts. In the last two years, we have published three papers,[3] in which Ewing's theory of molecular magnetism is treated quantitatively and extended so as to include the theory of hysteresis-loss, as well as the effect of temperature on magnetization. We have obtained the curve of magnetization possessing all the characteristics belonging to iron. Not only has the well-known hysteresis-loop been obtained, but the loss is expressed in terms of the magnetic field and the two constants characteristic of a substance The effect of temperature on the magnetization, that is, the complicated course of magnetization-temperature curves, is also explained on the basis of the same theory.

2. The theory of magnetism in general may thus be considered as established in the main; but there still remains a great advance to be made. As to Langevin's theories of paramagnetic and diamagnetic substances, some modifications are necessary in order to account for the observed facts. For example, take the case of diamagnetic substances: According to Langevin, the diamagnetic susceptibility k per unit of volume of a substance is given by

$$k = -\frac{\rho}{12}\left(\frac{e}{m}\right)^2 r^2,$$

where ρ is the density of electrons, e/m the ratio of the charge to its mass. and r the radius of the orbit of electrons. The susceptibility x per one gram atom or molecule is given by

$$x = -\frac{W\rho}{12D}\left(\frac{e}{m}\right)^2 r^2,$$

where W is the atomic or molecular weight of the substance and D its density. Now, for the density of electrons, we have

$$\rho = \frac{DN}{1800W},$$

---

[1] Phil. Mag., (5), 30, 1891, 205; Magnetic Induction in Iron and other Metals.

[2] Gött. Nachr., 1910, 197; 1911, 118. R. Gans & P. Hertz, Zeitsch. für Math. u. Phys. 61, 1913, 13.

[3] Sci. Rep., 5, 1916, 153; 5, 1916, 325; 6, 1917, 183.

provided each atom or molecule contains N electrons. Hence for different elements, we have

$$x = -\frac{N}{12 \times 1800}\left(\frac{e}{m}\right)^2 \Sigma r^2,$$

where Σ is to be extended for all electrons. Here we may take N as the atomic number of the elements. Thus, according to Langevin, since Σr^2 is to be considered as increasing with atomic weight, the diamagnetic susceptibility of the elements must increase with their atomic number. But, actually this is not the case; in fact, the susceptibility varies periodically with the atomic weight of the elements. This discrepancy is, however, partially reconciled, if we consider the diamagnetism actually observable to be the differential effect of the paramagnetic and diamagnetic phenomena;[1] that is, denoting the susceptibility observed by x, and Langevin's paramagnetic and diamagnetic susceptibilities by x_p and x_d respectively, we obtain

$$x = x_p + x_d;$$

hence though x_d increases proportionally with the atomic number, x may change periodically, provided x_p makes a similar change.

3. Next, consider the paramagnetic theory of Langevin. According to the kinetic theory of gases, the molecules of a gas are continuously making translational and rotational motions. We suppose each molecule to be a magnet composed of a set of revolving electrons. At a given instant, we may suppose the directions of the axes of these magnetic molecules to be uniformly distributed in all directions, if acted on by no external field. At the next instant, all the magnets change the directions of their axes; but as a whole, these axes are again equally distributed in all directions, and so on. In the case, when an external field acts on them, Langevin considers that the axes of these molecules tend to direct themselves toward the direction of the field; but the thermal impacts prevent this tendency. These two opposing effects make the distribution of the axes of the molecular magnets a little denser in the direction of the field than in the opposite direction. The directions of the magnetic axes are of course changing from instant to instant, but as a whole, the above distribution may be assumed to remain unchanged. By following the analogy of the distribution of the density of a gas under the action of gravity, he assumed the distribution of the axes of the molecular magnets to be given by Maxwell-Boltzmann's law. Resolving their magnetic moments in the direction of the field, and summing up these components, he obtained the well-known expression for the paramagnetic susceptibility.

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[1] K. Honda, Sci. Rep., 3, 1914, 171; J. Kunz, PHYS. REV., 6, 1915, 113.

Now, since the molecules are continuously rotating, things are not so simple as Langevin considers. Generally the axis of rotation of a molecule does not coincide with its magnetic axis; hence we consider the magnetic moment to be decomposed into two components parallel and perpendicular to the axis of rotation. The former component is here termed the axial component of the moment and the latter the transverse component. When a magnetic field is suddenly applied to the molecule, in virtue of the axial component of the magnetic moment, its axis of rotation makes a small gyrostatic motion, that is, it makes procession and nutation round the direction of the field. This motion is very little affected by the presence of the transverse component of the moment. The amplitude of the nutational motion measured from the direction of the field extends from an initial angle α to an angle θ towards the field. This motion must therefore make a certain contribution to the increase of the magnetic moment in the direction of the field, while the precessional motion itself does not cause any change in the magnetic moment. If we suppose that by continuous thermal impacts with other molecules, the nutational motion, whose frequency is much larger than that of the precession, rapidly decreases, while the precessional velocity does not considerably alter, a simple precessional motion results with an angle θ, which is greater than the initial angle α and less than the angle θ, and causes also an increase of the magnetic moment in the direction of the field. If the field, instead of being applied suddenly, be gradually applied and increased extremely slowly to its final value, the molecules will make such slow nutational and precessional motions that the first precessional motion at the lower limit may be considered as statical. This case is analogous to that of a pendulum acted on extremely slowly by the force of gravity, in which case, the pendulum may be assumed to remain at rest at its lowest point. If, under the action of a magnetic field, the molecules make a simple precessional motion or take definite orientations, as explained above, it is evident that his case is almost the same as that treated by Langevin. Taking the sum of the magnetic moments in the direction of the field due to all molecules, we obtain usually a very small moment, that is, the paramagnetic moment. By increasing the field, the angular distance α–θ increases for all molecules, and therefore the paramagnetism of the gas increases with the field. As will be shown later, this result almost coincides with that of Langevin.

4. Consider next the transverse component of the magnetic moment. This component of magnetism is continuously rotating with the molecule in its equatorial plane. Let us at first suppose that this plane coincides with the plane of the magnetic field. Initially its angular velocity is

uniform for all directions during a complete revolution; but if a magnetic field acts on the molecule, the revolution is now not uniform. Toward the direction of the field, the motion is always accelerated; but in the opposite direction, it is always retarded. Hence, if we wish to find its magnetic effect, it is necessary to take the time-mean of the magnetic moment during a complete period of revolution. Since during a revolution, the plus pole of the molecule is a shorter time in the direction of the field than in the opposite direction, the time-mean of the magnetic moment becomes negative, that is, the transverse component of the magnetic moment of rotating a molecule produces a diamagnetic effect. If the equatorial plane of the molecule makes some angle with the plane of the field, it is only necessary to resolve the above effect in the plane of the field. Hence, if there is a number of molecules, the directions of the equatorial planes of which are uniformly distributed between the angles 0 and π, we must take the component moments for these molecules in the direction of the field and integrate them; the result is a diamagnetism. Langevin did not in the least consider the effect corresponding to this kind of diamagnetism. The total magnetic effect of rotating molecules is then the differential effect due to axial and transverse components of magnetic moment, and therefore it may be paramagnetic or diamagnetic, according as the effect due to the axial component is greater or less than that due to the transverse component.

If the above view be correct, Langevin's theory of paramagnetic gas is not valid; and therefore the theory of magnetons, as well as that of molecular field put forward by Prof. Weiss, which are based on the Langevin theory, have no foundation. For this reason, the above view of the magnetization of a gas is very important.

5. We shall next consider the results qualitatively explained above, more in detail. Let us suppose that the magnetic field is acting on a molecule of a gas rotating about its magnetic axis; then it will make precessional and nutational motions about the direction of the field. Let o be the center of the magnetic molecule, its pole-strength and pole-distance being m and r respectively. Let α be the initial angle of a molecular magnet, which is acted on by an external field H. Then the

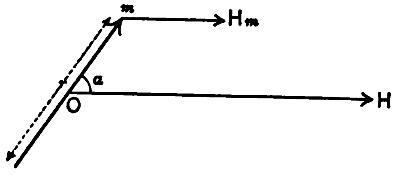

Fig. 1.

magnetic moment M is

$$M = mr.$$

The following notations are here used:

K = moment of inertia about the magnetic axis,

ω = angular velocity about the magnetic axis,

$\dot{\psi}$ = precessional velocity,

$\dot{\theta}$ = nutational velocity.

From the theory of the gyroscope, we know that

(*i*) the angular momentum about oH remains constant,

(*ii*) the sum of the kinetic and potential energies of the gyro-molecule is constant;

that is,

$$K\omega\cos\theta + K\dot{\psi}\sin^2\theta = K\omega\cos\alpha,$$

$$\tfrac{1}{2}K\omega^2 + \tfrac{1}{2}K\dot{\psi}\sin^2\theta + \tfrac{1}{2}K\dot{\theta}^2 - MH\cos\theta = \tfrac{1}{2}K\omega^2 - MH\cos\alpha;$$

Hence

$$\dot{\psi} = \frac{K\omega(\cos\alpha - \cos\theta)}{K\sin^2\theta}$$

and

$$K\omega^2(\cos\theta - \cos\alpha)^2 + K^2\dot{\theta}^2\sin^2\dot{\theta} = 2MHK\,(\cos\theta - \cos\alpha).$$

The limits of the nutational angle θ and θ_1 are given by

$$\mathrm{Cos}\,\theta = \mathrm{Cos}\,\alpha,$$

$$\mathrm{Cos}\,\theta_1 = -\frac{1}{a} \pm \sqrt{1 + \frac{2}{a}\cos\alpha + \frac{1}{a^2}},$$

where

$$a = \frac{MH}{\tfrac{1}{2}K\omega^2}.$$

In the last equation, $\cos\theta$ must always be less than 1; hence the plus sign must be taken for the double sign. We have then

$$\theta = \alpha,$$

$$\cos\theta_1 = -\frac{1}{a} + \sqrt{1 + \frac{2}{a}\cos\alpha + \frac{1}{a^2}}.$$

Hence the gyro-molecule makes a nutational motion between α and θ_1.

Suppose at first that the magnetic field is applied infinitely slowly, or that the field is suddenly applied, but the nutational motion is assumed to subside very rapidly, the angle θ for the precessional motion being the same as in the above case. This last supposition may not be strictly true, but as we shall see later, if a be very small, the result does not materially differ from that of a case free from the above supposition. In a case, where a is small, the above supposition gives the result, which exactly coincides with that of Langevin. The magnetic moment in the direction of the field is then $M \cos \theta_1$. The total magnetic moment per one gram molecule is

$$\sigma = \frac{Mn}{2}\int_0^{\pi} \cos \theta_1 \sin \alpha d\alpha$$

$$= \frac{Mn}{2}\int_0^{\pi} \left\{ -\frac{1}{a} + \sqrt{1 + \frac{2}{a}\cos \alpha + \frac{1}{a^2}} \right\} \sin \alpha d\alpha$$

where n is the number of molecules per one gram molecule of the gas. Writing $Mn = \sigma_0$, we have

$$\frac{\sigma}{\sigma_0} = \frac{1}{2}\int_0^{\pi 0} \left\{ \sqrt{1 + \frac{2}{a}\cos \alpha + \frac{1}{a^2}} - \frac{1}{a} \right\} \sin \alpha d\alpha$$

$$= \frac{1}{2}\int_{-1}^{+1} \sqrt{1 + \frac{1}{a^2} + \frac{2}{a}x}\, dx - \frac{1}{a} = \frac{a}{6}\left| \sqrt{1 + \frac{2}{a}x + \frac{1}{a^2}} \right|_{-1}^{+1} - \frac{1}{a}$$

$$= \frac{a}{6}\left| \left\{ \left(1 + \frac{1}{a}\right)^2 \right\}^{\frac{3}{2}} - \left\{ \left(1 - \frac{1}{a}\right)^2 \right\}^{\frac{3}{2}} \right| - \frac{1}{a}.$$

Hence if $a < 1$, we have, taking the minus sign for the square root of $\left(1 - \frac{1}{a}\right)^2$,

$$\frac{\sigma}{\sigma_0} = \frac{a}{3}. \tag{1}$$

but if $a > 1$, we get, taking the plus sign for the square root,

$$\frac{\sigma}{\sigma_0} = 1 - \frac{1}{a} + \frac{1}{2a^2}. \tag{1^1}$$

Since $\frac{1}{2}K\omega^2$ is the rotational kinetic energy of the molecule, a has the same meaning as that in the Langevin theory, that is,

$$a = \frac{MH}{rT},$$

where r is the gas constant referred to one molecule.

According to Langevin,

$$\frac{\sigma}{\sigma_0} = \operatorname{Coth} a - \frac{1}{a}; \tag{2}$$

if a be small, we have, by expansion,

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a - \frac{2}{90}a^3 + \cdots \tag{3}$$

which almost coincides with equation (1). In the following table, the two sets of values for σ/σ_0, as calculated from equations (1) and (2), are given side by side:

a	σ/σ_0 (Honda & Ôkubo).	σ/σ_0 (Langevin).	a	σ/σ_0 (Honda & Ôkubo).	σ/σ_0 (Langevin).
0.0	0.0000	0.0000	5.0	0.8133	0.8000
0.5	0.1667	0.1640	6.0	0.8426	0.8334
1.0	0.3333	0.3130	7.0	0.8638	0.8571
2.0	0.5833	0.5373	8.0	0.8802	0.8750
3.0	0.7037	0.6717	9.0	0.8930	0.8889
4.0	0.7708	0.7507	10.0	0.9033	0.9000

From the above table, we see that for small values of a, our formula exactly coincides with that of Langevin; but for a moderate value of a, these two slightly deviate from each other. In Fig. 2, these results (curves 1) are graphically given.

6. Next, we shall consider the case, when the magnetic field is applied impulsively and the consequent nutational motion continues for a long time.

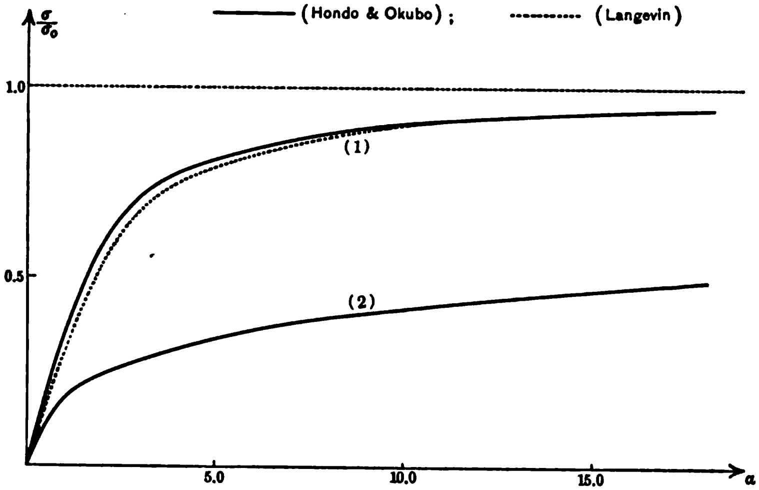

Fig. 2.

Now, the nutational velocity is given by

$$K\sin^2\theta\left(\frac{d\theta}{dt}\right)^2 = MH(\cos\theta - \cos\alpha)(\cos\theta - \cos\theta_1)(\cos\theta_2 - \cos\theta).$$

Putting

$$\cos\theta = \cos\theta_1\cos^2\varphi + \cos\alpha\sin^2\varphi$$

and changing the variable from θ to φ, we get

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{MH}{4K}(\cos\theta_1 + \cos\theta_2)\,(1 - \kappa^2\sin^2\varphi),$$

where

$$\kappa^2 = \frac{\cos\theta_1 - \cos\alpha}{\cos\theta_1 + \cos\theta_2}.$$

Hence

$$\frac{dt}{p} = \frac{d\varphi}{\sqrt{1 - \kappa^2\sin^2\varphi}},$$

where

$$p = \sqrt{\frac{MH}{4K}(\cos\theta_1 + \cos\theta_2)}.$$

Let us suppose that

$$\text{at } t = T, \quad \theta = \alpha, \quad \varphi = \frac{\pi}{2},$$

and

$$\text{at } t = 0, \quad \theta = \theta_1, \quad \varphi = 0,$$

then

$$\frac{T}{p} = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - \kappa^2\sin^2\varphi}} = K(\kappa),$$

where $K(\kappa)$ is the complete elliptic integral of the first kind. θ or φ at any instant is given by

$$\frac{t}{p} = \int_0^{\phi} \frac{d\varphi}{\sqrt{1 - \kappa^2\sin^2\varphi}} = F(\varphi, \kappa).$$

$F(\varphi, \kappa)$ is the Legendre elliptic integral of the first kind, that is,

$$\varphi = am\frac{t}{p}$$

and

$$\cos\theta = \cos\theta_1\, cn^2\frac{t}{p} + \cos\alpha\, sn^2\frac{t}{p}.$$

Hence the time-mean of the magnetic moment is

$$\begin{aligned} M_m &= \frac{1}{T}\int_0^T M\cos\theta\, dt \\ &= \frac{M}{T}\left\{\cos\theta_1\int_0^T cn^2\frac{t}{p}dt + \cos\alpha\int_0^T sn^2\frac{t}{p}dt\right\} \\ &= \frac{Mp}{T}\Bigg|\cos\theta_1\left\{\frac{1}{\kappa^2}\left[E\left(am\frac{t}{p},\ \kappa\right) - \kappa'^2\frac{t}{p}\right]\right\} \\ &\quad + \cos\alpha\left\{\frac{1}{\kappa^2}\left[\frac{t}{p} - E\left(am\frac{t}{p},\ \kappa\right)\right]\right\}\Bigg|_0^T. \end{aligned}$$

But since

$$\frac{p}{T} = K(\kappa) \text{ and } amK = \frac{\pi}{2},$$

$$\therefore \frac{M_m}{M} = \frac{\cos\theta_1 - \cos\alpha}{\kappa^2}\,\frac{E(\kappa)}{K(\kappa)} + \frac{1}{\kappa^2}(\cos\alpha - \kappa'^2\cos\theta_1)$$

$$= (\cos\theta_1 + \cos\theta_2)\frac{E(\kappa)}{K(\kappa)} - \cos\theta_2.$$

If the axis of the molecular magnets are initially distributed uniformly in all directions, the resultant magnetic moment in the direction of the field is

$$\frac{\sigma}{\sigma_0} = \frac{1}{2}\int_0^\pi \left\{(\cos\theta_1 + \cos\theta_2)\frac{E(\kappa)}{K(\kappa)} - \cos\theta_2\right\}\sin\alpha d\alpha. \tag{4}$$

Since $\cos\theta_1$ and $\cos\theta_2$ are all functions of α, the above integral cannot be evaluated in a rational form. If a be small, $\cos\theta_1$, κ, $E(\kappa)$, $K(\kappa)$ can be expanded in the ascending power series of a and then integrated, we have

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a - 0.0334a^3 + \cdots. \tag{5}$$

In a case when a is very large, these quantities can be expanded in an ascending power series of $\lambda = 1/a$; then we get

$$\frac{\sigma}{\sigma_0} = 0.4818 - 0.6875\lambda + 0.6476\lambda^2 + \cdots. \tag{6}$$

This equation shows that even in an infinitely large field, the magnetic moment does not attain its saturation value; because for $H = \infty$, $\lambda = 0$. Hence

$$\frac{\sigma}{\sigma_0} = 0.4818.$$

The reason for this is evident, since according to our first supposition, all the molecules are making continuously nutational oscillations like a pendulum about the direction of the field.

By using equations (5) and (6) for small and large values of a, and also by mechanical integration of equation (4) for moderate values of a, we calculated the values of σ/σ_0 and the result is given in Fig. 2, curve 2. In ordinary cases actually realizable, in which a is a very small quantity, equations (3) and (5) coincide with each other; but as the values of a becomes large, the deviation between them becomes always greater. To decide which one of these equations for a large value of a actually corresponds to the observed facts, requires a further experimental investigation.

7. Next, we shall consider the case of a molecule rotating about an axis perpendicular to the magnetic axis. If the plane of rotation coincides with that of the field, the time-mean of the magnetic moment M_m in the direction of the field[1] is given by

$$\frac{M_m}{M} = \frac{2}{\kappa^2}\frac{E(\kappa)}{K(\kappa)} - \left(2\frac{\kappa'^2}{\kappa} + 1\right),$$

where

$$\kappa^2 = \frac{4MH}{K\omega^2} = 2a.$$

If the plane of rotation makes an angle θ with the direction of the field, its effective moment is $M_m \cos\theta$. Hence if the distribution of the planes of rotation for n molecules be uniform, the resultant moment in the direction of the field will be

$$\sigma = \frac{M_m n}{2}\int_0^\pi \cos\theta \sin\theta d\theta = \frac{M_m n}{2},$$

or

$$\frac{\sigma}{\sigma_0} = \frac{1}{2}\left\{\frac{2}{\kappa^2}\frac{E(\kappa)}{K(\kappa)} - \left(2\frac{\kappa'^2}{\kappa^2} + 1\right)\right\}. \tag{7}$$

If a or κ be small, we have by expansion

$$\frac{\sigma}{\sigma_0} = -\frac{a}{8} - \frac{a^2}{8} - \cdots. \tag{8}$$

This shows that the molecules rotating about the axis perpendicular to the magnetic axes produce the diamagnetism. Since $\frac{1}{2}K\omega^2$ in the expression for a is the rotational energy of the molecules, its value is of the same order of magnitude as rT. Hence even in the strongest field available at the present day, the value of a is very small, unless the temperature is very low. Hence we may safely neglect a^2 in comparison with a, and we obtain

$$\frac{\sigma}{\sigma_0} = -\frac{a}{8}. \tag{9}$$

8. Having thus found the magnetic effects of the gas molecules for the two cases above mentioned, we now proceed to consider the case of the magnetization of an actual gas. Since the molecules are always rotating about axes whose directions are continuously changing, the axis of rotation and the magnetic axis do not generally coincide with each other. Now, at any given instant, among n molecules, there is a certain number of molecules

$$dn = \frac{n}{2}\sin\varphi d\varphi,$$

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[1] K. Honda & J. Okubo, Sci. Rep., 5, 1916, 328.

whose magnetic axis make with the axis of rotation an angle lying between φ and $\varphi + d\varphi$. Resolving the magnetic moment of these molecules in the axial and transverse directions, we get $M \cos \varphi$ and $M \sin \varphi$ for these two components respectively. On the one hand, these molecules have then the magnetic moment $M \cos \varphi$ in the direction of the axis of rotation, and therefore by applying a magnetizing field, their magnetic polarization is paramagnetic. Its intensity of magnetization $d\sigma$ is obtained from equation (1), since the quantity corresponding to a is very small; that is,

$$a' = \frac{MH \cos \varphi}{\frac{1}{2}K\omega^2} = a \cos \varphi$$

and

$$d\sigma = M \cos \varphi dn \times \frac{a'}{3} = \frac{Mna}{6} \cos^2\varphi \sin \varphi d\varphi.$$

φ may take any value between 0 and π; hence integrating this expression for φ, we get the total effect, that is,

$$\frac{\sigma}{\sigma_0} = \frac{a}{6}\int_0^\pi \cos^2 \varphi \sin \varphi d\varphi = \frac{a}{9}. \tag{10}$$

On the other hand, the magnetic effect due to the transverse component of magnetic moment for the molecules is obtained from equation (9). Thus,

$$a' = \frac{HM \sin \varphi}{\frac{1}{2}K\omega^2} = a \sin \varphi$$

and

$$\begin{aligned} d\sigma &= - M \sin \varphi dn \times \frac{a'}{8} \\ &= - \frac{Mna}{16} \sin^3 \varphi d\varphi. \end{aligned}$$

Integrating this expression for φ between 0 and π, we get the total effect, that is,

$$\frac{\sigma}{\sigma_0} = - \frac{a}{16}\int_0^\pi \sin^3 \varphi d\varphi = - \frac{a}{12}. \tag{11}$$

Hence, if the principal moments of inertia are equal to each other, that is, the shape of the molecules is spherical, the resultant moment of n molecules is the sum of the expressions (10) and (11). That is,

$$\frac{\sigma}{\sigma_0} = \frac{a}{3}\left(\frac{1}{3} - \frac{1}{4}\right) = 0.0278a. \tag{12}$$

Thus the resultant is paramagnetic. This differs quantitatively from the result of Langevin.

It is commonly assumed that the molecules of a monoatomic gas have a spherical shape and therefore thermal impacts cannot produce any rotation of the molecules and they have no energy of rotation. But according to a theory[1] proposed by one of the present writers, the molecules of the monoatomic gas are also continuously rotating with great velocity, which is acquired in a liquid state, but this velocity does not depend on the temperature of the gas. Hence, even for the monoatomic gas, the above result is equally applicable.

If the form of the molecules is a body of revolution about the magnetic axis, we have

$$a = \frac{MHK'}{\frac{1}{2}K^2\omega^2} = \frac{\sigma_0 H}{q}\frac{K'}{K}$$

and

$$a' = \frac{MH}{\frac{1}{2}K'\omega'^2} = \frac{\sigma_0 H}{q},$$

where K and K' are respectively the moments of inertia about the magnetic axis and the axis perpendicular to it, and

$$q = \frac{n}{2}K\omega^2 = \frac{n}{2}K'\omega'^2$$

is the kinetic energy of rotation, the equipartition of the rotational energy being assumed. Hence,

$$\frac{\sigma}{\sigma_0} = \left(\frac{K'}{3K} - \frac{1}{4}\right)\frac{\sigma_0 H}{3q}. \tag{13}$$

The gas may therefore be paramagnetic or diamagnetic, according as

$$4K' - 3K \gtrless 0.$$

As an example, take an ellipsoidal molecule, whose form is defined by

$$\frac{x^2}{A^2} + \frac{y^2 + z^2}{B^2} = 1$$

and whose major axis x coincides with that of its magnetic axis; then we have

$$K = \frac{M}{5} \times 2B^2 \text{ and } K' = \frac{M}{5}(A^2 + B^2).$$

Hence the gas is paramagnetic or diamagnetic according as

$$4A^2 - 2B^2 \gtrless 0,$$

or

$$\frac{A}{B} \gtrless 0.7072,$$

or

$$\text{eccentricity} \gtrless 0.707.$$

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[1] Honda, Sci. Rep., 7, 1918.

It is an interesting fact that the sign of the magnetization of a gas depends only on the shape of the molecules, and not in the least on their the magnetic moment.

9. It is generally admitted that the molecular magnets consist of certain sets of electrons rapidly revolving about positive nuclei, and that this revolution of electrons does not vary with temperature, its velocity always remaining the same, even at absolute zero. This rotational energy may be called the zero point energy; we denote it by $\frac{1}{2}R\Omega$, where R is the gas constant referred to one gram molecule. Besides, at the temperature T, we have also the axial component of rotation of the molecules as a whole; its energy may be given by $\frac{1}{2}RT$. We have then

$$\frac{n}{2}K\omega^2 = \frac{1}{2}RT + \frac{1}{2}R\Omega = \frac{1}{2}R(T + \Omega).$$

Hence equation (13) becomes

$$\frac{\sigma}{\sigma_0} = p\frac{\sigma_0 H}{R(T + \Omega)},$$

where

$$p = \frac{2}{3}\left(\frac{K'}{3K} - \frac{1}{4}\right);$$

p may be positive or negative; or

$$x = \frac{\sigma}{H} = \frac{p\sigma_0^2}{R(T + \Omega)}. \tag{14}$$

Thus, the paramagnetic or diamagnetic susceptibility is proportional to the magnetic field. From the last equation, we have

$$x(T + \Omega) = \frac{p\sigma_0^2}{R} = \text{const.}$$

If $p > 0$ and Ω is very small as compared with T, we have

$$xT = \text{const.},$$

which is the Curie law for paramagnetic gas. On the other hand, if $p < 0$ and Ω is very large, we have

$$x = \text{const.},$$

Hence for a gas satisfying these conditions, its diamagnetic susceptibility is independent of temperature.

Here the following remarks are to be made. So far we have assumed the molecular magnets to be perfectly rigid; but as shown by Langevin, the motion of the electrons is slightly modified by applying a strong magnetic field, and this change also contributes to the diamagnetism of the substance. Hence to equation (14), we must add the diamagnetic susceptibility as given by Langevin, that is,

$$x = -\frac{N}{12 \times 1800}\left(\frac{e}{m}\right)^2 \Sigma r^2.$$

As an application of the above theory, it is interesting to calculate the magnetic moment of a hydrogen atom or molecule. According to Bohr, the atom of hydrogen consists of an electron revolving about its nucleus. Since Bohr's value for the smallest radial distance of the revolving electron seems to be too small, we take its next value for r and the corresponding angular velocity, that is,

$$r = 2.2 \times 10^{-8} \text{ cm.}, \qquad \omega = 0.755 \times 10^{15} \text{ per sec.}$$

Then the magnetic moment of an atom is

$$M = \tfrac{1}{2}er^2\omega = 2.95 \times 10^{-21}$$

and

$$\sigma_0 = 1806,$$

which is a little greater than half the magnetic moment of the nickel atom.

The zero point energy E_0 of rotation amounts to

$$E_0 = 6.2 \times 10^{11};$$

but at 27° C.,

$$\tfrac{1}{2}RT = 1.24 \times 10^{10},$$

which is only a small fraction of the zero point energy. Hence, we have

$$\frac{n}{2}K\omega^2 = E_0 + \tfrac{1}{2}RT = 0.63 \times 10^{11}.$$

In the well known Bohr model of a hydrogen molecule, the two electrons are revolving about the line joining two positive nuclei. Here the moment of inertia K about the magnetic axis is very small as compared with the moment of inertia K' about an axis perpendicular to it; hence the ratio K'/K is very large and therefore the susceptibility of hydrogen must be paramagnetic. This inference, however, contradicts the observed fact. In order therefore that the hydrogen molecule may give the observed diamagnetic susceptibility, the positive nuclei must approach very near to each other.

Let us next calculate the distance d between two positive nuclei of a hydrogen molecule composed of two atoms, the planes of the electron orbits being parallel to each other, under the condition that the susceptibility of the gas may give the value[1] actually observed, that is,

$$x = -1.89 \times 10^{-6}.$$

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[1] The measurement of the susceptibility of different gases is now being made in this laboratory by Mr. J. Soné.

In this case, Langevin's diamagnetic susceptibility amounts to

$$x = -7.0 \times 10^{-6}.$$

Hence by equation (13), we have the relation

$$-1.8 \times 10^{-6} = \frac{1}{3}\left(\frac{K'}{3K} - \frac{1}{4}\right)\frac{\sigma_0^2}{q} + xd;$$

or

$$\frac{K'}{3K} = \frac{1}{12} + 5.4 \times 10^{-6}\frac{q}{\sigma_0^2} = 1.17.$$

$$\therefore \quad \frac{r}{d} = 13.12.$$

Hence the two positive nuclei should be only distant from each other by an amount less than one thirteenth of the radius of the electron orbits.

10. The case of solid substances will be next considered. According to the prevailing theory of the solid state, the molecules are arranged according to a certain space-lattice and are continuously making rectilinear oscillations, but not any rotations. But as we have already remarked, the electrons in each atom are rapidly revolving about the centers of mass of the electrons and the positive nuclei; its axis of rotation coincides evidently with that of the magnetic moment. The energy of this rotational motion, that is, the zero point energy, may be expressed by $\frac{1}{2}\epsilon R\Omega$, where Ω is a constant and ϵ a small fraction of unity. With the rise of temperature, the angular velocity of this rotation may probably increase, because of the axial rotation of the molecules as a whole due to thermal impacts. In a solid, this increase of rotation is not independent of the rectilinear vibrations and therefore the increase of energy is far less than that corresponding to the energy of one independent freedom, that is, than $\frac{1}{2}RT$; hence it may be expressed by $\frac{1}{2}\epsilon RT$. For the molecules of a solid, a simultaneous rotation of the molecules perpendicular to the above rotation can not possibly exist.[1] Hence assuming the molecules to form a numerous number of minute groups, in each of which the axis of magnetic molecules take the same direction, but the magnetic axes of these groups are uniformly distributed in all directions, the expression for the susceptibility of a solid substance is given by equation (1). Since

$$\frac{n}{2}K\omega^2 = \frac{\epsilon}{2}R(T + \Omega),$$

$$\frac{\sigma}{\sigma_0} = \frac{a}{3} = \frac{2}{3}\,\frac{\sigma_0 K'H}{KR\epsilon(T + \Omega)};$$

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[1] K. Honda, Sci. Rep., 7, 1918.

$$x = \frac{\sigma}{H} = \frac{2}{3}\frac{\sigma_0^2 K'}{KR\epsilon(T+\Omega)}; \qquad (15)$$

$$\therefore \quad x(T+\Omega) = \frac{2}{3}\frac{\sigma_0^2 K'}{KR\epsilon} = \text{const.}$$

There is a considerable number of paramagnetic substances, whose susceptibility satisfies the above relation. Strictly speaking, to the above value of susceptibility, we must also add the small diamagnetic susceptibility given by Langevin.

According to the theory of fusion[1] proposed by one of the present writers, the rotational motion of the molecules during fusion begins to take place with the energy corresponding to the degree of freedom at the melting point, so that the liquid molecules are making axial and transverse rotations like those of a gas. Hence during fusion, the susceptibility of a substance discontinuously changes from a value expressed by equation (15) to that expressed by equation (14), and may in some cases change its sign. The remarkable fact[2] that during melting, the susceptibility of tin changes from paramagnetic susceptibility to the diamagnetic, is explained in this way.

11. The case of a ferromagnetic substance comes next for our consideration. According to our theory of magnetism,[3] which explains the observed fact in a very satisfactory manner, the molecules of a ferromagnetic substance are arranged in a definite space-lattice and exert mutual magnetic action upon one another. In the light of the present theory, this means that the energy of rotation about the magnetic axis, that is $\frac{1}{2}K\omega^2$, is very small; for in this case, the gyrostatic action of the molecules is negligably small in comparison with the mutual magnetic action, and the molecules are now in equilibrium under the action of this magnetic force, a condition required in our theory.

Now, take equations (1) and (1′) as our starting point; it appears at first sight that the expression

$$\frac{\sigma}{\sigma_0} = \frac{a}{3} \text{ for } a < 1,$$

$$\frac{\sigma}{\sigma_0} = 1 - \frac{1}{a} + \frac{1}{2a^2} \text{ for } a > 1,$$

where

$$a = \frac{\sigma_0^2 HK'}{\frac{n}{2}K^2\omega^2},$$

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[1] K. Honda, Sci. Rep., 7, l. c.

[2] K. Honda, Sci. Rep., 1, 1911, 1.

[3] K. Honda and J. Okubo, Sci. Rep., 5, *l. c.*

may be looked on as the law of magnetization for a ferromagnetic substance, provided we consider σ to be a function of H and $(n/2)K\omega^2$ to be a very small quantity depending on the temperature of the substance. The form of the σ_1H curve is the same as curve 1 in Fig. 2. The curve possesses two characteristic points of the curve of magnetization, that is, the proportionality between the initial magnetization and the field, and the gradual approach of magnetization to saturation; yet it is only a rough representation of the fact; moreover it cannot explain the important phenomenon of hysteresis. These discrepancies are the consequences of the present theory, which neglects the mutual magnetic action and takes only the axial rotation of the molecules into account. As we have shown in our former papers, in the case of ferromagnetic substances, we must consider the mutual magnetic action as the most important factor in determining the law of magnetization, and therefore equations (1) and (1′), which take only the gyrostatic action of the axially rotating molecules into account, cannot be considered as the true law of magnetization.

The effect of temperature on magnetization will be next considered. The temperature affects magnetization in two opposite ways; the first effect, which is especially conspicuous in weak fields, but which completely vanishes in strong fields, is to increase the magnetization, while the second is always to decrease the magnetization. In our former paper,[1] these two effects are explained on the basis of small vibratory motions of the molecular magnets, about their mean orientations. That the magnetization in weak fields is considerably increased by the thermal vibrations of the molecules, is very satisfactorily explained; but the explanation of the second effect by the same vibrations is met with the great difficulty that if the half-amplitude of this vibration exceeds about 131°, the magnetization in a strong field is reversed; this, however, is an abnormal result, which has never been observed.

In view of the present theory, the following explanation for the temperature-effect in question will be very natural and agrees satisfactorily with the observed facts: The molecules of a ferromagnetic substance are continually making axial rotations about their magnetic axis, and at the same time, small vibratory motions about the axes perpendicular to it. The first motion explains the diminution of magnetization, while the second motion explains its increase in weak fields, as shown in our former paper. At ordinary temperature, the energy of the axial rotation is considered to be very small, but as the temperature rises, it rapidly increases, its rate of increase becoming always greater. At the critical point, the energy is considered to be greatly increased, but it has still a small fraction of the value $\frac{1}{2}RT$ corresponding to one degree of freedom at the temperature T. The energy-content of the axial rotation is then given by an expression of the form

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[1] Sci. Rep., 5, 1916, 325.

$$\frac{n}{2}K\omega^2 = \frac{m}{2}RT,$$

where m is a function of temperature increasing rapidly with the latter, the zero point energy being neglected. An expression m of the form

$$m = \frac{\epsilon}{\sqrt{1 - \frac{T}{\theta} + \epsilon}},$$

where θ is the critical temperature; e and e_1 being very small quantities, will accord with the observed facts very satisfactorily. The law of variation of magnetization with temperature is then given by equations (1), (1′) and

$$a = \frac{\sigma_0^2 HK'}{\frac{1}{2}KmRT}.$$

In order to show that these expressions are a good representation of the observed facts, we have calculated the constants entering into the expression of a for iron and obtained the following results:

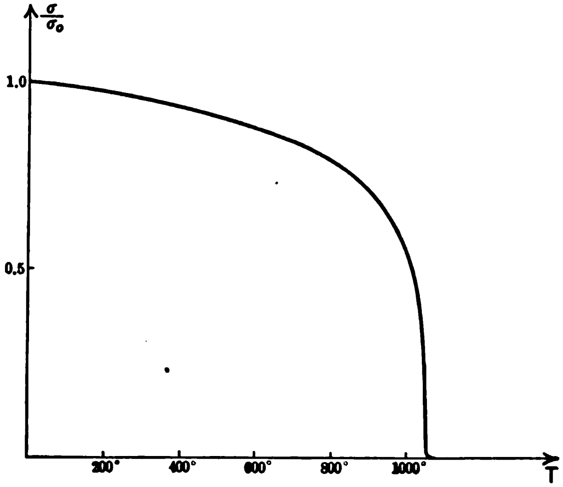

Fig. 3.

$$\theta = 790° \text{ C.},$$

$$\epsilon = 0.431 \times 10^{-5},$$

$$\epsilon_1 = 4.96 \times 10^{-8}.$$

The curve is also given in Fig. 3; we see that it is very similar to the I_1T curve in a strong field. Hence we conclude that the effect of temperature, which diminishes the magnetization for all fields, is due to the increase of the axial rotational energy of the molecules. This inference is also confirmed by the fact that during heating or cooling through the critical range of magnetic transformation, the heat is absorbed or evolved, that is, through the same range, the specific heat abnormally changes.

If the above view be correct, the rotational energy gained during heating through the range of magnetic transformation up to the critical point, is equal to the total heat absorbed during the transformation. If we neglect a small rotational energy at room temperature, the total energy q gained is equal to $(n/2)K\omega^2$. Hence, for one gram-molecule of a ferromagnetic substance, we have

$$x = \frac{2}{3}\frac{K'\sigma_0^2}{nK^2\omega^2} \text{ or } = \frac{K'\sigma_0^2}{3Kq};$$

$$q = \frac{K'\sigma_0^2}{3KX}.$$

This relation admits of the calculation of the heat absorbed or evolved during heating or cooling, provided the shape of the molecules be known. But at present, we have no data for determining the ratio K'/K for ferromagnetic substances, and therefore the verification of the above relation is left for a future occasion.

The following remark deserves also to be made:—The magnetic (or A_2) transformation of iron has been the subject of a long dispute among metallurgists, viz. as to the nature of this transformation.[1] The present theory gives, however, an exact mechanism of this transformation; that is, the A_2 transformation is not a change of molecular configuration or a change in the molecules, as is the case in an allotropic change; but it is the process of acquiring the rotational energy of the molecules, whose amount is a definite function of temperature. Hence the transformation cannot be considered as an allotropic change, as has repeatedly been stated by one of the present writers.[1]

10. Lastly let us consider the susceptibility of a ferromagnetic substance above its critical point. In this region, the kinetic energy of axial rotation, that is, $(n/2)K\omega^2$, is very large in comparison with that in the magnetic region, but it is still far less than the energy corresponding to one degree of freedom at the temperature under consideration, that is, $\frac{1}{2}RT$, which value is only attained by the fusion of the substance. We may express the above fact by the expression

$$\tfrac{1}{2}K\omega^2 = \tfrac{1}{2}RT - \tfrac{1}{2}R\Omega$$

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[1] K. Honda, Sci. Rep., 4, 1915, 169; 6, 1917, 213.

and therefore equation (1) becomes

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a = \frac{2\sigma_0 H K'}{KR(T - \Omega)},$$

or

$$x(T - \Omega) = \frac{2\sigma_0^2 K'}{RK} = \text{const.} \tag{16}$$

This is the relation, which holds good approximately above the critical point. This relation was first obtained by Prof. P. Weiss from his theory of molecular field and given in his paper as a confirmation of his theory. But as was shown by one of the present writers, it can also be obtained from another theory,[1] which has no connection with the molecular field. The above deduction of the same relation affords also a similar example.

The relation (16) may also be written as

$$I(T - \Omega) = \text{const.} \times H.$$

This represents a hyperbola, whose parameter is H; curves 1, 2, 3, in Fig. 4 are those corresponding to successively increasing fields. Curie, in his experimental research on the susceptibility of iron at high temperatures, first noticed this result and remarked that the course of the curves resembles the p, v curve of a gas near its critical point. In recent years, this remark has attracted the attention of many physicists[1]; they have attempted to explain the relation between ferromagnetic and paramagnetic states by the analogy of the relation of the liquid state to the gaseous. But as we have shown above, the fact is a natural consequence from the present theory, and we think it has not the important significance which other physicists assign to it.

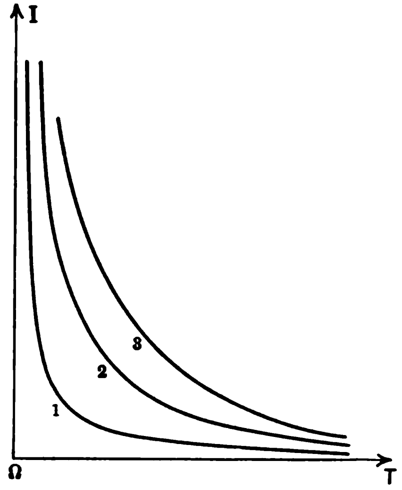

Fig. 4.

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[1] P. Weiss & P. N. Beck, Jour. de Phys., 7, 1908, 249. Ashworth, Phil. Mag., 27, 1915, 357; 30, 1915, 711; 33, 1917, 334. E. Bruins, Phil. Mag., 34, 1917, 380.

ON THE THEORY OF SUPERPOSED DIFFRACTION-FRINGES.

By Chandi Prasad.

IN the Proceedings of the American Philosophical Society for 1913 (pp. 276–282) C. F. Brush[1] has published various interesting observations of diffraction phenomena which he has attempted to group together under a common explanation by regarding them as "superposed fringes." Among the effects observed by him were those due to the diffraction of light by a cylindrical edge which have recently been further investigated and explained in an entirely different manner by N. Basu.[2] In the present paper, I am concerned with what were perhaps the most interesting and original observations recorded by Brush, that is, the diffraction-effects produced by a row of straight edges placed in echelon order. The arrangement adopted by Brush is shown diagrammatically in Fig. 1. A row of Gillete razor-blades (sometimes as many as twenty-four) were clamped together, so that their edges lay as nearly as possible in one plane which was placed very obliquely in the path of the train of light waves. The phenomena in the immediate neighborhood of the system were observed through a microscope. Brush noticed that the fringes showed contrasts between the maxima and minima of illumination which were much more marked than in the diffraction fringes of the Fresnel type due to a single straight edge, and explained this as due to the superposition of the fringe-systems produced by the successive edges. I propose in the present paper to show how the principle of superposition suggested by Brush may be formulated mathematically and its validity tested in experiment. The value of the principle is that it simplifies the treatment of the problem of diffraction by a succession of edges which

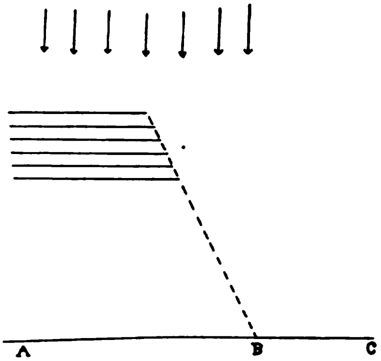

Fig. 1.

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[1] See also Science Abstracts (1913), No. 1810.

[2] Phil. Mag., Jan., 1918, p. 79.

would otherwise be very laborious.[1] Incidentally I also describe some observations of my own which appear to be of interest from the point of view of the general theory of diffraction.

Sommerfeld has shown in his well-known investigation on the mathematical theory of diffraction[2] that the diffraction fringes due to a semi-infinite screen may be regarded as due to the interference of a system of a series of cylindrical waves emitted by the edge of the screen with the incident plane waves. In the case of a number of semi-infinite screens placed parallel to each other in echelon order, it would obviously *not* be correct to assume that in the region to the rear of the system, each of the edges emits cylindrical waves, the amplitude of which, in any direction is the same as if the other screens were absent. The assumption would certainly be incorrect in respect of the region lying on one side of the plane containing the edges (*AB* in Fig. 1); for, in this region each of the screens would obviously intercept and cut off the radiations emitted by the edges in front of it. But in the region lying on the other side (*BC* in Fig. 1), an assumption of the kind stated above would be justified as a first approximation, provided each edge in the echelon is sufficiently in advance of the preceding edge in relation to its distance from it. For, the illumination in the region at and near each edge would then be practically the same as that due to the incident waves alone, and the edge of each screen therefore emits a radiation, the amplitude of which in the region considered is practically the same as in the absence of the other screens. By superposing the effects of the cylindrical waves emitted by the edges upon that due to the incident waves, a mathematical treatment of the phenomena observed by Brush would be possible.

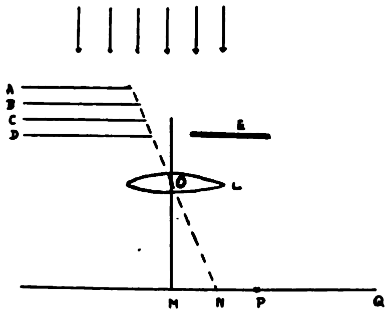

Fig. 2.

In order to test the preceding views and to obtain a definite confirmation, I have devised the experiment shown diagrammatically in Fig. 2 in which the effects due to the echelon of edges may be examined separately from that due to the incident waves.

A row of razor edges is placed on an optical bench, the distances between successive edges being about 10 cm. The edges are brought carefully into parallelism with each other and placed in echelon order.

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[1] On this point, see Mascarat's Traite de Optique, Vol. I., p. 287.

[2] Math. Annalen, Vol. XLVII., 1896, p. 317.

The incident plane waves of light are diffracted by the edges and pass through the lens *L*, the observations being made in the focal plane. The superfluous light was cut off by the screen *E*. The lens may be dispensed with, if the fringes are examined at a great distance behind the system. Some interesting effects are then observed. First, we have the direct light which comes to a focus at or near the point *M*, on either side of which we have the diffracted light due to the limitation of the aperture of the lens. In the region *NPQ*, we have another system of fringes which are due to the superposition of the light diffracted by the series of edges. These fringes appear only to the right of the plane containing the edges, *ON* being the line drawn through the center of the lens parallel to this plane. The center of this system of fringes (as seen in white light) is at *P* where $MN = NP$. Fig. 3*A* and Fig. 3*B* reproduce photographs of the system of fringes observed with an echelon of two edges only, the second edge being further in advance of the first edge in the latter case than in the former. It will be seen that the fringe-widths are asymmetrical, that is, the fringe width decreases continuously as we move from one side of the system to the other. The positions of the fringes are approximately the same as that due to the interference of two cylindrical sources of light placed respectively at the two diffracting edges. This is shown by the following measurements of the width of the fringes on either side of the center *P*.

	Positions of the Maxima Read on a Micrometer Scale.	Fringe Widths Observed.	Fringe Widths Calculated.
First Maxima	3.283	.125	.124
Second Maxima	3.158	.105	.098
Third Maxima	3.053	.094	.084
Fourth Maxima	2.959	.071	.075
Fifth Maxima	2.888	.069	.068
Sixth Maxima	2.819 (the central fringe)	.062	.063
Seventh Maxima	2.757	.056	.058
Eighth Maxima	2.701	.051	.054
Ninth Maxima	2.650	.051	.052
Tenth Maxima	2.599	.045	.052
Eleventh Maxima	2.554	.045	.052
Twelfth Maxima	2.509		

The figures in the last column were calculated from the formula

$$a(\cos\theta - \cos\psi) = \pm n\lambda,$$

where a is the distance between the two edges; θ, ψ, are the small angles made by the incident and diffracted rays, respectively, with the plane containing the edges.

Fig. 3*C* reproduces a photograph of the fringes obtained with an echelon of three edges and Fig. 3*D* reproduces one obtained with four edges. It will be seen that in the former case, we have one faint secondary maxima between each pair of primary maxima, and in the latter case, we have two secondary maxima between the principal maxima. In fact, the results are somewhat though not completely analogous to those due to a diffraction grating composed of a small number of elements. The difference in this case is that the light is diffracted by simple edges, while in an ordinary diffraction grating we are concerned with either a corrugated surface or the strips of a plane surface. Photographs of the diffraction spectra formed by a very obliquely held surface consisting of two or three reflecting strips in a plane have recently been published by Mitra,[1] who has observed that the corresponding bands on either side of the pattern are of unequal brightness, the wider fringes on one side being of very feeble intensity in comparison with the narrow fringes on the other. The photographs of the diffraction effects produced by a row of straight edges published in the present paper (Figs. 3*A* to *D*) do not show this effect, thus indicating an essential difference between the two cases. In fact, in the diffraction spectra due to a row of straight edges, the broader fringes on one side are, if anything, actually brighter than the narrow fringes on the other.

With a view to further study of the effects noticed by Brush (in which the radiations from the edges are superposed on the transmitted waves), I have also measured and photographed the diffraction-fringes of the Fresnel type due to two edges lying in nearly the same line as the direction of propagation of the incident light. Four photographs are reproduced in Figs. 3 (*E* to *H*), the second edge being in advance of the first edge by different distances in the four pictures. The diffraction-fringes due to a single edge are shown in Fig. 3*I* for comparison. Some of the minima of illumination due to the superposition of the effects of the two edges are seen in the photographs to be much darker than the minima in the fringes of the Fresnel type due to a single edge. If we assume as a first approximation that the effects due to the two edges are practically additive, the illumination at any point in the fringe system may be calculated mathematically as follows:

The problem of the diffraction of light by a semi-infinite screen has been solved by Sommerfeld whose solution of the equation

$$\frac{\partial^2 s}{\partial t^2} = a^2 \left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} \right)$$

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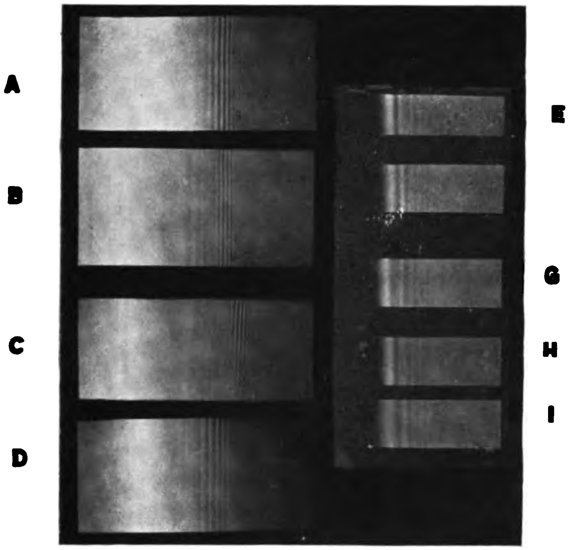
[1] Philosophical Magazine, Jan., 1918, Plate V.

Physical Review, Vol. XIII., Second Series.
January, 1919.

Plate I.
To face page 30.



Superposed Diffraction Fringes.

Fig. 3.

C. PRASAD.

is

$$s = \frac{e^{i(nt+\pi/4)}}{\sqrt{\pi}}\left[e^{-ikr\cos(\phi-\phi')}\int_{-\infty}^{\sqrt{2kr}\cos(\phi-\phi')/2} e^{-iT^2}dT \right.$$
$$\left. \pm e^{-ikr\cos(\phi+\phi')}\int_{-\infty}^{\sqrt{2kr}\cos(\phi+\phi')/2} e^{-iT^2}dT\right].$$

This solution may be simplified by expanding the integrals in a semi-convergent series, as shown by Sommerfeld. In the region of light where $\pi - \varphi' < \varphi < \overline{\pi + \varphi'}$, and provided $\sqrt{2kr}\cos(\varphi - \varphi')/2 > 1$, we have

$$s = \cos(kr\cos\overline{\varphi - \varphi'} + nt)$$
$$+ \frac{1}{4\pi}\sqrt{\frac{\lambda}{r}}\cos\left(kr - nt + \frac{\pi}{4}\right)\left\{\frac{\pm}{\cos\frac{\varphi+\varphi'}{2}} - \frac{1}{\cos\frac{\varphi-\varphi'}{2}}\right\}.$$

In the case of normal incidence, $\varphi' = \pi/2$, and writing $\varphi' = 3\pi/2 - \delta$, where δ is small, we find on reduction and simplification

$$s = \cos(kd - nt) + \frac{\sqrt{\lambda d}}{2\pi x}\cos\left(kd - nt + \frac{kx^2}{2d} + \frac{\pi}{4}\right),$$

which gives the usual maxima and minima when

$$\frac{kx^2}{2d} + \frac{\pi}{4} = 2n\pi \quad \text{or} \quad (2n+1)\pi,$$

respectively. Now, if another edge were placed at a distance a cm. and projecting b cm. from the line of the geometrical shadow of the first edge, and its effect is added to that of the latter,

$$s = \cos(kd - nt) - \frac{\sqrt{\lambda d}}{2\pi x}\cos\left(kd - nt + \frac{kx^2}{2d} + \frac{\pi}{4}\right)$$
$$- \frac{\sqrt{\lambda(\overline{d-a})}}{2\pi(x-b)}\cos\left(kd - nt + \frac{k\,\overline{x-b}^2}{2(d-a)} + \frac{\pi}{4}\right).$$

This gives for the intensity of illumination at any point

$$J = 1 + \frac{\lambda d}{4\pi^2x^2} + \frac{\lambda(d-a)}{4\pi^2(x-b)^2} - \frac{\sqrt{\lambda d}}{\pi x}\cos\left(\frac{kx^2}{2d} + \frac{\pi}{4}\right)$$
$$- \frac{\sqrt{\lambda(d-a)}}{\pi(x-b)}\cos\left(\frac{k(x-b)^2}{2(d-a)} + \frac{\pi}{4}\right)$$
$$+ \frac{\lambda\sqrt{d(d-a)}}{2\pi^2x(x-b)}\cos\frac{k}{2}\left(\frac{x^2}{d} - \frac{\overline{x-b}^2}{d-a}\right).$$

Of these six terms, the second, third and sixth are small compared with the first, third and fourth and may in calculating the position of the maxima and minima, be left out of account.

To test the results obtained from the formula, measurements have been made of the position of the maxima and minima in several cases. The experimental data and the theoretical values are shown for comparison in the table, the latter being taken from the illumination curves plotted from the formula. Fig. 4 shows three illumination curves and illustrates the general effect of the superposition of the fringes due to two edges.

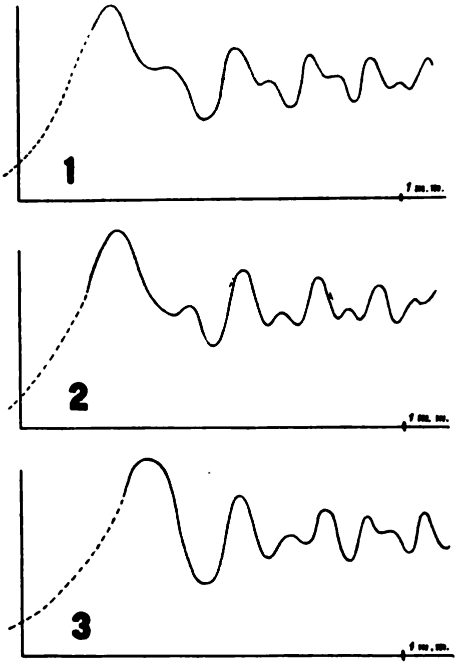


Fig. 4.

Illumination curves for two edges.

1. b. = .000 G.m. 2. b. = .002 G.m. 3. b. = .014 G.m.

I would like to express my indebtedness to Professor C. V. Raman for the help and encouragement he gave me during the course of this investigation, which was carried on last summer in the Laboratory of the Indian Association for the Cultivation of Science, Calcutta.

Positions of the maxima and minima of illumination (measured from the geometrical edge of the shadow of the edge nearest the plane of observation.

1. *d* = *20 cm., a* = *10 cm. and b* = *.002 cm.*

Bands.	Observed.	Calculated.
First maximum	+.024	.025
First minimum	+.040	.040
Second maximum	+.046	.044
Second minimum	+.052	.050
Third maximum	+.058	.058
Third minimum	+.065	.065
Fourth minimum	+.073	.073
Fifth minimum	+.083	.083
Sixth minimum	.....	.089
Seventh minimum	+.096	.098
Eighth minimum	+.108	.105

2. *d* = *20 cm.*	*a* = *10 cm.*	*b* = *.014 cm.*
First minimum	+.0378	.034
Second minimum	+.0526	.051
Third minimum	+.0617	.060
Fourth minimum	+.0712	.072
Fifth minimum	......	.081
Sixth minimum	+.0892	.088
Seventh minimum	+. 102	...

3. *d* = *20 cm.*	*a* = *10 cm.*	*b* = *.022 cm.*
First minimum	.021	.019
Second minimum	.032	.032
Third minimum	.042	.042
Fourth minimum	.051	.050
Fifth minimum	.059	.059

The agreement is fairly satisfactory.

Queen's College,
Benares, India,
March 28, 1918.

ON THE MECHANICAL AND ELECTRODYNAMICAL PROPERTIES OF THE ELECTRON.

By Megh Nad Saha.

THE object of the present paper is to extend Minkowski's method[1] of four-dimensional analysis to the investigation of the mechanical and electrodynamical problems connected with the electron. As is well-known, Minkowski's four-dimensional analysis is based on the principle of relativity, and we have thereby to abandon two time-honored concepts of physics, *i. e.*, absolute independence of time and space, and the constancy of mass. The correctness of these two principles is no longer a matter of hypothesis, but is founded on experiments. It is therefore to be hoped that the results of these investigations will be helpful to us for the elucidation of the mechanical and electrical problems connected with the electron, though sometimes difficulty may be encountered in putting proper interpretation on these results.

The notation is the same as that adopted by Minkowski, and for the convenience of the reader, it is explained at the very outset.

I.

$(x, y, z, l \pm ict)$ denotes the space and time coördinates of any point in the four-dimensional world

$$(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}\left[\frac{u_1}{c}, \frac{u_2}{c}, \frac{u_3}{c}, i\right]$$

denotes the velocity four-vector of the point.

We put

$$ds^2 = -(dx^2 + dy^2 + dz^2 + dl^2)$$

therefore we have

$$(\omega_1, \omega_2, \omega_3, \omega_4) = \left(\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}, \frac{dl}{ds}\right), \text{ and } \sqrt{-}\,(\omega_1, \omega_2, \omega_3, \omega_4)$$

---

[1] Minkowski's method of four-dimensional analysis is expounded in two papers: (1) Raum und Zeit, published in the Phys. Zeits., and (2) Die Grundgleichungen für die Electro-magnetischen Vorgänge in bewegten Körpern-Gött. Nach, 1908. These two papers have been translated by me, and are being published by the Calcutta University.

denote the direction cosines of the four-dimensional tangent to the path of the particle. c = velocity of light in space.

We put

$$(f_{23}, f_{31}, f_{12}) = (Hx, Hy, Hz),$$

the components of the magnetic field, and

$$(f_{14}, f_{24}, f_{34}) = -i[Ex, Ey, Ez],$$

the components of the electric field. Minkowski has shown that f constitutes a six-vector.

$$(a_1, a_2, a_3, a_4) = [F, G, H, l\phi],$$

are the components of the potential four-vector; (F, G, H) are the vector potentials Φ is the scalar potential.

ρ = electrical space-density;

$$\rho\left[\frac{u_1}{c}, \frac{u_2}{c}, \frac{u_3}{c}, i\right] = \rho_0(\omega_1, \omega_2, \omega_3, \omega_4)$$

are the components of the stream four-vector s;

$$\rho_0 = \rho\sqrt{1 - \frac{u^2}{c^2}}$$

is known as the rest-density of electricity.

The vector operator

$$\Box = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} + \frac{\partial}{\partial l}\right),$$

is known as the lor and the scalar operator

$$\Box^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial l^2}\right)$$

is known as the generalized D'alembertian.

The equations of electrodynamics can be written in the forms

$$\text{lor}\, f = s, \qquad \text{lor}\, f^* = 0,$$

$$f = \text{Curl}\, a; \qquad \Box^2 a = -s, \qquad \Box a = 0.$$

2. The Scalar and Vector Potentials of a Moving Electron.

Lienard,[1] and almost simultaneously Wiechert[2] showed that the scalar and vector potentials are given by the expressions

$$\phi = -\frac{e}{r\left(1 - \frac{ur}{c}\right)}, \qquad (F, G, H) = -\frac{e(u_1, u_2, u_3)}{cr\left(1 - \frac{ur}{c}\right)}, \qquad (1)$$

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[1] Lienard, L'eclairage electrique, 16 (1898), pp. 5, 53 and 106.

[2] Wiechert, Arch. Neérl., (2), 5 (1900).

If P be the point at which the potentials are calculated at the time t and M be the position of the electron at the time t_0, where $MP = c(t-t_0)$, the distance MP is denoted by r and $[u]$ denotes the velocity in the position M, and (ur) its component in the direction of r.

The formulæ are deduced from the theory of retarded potential and do not involve the principle of relativity.

Several investigators[1] have shown that the formulæ can also be deduced from the theory of relativity and can be thrown into the compact form

$$a = \frac{e[\omega]}{[R\omega]},$$

R being the four-vector joining the two points, $[R\cdot\omega]$ denoting the scalar product of R and ω.

It is quite clear that the forms (1) and (2) are quite equivalent.

In a paper published elsewhere, it has been shown that from Mimkowski's four-dimensional analysis we obtain

$$a = \frac{e[\omega]}{P}. \tag{3}$$

In this formula, (x, y, z, l) denote the time-space coördinates of the electron (A), $(\omega_1, \omega_2, \omega_3, \omega_4)$ its velocity-components, $(x'\ y'\ z'\ l')$ denote the space-time coördinates of the point B at which the potentials are estimated,

P denotes the four-dimensional perpendicular distance of B from the axis of motion of (A); since the direction-cosines of this axis are $-i\ (\omega_1, \omega_2, \omega_3, \omega_4)$, we have

$$\begin{aligned} P^2 &= (x-x')^2 + (y-y')^2 + (z-z')^2 + (l-l')^2 \\ &\qquad + [(x-x')\omega_1 + (y-y')\omega_2 + (z-z')\omega_3 + (l-l')\omega_4]^2 \\ &= R^2 + [R\omega]^2. \end{aligned}$$

Now if we make the assumption that the time coördinates are so chosen that

$$R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2 + (l-l')^2 = 0$$

i. e.,

$$c^2(t-t')^2 = r^2, \tag{4}$$

i. e.,

$$c(t-t') = r,$$

the formula (3) becomes the same as (2) and therefore (1). Also the assumption which we make here about the interval between the time-coördinates is identical with the premises of Lienard and Wiechert.

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[1] Sommerfeld, Über die Relativitäts-theorie, Ann. d. Physik, Vols. 32 and 33.

I am not quite certain whether this assumption (4) which is made here is at all essential. I am inclined to think that it is not essential, but necessary only for the interpretation of the result to those three-dimensional beings whose senses are not sharpened enough to enable them to grasp a result expressed in four-dimensional figures.

3. The Electric and Magnetic Fields Due to a Moving Electron.

If a denote the potential four-vector, the components of the six-vector f giving the electric and magnetic fields are given by

$$f = \text{Curl } a = \begin{vmatrix} \dfrac{\partial}{\partial x'} & \dfrac{\partial}{\partial y'} & \dfrac{\partial}{\partial z'} & \dfrac{\partial}{\partial'} \\ a_1 & a_2 & a_3 & a_4 \end{vmatrix}.$$

Thus

$$f_{12} = \frac{\partial a_2}{\partial x'} - \frac{\partial a_1}{\partial y'} = -\frac{e}{P^3}[\omega_3\alpha_2 - \omega_2\alpha_3], \text{ etc.} \tag{5}$$

where

$$\alpha_1 = P\frac{\partial P}{\partial x'}, \qquad \alpha_2 = P\frac{\partial P}{\partial y'} \ldots \text{ etc.}$$

we can easily verify that if we put $c(t - t') = r$, we have

$$Hx = \frac{e\beta^2}{r^3\lambda^3}\left[\frac{u_2}{c}(z - z') - \frac{u_3}{c}(y - y')\right],$$

where

$$\lambda = \left(1 - \frac{Ur}{c}\right),$$

$$= \frac{e\beta^2}{r^3\lambda^3}[\mathbf{u} \times \mathbf{r}]. \tag{6}$$

The electric forces are given by

$$f_{14} = -iEx = \frac{\partial a_4}{\partial x'} - \frac{\partial a_1}{\partial l'},$$

$$= -\frac{e}{P^2}[\omega_1(l - l') - \omega_4(x - x')],$$

$$= -i\frac{e\beta^2}{r^3\lambda^3}\left[r\frac{u_1}{c} - (x - x')\right], \tag{7}$$

and generally

$$Ex = \frac{e\beta^2}{r^3\lambda^3}\left[(x - x') - \frac{ru_1}{c}\right], \qquad E = \frac{e\beta^2}{r^3\lambda^3}\left[\mathbf{r} - \frac{r\mathbf{u}}{c}\right].$$

These values are widely different and simpler than the values obtained from the older theories, for example, compare the values given by Crehore.[1]

The discrepancy is due to the fact that in these older theories, we always assume that the equation

$$(x - x')^2 + (y - y')^2 + (z - z')^2 + (l - l')^2 = 0,$$

is an essential condition. But in performing differentiations with regard to (x', y', z', l') we here assume that they are quite independent of (x, y, z, l). I am not quite definite as to which of these two standpoints is correct but I am inclined to think that my standpoint is more in accordance with Minkowski's ideas of time and space. However it is preferable to keep an open mind on this point.

3. MAXWELL'S STRESSES, POYNTING-VECTOR, ETC.

Minkowski has shown that if we multiply f by its own matrix, we obtain a matrix

$$ff = \begin{vmatrix} S_{11} - L^2 & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} - L^2 & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} - L^2 & S_{34} \\ S_{41} & S_{42} & S_{43}\ , & S_{44} - L^2 \end{vmatrix}$$

where

$$S_{11} = \tfrac{1}{2}[f_{23}^2 + f_{34}^2 + f_{42}^2 - f_{12}^2 - f_{13}^2 - f_{14}^2],$$

$$S_{12} = [f_{13}f_{32} + f_{14}f_{42}],$$

$$L = \tfrac{1}{2}[f_{23}^2 + f_{32}^2 + f_{12}^2 + f_{14}^2 + f_{24}^2 + f_{34}^2],$$

and the matrix

$$\frac{1}{4\pi}\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{vmatrix} = \begin{vmatrix} X_x & X_y & X_z \\ Y_x & Y_y & Y_z \\ Z_x & Z_y & Z_z \end{vmatrix}$$

denote the Maxwellian stresses, $i(S_{14}, S_{24}, S_{34})$ denote the components of the Poynting vector, and S_{44} is the energy function. We have generally

$$X_x = \frac{1}{8\pi}[f_{23}^2 + f_{34}^2 + f_{42}^2 - f_{12}^2 - f_{13}^2 - f_{14}^2],$$

$$X_y = \frac{1}{4\pi}[f_{13}f_{32} + f_{14}f_{42}], \tag{8}$$

etc.

Now on the standpoint taken up by me, it is quite easy to calculate these qualities. It can be shown that

$$X_x = \frac{e^2}{8\pi P^2}[-P^2(1 + 2\omega_1^2) + \alpha_1^2], \quad X_y = \frac{e^2}{8\pi P^6}[-\omega_1\omega_2P^2 + \alpha_1\alpha_2], \tag{9}$$

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[1] PHYS. REV., July, 1917, p. 448.

The Poynting-vector

$$(X_t,\ Y_t,\ Z_t) = \frac{e^2}{8\pi P^6}[(-\ \omega_1\omega_4P^2 + \alpha_1\alpha_4),\quad (-\ \omega_2\omega_4P^2 + \alpha_2\alpha_4),\quad (-\ \omega_3\omega_4P^2 + \alpha_3\alpha_4)]$$

and the energy function

$$S_{44} = L = \frac{e^2}{8\pi P^6}[-\ P^2(1 + 2\omega_4^2) + \alpha_4^2]$$

where

$$\alpha_1 = P\frac{\partial P}{\partial x'},\quad \alpha_2 = P\frac{\partial P}{\partial y'},\quad \text{etc.}$$

and

$$\alpha^2 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 = P^2.$$

4. THE LAW OF ATTRACTION BETWEEN TWO MOVING ELECTRONS.

We can now proceed to find out the attraction which one moving electron exerts upon another moving electron.

According to Lorentz's theorem the components of the force acting on an electron (A) moving in any electromagnetic field are

$$\begin{aligned} X &= e[\omega_2 f_{12} + \omega_3 f_{13} + \omega_4 f_{14}],\\ Y &= e[\omega_1 f_{21} + \omega_3 f_{23} + \omega_4 f_{24}],\\ Z &= e[\omega_1 f_{31} + \omega_2 f_{32} + \omega_4 f_{34}], \end{aligned} \tag{10}$$

and we can also add the fourth or the time component

$$L = -\frac{ie}{c}[Xu_1 + Yu_2 + Zu_2],\quad \beta = \sqrt{1 - \frac{v^2}{c^2}}$$

which is proportional to the rate at which work is done by the moving charge,—we have

$$L = e[\omega_1 f_{41} + \omega_2 f_{42} + \omega_3 f_{43}].$$

In this case, the field is due to the second electron (charge e', position $x'\ y'\ z'\ l'$, velocity components $\omega_1'\ \omega_2'\ \omega_3'\ \omega_4'$).

According to the last section, the potential four-vector

$$a = \frac{e'[\omega']}{P'},\quad \text{where}\quad P'^2 = R^2 + [R\omega']^2.$$

We have now, since $f = \text{Curl}\ a$,

$$\begin{aligned} X = ee'\Bigg[\omega_2\left\{\frac{\partial}{\partial x}\left(\frac{\omega_2'}{P'}\right) - \frac{\partial}{\partial y}\left(\frac{\omega_1'}{P}\right)\right\} + \omega_3\left\{\frac{\partial}{\partial x}\left(\frac{\omega_3'}{P'}\right) - \frac{\partial}{\partial z}\left(\frac{\omega_1'}{P'}\right)\right\}&\\ + \omega_4\left\{\frac{\partial}{\partial x}\left(\frac{\omega_4'}{P'}\right)\frac{\partial}{\partial l}\left(\frac{\omega_1'}{P'}\right)\right\}&\\ = ee'\Bigg[\frac{\partial}{\partial x}\left(\frac{\omega_1\omega_1' + \omega_2\omega_2' + \omega_3\omega_3' + \omega_4\omega_4'}{P'}\right)&\\ -\left(\omega_1\frac{\partial}{\partial x} + \omega_2\frac{\partial}{\partial y} + \omega_3\frac{\partial}{\partial z} + \omega_4\frac{\partial}{\partial l}\right)\left(\frac{\omega_1'}{P'}\right)\Bigg],& \end{aligned}$$

Now putting

$$\Phi = ee'(\omega_1\omega_1' + \omega_2\omega_2' + \omega_3\omega_3' + \omega_4\omega_4')/P',$$

we find that

$$X = \frac{\partial\Phi}{\partial x} - \frac{d}{ds}\left(\frac{\partial\Phi}{\partial\frac{dx}{ds}}\right),$$

where $\partial/\partial x$ denotes differentiation in which x is explicitly involved, similarly with

$$\frac{\partial}{\partial\omega_1} = \frac{\partial}{\partial\frac{dx}{ds}}.$$

$$\frac{d}{ds} = \left(\omega_1\frac{\partial}{\partial x} + \omega_2\frac{\partial}{\partial y} + \omega_3\frac{\partial}{\partial z} + \omega_4\frac{\partial}{\partial l}\right),$$

as is easily seen. We have similarly

We have similarly

$$Y = \frac{\partial\Phi}{\partial y} - \frac{d}{ds}\left(\frac{\partial\Phi}{\partial\frac{dy}{ds}}\right), \qquad Z = \frac{\partial\Phi}{dz} - \frac{d}{ds}\left(\frac{\partial\Phi}{\partial\frac{dz}{ds}}\right),$$

$$L = \frac{\partial\Phi}{\partial l} - \frac{d}{ds}\left(\frac{\partial\Phi}{\partial\frac{dl}{ds}}\right). \qquad (11)$$

We can say that Φ is the kinetic-potential of the electron (A) in the field of the electron (B). Similarly if Φ' denote the Kinetic-Potential of the electron (B) in the field of (A),

$$\Phi' = ee'(\omega_1\omega_1' + \omega_2\omega_2' + \omega_3\omega_3' + \omega_4\omega_4')/P,$$

$$P^2 = R^2 + (R\omega)^2.$$

and we have similarly

$$X' = \frac{\partial\Phi'}{\partial x'} - \frac{d}{ds'}\left(\frac{\partial\Phi'}{\partial\frac{dx'}{ds'}}\right). \qquad (12)$$

Let us now interpret the results in three dimensions. We have

$$X = \frac{ee'\beta'^2}{r^3\lambda^3\beta'}(x - x')\left(1 - \frac{uv\cos\theta}{c^2}\right) - \frac{ee'\beta'^2}{r^2\lambda^3\beta c}\left(1 - \frac{Ur}{c}\right)u_1', \qquad (13)$$

where

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}, \qquad \beta' = \sqrt{1 - \frac{v'^2}{c^2}}, \qquad \lambda = \left(1 - \frac{Ur'}{c}\right).$$

In three dimensions, the forces are equivalent to a force of repulsion

$$\frac{ee'\beta'^2}{r^3\lambda^3\beta}\left(1 - \frac{uv\cos\theta}{c^2}\right)\mathbf{r},$$

in the direction of the line joining the two points, and a force

$$\frac{ee'\beta'^2}{r^2\lambda^3\beta c}\left(1 - \frac{Ur}{c}\right)\mathbf{u}', \tag{14}$$

in the direction of the velocity of the second or the attracting point.

We thus perceive that the force which comes out in a very simple form in four dimensions takes a very complicated form in three dimensions.

The kinetic potential

$$\phi = \frac{ee'\left(1 - \dfrac{vv'\cos\theta}{c^2}\right)}{r\left(1 - \dfrac{U'r}{c}\right)\beta}. \tag{15}$$

This kinetic potential is practically coincident with the kinetic potential assumed by Clausius in order to find out the law of attraction between two moving charges of electricity; Clausius has shown that this kinetic potential leads us to the celebrated electrodynamic laws of Ampere. A short résumé of the work done in this connection is given below for the purpose of comparison. The problem was first enunciated by Gauss in the year 1835, and was called by him the fundamental keystone of electrodynamics.[1]

1. Gauss (1835): The forces are the derivatives with regard to (x, y, z) of the potential function,

$$\phi = \frac{ee'}{r}\left(1 - \frac{3}{2c}\frac{d^2r}{dt^2}\right).$$

2. Weber (1843) takes the potential function

$$\phi = \frac{ee'}{r}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt}\right).$$

Both of these forms have been long discredited. Later writers have pointed out that the force cannot be simply the derivations with regard to (x, y, z) of some potential function, but are the Lagrangian derivatives of a certain kinetic-potential. We give the form of this kinetic potential according to different investigators.

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[1] For the literature on the subject, see Maxwell, Electricity and Magnetism, Vol. 2, Chap. XXIII., and J. J. Thomson, Application of Dynamics to Problems of Physics and Chemistry, pp. 35.

1. Clausius (1881);

$$\phi = \frac{ee'}{r}\left(1 - \frac{uu' \cos\theta}{c^2}\right),$$

where v and u' are the velocities of the two electrons, and θ is the angle between their lines of motion. In two papers communicated to the Crelle's journal,[1] Clausius deduces Ampere's laws of electrodynamical action between two currents from this law.

2. J. J. Thomson

$$\phi = \frac{ee'}{r}\left(1 - \frac{\mu}{3c^2}uu' \cos\theta\right) \qquad (\mu = \text{magnetic permeability} = 1).$$

Crehore[2] has calculated the forces components according to J. J. Thomson's theory.[3] He finds that the forces are equivalent to

$F_1 = \frac{ee'}{r^2}$, a repulsion along the line joining the centers.

$F_2 = \frac{ee'}{c^2r^2}uu' \cos\theta$, an attraction along the same line.

$F_3 = \frac{ee'}{r}u'$, a force in the direction opposite to the acceleration of the second charge.

$F_4 = ee'u'\frac{d}{dt}\left(\frac{1}{r}\right)$, a force in a direction opposite to the velocity of the second charge.

3. Sommerfeld[4] has also calculated the ponderomotive forces, assuming that the value of the potential four-vector

$$a = \frac{e[\omega]}{(R\omega)},$$

and using the condition

$$(x - x')^2 + (y - y')^2 + (z - z')^2 + (l - l')^2 = 0$$

in course of differentiation. Their forms are a bit too complicated.

5. EQUATIONS OF MOTION OF THE ELECTRON.

Minkowski[5] deduces the equations of motions of a ponderable particle by means of a variational process in which the function

$$\int m_0c^2ds, \text{ where } ds^2 = -(dx^2 + dy^2 + dz^2 + dl^2) = c^2dt^2\left(1 - \frac{u^2}{c^2}\right)$$

is used instead of the three-dimensional $\int Tdt$.

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[1] Vols. 82 and 83.

[2] It will be seen that forces F_1, F_2, F_4 are, but for some minor details, represented in our formula. Force F_3 does not occur at all.

[3] Phil. Mag., 1913.

[4] Ann. d. Phys., vols. 32 and 33, Über die Relativitäts theories.

[5] Minkowski, *loc. cit.* Anhaup, Mechanics.

He obtains

$$m_0c^2\frac{d^2x}{ds^2} = X, \qquad m_0c^2\frac{d^2y}{ds^2} = Y, \qquad m_0c^2\frac{d^2z}{ds^2} = Z, \qquad m_0c^2\frac{d^2l}{ds^2} = L. \tag{16}$$

Now we have

$$X = e[\omega_2 f_{12} + \omega_3 f_{13} + \omega_4 f_{14}],$$

according to Lorentz's theorem. We have also

$$\begin{aligned}\frac{d^2x}{ds^2} &= \left(\omega_1\frac{\partial}{\partial x} + \omega_2\frac{\partial}{\partial y} + \omega_3\frac{\partial}{\partial z} + \omega_4\frac{\partial}{\partial l}\right)\omega_1\\ &= \omega_1\frac{\partial}{\partial x}[\tfrac{1}{2}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)] + \omega_2\left[\left(\frac{\partial\omega_1}{\partial y} - \frac{\partial\omega_2}{\partial x}\right)\right.\\ &\qquad\qquad \left. + \omega_3\left(\frac{\partial\omega_1}{\partial z} - \frac{\partial\omega_3}{\partial x}\right) + \omega_4\left(\frac{\partial\omega_1}{\partial l} - \frac{\partial\omega_4}{\partial x}\right)\right]\\ &= -(\omega_2\Omega_{12} + \omega_3\Omega_{13} + \omega_4\Omega_{14}), \text{ putting } \Omega_{hk} = \frac{\partial\omega k}{\partial xh} - \frac{\partial\omega h}{\partial xk}.\end{aligned}$$

Hence we have the four equations, putting $\mu = c^2m_0/e$,

$$\left.\begin{array}{llll} & \omega_2(f_{12} + \mu\Omega_{12}) + \omega_3(f_{13} + \mu\Omega_{13}) + \omega_4(f_{14} + \mu\Omega_{14}) &= 0\\ \omega_1(f_{21} + \mu\Omega_{21}) & + \omega_3(f_{23} + \mu\Omega_{23}) + \omega_4(f_{24} + \mu\Omega_{24}) &= 0\\ \omega_1(f_{31} + \mu\Omega_{31}) + \omega_2(f_{32} + \mu\Omega_{32}) + & + \omega_4(f_{34} + \mu\Omega_{34}) &= 0\\ \omega_1(f_{41} + \mu\Omega_{41}) + \omega_2(f_{42} + \mu\Omega_{42}) + \omega_3(f_{43} + \mu\Omega_{43}) & &= 0\end{array}\right\}.$$

Of these, only three are independent; the fourth can be deduced from the first three.

We have now identically

$$\begin{vmatrix} & f_{12} + \mu\Omega_{12} & f_{13} + \mu\Omega_{13} & f_{14} + \mu\Omega_{14}\\ f_{21} + \mu\Omega_{21} & & f_{23} + \mu\Omega_{23} & f_{24} + \mu\Omega_{24}\\ f_{31} + \mu\Omega_{31} & f_{32} + \mu\Omega_{32} & & f_{34} + \mu\Omega_{34}\\ f_{41} + \mu\Omega_{41} & f_{42} + \mu\Omega_{42} & f_{43} + \mu\Omega_{43} & \end{vmatrix} = 0,$$

i. e.,

$$\begin{aligned}(f_{12} + \mu\Omega_{12})(f_{34} + \mu\Omega_{34}) + (f_{23} + \mu\Omega_{23})(f_{14} + \mu\Omega_{14})&\\ + (f_{31} + \mu\Omega_{31})(f_{24} + \mu\Omega_{24}) &= 0.\end{aligned} \tag{17}$$

The condition is evidently satisfied if we put

$$-\mu = \frac{f_{12}}{\Omega_{12}} = \frac{f_{23}}{\Omega_{23}} = \frac{f_{31}}{\Omega_{31}} = \frac{f_{14}}{\Omega_{14}} = \frac{f_{24}}{\Omega_{24}} = \frac{f_{34}}{\Omega_{34}}. \tag{18}$$

If of these equations, any three are satisfied the remaining three come out automatically from the equations of motion. But we cannot possibly be sure of the authenticity of these relations unless it can be deduced from an independent source. For this purpose let us take the original variational equations.

Let (X, Y, Z, L) represent the components of the force four-vector at any point, which is subjected to a virtual displacement δx, δy, δz, δl.

Then

$$\delta W = X\delta x + Y\delta y + Z\delta z + L\delta l,$$

i. e., if we call

$$W = \frac{\partial A}{\partial s}, \qquad A = \int W ds,$$

$$\begin{aligned}\delta A &= \int \delta W ds = \int (X\delta x + Y\delta y + Z\delta z + L\delta l) ds \\ &= e\int [f_{12}(dy\delta x - \delta y dz) + f_{23}(dz\delta y - \delta z dy) + f_{31}(dx\delta z - \delta x dz) \\ &\qquad + f_{14}(dl\delta x - \delta l dx) + f_{24}(dl\delta y - \delta l dy) + f_{34}(dl\delta z - \delta z dl)].\end{aligned}$$

Now the function $\int m_0 c^2 ds$ can also be subjected to a variational process. Since

$$ds = \omega_1 dx + \omega_2 dy + \omega_3 dz + \omega_4 dl,$$

we find

$$\begin{aligned}\delta \int m_0 c^2 ds = - m_0 c^2 \int [\Omega_{12}\delta S_{12} + \Omega_{23}\delta S_{23} + \Omega_{31}\delta S_{31} \\ + \Omega_{14}\delta S_{14} + \Omega_{24}\delta S_{24} + \Omega_{34}\delta S_{34}],\end{aligned}$$

where

$$\delta S_{hk} = dx_k \delta x_h - \delta x_k dx_h.$$

Thus from the variational equation

$$\delta \int m_0 c^2 ds + \int \delta W . ds = 0;$$

i. e., from the principle of least action, keeping the initial and final points fixed, we obtain the original equation

$$\int [(f_{12} + \mu \Omega_{12})\delta S_{12} + \ldots] = 0.$$

The relations (18) thus seem to be borne out by independent evidence.

Difficulty is encountered here about the interpretation of the terms $(\Omega_{12}, \Omega_{23}, \ldots)$ in three dimensions Ω is evidently a six-vector being the four-dimensional curl of the velocity four-vector. The components $[\Omega_{23}, \Omega_{31}, \Omega_{12}]$ are evidently connected with rotations

$$\left[\frac{\partial u_3}{\partial y} - \frac{\partial u_2}{\partial z}, \quad \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x}, \quad \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right]$$

and $[\Omega_{14}, \Omega_{24}, \Omega_{34}]$ are connected with the accelerations

$$\left[\frac{d^2x}{dt^2}, \quad \frac{d^2y}{dt^2}, \quad \frac{d^2z}{dt^2}\right]$$

but the exact interpretation in three dimensions has not yet been obtained. We can style Ω as the acceleration six-vector.

On a future occasion, I hope to communicate the result of my investigations on the orbits of the electron under different conditions.

In conclusion, I wish to express my best thanks to my friend, Mr. S. N. Basu, for much help, and useful criticism.

UNIVERSITY COLLEGE OF SCIENCE,
CALCUTTA, INDIA,
May 11, 1918.

A NEW EXPERIMENTAL DETERMINATION OF THE BRIGHTNESS OF A BLACK BODY, AND OF THE MECHANICAL EQUIVALENT OF LIGHT.

By Edward P. Hyde, W. E. Forsythe and F. E. Cady.

Part I. Brightness of a Black Body as a Function of its Temperature.

THE brightness or candlepower per cm^2 of a black body at various temperatures has long been a matter of interest. Lummer and Kurlbaum,[1] in connection with their work on radiation, determined the relation between brightness and temperature for radiating platinum and expressed the opinion that the exponential relation found would hold at least approximately for a black body. Subsequently[2] Lummer and Pringsheim determined the brightness of a black body at three temperatures (1448° K., 1598° K. and 1708° K.) and extrapolated their observed curve to give the brightness up to 2073° K., at which temperature they ascribed the brightness of 100 Hefner candles per cm^2.

A couple of years later Nernst,[3] starting from one of the extrapolated points given by Lummer and Pringsheim, found a brightness of 91 Hefner candles per cm^2 at the temperature of melting platinum (taken by him as 2018° K.), and a brightness of 1210 Hefner candles per cm^2 at the temperature of melting iridium, which latter he calculated to be 2621° K. Subsequently[4] Nernst undertook a new set of measurements on the brightness of a black body, carrying his determinations over a range of temperature extending from about 1460° K. to about 2280° K. His temperature measurements of the black body were made with a Wanner pyrometer, and were based on a temperature of 1337° K. as the melting point of gold, and a value of 14600 μ deg. for the constant C_2 in the Planck equation.

From the observed data Nernst computed the values of the constants A and B in the equation

$$\log K = -\frac{A}{T} + B,$$

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[1] Verh. d. Deut. Phys. Gesel., 2, p. 90, 1900.

[2] Phys. Zeit., 3, p. 97, 1901.

[3] Ibid., 4, p. 733, 1903.

[4] Ibid., 7, p. 380, 1906.

which had first been given by Rasch[1] and subsequently deduced on what Nernst considers more justifiable theoretical grounds by Haber[2] and Lucas[3] as expressing the relation between brightness, K, and temperature, T. Nernst apparently relies on the values deduced from this equation as more accurate than the individual direct determinations. With the use of this equation he computes the brightness of the black body at the temperature of melting platinum, taken as 2018° K., to be 63.4 Hefner candles per cm^2, rather than 91 as given in his original paper. He offers an explanation of the cause of error in the earlier work.

It should be noted in passing that Rasch's equation is deducible from the Planck equation as the variation in brightness in some one wave-length, and so is the theoretical foundation for the method of heterochromatic photometry proposed some time earlier by Crova.[4] But it has been recognized for a long time that Crova's method is only approximate over more than a small temperature interval, since the effective wave-length varies with the temperature.[5]

Eisler[6] in 1904, assuming Planck's equation for the distribution of energy in the spectrum of a black body and using Langley's original data on visibility computed the relative brightnesses of a black body at different temperatures, obtaining a relation of brightness with temperature from which he computed the exponent x in the equation

$$\frac{H_1}{H_2} = \left(\frac{T_1}{T_2}\right)^x,$$

where H_1 and H_2 are the relative brightnesses at the two respective temperatures T_1 and T_2 taken close together. The curve of x as a function of temperature passes fairly closely through the points plotted from the observations of Lummer and Pringsheim.

Various computations[7] after the method of Eisler have been made in recent years, using visibility data obtained in more accurate ways and unquestionably much nearer the truth than the original data of Langley, but these computations are founded on the original experimental brightness values of Lummer and Pringsheim, and the subsequent values of Nernst, and there is reason to believe that these experimental values are subject to some error, and that, moreover, in each case there is some question about the relative computed brightnesses on account either of the temperature scale or of the visibility data assumed. The only recent experimental data available are those of Langmuir,[1] and those of Ives and Kingsbury.[2] The former conducted his measurements on a tungsten filament, and his results are subject to some modification largely on account of the values assumed for the emissive power of tungsten which more precise determinations have shown to be in error. The experimental results of Ives and Kingsbury[3] determined both by the use of a black-body furnace and by the application of the wedge method in measuring black-body brightness at the melting points of platinum and gold respectively, are not very concordant, and are given but little weight by the authors themselves.

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[1] Ann. d. Phys., 14, p. 193, 1904.

[2] Thermodynamik technisches Gasreactionen (Munich), 1905.

[3] Phys. Zeit., 6, p. 19, 1905.

[4] Comptes Rendus, 93, p. 512, 1881.

[5] PHYS. REV., 32, p. 320, 1911.

[6] Elek. Zeit., 25, p. 188, 1904.

[7] Elec. World, 57, p. 1565, 1911. Verh. d. Deut. Phys. Gessell., 17, p. 219, 1915.

It therefore seemed opportune to the authors to undertake a new series of experimental determinations of black-body brightness as a function of temperature, and to compare these experimental results with the relative values computed on the assumption of the most probable visibility curve and of the Planck equation for spectral energy distribution after the manner of Eisler. Within the past few years much progress has been made in the establishment of the high temperature scale, and black-body furnaces may now be operated up to temperatures of at least 2600° K. under reasonably favorable conditions for brightness determinations, and with a considerable degree of confidence in the precision of the temperature measurements. Of course, the temperature scale assumed plays an important rôle. This question will be dealt with in some detail in a subsequent section.

In like manner much progress has recently been made in determining the visibility curve of the average eye, but, in the opinion of the authors, since all the recent determinations had been made by the flicker method, there seemed a necessity for a determination of visibility by the direct comparison method. The findings of the flicker method in the investigations of several observers were not very concordant, and the method itself was open to question. The present investigation was therefore interrupted to make a new determination of the visibility curve by the direct comparison method, as will be discussed in a separate section. Fortunately uncertainties in this function produce only second order errors in the relative brightness curve of a black body, and since the range of temperature that can be employed is not extremely large the errors in the relative computed brightnesses are quite small if the chosen visibility curve even roughly expresses the facts. To what extent these considerations are justified will become apparent in the presentation and discussion of the experimental data.

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[1] Phys. Rev., (2), 7, p. 322, 1916.

[2] Phys. Rev., (2), 8, p. 177, 1916.

[3] Since this paper was written, Dr. Ives has published a note in the Journal of the Franklin Institute (186, p. 122, 1918), giving data which seem to verify this original value.

The second part of the paper deals with the evaluation of the mechanical equivalent of light based upon the observed values of black-body brightness and upon the assumption of the constant σ ($E = \sigma T^4$) in the Stefan-Boltzmann law, and of the visibility curve. In this case errors in the visibility curve enter as first order effects and so the precise knowledge of this function becomes of much greater importance.

Temperature Scale.—The relation between observed brightness and temperature of a black body obviously depends upon the chosen scale of temperature. It is customary, in establishing a temperature scale, to assume certain fixed points, such as the melting points of gold and palladium, and to determine other points of the scale on the assumption of some radiation law such as the Stefan-Boltzmann law or the Planck equation. The former of these is accepted without reservation, but there is some slight question regarding the latter, though it is generally believed to represent the facts and is assumed as the basis of optical pyrometry. The values of certain of the constants entering into these equations are still, however, the subject of much discussion and investigation. This is particularly true of the constant C_2 in the Planck equation

$$E_\lambda = C_1 \lambda^{-5} \frac{1}{\epsilon^{\frac{C_2}{\lambda T}} - 1}.$$

As already stated it is customary to assume certain fixed points, such as the melting-points of gold and palladium as measured with the gas thermometer. The best determinations of these values are those of Day and Sossman,[1] but recent investigations would seem to show that the two melting-point temperatures are not consistent with the most probable value of C_2 in the Planck equation.[2] In order to have a definite temperature scale on which to express measurements of temperature the laboratories of the General Electric Company agreed upon the adoption of the gold point as given by Day and Sossman (1336° K.), and of the value 14350 μ deg. for the constant C_2. On this scale the temperature of melting palladium must be changed from 1823° K., the value found by Day and Sossman, to 1828° K. if the earlier measurements of the present authors[3] are correct, since it was found that the value of Day and Sossman was confirmed only upon the assumption of the value of 14460 μ deg. for C_2, if the gold point was accepted as 1336° K.

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[1] Am. Jour. of Sci., (4), 29, p. 93, 1910.

[2] Gen. Elec. Rev., 20, p. 819, 1917.

[3] Astrophys. Jour., 42, p. 300, 1915.

Visibility Data.—As stated in an earlier paragraph, there was much doubt in the minds of the authors regarding the acceptability of the various visibility data available when the present investigation was begun. This doubt arose partly from the inconsistency of the various published results but more largely from an inherent skepticism regarding the applicability of visibility data obtained with the flicker photometer to practical problems in heterochromatic photometry in which the method of direct comparison was employed, as in the present investigation.

It is true that only gross differences in the visibility curve can produce appreciable errors in the computed relative brightnesses of a black body at two temperatures differing by only a few hundred degrees, but uncertainties in the visibility curve produce first-order effects in the evaluation of the mechanical equivalent of light. And when the authors found that even the second-order errors in the computed relative brightnesses of a black body on the assumption of the most recently published visibility data[1] with the flicker photometer, as compared with the experimental values found using the direct-comparison method of photometry, were larger than might readily be justified on the ground of experimental error, it was decided to delay the publication of the present investigation until a new determination of the visibility of radiation, using the direct-comparison method, might be made. The results of this investigation, together with a more complete discussion of the question, have already been published.[2]

The visibility data obtained in that investigation will be used in computing the relative brightnesses of a black body at different temperatures, for comparison with the values experimentally observed, and in evaluating the mechanical equivalent of light. For the extreme red end of the spectrum not included in the investigation referred to, use will be made of the visibility data obtained by two of the present authors in an earlier study[3] of visibility in this region; and for the extreme blue end the data of Hartman[4] obtained also in this laboratory will be employed. The complete visibility curve used is that recommended in appendix II. of the recent paper to which reference has just been made.

Apparatus and Method.—The measurements of the brightness of a black body at various temperatures were carried out with two electrically heated black-body furnaces. Most of the measurements were made with a specially designed water-cooled carbon-tube furnace, the remaining ones being made with a platinum-wound porcelain furnace provided with a Lummer-Kurlbaum black-body tube. The set-up, with the carbon furnace,[1] is shown in Fig. 1, in which *F* is the carbon furnace supplied with a limiting diaphragm. Three different sizes were used with apertures varying from 4 to 5 mm. in diameter. For details of construction and operation of this furnace the reader is referred to a previous paper by the authors on "Color Temperature Scales for Tungsten and Carbon."[2] *P* is a Lummer-Brodhun contrast photometer of special design, in which the contrast is only about 3.5 per cent. on the average and is graduated in either direction from the center—the two features conducing to higher sensibility. *L* is the comparison lamp,—at low temperatures a carbon lamp and at the higher temperatures a vacuum tungsten lamp. A series of black velvet screens *D* with suitable openings shut off stray light from the photometer.

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[1] Bul. Bur. of Stds., 14, p. 167, 1917.

[2] Astrophys. Jour., 48, Sept., 1918.

[3] Ibid., 42, p. 285, 1915.

[4] Ibid., 47, p. 83, 1918.

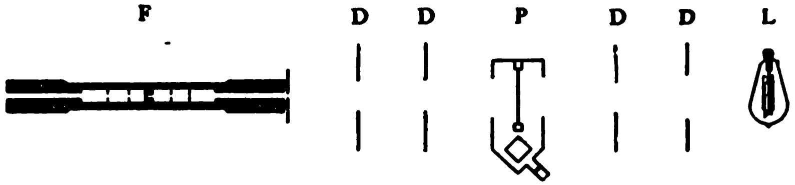

Fig. 1.
Diagrammatic Sketch of Apparatus.

The brightness measurements of the black body were always carried out with the comparison lamp at an approximate color match, except beyond 2400° above which temperature it was considered unsafe to operate the comparison lamp. The calibration of the comparison lamp at the various voltages, involving the difficulties of heterochromatic photometry, was made subsequently. In order to avoid the possible errors arising from individual idiosyncrasies of vision, the candlepower scale chosen was that given for a tungsten lamp by Middlekauff and Skogland.[3] The procedure of calibration consisted in transferring the photometer and comparison lamp to a standard photometer bench, and substituting a tungsten vacuum lamp for the black body. The comparison lamp was then brought successively to the various voltages used against the black body and readings were made against the tungsten lamp. In cases of large color difference blue screens calibrated at the Bureau of Standards were used. The relative candlepowers of the standard tungsten lamp were computed from the data of Middlekauff and Skogland, and hence the candlepowers of the comparison lamp, and so the brightnesses of the black body were obtained on this same scale.

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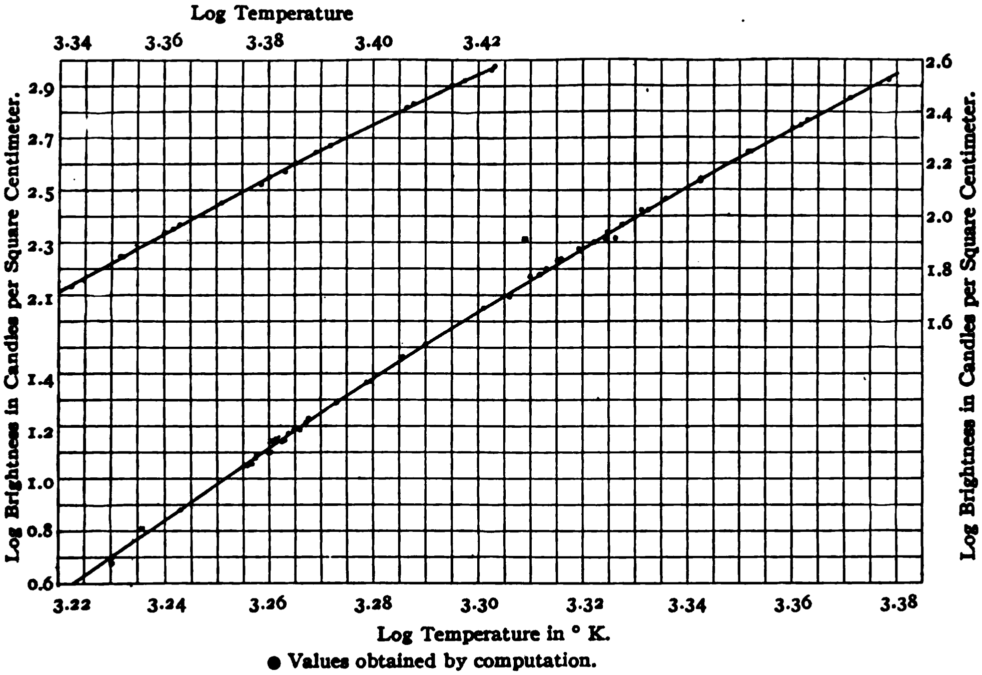
[1] Not as much weight is attached to the results obtained with the platinum-wound furnace because of the photometric difficulties at the low temperatures and with the necessarily small limiting diaphragm which the dimensions of the furnace demanded.

[2] PHYS. REV., (2), 10, p. 395, 1917.

[3] Bul. Bur. of Stds., 11, p. 483, 1915.

The justification of the Middlekauff and Skogland candlepower scale as representative of the average eye may be arrived at through the intercomparison of the visibility curves of several members of this group with the average of the 125 observers employed in the investigation of Coblentz on the visibility of radiation. Such a comparison indicates that the mean of the eight observers employed by Middlekauff and Skogland is not markedly different from the mean of the 125 observers used by Coblentz. Other lines of reasoning would indicate that the scale of Middlekauff and Skogland is not far from correct, as representing an average eye, but that if any change were made it would be in the direction of raising the upper region of the candlepower curve slightly with respect to the curve for the lower temperatures. This change would be less than one per cent. and so, in the absence of more definite information, the Middlekauff-Skogland scale has been adopted in the present investigation.



● Values obtained by computation.
○ Observed values using carbon-tube furnace.
× Observed values using platinum-wound furnace.
☐ Values given by Nernst.

Fig. 2.

Brightness of a Black Body at Various Temperatures.

Experimental Results.—The results on black-body brightness expressed in candles per cm^2 as a function of the temperature in degrees K. are given in a logarithmic form in Fig. 2, and the corresponding numerical results, taken from the curve are given for every 50° in Table I. The circles on the plat are the observed points obtained with the carbon-tube furnace, and the x's are the observed points using the platinum-wound furnace. The curve is drawn according to a least-square solution applied to the 46 observed points and assuming the cubic equation,

TABLE I.

Brightness of a Black Body at Various Temperatures.

Temperature.	Brightness Candles per Square Centimeter.	Temperature.	Brightness Candles per Square Centimeter.
1700° K.	5.0_8	2200° K.	$137._6$
1750	7.6_4	2250	177.
1800	11.3	2300	226.
1850	16.3	2350	284.
1900	23.1	2400	354.
1950	32.2	2450	438.
2000	$44._3$	2500	537.
2050	$60._0$	2550	651.
2100	$80._1$	2600	785.
2150	$105._7$	2650	939.

$$u = a + bt + ct^2 + dt^3,$$

where u = log candles per cm^2,

$t = \log T - 3.2$,

T = temperature in degrees K.

The values of the constants were found to be as follows:

$$a = 0.2626, \qquad c = -12.38,$$

$$b = 14.959, \qquad d = -1.13.$$

The 46 points are so distributed with respect to the least square curve that 23 points lie on one side and 23 on the other, the maximum deviation of any point, as seen from the curve, being about 8 per cent. The two points furthest from the curve by Chauvenet's criterion could be discarded. Thirty points lie within 2.5 per cent. of the curve, the average deviation of all points being about 2.4 per cent. The upper one of the three points obtained by Lummer and Pringsheim is within about 3.5 per cent of the curve (the other two points are at lower temperatures than any included in our observations) and the later observations of Nernst, reduced to the present temperature scale, are indicated by □'s.

The dots are the points computed on the basis of the authors' recently published data on the visibility of radiation, as discussed above. These computed brightnesses are, of course, only relative, but the value at 2077° K., which is in the neighborhood of the color temperature of the 4-wpc carbon standard lamp, was taken from the least square solution, and the other points were plotted in terms of that. It is seen that the computed points lie on the observed curve within an error of less than 1 per cent. except at the extreme ends of the temperature range. This is better than might be expected owing to the inherent difficulties in the present experiment. Points computed from Coblentz's visibility data show a consistent difference from the observed curve, though the difference is nowhere large.

The accuracy indicated in the brightness measurements suggests the advisability of adopting tentatively the value 70.2 candles per cm^2 as the brightness of a black body at the temperature 2077° K. and as a value for a primary standard of light. The temperature chosen is approximately the color temperature of the standard 4-wpc carbon lamps.[1]

Part II. The Mechanical Equivalent of Light.

This term has come to be used to mean the watts per lumen of the monochromatic light of greatest visibility, corresponding, in the recent experiments of the authors, to a wave-length $\lambda = 0.556\ \mu$. It is a physical constant of theoretical interest, and also serves as a means of expressing the absolute efficiency of illuminants. It depends on the brightness of a black body, or other source with known emissive power, on the temperature scale chosen, also on the constant σ in the Stefan-Boltzmann law ($E = \sigma T^4$), and finally on the curve of relative visibility of radiation,—*i. e.*, provided it is evaluated in the way to be followed in this paper. It may be determined directly by measuring the radiant flux and the luminous flux from some monochromatic source in a wave-length at or near that of maximum visibility. Determinations have been carried out by this method,[2] but in the opinion of the authors the experimental difficulties are too great to justify much confidence in the results.

In the present investigation the mechanical equivalent M, following the usual procedure, has been evaluated in the following way. Let

$B_0 \equiv$ normal brightness of the black body at any temperature T expressed in candles per cm^2.

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[1] The data given above are somewhat different from those published in the advance note in the Journal of the Franklin Institute (Jour. of Franklin Inst., 181, p. 420, 1916.) These differences are due to the assumption of a different temperature scale and to the addition of several new observed points which produced a small change in the curve, particularly at the higher temperatures.

[2] Phys. Rev., (2), 5, p. 269, 1915.

E_λ = the radiant flux per cm^2 per unit wave-length at the wave-length λ.
V_λ = the relative visibility of radiation in any wave-length λ in terms of the maximum visibility taken equal to unity.

Then

$$B_0 = \frac{1}{\pi M}\int_0^\infty E_\lambda V_\lambda d\lambda \tag{1}$$

or

$$M = \frac{1}{\pi B_0}\int_0^\infty E_\lambda V_\lambda d\lambda. \tag{2}$$

In the present investigation the visibility function is assumed as extending from 0.40 μ in the blue to 0.76 μ in the red, the values assigned in the two extreme ends of the spectrum being taken from the previously published data of two of the present authors[1] in the red, and of Hartman[2] in the blue.[3] Beyond the chosen limits the added luminosity area would be entirely negligible.

The method employed in evaluating the integral in equation (1) was that of summating the product $E_\lambda V_\lambda \Delta\lambda$ for steps $\Delta\lambda = 0.01\ \mu$, and correcting for the errors of summation by the one third rule. The value of E_λ and therefore that of M depends upon the constant C_1 in Planck's equation, or its equivalent σ in the Stefan-Boltzmann equation, since these two constants are definitely related if C_2 is known. The latter is taken as 14350 μ deg., as discussed in the first part of the paper, and for σ the value $5.7 \times 10^{-12} \frac{\text{watts}}{\text{cm}^2\ \text{deg}^4}$, as found by Coblentz, has been assumed as the most probable value. On the basis of these values, C_1 comes out as 3.72×10^{-12} watts cm^2.

Table II.

Mechanical Equivalent M for Various Temperatures.

Temperature.	M in Watts per Lumen.	Temperature.	M in Watts per Lumen.
1700° K.	0.00147_3	2200° K.	$.00149_6$
1800	$.00149_1$	2300	$.00149_7$
1900	$.00149_8$	2400	$.00149_7$
2000	$.00149_8$	2500	$.00150_2$
2100	$.00149_7$	2600	$.00151_1$
Average.......	....	....	0.001496

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[1] *Loc. cit.*

[2] *Loc. cit.*

[3] The complete visibility curve used is given in Appendix II. of the paper by the present authors on "The Visibility of Radiation," Astrophys. Jour., Sept., 1918.

TABLE III.

Investigators.	Recommended Values of the Mechanical Equivalent of Light in Watts per Lumen.	Visibility Data.	Constants σ in $\frac{\text{Watts}}{\text{Cm.}^2}$ Deg.^{4} C_2 in μ Deg.	Method.
P. G. Nutting	0.00120±0.00005	Nutting	$\sigma=5.7\times10^{-12}$ (Presumably)	Measurements of total radiant flux and total luminous flux from acetylene. Coblentz' energy curve for acetylene used.
H. E. Ives	0.00160±0.00003	Ives & Kingsbury Curve from equation	$\sigma=5.7\times10^{-12}$	(1) Measurement of total radiant flux and total luminous flux from a monochromatic source. (2) Measurement of radiant flux through luminosity filter and integral luminous flux direct from the source.
	Secondary value 0.00154	Ives & Kingsbury	$\sigma=5.7\times10^{-12}$ $C_2=14350$	(3) Measurement of brightness of a platinum-wedge black body at the melting point of platinum taken as 2037° K.
W. W. Coblentz	0.00162±0.00005 Note: If in method (1) the final data of Hyde, Forsythe and Cady were used, the value derived by this method would be increased by approximately 1½ per cent.	Coblentz and Emerson	$C_2=14350$ Melting points as given by Day and Sossman	Several methods using observations by himself and others. (1) Computation using preliminary data on brightness of a black body by Hyde, Forsythe and Cady. (2) Direct measurement on monochromatic radiation, by Coblentz, Ives and Kingsbury. (3) Measurements on incandescent lamps by Coblentz and Emerson.
Hyde, Forsythe and Cady	0.00150±0.00005	Hyde, Forsythe and Cady	$\sigma=5.7\times10^{-12}$ $C_2=14350$ Melting point of gold 1336° K.	Direct measurement of brightness of carbon and platinum tube black bodies.

Substituting in equation (5) the brightness of the black body at any temperature, as given in the first part of this paper, a value of the mechanical equivalent M for any temperature is found. If the observed and computed curves of black-body brightness are the same throughout, then the value of M would be found to be the same for all temperatures, as might be expected. The differences between the two curves as presented in the first part of this paper are so small that the values for M computed for every 100°, as given in Table II., are sensibly constant, except at the extreme ends of the curve, the total range being only 2.2 per cent., and the range from 1800° to 2400° K. being only 0.5 per cent.

The average value from the present experiments is given in Table III., in comparison with the published values of other observers. In each case so far as is possible from the published reports, the method used, the visibility curve employed (if any) and the temperature scale involved are given.

SUMMARY.

1. A new set of experimentally determined values of the brightness of a black body from 1700° to 2600° K. are given on the assumption of a temperature scale based upon Planck's equation taking the gold point as 1336° K. and the constant C_2 as 14350 μ deg.

2. These values, as plotted in a curve obtained by a least-square solution, are compared with the relative computed values on the assumption of the visibility curve recently published by the authors.

3. A value of 70.2 candles per cm^2 as the brightness of a black body at a temperature of 2077° K. (color match with 4-wpc standard carbon lamps) is proposed tentatively as an absolute standard of light.

4. The mechanical equivalent of light for the wave-length of maximum visibility ($\lambda = 0.566\ \mu$) is computed to be 0.00150 ± 0.00005 watts per lumen.

APPENDIX I.

It is frequently desired to know the rate of change of candlepower or of brightness of a black body at any temperature with a change in temperature or a change in watts. Since the data presented in the body of this paper are somewhat different from other published data it seems advisable to compute these coefficients and at the same time to evaluate the Crova wave-length[1] at different points on the brightness-temperature scale.

Differentiating equation (1) for the brightness of a black body with reference to temperature, the following final equation is obtained:

$$\frac{dB_0}{dT}\frac{T}{B_0} = \frac{C_2}{T}\frac{\int_0^\infty \frac{1}{\lambda} E_\lambda V_\lambda d\lambda}{\int_0^\infty E_\lambda V_\lambda d\lambda}. \tag{3}$$

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[1] Loc. cit.

If now the summation method employed in the body of the paper in computing the mechanical equivalent of light be used the integral in the denominator is at once obtained; and if the respective terms at any temperature are multiplied by the reciprocals of the wave-length and the summation made, the numerator is also obtained. The ratio of these two integrals at any temperature, multiplied by C_2/T gives the desired percentage change in brightness corresponding to a change of 1 per cent. in temperature. These results are shown in Fig. 3. Since $E = \sigma T^4$ expresses the relation between temperature and total watts radiated it follows that if the ordinates of the curve are divided by 4 the resultant curve will show the percentage change of brightness corresponding to a change of 1 per cent. in total watts radiated.

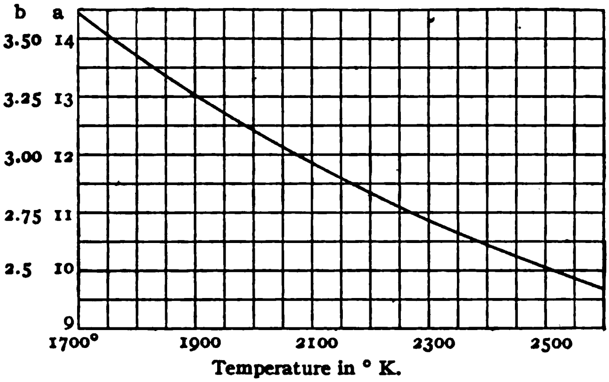

Fig. 3.

Percentage Changes in Brightness of a Black Body at Various Temperatures for (*a*) one per cent. change in temperature and for (*b*) one per cent. change in watts.

Moreover it follows from equation (3) that if λ_0 is the wave-length such that at some temperature the ratio of the two integrals in equation (3) is $1/\lambda_0$, then at that temperature

$$\frac{dB_0}{dT}\frac{T}{B_0} = \frac{C_2}{\lambda_0 T}, \tag{4}$$

or, the percentage change in brightness corresponding to a change of 1 per cent. in temperature is equal to $C_2/\lambda_0 T$.

But it follows immediately from Wien's equation that

$$\frac{dE_{\lambda T}}{dT}\cdot\frac{T}{E_{\lambda T}} = \frac{C_2}{\lambda T}, \tag{5}$$

or, the percentage change in watts radiated by a black body at any wave-length λ and at any temperature T, corresponding to a change of 1 per cent. in temperature is $C_2/\lambda T$. Hence if the particular value λ_0 be taken for λ, then from equations (4) and (5) it follows that the percentage change in brightness at any temperature, and the percentage change in watts emitted at that temperature in the wave-length λ_0, corresponding to a change of 1 per cent. in temperature, will be the same. By definition, therefore, λ_0 is the Crova wave-length for that temperature, and hence is readily evaluated since the two integrals in equation (3) are now known. The values of the Crova wave-length thus obtained are given in Fig. 4.

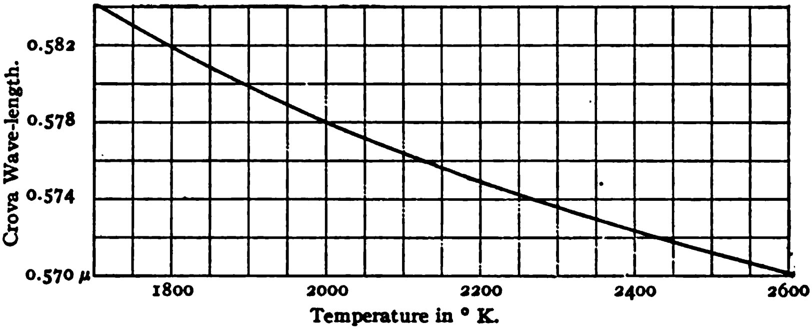

Fig. 4.

Crova Wave-length at Various Temperatures for Black Body Radiation.

It will be seen by comparison with the data published by Kingsbury[1] and by Coblentz[2] that the authors' values for the Crova wave-length are somewhat smaller. This difference is for the most part due to the difference in visibility curves since those obtained by the flicker method show larger values in the longer wave-lengths of the visible spectrum.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,
NELA PARK, CLEVELAND, OHIO,
September, 1918.

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[1] PHYS. REV., (2), 7, p. 167, 1916.

[2] Bul. Bur. of Stds., 14, p. 255, 1918.

IONIZATION AND RESONANCE POTENTIALS FOR ELECTRONS IN VAPORS OF ARSENIC, RUBIDIUM, AND CÆSIUM.

By Paul D. Foote, O. Rognley and F. L. Mohler.

WHEN electrons are accelerated in a metallic vapor, at least two types of inelastic impact between an electron and atom occur. The first of these results in an orbital shift of the electrons bound in the atom and the second in the complete removal of an electron or ionization of the atom. The potential differences through which an electron must fall to obtain sufficient velocity or kinetic energy for these two types of collision are known as the resonance and the ionization potentials for the particular metal in question. The resonance and ionization potentials of a series of metals have been determined in this laboratory.[1] The present paper is a report upon the work with arsenic, rubidium and cæsium.

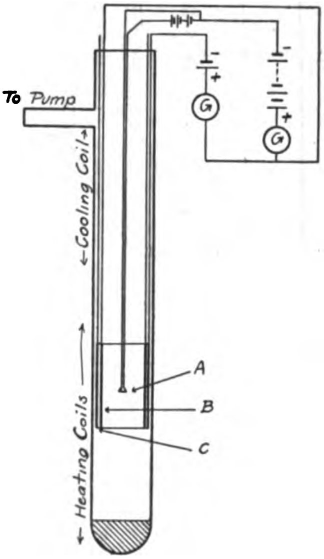

Fig. 1.
Diagram of apparatus used for arsenic.

The arrangement of apparatus for arsenic is shown in Fig. 1. Arsenic was vaporized at the bottom of the pyrex glass tube and the vapor, after passing through the superheated ionization chamber, condensed in the upper half of the tube. The ionization chamber was supported entirely by steel rods running from the top of the tube. The source of electrons was a hot tungsten wire cathode, *A*, of low resistance, coated with lime. Surrounding the cathode was a cylinder of iron net, *B*, and outside and coaxial with this a cylinder of sheet iron, *C*. The apparatus was evacuated by means of a Langmuir and Stimson condensation pump to a gas pressure of about 5.10^{-5} cm. Hg as registered by a McLeod gage. Sufficient vapor was obtained by heating the arsenic to about 400 to 500° C.

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[1] Na, Tate and Foote, J. Wash. Acad. Sci., 7, p. 517, 1917. Cd, Tate and Foote, Bur. Standards Sci. Paper No. 317. Na, Cd, K, Zn, Tate and Foote, Phil. Mag., 36, p. 64; 1918. Mg, Tl, Foote and Mohler, Phil. Mag. (in press).

A trap containing cadmium-tin alloy chips immersed in liquid air was used between the pumps and the ionization tube in order to preclude the possibility of the presence of mercury in the ionization chamber.

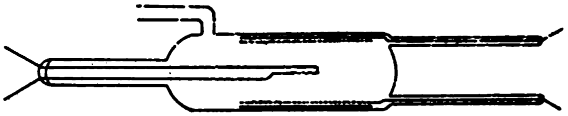

Fig. 2.
Ionization tube used for rubidium and cæsium.

The experimental procedure consisted in applying a constant retarding potential of 0.8 to 1.5 volts between the net and the outside cylinder and measuring both the total current from the hot wire, and that portion of the current which reached the outside cylinder against the retarding field, as functions of a variable accelerating potential applied between the hot wire and net. The apparatus used for rubidium and cæsium is illustrated by Fig. 2. This was constructed of soda glass, as it was found that the heated vapors of these metals do not noticeably attack soda glass while the reaction with lead glass is very pronounced. The net and cylinder were of nickel, and a platinum hot wire coated with lime served as the cathode. A retarding potential of about 0.5 volt was applied between the net and cylinder. Metallic rubidium from Kahlbaum was used and several grams of cæsium were obtained by heating metallic calcium and Kahlbaum cæsium chloride to about 400° C. at a very low pressure. The metal was distilled into a small test-tube, covered with a drop of benzol, and inserted into the pump intake, after which the apparatus was sealed and evacuated. The metal was then redistilled into the ionization chamber. During the observations the

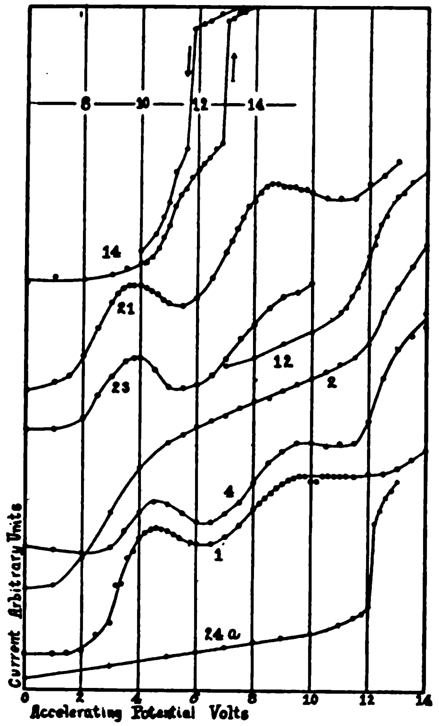

Fig. 3.
Total and partial current curves for arsenic.

entire ionization tube was uniformly heated to about 150° C., and the metal gradually distilled out, condensing in the exhaust tube. When necessary the metal was redistilled back into the ionization tube.

Table I.

Ionization and Resonance Potentials for Electrons in Arsenic Vapor. [Curves 1 to 12; Cadmium-tin alloy trap and liquid-air trap between pump and ionization tube for several hours before and during the time of observation. Curves 21 to 24a: All apparatus reconstructed and both traps used continuously from the time the apparatus was assembled.]

Curve.	Resonance.			Applied Potential at Ionization.	Initial Potential.	Ionization Potential.[1]	Resonance Potential.[2]	
	a.	*b.*	*c.*				*b–a.*	*c–b.*
1	3.6	8.4	. . .	. . .	. .	. . .	4.8	. .
2	. .	. .	. . .	10.4	1.0	11.4	. .	. .
3	. .	. .	. . .	10.5	1.0	11.5	. .	. .
4	3.8	8.4	12.6	. . .	. .	. . .	4.6	4.2
4*a*	3.8	8.4	12.8	. . .	. .	. . .	4.6	4.4
5	. .	. .	. . .	10.6	1.0	11.6	. .	. .
10	3.6	8.8	. . .	. . .	. .	. . .	5.2	. .
12	. .	. .	. . .	10.6	1.1	11.7	. .	. .
21	2.8	7.7	. . .	. . .	. .	. . .	4.9	. .
22	. .	. .	. . .	10.0	1.8	11.8	. .	. .
23	3.0	7.8	. . .	. . .	. .	. . .	4.8	. .
23*a*	3.2	. .	. . .	. . .	. .	. . .	. .	. .
24	. .	. .	. . .	9.6	1.6	11.2	. .	. .
24*a*	. .	. .	. . .	10.0	1.6	11.6	. .	. .

Fig. 3 represents some of the data obtained for arsenic. A complete analysis of these and other curves is given in Table I. The partial-current curves show pronounced changes in curvature at successive points *a*, *b*, *c*, which differ in potential by a constant amount. This constant difference gives the resonance potential directly, eliminating any consideration of initial velocities. The total-current curves show a rapid increase in slope at a point for which the potential, corrected for the initial velocity (potential) of the electrons as obtained from the partial-current curves, gives the ionization potential. The initial potential is the difference between the resonance potential and the applied potential corresponding to the first point of resonance. A detailed discussion of this method for determining the ionization potential has been given elsewhere.[3] Curve 14, Fig. 3, is of interest as an example of a discontinuous total-current curve. On increasing the accelerating potential the total current increases gradually up to about 13 volts. At this point the current increases discontinuously and upon decreasing the accelerating potential the current decreases discontinuously at about 12 volts. Similar curves have been obtained for mercury, sodium and rubidium. Curves of this type may be secured when the vapor density and the electronic current are high. In the present instance a measurement of the initial velocity of the electrons was not made but, in general, these discontinuous curves afford as accurate a determination of the ionization potential as those obtained with a smaller vapor density and initial velocity. The discontinuous change occurs considerably above the ionization potential and is probably due to a potential redistribution in the ionization chamber. As the potential is gradually increased the most intense part of the discharge, at least for rubidium, sodium, etc., for which metals the glow discharge is visible, moves suddenly at the point of discontinuity from the hot wire to the net.

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[1] Mean ionization potential = 11.5 volts.

[2] Mean resonance potential = 4.7 volts.

[3] Tate and Foote, J. Wash. Acad. Sci., 7, p. 520, 1917.

The final results with arsenic gave 4.7 volts for the resonance potential and 11.5 volts for the ionization potential. These values are quite similar to the corresponding voltages for mercury, namely, 4.9 volts and 10.3 volts, and for this reason it was at first suspected that mercury from the pumps and gage was present in the ionization chamber. Although both liquid air and cadmium-tin alloy were used as a mercury trap in the first series of experiments here described (curves 1 to 12) the apparatus was entirely reconstructed with every precaution again being taken to avoid contamination by mercury. The results of this second series of experiments (curves 21 to 24*a*) confirmed those of the first series. In work at this laboratory with eleven different metals no trace of mercury has ever been observed, when the metals are volatilizing rapidly, in the characteristic ionization and resonance curves, although heretofore we have never used either cadmium-tin or liquid-air traps. In the case of the gases and non-metallic vapors, however, considerable precaution must be taken to avoid the presence of mercury. It was therefore thought possible that contamination by mercury might still be present and show up prominently in the somewhat non-metallic arsenic vapor. The following experiment was accordingly performed.

It was noted that, contrary to our experience with other metals, no glow discharge could be detected in arsenic at the ionization point. A special discharge tube was constructed for observing the arsenic arc more carefully. A steel disk anode and a tungsten hot-wire cathode were mounted about 1 cm. apart in a glass tube lined with iron net in electrical contact with the hot wire. The tube was mounted in a special electric furnace having a small side opening through which the arc could be viewed. Above the distillation point of arsenic the electron current indicated marked ionization as before at the ionization potential of 11.5 volts. With a current of 30 milliamperes no arc or glow discharge whatever was visible. A few drops of mercury were then placed in the ionization tube. A visible arc was obtained with a current of 0.02 milliampere, and with a current of 20 milliamperes the arc was very intense. Thus the fact that a current of 0.02 milliampere is sufficient to show the presence of mercury as an impurity of arsenic, and that a current of 30 milliamperes in supposedly pure arsenic vapor does not show a trace of any glow whatever, must be taken as a proof of the purity of the vapor. It is of further interest to note that when arsenic is heated with a small amount of mercury the ionization due to the mercury will appear 200° C. below the point where arsenic vaporizes sufficiently to show ionization, and after prolonged heating at 500° C. the mercury is driven out of the tube so that finally the ionization of arsenic alone is observed and the glow discharge disappears.

It has been shown in the earlier papers referred to that for many metals the ionization and resonance potentials are determined by the quantum relation $h\nu = eV$ where h is Planck's constant of action, e the electronic charge, V the resonance or ionization potential, and ν a frequency determinable from spectroscopic data. The value assigned to ν appears to depend upon the column of the periodic table to which the particular metal under consideration belongs. Thus, for Group I. of the periodic table, ν is the frequency of the first line of the principal doublet series for resonance potential and the convergence of this series for ionization potential. For Group II. the resonance potential is determined by the first line of the combination series of single lines $1.5\,S{-}mp_2$, and the ionization potential is determined by the convergence of this series. Arsenic is the first metal in Group V. so far investigated. The quantum relation may be written $\lambda = 12334/V$ on substituting the proper values for h and e, where λ is the wave-length in Ångstrom units corresponding to the potential V expressed in volts. Thus, the resonance potential of 4.7 volts and the ionization potential of 11.5 volts correspond to the wave-lengths $\lambda = 2620$ Å. and $\lambda = 1070$ Å., respectively. No series having its first line at $\lambda = 2620$ Å. or its convergence at $\lambda = 1070$ Å. is known for arsenic. In fact, all the series relations so far observed in arsenic have been quite unsatisfactory, which is true also of the other metals in this family. Accordingly, the present work very likely indicates an undiscovered series in arsenic having its first term (or terms if a doublet series) near $\lambda = 2620$ Å. and converging near $\lambda = 1070$ Å., but as to the character of the series nothing can be inferred.

Fig. 4 represents the partial-current curves and Fig. 5 the total-current curves obtained with rubidium. As seen from Table II., the mean value of the resonance potential is 1.6 volts. This value and the point of resonance collision, *a*, permit the determination of the i velocities or equivalent potentials shown in column 9. The t current curves 1, 6, 11, 12 were taken under the same experim conditions as the partial-current curves 3, 4, 5, so that the initial p tial, V_0, to be added to the observed applied potential at ionizati give the ionization potential, is 1.2 volts. Curve 12 correspon curve 7 with an initial potential of 1.4 volts, the cathode being ope

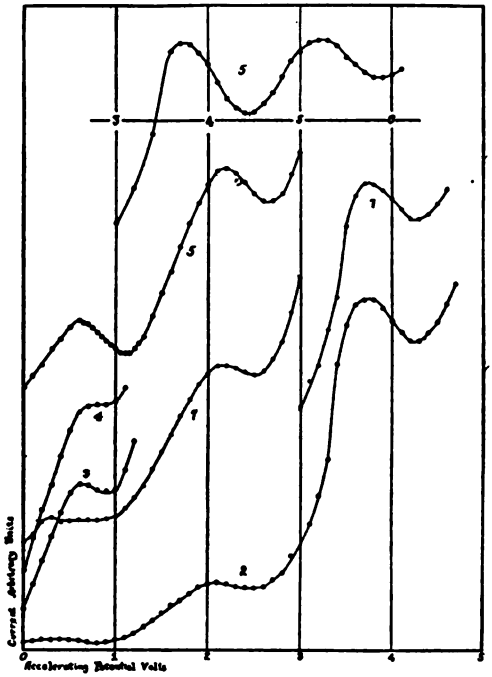

Fig. 4.

Partial-current curves for rubidium.

at a higher temperature when these two curves were obtained. I case of rubidium and cæsium the total current did not show satur below the ionization potential, so that the point at which ioniz begins is not indicated by as sharp a break as would otherwise o The following method for determining the break point proved satisfactory.

Langmuir[1] has derived an equation showing the relation bet the thermionic current and voltage for the case of a hot wire surrou by a cylindrical anode. If account is taken of the initial velocity c electrons, an additional constant appears in the equation derived by Langmuir. The kinetic energy of an electron at a distance r from the hot wire is:

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[1] PHYS. REV., 2, p. 457, 1913.

$$\tfrac{1}{2}mv^2 = (V + V_0)e, \tag{1}$$

where V_0 is the initial potential and $V = f(r)$ is the applied accelerating potential. Defining i as the thermionic current per unit length of the hot wire, one obtains directly from Poisson's equation:

$$r\frac{d^2V}{dr^2} + \frac{dV}{dr} = i\sqrt{\frac{2m}{e(V + V_0)}}, \tag{2}$$

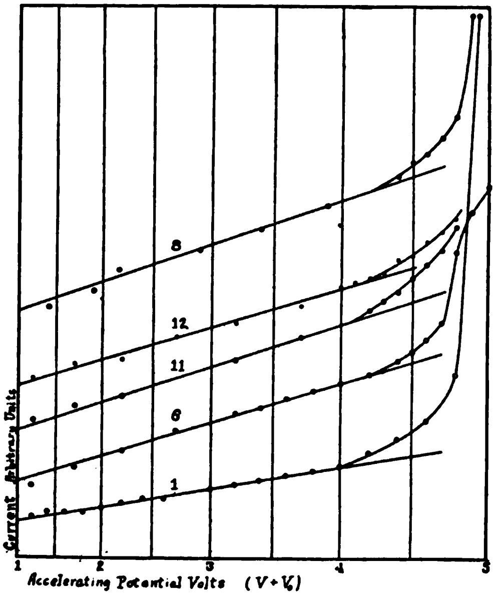

Fig. 5.
Total-current curves for rubidium.

the solution of which may be represented by the following relation where A is a constant for any definite configuration of the ionization chamber and temperature of the hot wire.

$$i = A(V + V_0)^{3/2}. \tag{3}$$

Hence, the relation between i and $(V + V_0)^{3/2}$ is linear for the purely thermionic current below its saturation value. When ionization begins the current increases more rapidly so that the ionization potential is given by the value of $(V + V_0)$ at the point where the curve i plotted against $(V + V_0)^{3/2}$ deviates from a straight line. Fig. 5 represents the total-current curves for rubidium plotted in this manner. The values of the applied potential were corrected by adding the initial potential V_0 as given in Table II. The linear relation is well satisfied except possibly at the lowest potentials. The agreement is even better than

TABLE II.

Ionization and Resonance Potentials for Electrons in Rubidium Vapor.

Curve.	Resonance.				Resonance Potential.[1]			Initial Potential.
	a.	*b.*	*c.*	*d.*	*b–a.*	*c–b.*	*d–c.*	
2......	..	1.65	3.35	...	...	1.70	...	1.2
3......	0.4	...	...	...	...	...	...	
4......	.4	...	...	...	...	...	...	
5......	.4	1.85	3.45	4.90	1.45	1.60	1.45	
7......	.15	1.70	3.50	...	1.55	1.80	...	1.4
					Mean = 1.6 volts.			
	V_0.	$V+V_0$.[2]						
1......	1.2	4.0						
6......	1.2	4.2						
8......	1.4	4.2						
11......	1.2	4.1						
12......	1.2	4.2						

might be expected from a more careful consideration of the applicability of equation (3). Strictly considered, this equation should not hold exactly for two reasons: (1) Because of inelastic collision of some of the electrons at the resonance potential, and (2) because of the potential drop along the hot wire. Thus, equation (3) states that when $V = 0$, $i = AV_0^{3/2}$, but, although V_0 is comparable with V, the observed current is practically zero when the applied field is zero. The reason for this is that the electrons with no accelerating field between the hot wire and net fall along the hot wire from points of lower to higher potential or along one of the leading-in wires. A measurable current is observed only when the applied potential is comparable with the electrostatic field along the wire. This fact accounts in part for the high initial velocities observed. Also it may be noted that our method of selecting the point of resonance gives greatest weight to the electrons

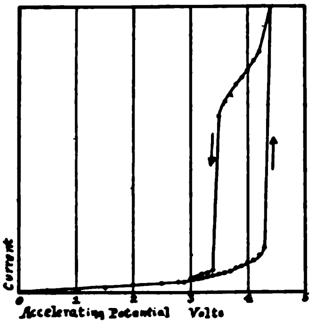

Fig. 6.
"Discontinuous" total-current curve for rubidium.

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[1] Resonance potential = 1.6 volts.

[2] Ionization potential = 4.1 volts.

of highest speed. A point of further interest is that practically the same values for the initial velocity are obtained when the partial-current curves are plotted directly and when they are plotted on the rectified coördinate system used for the total-current curves.

The mean value of the ionization potential of rubidium obtained from the curves shown in Fig. 5 is 4.1 volts. Fig. 6 illustrates a type of discontinuous total-current curve obtained when the density of the rubidium vapor is high.

TABLE III.

Ionization and Resonance Potentials for Electrons in Cæsium Vapor.

Curve.	Resonance.		Resonance Potential.[1]	Initial Potential.
	a.	*b.*		
3	0.60	2.10	1.50	0.88
5	.55	2.00	1.45	.93
7	.50	2.00	1.50	.98
8	.55	2.00	1.45	.93
10	.55	2.00	1.45	.93
11	.55	2.00	1.45	.93
12	.40	1.90	1.50	1.08
14	.40	1.90	1.50	1.08
Mean			1.48	1:0
	$(V+1.0)^{\frac{3}{2}}$.	Ionization Potential.[2]		
1	8.5	4.2		
4	7.7	3.9		
6	7.6	3.9		
9	7.1	3.7		
13	7.4	3.8		

Fig. 7 shows the partial-current curves and Fig. 8 the total-current curves for cæsium vapor. The method of determining the ionization and resonance potentials was that described above for rubidium. The final values, summarized in Table III., are 1.48 volts for the resonance potential and 3.9 volts for the ionization potential.

From the behavior of sodium and potassium one would expect the resonance and ionization potentials of rubidium and cæsium to be determined by the relation $V = 12334/\lambda$ where λ is the first line and convergence wave-length, respectively, of the principal series of doublets for each metal. The following table, to which for completeness have been added the data on the other alkali metals, confirms this fact.

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[1] Resonance potential = 1.48 volts.

[2] Ionization potential = 3.9 volts.

TABLE IV.

Resonance and Ionization Potentials for the Alkali Metals.

Metal.	λ (Å). ($\nu = 1.5s - 2p$).	Resonance Potential Volts.		λ (Å). ($\nu = 1.5\ s$).	Ionization Potential Volts.	
		Computed.	Observed.		Computed.	Observed.
Lithium....	6707.85[1]	1.839	....	2299.67[2]	5.363	....
Sodium.....	5895.94[1] 5889.97[1]	2.092 *2.094*	2.12[3]	2412.63[4]	5.112	5.13[3]
Potassium..	7699.01[1] 7664.94[1]	1.602 *1.609*	1.55[3]	2856.69[5]	4.318	4.1[3]
Rubidium..	7947.64[1] 7800.29[1]	1.552 *1.581*	1.6[6]	2968.40[5]	4.155	4.1[6]
Cæsium....	8943.46[1] 8521.12[1]	1.379 *1.447*	1.48[6]	3184.28[5]	3.873	3.9[6]

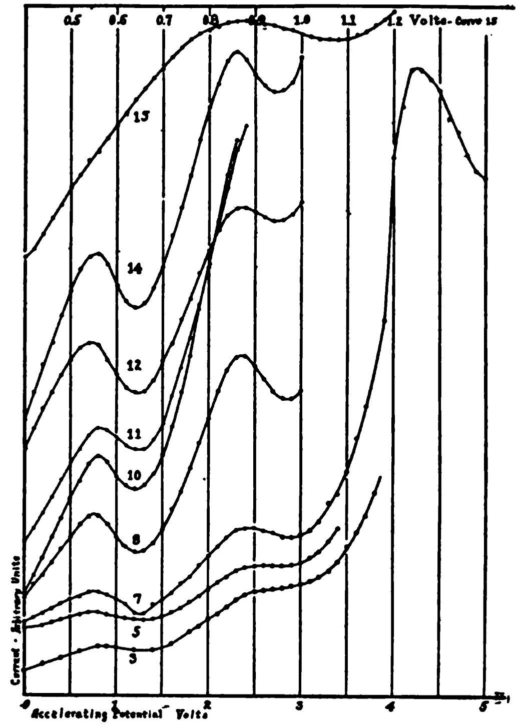

Fig. 7.

Partial-current curves for cæsium.

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[1] Meggers, Bur. Standards Sci. Paper No. 312.

[2] Dunz Tübingen Thesis, 1911.

[3] Tate and Foote, Phil. Mag., 36, p. 75, 1918.

[4] Wood and Fortrat, Astrophys. Jour., 43, p. 73, 1916.

[5] Watts, Phil. Mag., 29, p. 775, 1915.

[6] Foote, Rognley, and Mohler, this paper.

The first pair of doublets in the principal series of both rubidium and cæsium is rather widely separated. In the case of cæsium the separation is equivalent to about 0.07 volt. If resonance collision occurs at both 1.379 and 1.447 volts, corresponding to each member of the cæsium doublet, the partial-current curves should show two sets of maxima differing by 0.07 volt at the first point of resonance, by 0.14 volt at the second point of resonance, etc. None of the curves of Fig. 7 show double maxima and for curve 15, observations were made at every 0.02 volt. It is possible that the presence of double maxima could not be detected on account of the initial velocity distribution from the hot wire. However, it has been found by Foote and Mohler[1] that in the case of thallium the shorter wave-length member of the first doublet of the principal series appears to determine the resonance potential. Hence it seems quite likely that the proper theoretical values for the resonance potentials of rubidium and cæsium are 1.581 and 1.447 volts, respectively. This possibility suggests an analogy to the general behavior of the *K*

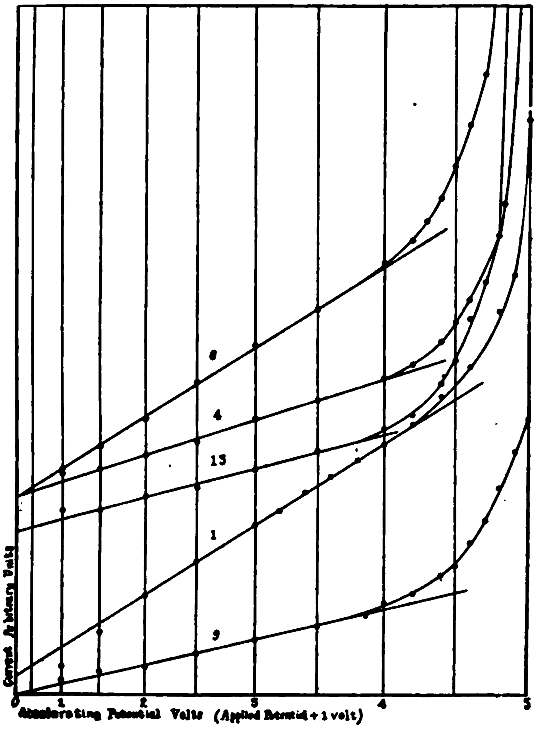

Fig. 8.

Total-current curves for cæsium.

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[1] Phys. Rev., 11, p. 487, 1918.

series in the X-ray spectrum of metals as has been pointed out by Foote and Mohler.[1] The present work indicates that the "single line" spectra of rubidium and cæsium are the doublets $\lambda = 7949$ and 7803 Å. and $\lambda = 8950$ and 8528 Å., respectively. At the ionization potential the principal series, the first subordinate series, and the second subordinate series appear. The following doublets were observed visually in the case of cæsium when the accelerating potential was equal to the ionization potential: Principal series, $m = 3$; first subordinate series, $m = 5, 6, 7, 8, 9, 10$; second subordinate series, $m = 4.5$.

SUMMARY.

Two types of inelastic collision between the electron and atom occur in vapors of arsenic, rubidium and cæsium. The potential differences through which an electron must fall to attain sufficient energy or velocity to produce these collisions are known as the resonance and ionization potentials for the particular metal in question. For rubidium and cæsium the resonance potential is determined by the quantum relation $h\nu = eV$ where ν denotes the frequency corresponding to the first doublet of the principal series. A similar relation holds for the ionization potential where ν denotes the convergence frequency of the principal series. Experimentally determined values of the resonance and ionization potentials for rubidium were 1.6 volts and 4.1 volts, respectively; for cæsium, 1.48 volts and 3.9 volts, respectively; for arsenic, 4.7 volts and 11.5 volts, respectively. By applying the quantum relation to the results obtained with arsenic we are able to predict the presence of an undiscovered spectral series in arsenic having its first term (or terms if a doublet series) near $\lambda = 2620$ Å. and converging near $\lambda = 1070$ Å. A Wehnelt tube discharge in arsenic possesses no luminosity indicating that the important series lines lie in the ultra-violet. At the ionization potential of rubidium and cæsium the principal, first, and second subordinate series appear. No ionization whatever could be detected at the resonance potentials of rubidium, cæsium, and arsenic.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
August 16, 1918.

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[1] J. Wash. Acad. Sci., Sept. 19, 1918.

ENERGY OF THE CHARACTERISTIC X-RAY EMISSION FROM MOLYBDENUM AND PALLADIUM AS A FUNCTION OF APPLIED VOLTAGE.

By Benjamin Allen Wooten.

Introduction.

SINCE the introduction of the X-ray spectrometer by W. H. and W. L. Bragg as an instrument for studying the various properties of X-radiation, much work has been done. Most of this work concerns itself with the general radiation and there is relatively little published matter on the intensity of the characteristic rays. In a recently published article[1] D. L. Webster finds that the rays of the K series of rhodium are produced only at voltages above that required by the Planck quantum hypothesis to excite general radiation of a frequency slightly greater than that of the γ line. He finds that above that potential all K lines increase in the same ratio for a given increase in potential. The rates of increase of intensity are continuously accelerated.

The following work was undertaken with a view of studying the relation between the energy of the lines of the K series of molybdenum and palladium and the voltage producing them. The spectrometer that was used was similar in general to the one used by Dr. Clayton T. Ulrey[2] in his work on the general radiation. Voltages were measured by means of an electrostatic voltmeter and energy was supplied for the X-ray tubes by a Kenotron rectifying set. During the progress of the work points concerning the probability of new lines, absorption coefficients, etc., presented themselves. They were dealt with and will be discussed.

Description of Apparatus.

Current at 125 volts and 500 cycles was taken from a motor generator set and was stepped up by a three-kilowatt oil-immersed transformer. The high voltage alternating current from the transformer was rectified by means of two General Electric Kenotrons and an oil-immersed glass-plate condenser. The energy supplied to the X-ray tube was taken from this condenser. Under normal conditions the voltage generated by the set was constant to one and one half per cent. The voltage was measured by an electrostatic voltmeter that was built in the laboratory. It consisted of a pair of fixed spheres and a pair of movable ones hung on a steel strip suspension and pivoted at the bottom on a needle. The four spheres were charged to the same potential, and the force of repulsion between the fixed and movable ones caused the instrument to show a deflection.

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[1] D. L. Webster, Phys. Rev., June, 1916, pp. 599–613.

[2] C. T. Ulrey, Phys. Rev., May, 1918.

The voltmeter was calibrated by means of a standard spark gap. This gap consisted of 12.5 cm. balls. The relation between sparking distance and voltage was obtained from published tables. The calibration curve of the voltmeter was very regular, almost approaching a straight line at high voltages. The voltmeter kept its adjustment perfectly.

Currents to the X-ray tube were measured on a Weston direct current milliammeter in all the work except that on absorption. In that part of the work the currents were read on a Leeds and Northrup portable galvanometer properly shunted. An accident to the milliammeter made this change necessary.

The X-ray tube that was used in the work on molybdenum was one that was kindly supplied by Dr. W. D. Coolidge, of the General Electric Company. The Palladium tube was built up in the laboratory. The cathode was of the Coolidge type and it was also furnished by Dr. Coolidge. The anode consisted of a cylinder of iron with a small piece of palladium set in it. The target face was inclined at an angle of 26 degrees with the plane perpendicular to the elements of the cylinder. The tube was evacuated by means of a diffusion pump of the Langmuir type, the mercury trap of which was cooled with a mixture of carbon dioxide snow and ether. This pump was in series with a Gaede rotary pump and a piston pump. The tube was baked at 300° Centigrade while the preliminary evacuation was in progress and the target was heated to dull redness from time to time by discharges at 25 kilo-volts. It was found necessary to keep the pump in operation whenever observations were being made.

The spectrometer was peculiar only in the fact that the ionization chamber was very long. Its length was 128 cm., and complete absorption through a long range of wave-lengths was possible with its use. This chamber contained the saturated vapor of methyl iodide. Methyl iodide was used because it does not show abnormal absorption at any point throughout the range of wave-lengths investigated.[1] The chamber and electrometer were freed of water vapor by the use of phosphorous anhydride. The gold-leaf electrometer was attached directly to the end of the ionization chamber, and the image of the leaf was projected on a ground glass screen with a scale marked on it. This screen was securely fastened to the ionization chamber. The leaf was charged negatively to 400 volts by means of a small dynamotor. The X-ray tube was surrounded by a heavy lead screen and a beam was taken off at about 26 degrees from the face of the target, this beam coming as nearly as possible from the center of the focal spot. After passing through an aperture in the screen this beam was passed through an adjustable slit, the jaws of which were of lead, and then reflected by the crystal. All circuits to the tubes, voltmeter, etc., carrying high potential current returned through ground. The Coolidge cathodes were heated by current from a small storage battery.

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[1] "X-rays," G. W. C. Kaye, p. 147.

The crystal used was of Montana calcite and was kindly lent to us by Professor A. J. Moses, of the Department of Mineralogy. Photographs of the back slit taken with the reflected beam showed remarkably clear and perfect lines, and it was not possible to detect any irregularities in them.

The natural leak of the electrometer when properly connected to the ionization chamber was such as to cause the leaf to move across the interval of the scale that was used in taking readings in about 625 seconds. This of course varied, but it was usually more than 600 seconds. The readings of X-ray intensity were taken in the usual way by allowing the reflected beam to pass into the chamber through an adjustable slit, and noting with a stop watch the time taken for the leaf to pass across a fixed part of the scale. The same scale interval was used in all the readings taken. The time taken for a reading varied from less than one second to 550 seconds. The projected image of the leaf on the ground glass screen was clear and sharp and readings could be repeated with an accuracy of one per cent.

Results.

The apparatus was adjusted and lined up so that a beam of X-rays from the back slit would pass over the center of the spectrometer circle. The position of the ionization chamber, when this beam was entering its slit was read on the circle and taken as its position for direct rays.

The crystal was then put in place and adjusted so that its reflecting face was as nearly as possible parallel to the beam of direct rays and over the axis of the spectrometer. The crystal was moved in steps of θ and the ionization chamber in steps of 2θ until strong reflection was observed. The crystal was then held stationary while the chamber was moved in small steps across the reflected beam. From the readings of intensity obtained for each step and the position of the chamber at each step a curve showing a sharp peak at the position of the chamber for maximum intensity was plotted. The position of the chamber corresponding to this peak was taken as its position when the reflected beam was passing correctly through its slit.

In obtaining the values of intensity from the readings of the rate of fall of the leaf, this rate was first corrected for natural leak and then reduced to what it would have been for a current of one milli-ampere in the tube.

As the nature of the work undertaken was not such as to demand extreme accuracy in wave-length measurements, the angular readings were not corrected for depth of penetration of the beam into the crystal. The natural leak was always taken when the apparatus was in operation with the chamber well out of line. The walls of the ionization chamber were thick enough to shield out all the secondary radiation at the highest voltages that were used in this work, and no correction for it was necessary.

VOLTAGE INTENSITY CURVES.

After properly lining up the apparatus data was taken on intensity for different wave-lengths at constant voltages up to and including 50 KV. The readings were taken at intervals of three minutes of arc and the intensities were corrected for natural leak and reduced to values for unit current. Curves were taken for different voltages and their general shapes noted for both molybdenum and palladium. Fig. 1 is such a curve taken for palladium and it is shown as being typical of the ones obtained. These curves were taken in order to locate the more prominent lines in the K series and to show the general nature of the spectra of the metals.

In order to study the intensity of the characteristic radiation it was necessary to subtract the intensity of the general radiation of the same wave-length as a given line from the intensity of the line as measured by the spectrometer. With a number of the constant voltage, intensity and wave-length curves at hand, a point in the general radiation on each side of a line (α or β) was taken. These points were chosen so that a straight line through them would pass through the line at the same place that a continuation of the curve of general radiation would cut it. It was found necessary to take a great many curves in order to choose these points correctly.

Having chosen these points for the α and β line of each of the metals, data was taken in the following manner: The spectrometer was set for the wave-length of one of the chosen points, care being taken to have the setting correct. In order to make certain of this, the crystal was set on the proper reading as determined from the wave-length intensity curves and the ionization chamber moved past the reflected beam in steps as described above. The results of reflected intensity were plotted against angular displacement of the chamber and the chamber was set at the reading corresponding to the peak of this intensity curve. In all settings made in this part of the work this procedure was gone through. The voltage on the tube was increased in steps to 50 KV., and with each change of potential the current through the tube was adjusted so as to keep the energy supplied to the tube constant. Determinations were made of intensity of reflection for each value of voltage and all intensities were reduced to unit current conditions. The spectrometer was then set for the wave-length of the other point and similar readings taken for it, giving the same values to potential and current that were used before, for each step. Then the spectrometer was set for the line under consideration and values for its intensity for the voltages and currents that

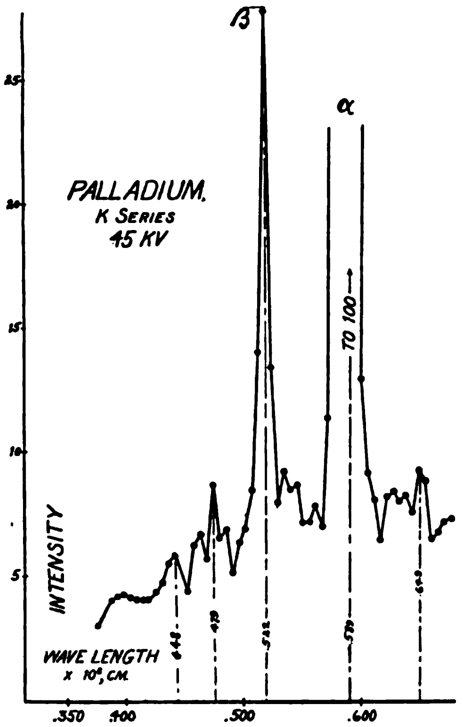

Fig. 1.

were used in the determinations on the two wave-lengths in the general radiation were obtained. This data was determined for each line that was studied.

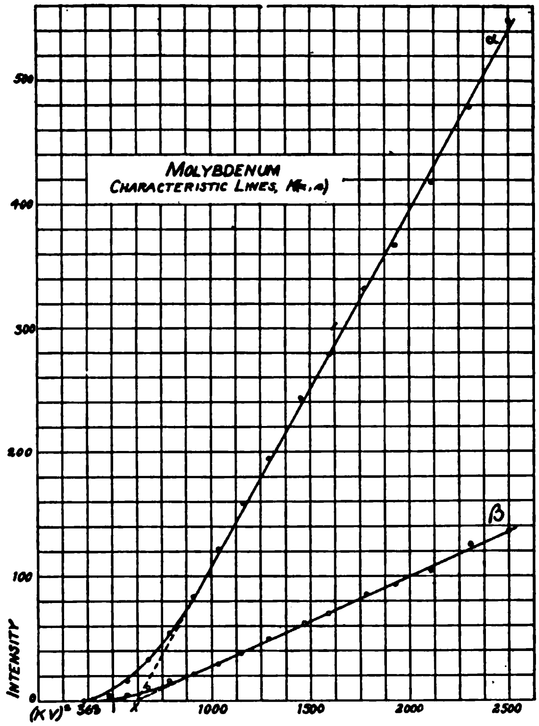

Fig. 2.

The three intensities for any potential, determined in this way give three points which determine a triangle with its acute angle very small. The distance between the vertex of this acute angle and the base line, as measured along the bisector of the angle, gives the intensity of the radiation of the line corrected for the general radiation of the same wave-length. The shape of the triangle thus formed is such as to make the length of this bisector sensibly equal to the length of either one of its sides except for low voltages.

It was found that the α and β lines for each metal appeared together at a certain minimum voltage. In the case of molybdenum this voltage was found to be 19.2 KV. and for palladium it was 24 KV. Below these potentials the wave-length curves were smooth and they intersected the wave-length axis at points corresponding to longer wave-lengths than the wave-length of the β line. It was shown by Duane and Hunt[1] that this point of intersection of the curve with the wave-length axis represented the minimum wave-length that the potential at which the curve was taken could give by the quantum relation. This was found to be true for these curves.

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[1] Duane and Hunt, American Physical Society Proc., April, 1915, PHYS. REV., August, 1915.

Curves were plotted between the kilo-volts squared to the tube and the intensity obtained as above of the lines for each metal. These curves are shown in Figs. 2 and 3 for molybdenum and palladium respectively. It will be noticed that the two sets of curves, each set consisting of a curve for the α and one for the β line of a metal, are of the same general shape and character. In each set the α and β curves rise from a common point on the voltage squared axis corresponding to the square of the lowest potential that will excite the line or the series, and quickly become straight lines. The intensity of the radiation in the α and β lines for the two metals thus appears to vary as the square of the voltage applied for voltages not too near the critical voltage; and the ratio of the intensity of the α to the intensity of the β line in each case quickly becomes constant for voltage variations.

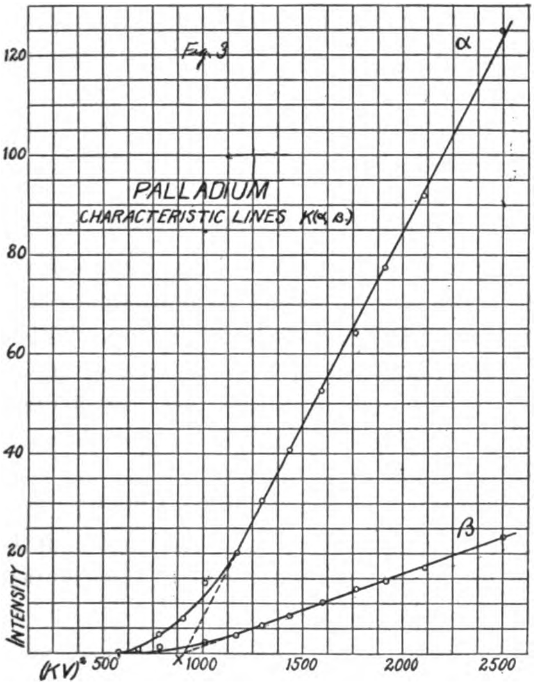

Fig. 3.

Below is a table (Table I.) of the results of various measurements taken in determining these curves, together with other data concerning them.

TABLE I.

	Molybdenum.	Palladium.
Atomic number	42	46
Wave-length of α line (cm. × 10^8)	.716	.589
Wave-length of β line (cm. × 10^8)	.641	.522
Minimum potential to produce K series, in kilo-volts	19.2	24.0
Point of intersection of straight line parts of curves with $(\text{voltage})^2$ axis, $(\text{kilo-volts})^2$	625	930
Ratio of intensity of α to β line as measured from data	3.99	5.28
Ratio of intensity of α to β line corrected for absorption in glass	5.55	6.25

ABSORPTION COEFFICIENTS.

A theoretical explanation of these voltage intensity curves has been developed by Professor Bergen Davis[1] so it was decided to correct the curves for absorption in the glass of the X-ray tubes and to compare the experimentally determined results with the curves obtained from his equation. An investigation of the absorption coefficients of the α and β lines of molybdenum and palladium in glass and in molybdenum and palladium respectively was undertaken.

The radiation was taken from a Coolidge tube with a tungsten anticathode and the wave-lengths desired were reflected from the same crystal that was used in the other part of the work. The voltage on the tube was made greater than the critical voltage of either metal for the *K* series and it was held constant. The collimator slit and the slit on the ionization chamber were narrowed to .2 mm. and the crystal and chamber were set at the proper angles to give the desired wave-length. An aluminum screen .3 mm. thick was placed between the crystal and the slit on the ionization chamber so that the reflected beam would pass through it. It was found necessary to use this screen in order to get consistent results. It absorbed practically all of the secondary radiation from the crystal, and thus allowed the reflected ray to pass into the ionization chamber as an homogeneous beam. This filtering out of the X-rays of long wave-length that came from the crystal was important.

The reflected beam after having passed through the aluminum filter was passed directly into the ionization chamber and its intensity determined. This value was used as the initial intensity in the computations. Then thin sheets of glass were placed in front of the slit in such a way that the reflected beam passed through them before it entered the ionization chamber. Determinations of intensity were made with the rays passing through various thicknesses of glass and the absorption coefficient calculated by the well-known exponential equation. The results obtained with different thicknesses of glass were averaged for the true absorption coefficient.

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[1] Bergen Davis, PHYS. REV., June, 1918.

Measurements were made in this way for the α and β lines of both molybdenum and palladium in glass. In addition absorption coefficients of the α and β line of molybdenum in molybdenum were determined, as were the absorption coefficients of the α and β lines of palladium in palladium.

The aluminum filter was kept in place in all this work and conditions were the same as regards voltage, current, etc., in all the determinations that were made. In one set of observations the potential across the tube was 35 KV. and in another it was held at 45 KV. The results shown in this paper were those obtained at 45 KV. The sheets of glass that were used were .022 cm. in thickness and the sheets of palladium were .0065 cm. thick. The molybdenum was .016 cm. thick and only two thicknesses were used. The absorption coefficients obtained are given in Table II.

TABLE II.

Line.	Wave-length Cm. × 10^8.	Absorption Coeff. in Glass.	Absorption Coeff. Mo.	Absorption Coeff. Pd.
Mo. α..........	.716	14.50	210	
Mo. β..........	.641	11.65	170	
Pd. α..........	.589	7.79		138
Pd. β..........	.522	5.37		100

WAVE-LENGTH INTENSITY CURVES.

During the early part of the work a great many curves were obtained between wave-length and intensity of radiation for molybdenum. The readings were taken for intervals of three minutes of arc on the spectrometer circle and in no case was the α line divided. The α line as obtained is wider than the β, and this widening at the base suggests that the α line consists of two lines. It has been shown by Professor Duane[1] that the α line of rhodium is a doublet consisting of an α_1 and an α_2 line. In this work no distinction is made between the α_1 and the α_2 lines and the radiation designated here as that of the α line for either metal consists of the combined intensities of the two lines of the α doublet.

In order to correct the intensities of the lines that were studied (α and

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[1] Not yet published.

β lines) for absorption in the glass of the X-ray tubes, the thickness of the glass was determined. A tube was placed under a microscope and the microscope was focused on an object on the inner surface of the glass and then on an object on the outer surface. From these readings and the index of refraction of the glass the thickness of the glass was determined. Determinations were made of the thickness of both tubes used and the results were:

Thickness of glass of molybdenum tube..............1125 cm.
Thickness of glass of palladium tube..................0706 cm.

Using these thicknesses and the absorption coefficients in Table II., the intensities within the tubes of the lines studied were found to be

Moα ....5.11 times the observed intensity.
Moβ ....3.67 " " " "
Pdα ....1.73 " " " "
Pdβ ....1.46 " ". " "

The curves between intensity of radiation and voltage squared (Figs. 2 and 3) show that for the straight line parts of the curves the ratio of the intensity of the α radiation to that of the β radiation is, in the case of molybdenum, 3.99; and in the case of palladium 5.28. When the intensities of the lines are corrected for absorption in the glass these ratios are changed for molybdenum to 5.55 and for palladium to 6.25. The curves in Figs. 2 and 3 are the experimentally determined curves and they are not corrected for this absorption.

THE RATIO OF THE ENERGY IN THE α AND β RADIATION AS EMITTED FROM THE ATOM.

The corrected values give the intensities of the α and β radiation inside the tube before coming out through the glass. *A still more important matter is the ratio of the intensities of these frequencies actually emitted by by the atom.* The theory developed by Professor Bergen Davis[1] makes the calculation of this ratio possible. The equation resulting from a mathematical treatment of this theory is

$$I_a = E_a\,(hn_a)\frac{BN}{b}\left[\frac{b}{cu}\left(1 - e^{-\frac{cu}{b}(V^2 - V_0^2)}\right)\right.$$
$$\left. - 2V_0(V - V_0) + 2/3\left(\frac{cu}{b}\right)V_0\{2V^3 - V_0(V^2 - V_0^2)\}\right.$$

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[1] *Loc. cit.*

$$-\frac{V_0}{15}\left(\frac{cu}{b}\right)^2\{8(V^5 - V^4V_0) - 4V^2V_0(V^2 - V_0^2)$$
$$- 3V_0(V^2 - V_0^2)^2\}\Big]$$

when expanded to four terms.

V_0 = minimum voltage required to produce the characteristic radiation.
u = the absorption coefficient of a line in the metal that produces it.
c = the ratio of the path of a ray through the metal to the depth of penetration of the electron producing it.
b = a constant corresponding to a in Whiddington's law.[1] This constant is expressed for use here in terms of potential.
V = any voltage applied to the tube (Kilo-volts).
I = intensity of the line (as written above it means the intensity of the α line).
N = the number of electrons striking the target per second.
B = the number of impacts of an electron with the atomic nuclei per unit distance.

($E_\alpha hn_\alpha$) is proportional to the energy emitted by one atom.

It will be noticed that the constants in the equation, with the exception of $E_\alpha hn_\alpha$ and u have the same values for all the lines from a particular metal. The absorption coefficients were determined experimentally for the α and β lines of both palladium and molybdenum and their values are given in Table II. The constants E_α and E_β are the fractions of the effective impacts that result in producing the corresponding radiations.

The combined constants $E_\alpha hn_\alpha$ and $E_\beta hn_\beta$ are proportional to the energies emitted as radiation of each frequency from one atom. The ratio $E_\alpha hn_\alpha/E_\beta hn_\beta$ is the ratio of the energy of the α frequency to that of the β frequency emitted by a single atom.

The experimentally determined values of I for the α and β lines of molybdenum and palladium, after having been corrected for absorption in the glass of the tubes, were plotted. Values of I for different voltages were computed for the α and β lines of both metals and plotted to the same scale. From these computations and from the experimental results the data in Table III. were obtained.

TABLE III.

Metal.	Observed I_α/I_β.	Inside Tube I_α/I_β.	Ratio of Intensities Emitted by Atom $E_\alpha hn_\alpha/E_\beta hn_\beta$.
Mo.	3.99	5.55	5.70
Pd.	5.28	6.25	7.18

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[1] Proc. Roy. Soc., Vol. 86, 1912.

Professor Bergen Davis's equation can be written:

$$I_\alpha = (E_\alpha h n_\alpha) f_\alpha(V)$$

and therefore

$$\frac{I_\alpha}{I_\beta} = \frac{E_\alpha h n_\alpha f_\alpha(V)}{E_\beta h n_\beta f_\beta(V)}.$$

The computations made in determining the points for the curves give the ratio $f_\alpha(V)/f_\beta(V)$ so that ratio $E_\alpha h n_\alpha / E_\beta h n_\beta$ can be determined. This ratio, the ratio of the energy emitted by an atom as α radiation to that emitted as β radiation, determined in this way is contained with other data pertaining to the comparison of the curves in Table III.

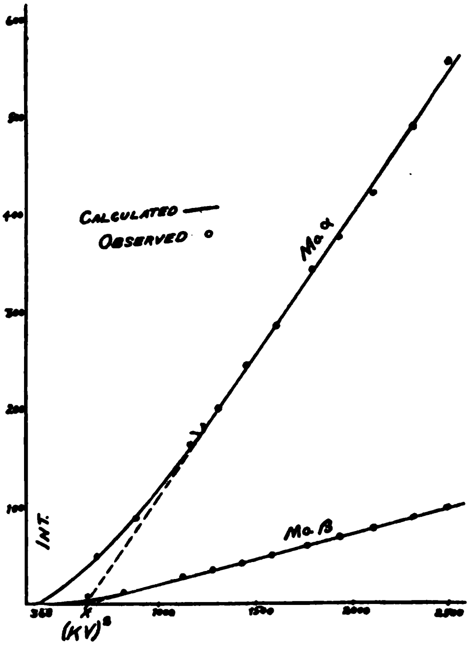

Fig. 4.

The curves given by Professor Bergen Davis's equation plotted to the same scale with the experimentally determined points corrected for absorption, are shown in Figs. 4 and 5. The very close agreement as to shape between the calculated curves and the experimental ones indicates that the assumptions underlying the theory are sound.

In these figures the calculated curves are given in full lines, and the experimentally determined points are marked with circles.

The above comparison is only true in so far as one may consider the reflecting power of the crystal the same for two frequencies not differing more than the α and β frequencies differ. If the reflecting power of a crystal does depend to a marked degree on the wave-length of the radiation it will be necessary to correct the above ratios for such variation in reflectivity when the same shall be experimentally determined.

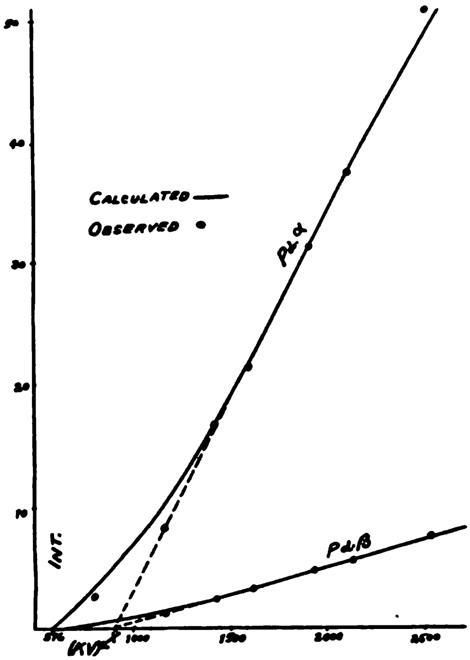

Fig. 5.

Discussion of Results.

In all the work that was done it was assumed that the crystal that was used reflected all wave-lengths with equal efficiency. W. H. Bragg[1] showed that the reflecting power of a crystal for a given wave-length varied with the angle of reflection. He varied the angle of reflection of the beam by taking the different orders of reflection. His results show that the reflecting power of a crystal varies inversely as the square of the order of reflection. This is the only experimental work that has been published as far as the writer knows, and the lack of published matter dealing experimentally with the subject of reflecting power in any single order of different wave-lengths has made the above assumption necessary.

Reference to Figs. 2 and 3 shows that the intensity of the radiation for the α and β lines of the metals used varies directly as the square of the voltage for voltages considerably higher than the critical voltage. In a paper recently published by Webster and Clark[1] it was shown that the lines of the K series of rhodium vary approximately as the 3/2 power of the difference between the potential applied to the tube and the critical potential of the series. This relation was tried with the data obtained in these experiments but it was found that the curves obtained did not fit the points as well as the ones in Figs. 2 and 3 fit them.

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[1] W. H. Bragg, Philosophical Magazine, 27, 1914, 881.

In these curves it will be seen that the linear parts of the curves when produced back cut the voltage squared axis at the same point. In each set of curves this point is marked X. It was found on computation that the following equation is true.

$$\frac{N_{\mathrm{Pd}}^2}{N_{\mathrm{Mo}}^2} = \frac{V_{0\mathrm{Pd}}}{V_{0\mathrm{Mo}}} = \frac{\sqrt{X_{\mathrm{Pd}}}}{\sqrt{X_{\mathrm{Mo}}}},$$

where

N_{Pd} = atomic number of palladium = 46.

N_{Mo} = atomic number of molybdenum = 42.

$V_{0\mathrm{Mo}}$ = critical voltage for K series of molybdenum = 19.2 Kv.

$V_{0\mathrm{Pd}}$ = critical voltage for K series of palladium = 24.0 Kv.

X_{Mo} = point on axis where straight lines intersect = 625.

X_{Pd} = point on axis where straight lines intersect = 930.

The square root of the value in Kv. squared represented by the point X then, bears the same relation to the metal producing the characteristic radiation that the critical voltage does, *i. e.*, it is proportional to the square of its atomic number.

The method that was used in determining the absorption coefficients is the same as that of the Braggs[2] except for the aluminum filter which was placed at the crystal to absorb its secondary radiation. They find the mass absorption coefficient for the α line of palladium in palladium to be 11.4. The coefficient given in Table I., when divided by the density of palladium gives 12.1. Their work shows that absorption coefficients in a given substance vary as the 5/2 power of the wave-length of the radiation. A. W. Hull and Marion Rice[3] find that absorption coefficients when corrected for scattering vary as the cube of the wave-length, and C. D. Miller[4] finds that they vary for long wave-lengths as the 2.77 power of the wave-length. Miller uses a beam that has passed through a great thickness of absorbing material and in that way absorbs out all but the shortest wave-length. The absorption coefficient is determined for this end beam.

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[1] D. L. Webster and E. Clark, Nat. Acad. of Science Proc., 3, pp. 181–185, March, 1917.

[2] W. H. and W. L. Bragg, X-rays and Crystal Structure, pp. 175–206.

[3] A. W. Hull and Marion Rice, Proc. Amer. Phys. Soc., PHYS. REV., Sept., 1916, pp. 326–328.

[4] C. D. Miller, PHYS. REV., Oct., 1916, pp. 329–343.

The results obtained here, while uncorrected for scattering in the thin sheets of absorbing material that were used, follow the cube of the wave-length law more consistently than they do either of the others. Their agreement with this law is fairly good. The coefficients for the molybdenum α and β radiation in molybdenum are not as trustworthy as the others, because it was not possible to use more than two thicknesses of molybdenum in the screen.

On substituting the observed values of the critical voltage in Planck's equation,

$$Ve = hn$$

in the case of molybdenum, the value of n is such as to correspond to a wave-length of $.643 \times 10^{-8}$ cm. The wave-length of the β line as measured is $.641 \times 10^{-8}$ cm. The measured voltage required to excite the K series characteristic radiation is therefore almost exactly the same as that given by the Planck relation for the β line of the K series of this metal.

For palladium the calculated result, using the observed critical voltage, is a slightly shorter wave-length than that of the β line of the K series of palladium as measured. The measured wave-length of the β line is $.552 \times 10^{-8}$ cm. while the calculated wave-length for the critical voltage is $.516 \times 10^{-8}$ cm.

In a paper published by Uhler[1] it is shown that while Moseley's law does not completely represent the X-ray series over the entire range of wave-lengths, it represents the series for wave-lengths corresponding to the atomic numbers of molybdenum and palladium very closely. Placing $\tau_1 = 2$ in Bohr's equation to determine the wave-length of the α line, the value obtained is, for molybdenum $.713 \times 10^{-8}$ cm., and for palladium $.581 \times 10^{-8}$ cm. The measured wave-length for the molybdenum α radiation is $.716 \times 10^{-8}$ cm. and for palladium α radiation it is $.589 \times 10^{-8}$ cm. In each case the agreement between calculated and observed wave-length is as close as Moseley's law agrees with the observed wave-lengths according to Uhler.

SUMMARY.

The K series characteristic X-radiation of molybdenum and palladium has been studied.

It was found that in each metal the α and β radiation appeared at a certain voltage. For molybdenum this voltage was found to be 19.2 Kv. and for palladium it was 24 Kv. For molybdenum this measured critical voltage is almost exactly that required by the Planck hypothesis to produce the β line. For palladium the measured critical voltage is that required by the same hypothesis to produce a wave-length slightly shorter than the β line.

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[1] H. S. Uhler, Phys. Rev., April, 1917, pp. 325–335.

Curves were plotted between the voltage squared and the intensities of the α and β radiation for both metals, and these curves show that the intensities of these radiations varied directly as the square of the voltage for voltages not too near the critical voltage. The ratio of the intensity of the α line to that of the β line was found to become constant as the voltage was increased, for each metal.

The straight line parts of the curves, if produced back cut the voltage squared axis in the same point, and this point was found to bear the same relation to the atomic number of the metal of the anti-cathode that the point at which the curves themselves cut the axis bears, that is, the square root of the coördinate represents a voltage proportional to the square of the atomic number of the radiator.

Absorption coefficients for wave-lengths of the α and β lines of molybdenum and palladium in glass and in molybdenum and palladium respectively were determined, and they were found to vary approximately as the cube of the wave-length of the radiation. The observed intensities of the α and β lines of each metal were corrected for absorption in the glass of the X-ray tube.

By application of the theory of Professor Bergen Davis it was possible to calculate the ratio of the intensity of the α radiation to that of the β radiation as given by a single atom.

The curves obtained by plotting the theoretical equation were found to agree excellently with the experimentally determined curves.

CONCLUSION.

In conclusion the writer wishes to thank Professor Bergen Davis, who suggested the research, for the active help and constant advice that he has furnished throughout the work. He is deeply indebted to Columbia University for the grant of the Tyndall Fellowship for the prosecution of the research.

He is also grateful to Dr. W. D. Coolidge for supplying the molybdenum tube, and to Dr. C. S. Brainin for the loan of the sheet palladium that was used.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
May 1, 1918.

NEW BOOKS.

The Science and Practice of Photography. By JOHN R. ROEBUCK. New York, D. Appleton and Co., 1918. Pp. xiv + 298.

This book was written for class room use in a course given by the author in the Department of Physics at the University of Wisconsin. The work consists of two parts; the first is a general treatise on photography, while the second part, about one fifth of the book, is a manual of experiments to be performed by the student. The book is planned for students who have had elementary courses in physics and chemistry; no mathematics beyond elementary algebra is needed.

While the book was written primarily as a text for students, many who are interested in photography will find it very instructive reading, especially those who are interested in a more scientific treatment than is usually given in books on this subject. Many references to recent researches are freely given and the results are well summarized. Most of this material, because it was so scattered through the literature of photography, was usually not available to the amateur. This book should find many eager readers.

O. M. S.

Radio Communication, Theory and Methods. By JOHN MILLS. New York, McGraw-Hill Book Co., 1917. Pp. xi + 205.

This book is based on a course of lectures given soon after the United States entered the war, to employees of the Western Electric Company who were being trained for radio work. Since then it has been adopted as a text by the War Department for those men who have had some training in electrical engineering.

Methods used in radio work have been improved so rapidly that the older treatises do not cover modern methods. This book contains much that cannot be found elsewhere except by going through a great many original papers. For example the explanation of the use of the vacuum tube both as a detector and as an amplifier of alternating currents without distortion, is not contained in any of the standard treatises.

The author tries to adapt the book to those who have not had calculus by using the vector operator j and the differential operator p and using only algebraic methods. However, short-cut mathematical processes are of doubtful value to the beginner and certainly one who has had calculus and knows something about the vector methods of solving differential equations as used by engineers will more quickly grasp the processes used. The author has purposely left some of the most important chapters, on the vacuum tube, on detection, on undamped waves, and on radio telegraphy and telephony almost entirely free from mathematical methods.

The author has aimed throughout to "present fundamental principles and methods rather than detailed instruction as to apparatus and its operation." And this aim is fulfilled excellently. In an appendix there is a chapter on transmission over wire circuits and some graded problems for the use of the student.

This interesting book on one of the most interesting branches of electricity will certainly be welcomed by those physicists and and engineers who have not had the time or opportunity to keep in touch with the principles involved in the modern methods.

O. M. S.

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THE

PHYSICAL REVIEW.

WITH FURTHER REFERENCE TO THE FORCE BETWEEN TWO REVOLVING ELECTRONS AT A GREAT DISTANCE.

BY ALBERT C. CREHORE.

IN a paper[1] published in June, 1917, reasons were given for the conclusion that the current form of electromagnetic equations requires modification in a fundamental manner. Briefly, it appears from a consideration of the mechanical force between two revolving electrons in small circular orbits, the centers being a great distance apart, that the force between two bodies of gross matter at a great distance apart must be immensely greater than the known force between them, namely the gravitational force, assuming, of course, that there was no error in the somewhat involved processes of applying the electromagnetic equations to the case.

In a recent paper[2] G. A. Schott has examined in much detail the investigation in the paper above referred to, and says in his opening paragraph "hence (Crehore) concludes, quite legitimately if his result be correct, that the fundamental equations of the accepted electron theory require substantial modification" and "It is obviously imperative that Crehore's result be either verified or disproved." In the second paragraph he says: "The following investigation is based on Crehore's equations for the electric part of the mechanical force,"[3] "which have been verified, except some obvious misprints,[4] *e. g.*, a_2 for a_1 in the last term of (49)." Above in the same paragraph Schott remarks: "Thus it is desirable to take the magnetic effect into account *ab initio*, although it will be found to be inappreciable on the average for an amorphous medium but not necessarily so for a crystal."

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[1] A. C. Crehore, PHYS. REV., Sec. Ser., Vol. IX., p. 445, June, 1917.

[2] G. A. Schott, PHYS. REV., Sec. Ser., Vol. XII., p. 23, July, 1918.

[3] *Loc. cit.*, pp. 453, 454—this referring to the June, 1917, paper above mentioned.

[4] It is hoped that Dr. Schott will publish all of the errors in the expression for the instantaneous force that he alludes to, in order that this important equation may be available in its complete form, having been verified.

In reply to this last criticism it should be stated that the complete instantaneous force, including the magnetic component, had been obtained and averaged in a similar manner to the electric component prior to the time of publishing the June, 1917, paper, but it was not published partly because of its length, and partly because each magnetic term in the final average was affected by a common factor in powers of β higher than the square. So long as terms were being obtained from the electric component having the greater factor β^2, they were large compared with any terms obtained from the magnetic component. The quantity, $\beta_1\beta_2^3$, is a common factor of the equation (48), p. 36 of the Schott paper,[1] which is his final expression for the time average of the magnetic component. The importance of the equation for the instantaneous force with no terms whatever omitted is so great that the equation for the magnetic component, supplementing the electric already published in equations (48), (49) and (50), June, 1917,[2] is given below in an appendix. In the case of coaxial rings of electrons, where the instantaneous and the average forces are identical, this equation has a wider application than to possible long range gravitational forces. As evidence that the complete force had been obtained, equation (92), p. 469, of the June, 1917, paper may be cited. This gives the complete force, including all terms from both the electric and the magnetic parts exerted upon any selected electron in a ring of electrons due to any other electron in the same ring.

The present paper is offered as a preliminary reply to the Schott paper referred to. It is submitted before there has been opportunity to make a careful study of the suggested corrections as to averaging, which at best is a laborious process, and is submitted principally because the deductions that Schott draws from his own formula fall short of the mark. In his concluding remarks, p. 37,[3] Schott states that "from an experimental point of view, however, the deviation is not of much moment, for it is easily seen by expanding the function of β_2 in (50) that the first power which occurs is the fourth, so that, if β_2 be as large as .01, the relative deviation is less than the hundred millionth. For this reason I shall not pursue the matter further here." This statement refers to the previous sentence, in which it is pointed out that the average force of the second electron acting upon the first differs from that of the first acting upon the second, the law of action and reaction not holding true, as Schott states, except to a first approximation.

The only interpretation that I can give to these remarks is that Schott is making a comparison between the magnitude of the terms containing β, which vary with the value of β, and the constant term, namely the electrostatic force, which does not vary with any change of speed. (See his equation for the average force repoduced in (1) below.) It is manifestly improper to make such a comparison when applying these results to the *atoms* of matter, as was done in the June, 1917, paper referred to. The obtaining of the force between two revolving *electrons* is but one step in the process of finding the force between two *atoms*, supposing these atoms to consist of rings of electrons revolving around a positive nucleus located at the center of the orbits, and having a charge equal to the sum of the charges on the electrons but of opposite sign, as in a neutral atom.

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[1] *Loc. cit.*

[2] *Loc. cit.*

[3] *Loc. cit.*

To place the equation under discussion before us, Schott's result, his equation (50) p. 37,[1] is reproduced here, after making an obvious correction in the denominator of the last term under the logarithm, making β^2 read β_2, namely

$$(\mathbf{F}\cdot\mathbf{r})/r = -\frac{e_1e_2}{r^2}\left\{1 + \beta_2^2\left(\frac{1}{1-\beta_2^2} - \frac{1}{2\beta_2}\log\frac{1+\beta_2}{1-\beta_2}\right)\right\}. \qquad (1)$$

Adopting Schott's suggestion, and expanding this in powers of β_2, we find

$$\frac{1}{2\beta_2}\log\frac{1+\beta_2}{1-\beta_2} = 1 + \frac{1}{3}\beta_2^2 + \frac{1}{5}\beta_2^4 + \frac{1}{7}\beta_2^6 + \dots \qquad (2)$$

and

$$\frac{1}{1-\beta_2^2} = 1 + \beta_2^2 + \beta_2^4 + \beta_2^6 + \dots, \qquad (3)$$

and, finally, the complete force equivalent to (1) is

$$(\mathbf{F}\cdot\mathbf{r})/r = -\frac{e_1e_2}{r^2}\left(1 + \frac{2}{3}\beta_2^4 + \frac{4}{5}\beta_2^6 + \frac{6}{7}\beta_2^8 + \dots\right). \qquad (4)$$

When the effect of the positive nuclei are taken into the account in summing up the forces between the various component parts of two *atoms*, all the electrostatic terms cancel out, and require no further consideration, the whole force at great distances between such atoms being due solely to the terms involving the speed of the revolving electrons, the β-terms in (1) or (4) above.

Referring again to the paper of June, 1917, top of page 456, it is stated that the electrostatic terms were omitted in taking the average force for the reason that they contribute nothing to the inverse square of the distance terms under discussion, when the effect of the positive charge of the atoms is included so as to get the force between atoms instead of electrons only. Schott, however, has carried these electrostatic terms through the averaging process from the beginning, and they give rise to the constant term, $-e_1e_2r^{-2}$, in (1) above, which should now be omitted in making any deductions about atoms or gross matter. Making this omission, and also making a sufficiently close approximation to the value of the force by omitting all powers of β higher than the fourth, we simplify Schott's result, as applied to a pair of electrons in neutral atoms, from (4) to be

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[1] *Loc. cit.*

$$(\mathbf{F}\cdot\mathbf{r})/r = -\tfrac{2}{3}e^2\beta_2^4r^{-2}. \tag{5}$$

Comparing this with the result obtained in June, 1917, the difference is principally in the sign and in the substitution of β_2^4 for β_2^2. As far as the law of action and reaction is concerned, the difference is greater with β_2^4 than with β_2^2. If we substitute β_1 for β_2, and take the ratio of the forces, it is evidently equal to $(\beta_1/\beta_2)^4$. If β_1 is nearly twice as large as β_2, the ratio is nearly 16 to 1, and the forces of Action and Reaction are not only not equal and opposite to a first approximation, as Schott states, but they are not even of the same order of magnitude. The ratio between them is greater than it would be if β^2 appeared in the formula instead of β^4.

An important criticism of the formula, however, is connected with the magnitude of the force between two pieces of matter at great distances. If we had β^2 instead of β^4, as formerly, the magnitude of the force at great distances exceeds the known force of gravitation in an immense ratio, about 10^{31}. The change from β^2 to β^4, assuming β^2 to be of the order of 10^{-4}, only reduces this ratio from 10^{31} to about 10^{27}, the two figures being so nearly alike that there is little choice between them. Whether the formula calls for a repulsive force or an attraction makes no difference in this argument as to the magnitude. We know that no such force exists, and the argument for a revision of the fundamental electromagnetic equations remains valid, taking Schott's results as to averaging to be correct.

This matter of the magnitude of the force may, perhaps, best be shown by a special example. In order not to raise the question of synchronism between the two electrons, let us select two atoms, the one having a single ring of two electrons and the other of four. If the two atoms were supposed to be alike, it is reasonable to take the speeds the same for each, but Schott does not claim that his formula applies to two synchronous electrons. It may be remarked, however, on this point I have found that, when complete rings of equally spaced electrons are concerned, the formula for synchronous electrons reduces to precisely the same form as for incommensurable velocities, since all the terms involving the phase angles disappear when rings are taken, and we obtain the same result as without synchronism. Not to raise the question, however, let us calculate the force on atom A, a two electron single ring atom, due to atom B, a four electron single ring atom, the speed of each electron in A being β_1, and in B, β_2. According to (5) the force of the four electrons in B upon one electron in A is 4 times the expression in (5). Similarly, the force of the four upon the second electron in A is 4 times (5), making a total of 8 times (5).

On the other hand, the force of the two electrons in A upon one in B is two times (5), writing β_1 instead of β_2. The total force of the two in A upon the four electrons in B is then eight times (5) writing β_1 instead of β_2. We have then

$$\text{Total force of } B \text{ on } A = -\tfrac{16}{3}\, e^2\beta_2^4 r^{-2}. \tag{6}$$

$$\text{Total force of } A \text{ on } B = -\tfrac{16}{3}\, e^2\beta_1^4 r^{-2}. \tag{7}$$

If it is permitted, in making an approximation to the speeds of the electrons, to adopt the approximate formula (3), page 14, Physical Review, July, 1918, for the speed of any ring of electrons, and a moderate error in the estimated speed cannot affect our argument at all, namely

$$\beta^2 = p\frac{\pi^2 e^4}{c^2h^2}, \tag{8}$$

we have for the fourth power of the speed of each electron in atom A

$$\beta_1^4 = 4\frac{\pi^4 e^8}{c^4h^4}, \tag{9}$$

and in atom B

$$\beta_2^4 = 16\frac{\pi^4 e^8}{c^4h^4}. \tag{10}$$

whence (6) and (7) become

$$F_{BA} = -\frac{256}{3}\,\frac{\pi^4 e^{10}}{c^4h^4}\,r^{-2}, \tag{11}$$

$$F_{AB} = -\frac{64}{3}\,\frac{\pi^4 e^{10}}{c^4h^4}\,r^{-2}. \tag{12}$$

We shall also make a further assumption that the two-electron atom, A, represents hydrogen, and the four electron atom, B, helium. This assumption also will not affect our argument where such a large ratio as 10^{37} is concerned. According to the law of gravitation the average force between one atom of hydrogen and one of helium is

$$F = km_H m_{He} r^{-2}. \tag{13}$$

The mass of the helium atom is approximately four times that of the hydrogen atom, so that, in terms of m_H, (13) becomes

$$F = 4km_H^2r^{-2}. \tag{14}$$

Taking the smaller of the two forces, namely that in (12), for the purpose of comparison with the magnitude of the gravitational force, and taking the ratio, we have

Ratio of the calculated force to the actual gravitational force

$$= \frac{16}{3}\frac{\pi^4e^{10}}{km_H^2c^4h^4}. \tag{15}$$

It was pointed out in the paper above referred to, equation (9), p. 15, July, 1918, that a very exact numerical value of the gravitational constant, k, is given by the expression,

$$k = \frac{p^4}{3K}\frac{m_0}{m_4^2}\frac{\pi^4e^{10}}{c^4h^4}, \tag{16}$$

in which $K = 1$, is the specific inductive capacity, and $p = 2$, the number of electrons in the hydrogen atom. m_0 is the mass of the negative electron at slow velocities. With these values, we have

$$k = \frac{16}{3}\frac{m_0}{m_H^2}\frac{\pi^4e^{10}}{c^4h^4}. \tag{17}$$

Upon substitution of this correct numerical value of the Newtonian constant, k, in (15), the expression reduces to

Ratio of calculated force to the gravitational force

$$= 1/m_0 = 1/.90 \times 10^{-27} = 1.11 \times 10^{27}. \tag{18}$$

The ratio would be four times greater than this if we had used the larger of the two forces as given in (11) above. This completes the special example, and shows that the force given by the Schott formula is greater than the gravitational force by a factor of about 10^{27}, as was pointed out before considering the example.

In conclusion, it has, we believe, been clearly shown that the application which Schott has made of his final result, equation (50) of his paper, or (1) above, to gross matter is incorrect. The positive nuclei of the atoms must be taken into the account as well as the revolving electrons. Whether or not the matter has been properly treated above in the simple omission of the constant, which is independent of the speed remains to be seen. It seems that any system of electromagnetic equations that does not make the mechanical force of one revolving electron acting upon a second exactly equal and opposite to that of the other on the one, thus strictly conforming to the law of action and reaction is pretty certain to lead to insuperable difficulties when applied to the accepted forms of atoms. It is difficult to believe that the average mechanical force on the first electron due to the second is entirely independent of the state of motion of the first electron. It seems as if its motion must in some way alter the average force upon it due to the second electron independent of any change that may be taking place in the motion of the second electron, and yet this is contrary to the Larmor-Lorentz form of theory.

Let us admit for the moment that a form of theory is possible that makes the average mechanical force between the two electrons a function of the product of the two speeds instead of the speed of one of the charges only. Then, when either of the two speeds vanishes the product vanishes, and the force between a revolving negative electron and a stationary positive charge is the same whether we consider the force of the one on the other or the other on the one, and is equal simply to the electrostatic terms. In the case of two revolving electrons the force of the one on the other would be the same as the other on the one, the Law of Action and Reaction being preserved way down to the ultimate constituent parts of the atoms and of gross matter, namely the electrons.

It seems as if the finding of a simple expression for the Newtonian constant given in a former paper, equation (9),[1] has some bearing upon this question, especially because all of the laws of gravitation have been shown to follow from the premises assumed. It is very evident that the least change in the fundamental equations of electromagnetic theory may easily switch the resulting force at great distances about from a repulsion to an attraction, the force being so small that the residual may take either a positive or a negative sign. The chief point that has now been established is that there are existing forces at great distances that vary inversely as the square of the distance, the gravitational law, as demanded by the present unmodified form of theory. Not much attention has been given to these forces in the past, and it seems as if it was not realized that they existed, as demanded by the accepted theory.

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[1] *Loc. cit.*

ON THE SPECULAR REFLECTION FROM ROUGH SURFACES.

By T. K. Chinmayanandam.

It is well known that matt surfaces behave nearly in the same way as polished reflectors, when the incident rays are of great wave-length or fall very obliquely on the surface. In a paper published in the Physical Review for January, 1916, A. F. Gorton has followed up the work of Lord Rayleigh[1] and T. J. Meyer[2] on this subject, and has given several curves showing the relative reflecting power of surfaces of ground glass as compared with that of a polished surface of the same material for various angles of incidence and for various wave-lengths of incident radiation. Though the primary object of Gorton's investigation appears to have been the development of an infra-red screen, the experimental results obtained by him are also of interest from the point of view of the general theory of reflection from rough surfaces. In the present paper, it is proposed to record the results of an attempt made to develop a math-

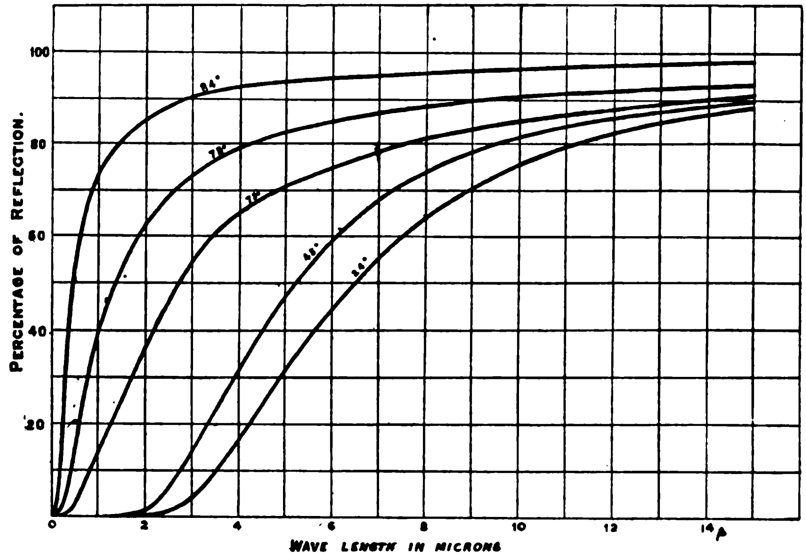

Fig. 1.

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[1] Rayleigh, Nature, 64, p. 385, 1901.

[2] T. J. Meyer, Verh. Deutsch. Phys. Ges., p. 126, Feb., 1914.

ematical formula which would quantitatively express the relative reflecting power of a rough surface over the range of incidences and wave-lengths studied by Gorton.

One important feature about the reflection from rough surfaces which is emphasized in Gorton's paper is the absence of decided interference minima in the reflection curves, such as would occur in the case of a plane grating or any surface of regular topography. This result is attributed to the fact that surfaces like those of groundglass are extremely irregular in structure. Starting on this idea, it seems possible, on certain hypotheses, to calculate the relative reflecting power of the surface for a given wave-length and a given incidence. It is convenient in the first place to make the simplifying assumption that the "relative reflecting power" of a rough surface depends only on its topography and not upon the nature of the material of which it is composed. *A priori*, it can be seen that such an assumption would not be very wide of the truth, particularly in the case of surfaces which, if polished, would have a fairly high coefficient of reflection at all incidences.[1] Further, the surface may be assumed to be "ideally rough," in other words, that different elements of it are distributed at different depths according to the probability law, the aggregate area of the elements lying between the distances x and $x + dx$ from a mean plane being proportional to $e^{-ax^2}dx$. The disturbance due to the reflected radiations is the resultant of the disturbances due to radiation from each of the surface elements, the proper phase of each being taken into account. Taking the mean plane as the reference plane, the relative phase of the disturbance sent out from an element of the surface at a depth x below it may be taken to be $2x \cos \theta \cdot 2\pi/\lambda$, where λ is the wave-length of the incident radiation, and θ the angle of incidence. Assuming now that, to a sufficient approximation, the surface is uniformly illuminated by the incident radiation, we get for the resultant disturbance due to the reflected radiation

$$y = \Sigma \cos (\omega t + 2x \cos \theta \cdot 2\pi/\lambda) ds, \tag{1}$$

the summation extending all over the elements. Since the distribution of the elements at different depths is assumed to be according to the probability law,

$$y = \frac{\int_{-\infty}^{\infty} e^{-ax^2} \cos (\omega t + [4\pi x \cos \theta]/\lambda) dx}{\int_{-\infty}^{\infty} e^{-ax^2} dx}. \tag{2}$$

Evaluating the integrals, the intensity of the reflected radiation is found to be given by

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[1] See also Gorton's remarks, Phys. Rev., Vol. VII., ser. 2, p. 75.

$$I = e^{-(8\pi^2 \cos^2 \theta)/a\lambda^2}, \tag{3}$$

and this will also determine the relative reflecting power of the surface.

It is easily seen that this formula represents curves of the same general form as those obtained experimentally by Gorton. The curves have an inflection point, the inflectional gradient being proportional to $a^{1/2}/\cos\theta$, so that the curves became steeper (1) as a increases, *i. e.*, as the surface becomes finer, and (2) as the angle of incidence increases; these results are in general agreement with Gorton's observations. The actual results calculated from the formula are given below for the different surfaces used by him. The surfaces are denoted by A, B, C, D, where A refers to curves I and II in Fig. 6 (p. 73) of his paper, B refers to curve III in the same figure, and C and D refer respectively to the curves in Figs. 7 and 8. It will be seen from Table I. that for moderate incidences the agreement is fairly good.

TABLE I.

Surface.	Angle of Incidence.	Relative Reflecting Power (as Percentages) for Wave-lengths (in μ).											
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	23°	Obsd.	—	0.8	4.0	13.9	26.6	38.2	47.4	56.7	63.8	68.5	71.6
		Calcd.	—	0.0	2.0	10.6	23.8	36.9	47.1	57.0	64.2	69.8	74.3
B	45°	Obsd.	—	15.9	33.8	50.3	62.7	71.5	77.7	81.7	—	—	—
		Calcd.	—	7.1	30.8	51.6	65.5	74.5	80.5	84.7	—	—	—
C	24°	Obsd.	2.0	3.2	7.9	19.1	32.1	44.5	54.1	63.7	69.8	75.0	—
		Calcd.	0.0	0.0	4.1	16.6	31.6	45.0	55.6	63.8	70.1	75.0	—
	45°	Obsd.	3.2	5.3	17.3	33.2	48.0	59.0	67.1	73.1	77.4	79.5	82.3
		Calcd.	0.0	1.3	14.7	34.0	50.2	61.9	70.3	76.4	80.8	84.2	86.7
D	54°	Obsd.	—	8.2	28.0	45.6	59.5	69.9	75.2	—	—	—	—
		Calcd.	—	4.0	24.0	44.8	59.8	70.0	76.9	—	—	—	—

But at oblique incidences, the simple single-constant formula fails to express the results quantitatively, as can be seen from the figures in Table II. below.

TABLE II.

Surface.	Angle of Incidence.	Percentage of Reflection for Wave-lengths (in μ).											
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	70°	Obsd.	4.0	25.3	39.8	48.1	55.3	60.2	63.7	66.9	70.1	72.6	75.2
		Calcd.	0.7	28.9	57.5	73.1	82.0	87.1	90.4	92.5	94.1	95.1	96.0
C	72°	Obsd.	17.0	36.9	51.0	62.3	69.3	74.4	77.9	80.9	83.1	84.7	—
		Calcd.	3.7	43.9	69.4	81.4	87.6	91.3	93.5	95.0	96.0	96.7	—
	84°	Obsd.	73.3	85.7	90.4	93.3	95.1	95.9	96.5	97.0	97.5	97.8	98.1
		Calcd.	68.9	91.1	95.9	97.7	98.5	99.0	99.2	99.4	99.5	99.6	99.7
D	70°	Obsd.	11.0	46.3	66.9	77.3	83.4	85.1	85.1	—	—	—	—
		Calcd.	1.3	33.7	61.7	76.2	84.0	88.6	91.5	—	—	—	—
	81°	Obsd.	82.3	92.5	94.7	96.3	97.7	98.7	99.3	—	—	—	—
		Calcd.	41.0	80.0	90.5	94.6	96.5	97.5	98.2	—	—	—	—

The constants for the different surfaces are as follows:[1]

Surface.	*A.*	*B.*	*C.*	*D.*
a $10^8 \times$	1.86	3.72	2.29	2.13
d μ	0.61	0.43	0.55	0.56

the height above or depth below the mean plane for which the proportional frequency of occurrence is one half being given by d in the second line.

The failure of the simple theory given above to account for the phenomena observed at very oblique incidences could have been anticipated; for, the assumption made that the elements of the rough surface are all equally illuminated by the incident radiation would obviously be wide of the mark at such incidences. The extent of the discrepancy evidently depends on the degree of rugosity of the surface. If this is very marked, and the radiation is of relatively short wave-length and is incident very obliquely, hardly anything more than the upper parts of the surface would be illuminated. These would act as diffracting sources of radiation, the permanent phase differences between them being, of course, much smaller than the phase differences between the radiations from the highest and deepest parts of the surface. On the other hand, with greater wave-lengths, even the deeper parts of the surface would contribute to the reflection, but the increase due to this may to some extent be set off by the larger phase differences that come into play.[2] In view of these complications, it seems very difficult to give even an approximate theoretical treatment for the case of very oblique incidences. An empirical formula may however be devised to fit the experimental results. This should obviously satisfy the following conditions indicated by theory. (*a*) For very large wave-lengths, the relative reflecting power should be unity. (*b*) For moderate obliquities and over a range of incidences depending on the rugosity of the surface, it should reduce to the simple single-constant formula (3) above. (*c*) For very oblique incidences, it should give results, which, for short wave-lengths, should be greater, but which increase with the wave-length less rapidly than as suggested by the single-constant formula (3). In divising such a formula, preference has naturally been given to terms of the exponential type suggested by the simple theory. The formula which has been found suitable is

$$I = e^{-a \tan^2 \theta} e^{-(b \cos^2 \theta)/\lambda^2} + (1 - e^{-a \tan^2 \theta}) e^{-(c \cos \theta + d \cos^2 \theta)/\lambda}.$$

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[1] The surfaces A and B had been silvered, C had been platinized cathodically, but D was unsilvered.

[2] The expression $2x \cos \theta \cdot 2\pi/\lambda$ for the phase difference between the radiations from the mean plane and a plane at a distance x from it will not, also, be valid in these circumstances.

This formula evidently satisfies the conditions stated above. For, when $\lambda = \infty$, I is unity. When θ is small, the factor $e^{-a\tan^2\theta}$ is practically unity, and the expression reduces to the formula (3). On the other hand, when θ approaches $\pi/2$, this factor becomes negligible, and the expression reduces to the form

$$I = e^{-(c\cos^{\frac{1}{2}}\theta + d\cos^2\theta)/\lambda}$$

which approaches unity more slowly than (3) as λ is increased. The actual results calculated from this empirical formula have been plotted out in Fig. 1 for the surface C. The numerical values have also been given in Table III. for detailed comparison with Gorton's observations. The constants used have the following values. It will be seen that of the four constants, a is the one most sensitive to alterations in the character of the surface.

Surface.	*A.*	*C.*	*D.*
a.	0.24	0.079	0.0115
b.	42.4	34.5	32.9 ($\times 10^{-8}$)
c.	2.5	1.43	0.0 ($\times 10^{-4}$)
d.	17.8	15.75	7.89 ($\times 10^{-4}$)

TABLE III.

Surface.	Angle of Incidence.		Percentage of Reflection for Wave-lengths (in μ).										
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	23°	Obsd.	—	0.8	4.0	13.9	26.6	38.2	47.4	56.7	63.8	68.5	71.6
		Calcd.	—	0.0	2.0	10.6	23.8	36.9	47.1	57.0	64.2	69.8	74.3
	70°	Obsd.	4.0	25.3	39.8	48.1	55.3	60.2	63.7	66.9	70.1	72.6	75.2
		Calcd.	5.3	23.0	37.5	48.0	55.6	61.3	65.7	69.2	72.1	74.5	76.5
C	24°	Obsd.	2.0	3.2	7.9	19.1	32.1	44.5	54.1	63.7	69.8	75.0	77.9
		Calcd.	0.0	0.0	4.1	16.6	31.6	44.9	55.6	63.8	70.1	75.0	78.8
	45°	Obsd.	3.2	5.3	17.3	33.2	48.0	59.0	67.0	73.1	77.4	79.5	82.3
		Calcd.	0.0	1.3	14.2	32.3	47.4	59.1	67.3	73.3	77.7	81.1	83.6
	72°	Obsd.	17.0	36.9	51.0	62.3	69.3	74.4	77.9	80.9	83.1	84.7	—
		Calcd.	14.5	36.1	55.5	64.9	71.0	75.3	78.4	80.9	82.9	84.5	—
	78°	Obsd.	38.2	55.6	64.7	70.3	73.3	76.8	79.6	81.7	83.8	85.8	87.7
		Calcd.	39.1	62.5	73.1	79.1	82.9	85.6	87.5	89.0	90.1	91.0	91.8
	84°	Obsd.	73.3	85.7	90.4	93.3	95.1	95.9	96.5	97.0	97.5	97.8	98.1
		Calcd.	73.3	85.6	90.2	92.5	94.0	95.0	95.6	96.2	96.6	97.0	97.2
D	54°	Obsd.	—	8.2	28.0	45.6	59.5	69.9	75.2	—	—	—	—
		Calcd.	—	4.5	24.0	44.5	59.5	69.6	76.3	—	—	—	—
	70°	Obsd.	11.0	46.3	66.9	77.3	83.4	85.1	85.1	—	—	—	—
		Calcd.	9.4	40.0	64.0	76.7	83.6	87.9	90.5	—	—	—	—
	75°	Obsd.	31.0	66.1	77.1	84.0	88.1	90.3	91.2	—	—	—	—
		Calcd.	32.6	66.4	80.8	87.3	90.8	92.9	94.2	—	—	—	—
	81°	Obsd.	82.3	92.5	94.7	96.3	97.7	98.7	99.3	—	—	—	—
		Calcd.	80.6	90.4	93.7	95.3	96.2	96.9	97.5	—	—	—	—

The agreement between the calculated and the observed values in Table III. seems fairly good, except in the case of Surface C for an incidence of 78° for which the corresponding experimental curve given by Gorton appears to be rather irregular. The fact that the empirical formula suggested is found to give fair agreement with observations on three different surfaces at different angles of incidence and for different wave-lengths over a wide range makes it probable that it is of fairly general application to rough surfaces.

AMPLIFICATION OF THE PHOTOELECTRIC CURRENT BY MEANS OF THE AUDION.

By Carl Eli Pike.

A METHOD has been outlined by Jakob Kunz[1] by means of which the photoelectric current may be amplified, thus making the photoelectric cell more useful as a photometer, especially in the region of ultraviolet light and also for technical purposes.

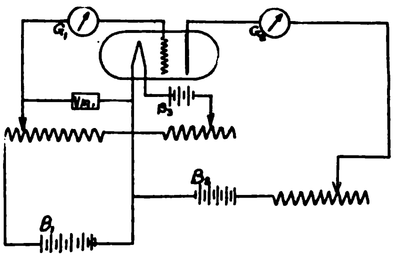

Fig. 1.

The amplification is produced by means of a vacuum tube with three electrodes, or the audion. In order to determine the best potentials to use in the primary and secondary circuits it is necessary to know the characteristic curves for the audions used. The characteristic curves of several audions have been determined by an arrangement of apparatus shown in Fig. 1, which is self explanatory. If we plot the grid potentials as abscissæ and the plate current as ordinates, the curve obtained is called the characteristic.

Due to the fact that the plate current as well as the grid current was so sensitive to small changes in the temperature of the filament, it was necessary to keep the heating current very constant. Large storage cells, well insulated from the ground, were used for this purpose. A large resistance was placed in the external circuit, so that a small variation in the resistance of the filament would not affect the current appreciably. The characteristic curves of three types of audions are shown in Figs. 3, 4 and 5. Audion no. 1 is an oscillion made by the DeForest Radio Telephone and Telegraph Co. Audion no. 2 of Fig. 4 is a W-type;

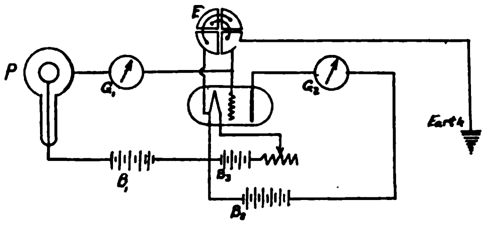

Fig. 2.

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[1] Phys. Rev., Vol. X., No. 2, p. 205.

Audion no. 3 of Fig. 5 is a V-type instrument made by the Western Electric Co. It is noted that the plate current in the oscillion reaches its saturation value more abruptly than it does in either of the other two instruments. In the oscillion it is necessary to heat the filament to incandescence before the electrons are emitted, while in audion W and V the light from the filament was scarcely visible.

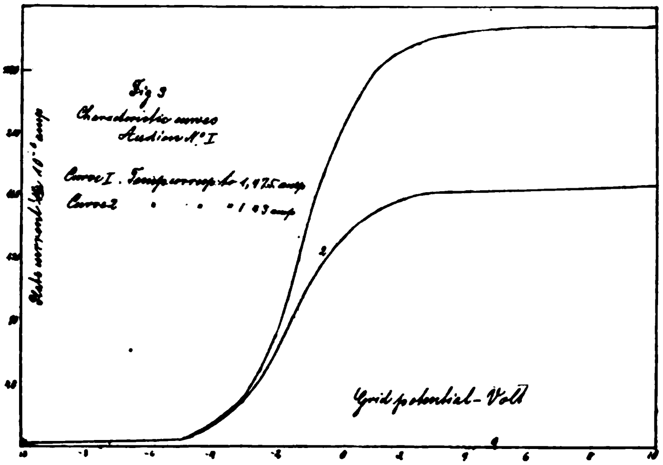

Fig. 3.

Audion no. 2, the W-type instrument, was used for the amplification of the photoelectric current, with an arrangement of apparatus shown in Fig. 2. Twenty-four volts were used in the secondary circuit and a hundred and twenty-five volts in the primary. The photoelectric cell used was a larger type of those made by Kunz in our laboratory. With 125 volts in the primary circuit, the drop of potential inside the audion was very small; measured with an electrometer it was found to be 0.56 volt for nearly the highest intensity of light incident on the photoelectric cell. Since the drop of potential between the grid and filament is very small in comparison to that across the terminals of the photoelectric cell, the photoelectric current is very nearly equal to what it would be if the audion were out of the circuit. The curve giving the relation between the intensity of light and the photoelectric current is shown in Fig. 6. It is unfortunately not a straight line. If this were a straight line and if the portion of the characteristic curve of the audion, used for the amplification, were straight, then we would expect a straight line relation between the intensity of light and the amplified current, and the amplification i_2/i_1, the ratio of the secondary to the primary current would be constant, represented by a straight line parallel to the hori-

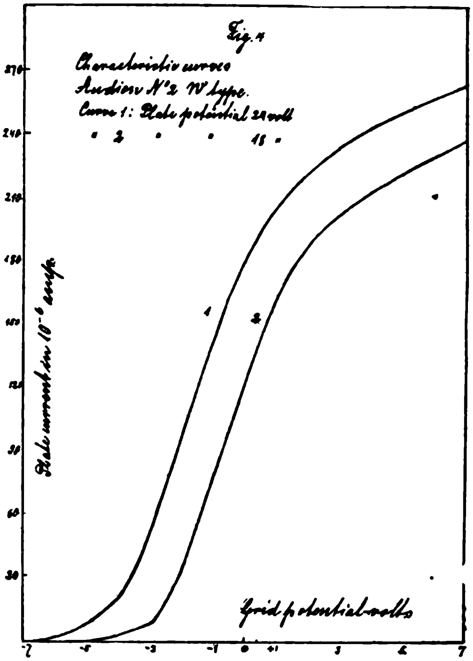
Fig. 4
Characteristic curves
Audion N°2 W type.
Curve 1: Plate potential 24 volt
" 2 " " 18 "
1
2
Plate current in 10^{-6} amp.
Grid potential volts
270
240
210
180
150
120
90
60
30
-7
-5
-3
-1
0
+1
3
5
7

Fig. 4.

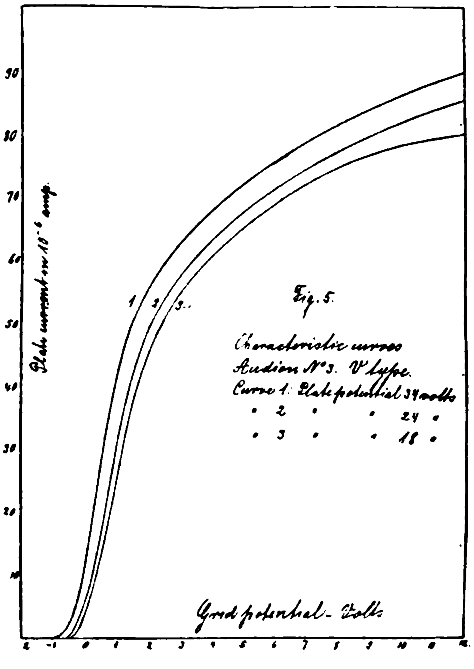
Fig. 5.
Characteristic curves
Audion N°3. V type.
Curve 1: Plate potential 34 volts
" 2 " " 24 "
" 3 " " 18 "
1
2
3
Plate current in 10^{-6} amp.
Grid potential - Volts
90
80
70
60
50
40
30
20
10
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
12

Fig. 5.

zontal axis of Fig. 7. Instead of this straight line, the curve of Fig. 7 has been obtained for intensities varying from 3 to 30 candle meters. For the highest intensity the amplification is about 1750, for the smallest intensity it is over 5,000; above an amplification of 4,000 the points appear somewhat scattered around the curve, but this was only so because the primary current deflections of the Leeds and Northrup galvanometer, G_1 (with a figure of merit $3.74 \cdot 10^{-9}$ for the scale distance used), were very small. If the primary or photoelectric current i is zero, there is already a large current through the secondary galvanometer with a

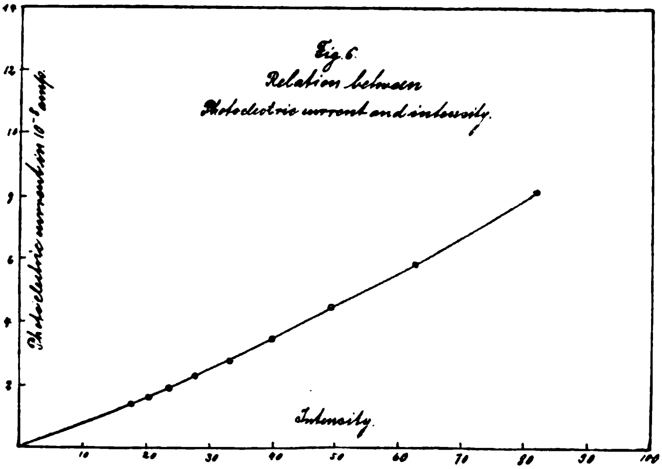

Fig. 6.

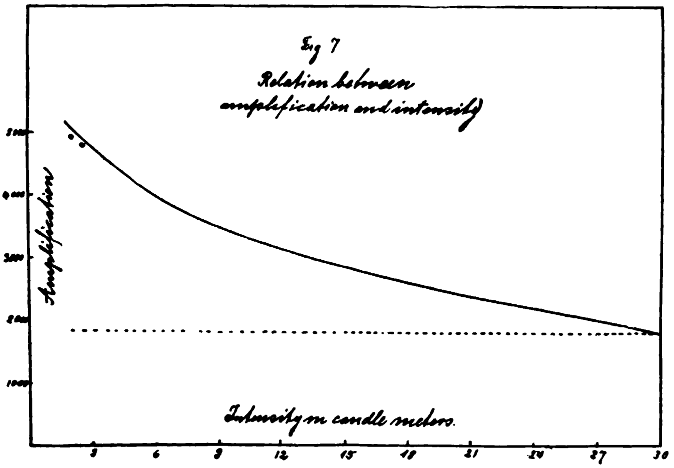

Fig. 7.

figure of merit $2.9 \cdot 10^{-6}$. It goes without saying that this "dark" current was subtracted from that current which was obtained in the galvanometer, G_2, when the photoelectric cell was under the action of light. The difference between the two deflections was proportional to the current i_2. The deflections of the galvanometers were very steady and could easily be repeated. Table I. gives the data that have been plotted in Fig. 7. A satisfactory theory of the audion, based upon the motion, accumulation and absorption of electrons has not yet been given. The current amplification can therefore not yet be determined theoretically. But we can find a simple expression for the amplification, namely, i_2/i_1 in the following way, which involves only Ohm's law and the experimental relation between the plate current and the grid potential; as long as we restrict the amplification to the straight portion of the characteristic $i_2 = Cp_1$, we get the following equations.

$$i_1 = \frac{E_1}{R_0 + R_1} = \frac{p_1}{R_1},$$

$$i_2 = \frac{E_2}{R_2 + R_3} = \frac{p_2}{R_2} = Cp_1 = Ci_1R_1,$$

$$\frac{i_2}{i_1} = C \cdot R_1.$$

The amplification is therefore constant if C and R_1 are constants, that is, if the straight part of the characteristic is used and if the resistance R_1 between the filament and the grid is constant. For large amplifications C and R_1 have to be large.

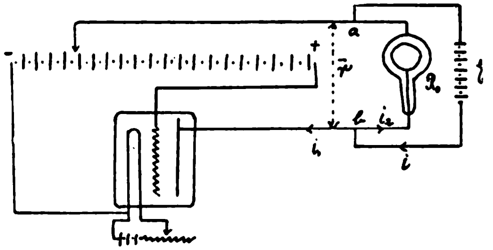

Fig. 8.

A different principle has recently been indicated by A. W. Hull,[1] of the General Electric Company, for the amplification of small currents. By a proper choice of the potentials, the electrons emitted from the incandescent filament pass through the grid and strike the plate where they are reflected. A system of this kind, shown in Fig. 8, presents a negative resistance $\bar{r}$ between the points a and b. If we place a photo-electric cell with the positive resistance R_0 in parallel with $\bar{r}$, then we get;

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[1] P. I. R. E., February, 1918.

TABLE I.

Increase in d_2.	i_2 Amperes.	d_1.	i_1 Amperes.	Amplification.	Intensity in Candle Meters.
73.5	213.0×10^{-6}	35.0	131.0×10^{-9}	1600	33.0
64.8	188.0	23.5	87.7	2100	24.0
56.5	164.0	17.0	63.5	2600	18.5
49.0	142.0	12.9	48.2	2900	14.8
41.0	119.0	10.0	37.4	3270	12.0
35.5	101.5	8.1	30.2	3400	9.9
31.4	91.0	6.8	25.4	3570	8.3
28.0	81.2	5.7	21.3	3800	7.1
23.8	69.0	4.8	17.9	3840	6.1
22.5	65.2	4.1	15.3	4250	5.3
19.5	56.6	3.5	13.1	4320	4.7
17.5	50.7	3.1	11.6	4370	4.2
16.5	47.8	2.8	10.5	4570	3.7
15.1	43.7	2.5	9.4	4680	3.3
13.0	37.7	2.1	7.9	4800	3.0
12.0	34.8	1.9	7.1	4890	2.7
10.9	31.6	1.8	6.7	4720	2.5
11.5	33.3	1.7	6.4	5100	2.3
10.0	29.0	1.6	6.0	4840	2.1
10.0	29.0	1.5	5.6	5200	1.9

"W" type audion, no. 2. Lamp current, 3.75 amperes; candle power, 3.0. Heating current, 0.665 amperes. B^1 equaled 125 volts. B^2 equaled 25 volts. Figure of merit of G^1 3.74×10^{-9}. Figure of merit of G^2 2.9×10^{-6} when shunted with 1.7 ohm resistance. Ratio of figure of merit of G^2 to that of G^1 equaled 775.

$$i = i_1 + i_2 = E\left(\frac{R_0 + \bar{r}}{R\bar{r}}\right),$$

and

$$i_1 = \frac{E}{\bar{r}}, \quad i_2 = \frac{E}{R},$$

hence the amplification

$$\frac{i_2}{i} = \frac{\bar{r}}{R_0 + \bar{r}}$$

may be made very large by making r and R_0 nearly equal in absolute values.

One application of the amplification of photoelectric currents in wireless telegraphy may be pointed out. J. Kunz and J. Kemp[1] have at first used the photoelectric cell as receiver in wireless telegraphy. Their method has been modified by H. Behnken[2] who showed that the photoelectric cell with a string electrometer forms a constant detector of high sensitiveness, especially useful for photographic registrations. With the amplification of the weak currents here involved it should be possible to increase the usefulness of the photoelectric detector. It was intended to continue this investigation with ultra-violet and interrupted light, with alternating currents, and with a differential galvanometer in the secondary circuit.

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[1] Jahrbuch d. drahtlosen Telegraphie, 6, 405, 1913.

[2] Verhdlg d. deutschen phys. Ges., 16, p. 668, 1914.

SUMMARY.

It has been shown that photoelectric currents can be amplified by means of the audion from 1,600 to 5,000 times. The weaker the light the smaller the primary photoelectric current and the larger the amplification. With different audions amplifications of 18,000 have been obtained.

In conclusion the writer wishes to express his appreciation to Jakob Kunz for suggesting the problem and directing the work.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
October, 1918.

THE EMISSION AND ABSORPTION OF PHOTOELECTRONS BY PLATINUM AND SILVER.[1]

By Otto Stuhlman, Jr.

As the result of light passing through very thin metal foil, photoelectrons will be emitted from both sides of the metal simultaneously. Since it will be necessary to distinguish between those electrons which are liberated from the side upon which the light is incident and those that come from the side from which the light emerges, we shall refer to them as "incidence" and "emergence" rays respectively.

It was hoped that some relations could be established between the atomic properties of metals and their photoelectric emissivity. Secondly to establish the magnitude of the coefficient of absorption of the electrons as a function of their speed and atomic properties of the metal in which the absorption was to take place. Finally to use the above information, if adequate, to throw some light on the structure of the atom.

All previous work by the writer and other investigators[2] who have been interested in the photoelectric properties of very thin metallic foil, or still thinner and transparent films of metal was undertaken with cathode deposited films supported by quartz.

The photoelectric currents and velocity investigations were, however, found to depend too much upon the previous history of the cathode and the potential gradient under which the cathodic deposit had been made to warrant any quantitative conclusions.

Thus in many cases, as in those investigations by the writer and K. T. Compton, both the magnitude of the current and the velocity of emission of the electrons could be altered at will, depending upon the impurities that are present through occlusion, in the cathode. The most serious difficulty, however, was found in mounting the very thin transparent films in such a way as to insure perfect electrical contact. These sources of error were so great that no quantitative conclusions could be drawn from the data at that time.

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[1] Presented at the Amer. Phys. Soc. meeting, December 27, 1917.

[2] Stuhlman, Phil. Mag., 20, p. 331, 1910; 22, p. 854, 1911; Phys. Rev., 4, p. 195, 1914. Stuhlman and Compton, Phys. Rev., 2, Sept., 1913. Kleeman, Proc. Roy. Soc., 84, p. 92. 1910. Robinson, Phil. Mag., 23, p. 542, 1912; p. 115, 1913; V., 32, p. 421, 1916. Werner, Ark. Mat. Ast. Fys., 8, No. 27, p. 1, 1913. Partzsh and Hallwachs, Annalen XLI., p. 247, 1913. Compton and Ross, Phys. Rev., 9, p. 559, 1917.

In the present work it was found necessary to investigate metallic films of less than 10^{-6} cm. in thickness with a degree of accuracy much beyond that heretofore attained in the above experiments. It was necessary to provide films of extreme purity and make electrical contact with them which would exclude the errors introduced through direct contact clamping between these very thin films and their supports leading to the electrometer. Heretofore successive thicknesses of films were produced either through successive additions of metal to an initial thin film, or successive plates were covered with various thicknesses and then separately investigated. This necessitated the investigation of many films of varying thickness deposited on the assumption that a linear relation existed between time of sputtering, and thickness of metal deposited. To attain this experimental linear relation by keeping the working conditions constant enough in the cathode-depositing-chamber through control of vacuum and cathode fall is itself quite a difficult undertaking.

TRANSPARENT METALLIC WEDGES WERE DEPOSITED.

In order to obviate the necessity of examining a large number of transparent films of successive increasing thickness, or of examining one film which had been built up of successive layers which had been photoelectrically fatigued through long use, the feasibility of substituting a transparent or semi-transparent metallic wedge was considered.

The deposition of such a wedge by an electrolytic method was however rejected. The difficulties of this method of deposition may be appreciated since we learn from Kundt's[1] well-known experiments on the optical constants of metals, that out of several thousand attempts only a score of wedges were produced which were sufficiently perfect to warrant investigation. A further objection however arises. A chemically deposited metal would contain impurities through occlusion of the electrolyte with the subsequent formation of polarized electrical double layers, which may become so prominent as to prevent all electronic emission.

A metallic wedge deposited by evaporation in vacuo from a fine wire, heated to luminescence, was eventually found to meet the above requirements and objections.

The method[2] essentially consists of a wire, heated to incandescence by means of an electric current, while the quartz plate upon which the wedge is to be deposited is placed below and to one side of it. If the whole is now placed in vacuo, the metal will vaporize and condense upon the quartz surface and surrounding objects.

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[1] See Kayser Handbuch der Spectroscopie, IV. Band. p. 542.

[2] Stuhlman. Presented before the Amer. Phys. Soc., New York meeting, December, 1916. Also Jour. Optical Soc. of Amer., V., 1, p. 78, 1917.

Under similar conditions Langmuir[1] has shown that the rate of evaporation m of an incandescent body is expressed by the relation

$$m = \sqrt{\frac{M}{2\pi RT}} \cdot p,$$

where M is the molecular weight of the vapor and p its vapor pressure. He has also shown that no reflection of molecules of the metal takes place at the surface upon which they initially strike, unless the surface is so hot as to cause reëvaporation. Under the above conditions we can therefore conclude that the total amount condensed per second is equal to the loss per second which the wire undergoes through evaporation.

Hence given a wire whose cross-sectional area is small compared with its distance above the plane, over which it is placed horizontally, it may be treated as a linear source which is sending out metallic vapor in all directions at right angles to its surface. Since no reflection takes place from the horizontal plane under the working conditions, then $dm/2\pi$ can represent the amount of evaporated metal passing through unit angle per second and subsequently condensed from a length dl of wire. Let the wire be stretched parallel to and at a height h cm. above the horizontal plane in which the quartz plate is placed, upon which the deposition is to form. After n seconds let t_0 represent the thickness of the metal condensed on the plane just below the wire. The flux per second of metal through an angle $d\theta$ condensing just below the wire producing this thickness t_0 is then

$$t_0 h d\theta = \frac{dm}{2\pi} d\theta.$$

The simultaneous condensation at any other point in the plane at a perpendicular distance r from the wire is given by

$$t_n ds = \frac{dm}{2\pi} d\theta.$$

Since

$$t_0 h d\theta = t_n ds = \frac{t_n r d\theta}{\cos\theta}$$

it follows that the thickness of deposit at any point in the plane is

$$t_n = \frac{t_0 h^2}{h^2 + x^2}, \tag{1}$$

where x represents the distance of the point n from the foot of the perpendicular h let fall from the wire to the plane. If we set $h = 2a$ and $t_n = y$ the equation reduces to the well-known curve

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[1] PHYS. REV., V., 2, p. 340, 1913.

$$y = \frac{8a^3}{x^2 + 4a^2}, \tag{2}$$

the "Witch of Agnesi," so that a section through the solid deposit formed from an evaporating wire, at constant temperature, with vacuum at constant pressure can be represented by the above curve. The deposit just below the wire is however not a wedge. It has a pronounced curvature concave downward, followed by a curvature concave upward. Further along, that is for the values of x ten times larger than values of y, the curve approaches a straight line. It was in this region that the quartz plate was placed, which was to receive the metal, where the deposit would approach nearest to the desired form of a true wedge.

Apparatus used to Deposit Wedges.

Fig. 1 shows a diagrammatic representation of the apparatus. Although originally[1] designed to make interferometer or opaque mirrors of uniform texture and optical purity, it was found to answer the requirements for the present purpose. The working parts are mounted on a thick plate of rolled brass, grooved to hold the bell jar *BJ*. Hard sealing wax floated in this groove formed the seal for the vacuum which was maintained by means of a Gaede rotary mercury pump. The carriage *HE* supports the incandescent wire. At *H* the wire is attached to a screw at the top of the rod connecting the two wheels, from where it passes down and then through a large hole in the rod between the wheels, then across through a similar hole in the rod at *E*, thence to the spring brass hook, *E*. This spring hook supplies the necessary tension to the wire. Upon heating the wire, it expands and the slack is further taken up by the spring in the hook. For the heating current a 110-volt alternating current was used. The quartz plate was placed in the plane *P* below and to one side of the wire.

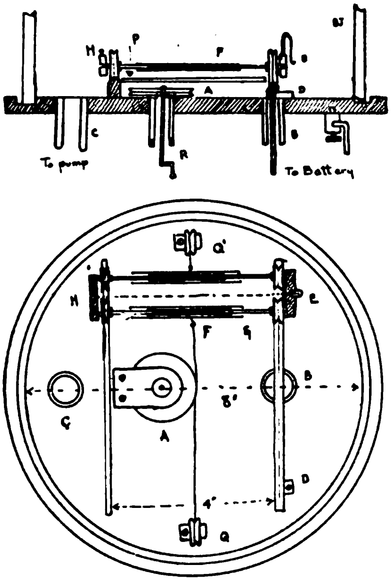

Fig. 1.

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[1] Read as a paper before the Amer. Phys. Soc., New York, December, 1916. Jour. Optical Soc., V., 1, p. 78, 1917.

The heating-current enters the base plate at the binding post to the right. It then passes into the metallic track at H which is screwed to the base plate. From here it passes through the left side of the carriage, thence through the wire across to E into the right-hand track. This track is insulated from the base-plate by means of two fiber L-shaped supports. The current passes from this insulated track through the rod B back to the battery. The rod B passes through a hole in the base-plate and is sealed in place with hard red sealing wax, which serves to insulate it from the plate and at the same time makes the opening air tight.

To prevent the carriage supports from short circuiting the glowing wire, they are separated by hard fiber rods. This joint F was then covered with a tight-fitting glass tube extending one centimeter beyond each end of the fiber insulation. This prevented the fiber from being coated with a metallic deposit which would otherwise short circuit the glowing wire. In order to shift the carriage it was necessary to have a mechanical control outside of the apparatus. (A magnetic control would be simpler for the present purpose.) It was supplied by means of a small crank which passed through a ground glass stopper sealed in the tube at R. This crank turned the horizontal V-grooved wheel A, around which passed a belt to the small wheel Q, from where it passed to a hook on the carriage at F. The other end of the belt passed over a similar wheel at Q' and thence to a similar hook on the other side of the carriage. By turning the crank R, the carriage and wire could be rolled to any position above the plane P which contained the quartz plate upon which the metallic wedge is deposited.

In order to insure a very pure deposit it was found necessary to acid clean the wire thoroughly. It was then mounted and heated in vacuo, but not so high as to produce evaporation. This removed most of the occluded gases and such impurities which would otherwise have been condensed on the quartz plate. The wire is now set at the proper distance from the quartz plate and allowed to vaporize until the desired thickness of metal has been attained.

A metallic deposit condensed as outlined above, was first examined for change in thickness as the distance from the wire increased; with a view of ascertaining how near photometric measurements could determine changes in thickness. A glass microscope slide 10 × 1 cm. was placed directly beneath and 1 cm. below the wire and the metal was deposited until the region just below the wire was opaque. The deposit was next examined by means of a microphotometer with unresolved light from a tungsten filament lamp. A beam 1 × 10 mm. was directed normally on the metallic side and after passing through the wedge was focussed on a linear thermopile in vacuo. The throw of a ballistic galvanometer in series was used to measure the current, which was assumed to indicate, in a rough way, the change in thickness of the wedge as successive millimeter portions were passed before the slit of the thermopile.[1] No quantitative conclusions were expected from these data due to the change in the optical constants with change in thickness of the film.

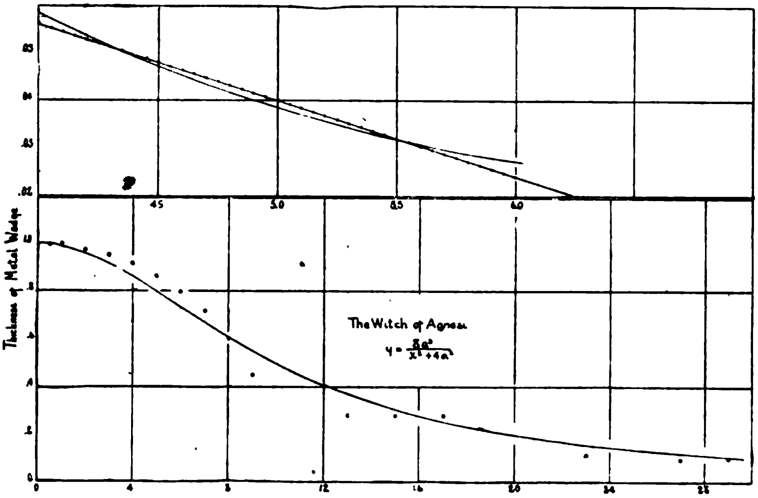

Fig. 2.

These more or less approximate results are shown in the lower curve of Fig. 2. The agreement between the observations as recorded and the theoretical equation $y = 8a^3/(x^2 + 4a^2)$ plotted, as a full line, are as close as might be expected under the circumstances.

In order to determine how near to the wire the best approach to a true wedge could be expected, the following graphical method was resorted to. The theoretical curve was plotted on large scale coördinate paper and that part of the curve chosen which fitted nearest to a straight line within an average of five per cent. The upper graph in Fig. 2 shows the portion of the curve chosen. The solid line represents values computed from the theoretical equation, for the thick end of the wedge at a distance of four centimeters from the foot of the normal let fall from the wire to the plane. The broken graph is a straight line drawn through the average value of the points. Although the metal deposited at this distance from the wire has a slightly concave surface, yet it approaches a straight line wedge with not more than a maximum of five per cent. variation at any point. This five per cent. error, contributed by the straight line assumption, is within the error of observation contributed by the subsequent photoelectric current measurements.

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[1] I am indebted to Dr. Ives, of the United Gas Improvement Co., Philadelphia, for the use of this apparatus, and to Dr. Eckhardt for assistance with these observations.

The metallic wedges investigated in this paper were all deposited in this region.

THE WEDGES.

The metals investigated were platinum and silver. They were deposited from chemically pure wires .043 cm. in diameter. The wire was 12 cm. above the plane which contained a 2.5 × 1.0 × 0.1 cm. quartz plate placed four centimeters away from, with its narrow side parallel to, the incandescent wire.

The platinum wedges, when examined by reflected light, showed for the opaque deposit at the edge nearest to the glowing wire, the characteristic metallic lustre of the metal. Semi-opaque thicknesses transmitted red light and reflected blue. As the thickness decreased the reflected light changed from blue to brown, to yellow, to blue and at the very thinnest end to indigo. The colors of the transmitted light for the same thicknesses were gray brown, greenish blue and reddish brown.

Silver wedges which were opaque at one end and shaded to zero thickness at the other end in 4 cm. of length showed the following variation in color. Deep violet was transmitted through the nearly opaque thicker end, shading into violet, blue, light blue, deep blue, purple, red, brown and into yellow at its thinnest end. The corresponding points when viewed by reflected light showed at the opaque end the metallic luster of silver followed by deep green, green, green blue, red green, red, blue red, blue and a lighter blue at its thinnest end.

An absorption band might hence be expected to exist in the green part of the spectrum about λ 4500. An absorption spectrum photograph obtained by means of a Hilger monochromatic illuminator through one of the silver wedges, showed a continuous gradually increasing absorption through the red and yellow, terminating at about λ 5300, from here a weak broad green blue absorption band exists terminating at about λ 4000 from where the absorption again drops off regularly into the violet but at a greater rate than the ascent in the red end.

While the thick and nearly opaque ends of some of the platinum wedges are transparent to red light (in fact the glowing filament of a tungsten lamp was visible through the thick end as a brilliant red image); the thick or nearly opaque end of the silver wedge showed the same filament as an indigo-blue image. Showing that the shorter wave-lengths penetrate further into a silver surface than into platinum. What evidence the photoelectric observations throw on this topic will be discussed in a separate chapter.

THE ARRANGEMENT OF THE APPARATUS.

The metallic wedge (1 × 2.5 cm.) supported by the quartz plate is shown in Fig. 3 as *Q*. It was mounted with the thick end of the wedge attached to the brass holder *E*. A countersunk screw pressed the metal-covered side of the quartz plate rigidly against the opposite facet of the clamp at *E*. This clamp was screwed to a rod ending in a rack *B*, which slid under V-shaped grooves in the fixture *A*. The pinion *N* engaging the rack, pushed the wedge into any desired position with respect to the beam of light *T* which was used to excite the photoelectric phenomena. The position with respect to the illumination was indicated on the steel millimeter scale *S* fastened to the fixture at *A*. This motion of the wedge allowed one to expose successively, increasing or decreasing thickness of the metal to the beam of ultra-violet monochromatic illumination passing through a Hilger Monochromatic Illuminator placed in front of the slit *R*. A 110-volt Cooper Hewitt quartz mercury arc running under constant power supplied the illumination. The slit at *R* was so adjusted that a clear cut image 8 mm. long and 0.23 mm. wide fell on the quartz plate at *T*. The degree of sharpness of the image and its position was at all times controlled by means of a fluorescent plate which could be substituted for *Q* and by means of a fluoroscope eye piece placed at *O*. The photoelectric chamber *D* is an oxidized copper cylinder covered with camphor soot whose photoelectric constants were well known, having been repeatedly used in former experiments. This cylinder D was mounted coaxially with the outer brass casing *P*. It was connected as shown, to an electrometer after passing through the hard sealing wax support at *P*. The electrometer showed a consistent sensibility of 1,220 divisions per volt which did not vary more than 1.3 per cent. during the course of the experiments.

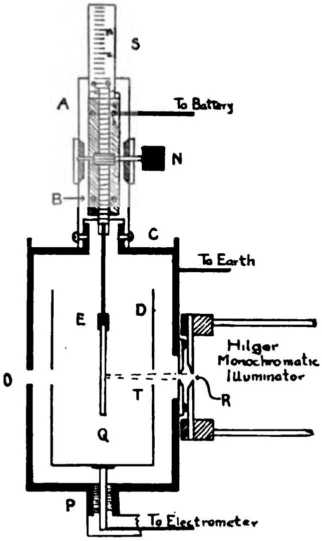

Fig. 3.

The present method differs fundamentally from previous measurements on thin metal films in so far as photoelectric current can be measured from one film, possessing various thicknesses and having the same history of production and ageing. An additional improvement consists in being able to examine the very thinnest film without introducing special appliances or precautions for electrical contact. It also allows one to measure the thickness of the very thin films to a greater degree of accuracy through extrapolation from the thicker films, which in turn lend themselves more easily to accurate measurements.

The wedge was exposed at millimeter intervals through its entire length. The present paper deals with the photoelectric current obtained from such a wedge when the metal side faced away from the light. Under these conditions the illumination was allowed to fall under normal incidence on the quartz side of the plate, it then passed through the transition layer quartz-metal and liberated photoelectrons in the metal. The electrons leaving the metal under these conditions are classified as "emergence." To measure the "incidence" effect all that was necessary was to rotate the superstructure in the groove *C* through 180° and repeat the measurements for thicknesses as relatively determined by the scale *S*.

Film Thickness.

The determination of the thickness of the film contributed the largest error to the final results. Optical methods as developed by Wiener[1] were given up as too unreliable. A very promising method, though not used on the data presented below, and especially adaptable to silver films, is that in which the silver film is locally converted into silver iodide by means of the proximity of a minute piece of iodine. A convex lens placed upon the resulting hump allows one to use the Newtonian ring interference method for computing the thickness of the hump and from it a value of the thickness of the original silver.

Here either mechanical or electrical methods were resorted to. As in the case of platinum, when the specific resistance of the material is known in terms of thickness,[2] it was found feasible to determine the thickness at several points along the wedge by using the following indirect method.

A quartz plate, similar in size to the one upon which a wedge was deposited, was covered with a uniform film, of thickness comparable to the thick end of the wedges. This uniform film can be deposited in the apparatus discussed above by simply moving the carriage, supporting the incandescent wire, back and forth across the plate with uniform speed. After attaining its equilibrium value the film was examined for its specific resistance. This resistance data could be used to compute its thickness. A photoelectric examination of this uniform surface made under similar conditions as existed when the wedge was investigated, determined the thickness of that part of the wedge which showed an equal photoelectric current.

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[1] Wied. Annalen, V., 31, p. 629, 1887.

[2] Patterson, Phil. Mag., IV., p. 663, 1902. Longden, Phys. Rev., II., p. 40, 1900. I. Stone, Phys. Rev., VI., p. 1, 1898.

When the thickness of the plane film was large enough, the above method was checked by direct weighing. The plane silver films could be changed to the iodide form, and then weighed, and from these results the thickness can be computed.[1]

EMERGENCE PHOTOELECTRIC CURRENT OBSERVATIONS.

The photoelectric currents for wave-lengths 2260, 2301, 2536 and 3131 were obtained for platinum and silver with variations in thickness of the metal exposed. Since the various spectral lines are not of equal intensity the observations were reduced to represent current per unit incident light intensity, by dividing the number of electrons liberated per unit time by the energy of the light of the corresponding wave-length.

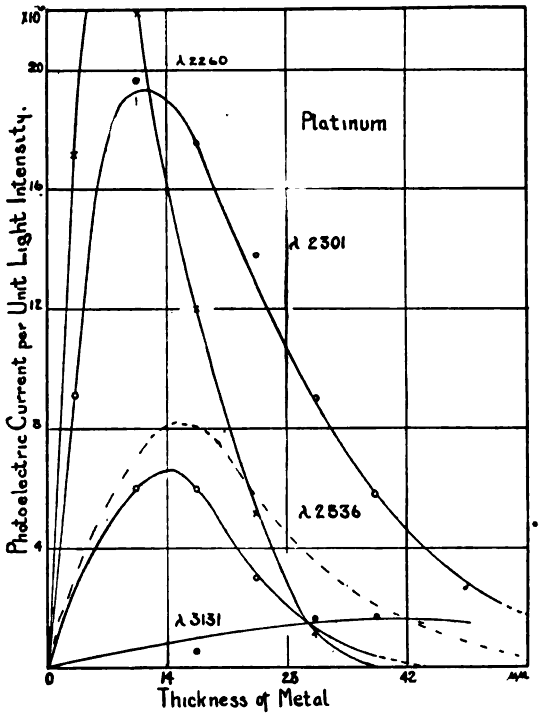

Fig. 4.

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[1] Quincke, Pogg. Ann., 129, p. 178, 186 .

The ordinates thus plotted in subsequent curves represent the relative number of electrons liberated in unit time by a unit amount of light-energy for the wave-length indicated.

In the case of platinum an attempt was made to get some results from λ 3131 but with little success, since platinum is no longer photoelectric sensitive for this wave-length. These latter results are only introduced for qualitative comparison, and since no light filtèr was used, no doubt the observed current was not really due to the excitation from this wave length, but from a somewhat shorter wave-length.

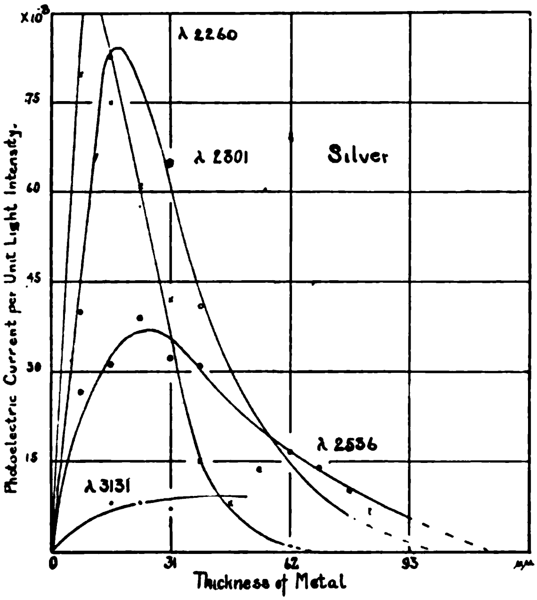

Fig. 5.

Figs. 4 and 5 show a typical set of observations for each of the metals. To avoid the error that might be introduced by photoelectric fatigue it was found necessary to prepare a separate wedge of metal for the observations obtained for each wave-length examined.

The photoelectric observations could be duplicated with a mean error of five per cent., but since the thickness determinations possessed an average error of ten per cent., no additional precautions were introduced into the photoelectric measurements to insure a greater accuracy. The constants computed from these curves show a degree of consistency in agreement with the above errors, which serve as a good check on the observational values.

In Fig. 4 the dotted line for λ 2536 gives the unfatigued values of the photoelectric current computed from the solid fatigued curve just below it. The ordinates of this curve were obtained by increasing the observed fatigued values by an amount experimentally determinable and proportional to the time elapsing between the removal of the wedge from its vacuum chamber and its subsequent photoelectric examination.

The curves for both metals have this in common; all make finite angles with the abscissa. There is no sign of inflexion on the ascent of the curve, so that the tangent to the curve at the origin is more inclined to the abscissa than the tangent at any other point. This means that for the very thinnest part of the wedge (atomic thickness) the photoelectric effect occurring there for any frequency of the exciting light, bears a definite ratio to the mass per square centimeter of the film. The slopes of the curves at the origin are relative measures of the energy absorbed by a film thickness comparable to the diameter of one molecule.

The curves can, for the sake of discussion, be divided into two parts. The initial rise which appears to be a type of saturation curve, and superimposed upon this a second phenomena of a more complex nature approximately represented by an exponential drop followed by a linear relation ending in another less rapid fall.

These two phenomena will be treated separately under the subject of scattering and absorption of photoelectrons.

The photoelectric current is excited by a monochromatic beam of light falling normally on the metal surface, and if no electronic absorption takes place the photoelectric current should be proportional to the light energy absorbed. If the optical constants of the metal do not change with change in film thickness for the wave-lengths used, it would follow that as the thickness of the metal increases linearly the photoelectric current should increase exponentially. This would continue until a thickness is attained beyond which the monochromatic light could no longer penetrate. This does not however hold true. When we measure the emergence effect, the curves seem to indicate that the light gives rise to electrons in amount apparently greater than the amount of light absorbed. It becomes therefore necessary to determine the cause of this additional electronic activity.

THEORY.

For the sake of simplicity let us consider only those electrons which leave the atom in the emergence direction, possessing energy proportional to the frequence of the exciting light but independent of the kind of atom from which they initially escape.

As the liberated electrons pass through successive layers of matter, let us further assume for simplicity that they are absorbed according to an exponential law.

If then a beam of light of frequency ν strikes a metal film, and if I_0 is the intensity penetrating the surface normally, if the intensity has fallen to I after passing through a thickness dx, then

$$I = I_0 e^{-\epsilon x}.$$

If all of the absorbed energy is used in liberating photoelectrons in such a way that the energy absorbed is equal to the photoelectric energy liberated then

$$I_0 - I = I_0(1 - e^{-\epsilon x}), \tag{3}$$

where ϵ is a constant, denoting the rate of appearance of electrons. It depends on the frequency of the light used and may be identified with the coefficient of absorption of the light. Since electronic density and light intensity are proportional under these circumstances, I_0 can be considered a variable, subsequently to be absorbed exponentially in the distance $t - x$. Then

$$\frac{dI_0}{dx} = -\alpha I_0$$

where α is the coefficient of absorption of the electrons passing through thickness $t - x$. The solution of this relation is given by

$$I_o = I' e^{-\alpha(t-x)}.$$

Since all the electrons due to any frequency ν of the light have the same initial velocity and hence may initially be considered a homogeneous beam coming from a layer of metal between x and $x + dx$, it follows if we assume that only those electrons are considered that move off in the emergence direction, that the intensity of the emergence radiation is proportional to

$$\frac{dN}{dx} = \epsilon I_0 e^{-\epsilon x} = \epsilon I' \int_0^t e^{-\epsilon x} e^{-\alpha(t-x)} dx$$

and

$$N = \frac{\epsilon I'}{\alpha - \epsilon}(e^{-\epsilon t} - e^{-\alpha t}). \tag{4}$$

It is seen that for small thicknesses the photoelectric current is proportional to the thickness or number of atoms per cubic centimeter. With increased thickness the curve passes through a maximum when

$$t_m = \frac{1}{\alpha - \epsilon} \log_e \frac{\alpha}{\epsilon} \tag{5}$$

and apparently decreases exponentially with the intensity $I' e^{-\epsilon t}$.

In order to determine to what extent the above curves check this theory the discussion is for convenience divided into two parts, the initial absorption of light as a function of the thickness and the absorption of the photoelectrons as determined by that part of the curve showing the decrease of the photoelectric effect with its subsequent disappearance as the thickness increases.

ABSORPTION OF LIGHT WITH VARIATION IN THICKNESS.

If under the experimental conditions outlined above, the photoelectric current is represented for any thickness t, by the relation $I_0(1 - e^{-\epsilon t})$ then this is equivalent to saying that the intensity of the electronic radiation from a unit area of the metal is always proportional to the number of atoms present in one cubic centimeter. The values of ϵ therefore obtained from these curves would help to determine the validity of this relation. If the number of electrons liberated are proportional to the amount of light absorbed, and if the metal film is so thin that the absorption of the electrons as they pass through the successive layers of molecules can be neglected, then successive similar layers of the metal will absorb equal fractions of the amount of light energy they receive, provided no scattering takes place, and emit a quantity of electronic energy which is a measure of the light energy absorbed. Under these conditions we can measure the electronic energy and use it as a measure of the light energy absorbed. If then we let ϵ represent this coefficient as measured

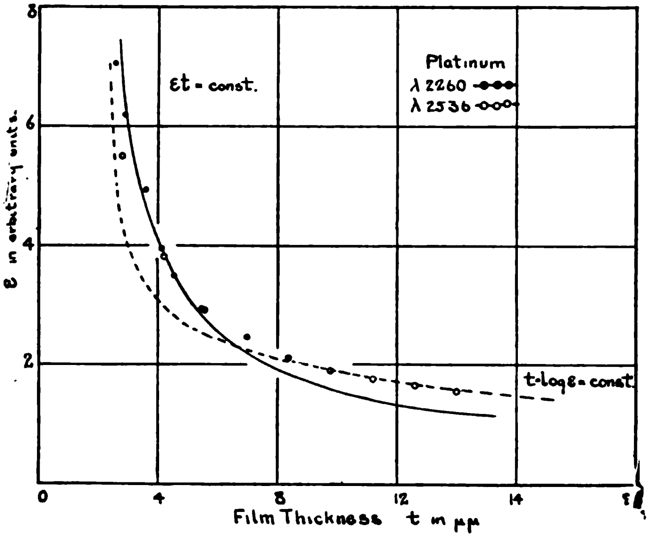

Fig. 6.

by the electronic results, I_0 the intensity of the energy entering the slab of material, I the intensity leaving, then $I = I_0e^{-\epsilon t}$ where t is the thickness of the slab. This may be rewritten in the form

$$\epsilon = \frac{2.3}{t}\log\frac{I_0}{I}$$

If now $\log I_0/I$ is plotted against t, the graph should be a straight line (for ϵ constant) and ϵ may be determined from the relation $\epsilon = 2.3 \times$ slope.

The curves thus obtained were however not linear, they showed a continuous decrease in the coefficient ϵ (Fig. 6) as the thickness increased. The coefficients were then computed from the photoelectric curves using the relation $\epsilon = 0.693/D$ where $e^{-0.693} = 0.5$ and D is the thickness which reduces the radiation to half value. Here $1/\epsilon$ represents the distance to which the energy penetrates before its intensity is reduced $1/e$ of its original value.

Table I.

Platinum, λ2301.			Silver, λ2301.		
t.	ϵ.	ϵt.	t.	ϵ.	ϵt.
1.40 $\mu\mu$	.990 $\mu\mu^{-1}$	1.39	1.70 $\mu\mu$	.393 $\mu\mu^{-1}$	.668
2.10	.660	1.39	2.54	.260	.660
2.70	.520	1.40	3.30	.206	.679
3.50	.396	1.38	4.25	.160	.680
4.20	.330	1.39	5.00	.135	.665
5.05	.283	1.43	5.77	.114	.657
5.90	.230	1.35	6.90	.100	.690

Table II.

Platinum, λ 2536.

t.	ϵ.	ϵt.	$t \log \epsilon$.
2.8 $\mu\mu$	.550 $\mu\mu^{-1}$	.154	264
4.2	.381	.160	244
5.6	.291	.163	260
7.0	.247	.173	275
8.4	.211	.177	273
9.8	.190	.186	273
11.2	.176	.197	276
12.6	.165	.208	273
14.0	.157	.216	271

The above extended values give some idea of the general trend of the results. From the samples cited in Table I. it is seen how very close the product ϵt approaches a constant for small values of thickness. The constancy of this product was found to hold, in the case of both platinum and silver for λ 2260 and 2301, so long as the values thus obtained remained associated with very small values of t. When the thickness became larger ϵt no longer remained constant but increased with increasing values of t. A characteristic set of observations illustrating this condition is cited in Table II. If however, the relation $t \log \epsilon$ is computed from the data it is found that this relation is constant for large values but not for small values of t. As an example the case of Pt λ 2536 is cited, the change taking place at thickness 7 $\mu\mu$.

One characteristic common to all these results is that ϵt seems constant under two circumstances, either for quite a range in values of t for large values of electronic velocities, excited by means of λ 2260 or 2301, or for a very short range in the values of t (about 7 $\mu\mu$) when slower electronic velocities are used, as for example those excited by λ 2536 or 3131. Although the curve for platinum λ 3131 is shown in Fig. 4, yet the coefficients calculated from this curve were omitted owing to insufficient points on the curve to determine its initial slope. In addition the minimum wave-length sensibility for platinum is just inside the wave-length used so that the above curve must have been obtained through part of the shorter wave-lengths in the spectrum overlapping this region. The value for silver λ 3131 has only been introduced to show the order of magnitude of the value ϵt.

TABLE III.

Mean Values of ϵt *= const. for small values of* t.

λ.	*Pt.*	*Ag.*	Ratio.
2260	1.41	.681	2.07
2301	1.39	.671	2.07
2536	1.61	.983	1.64
3131		1.80	
Atomic weights	195.2	107.9	1.82

A summary of these results for values of t small, representing mean values similar to the extended data given above are shown in Table III. The last column gives the ratio of the values for this relation. It is of the same order of magnitude as the ratio of the atomic weights. We might conclude that the light energy absorbed per atom, is proportional to its atomic weight.

Before going into the possible origin of the above relations it may be of interest to examine the constancy of the relation ϵt further. The values of t were therefore plotted against values of $1/\epsilon$ (Fig. 7). Under these conditions one would expect to get a straight line whose slope would give a relative measure of the absorption of the light in passing through a molecular layer.

Since the amount of light energy absorbed is proportional to the photoelectric energy liberated it follows that the values of ϵ determined for changes in thickness, are also measurements of the photoelectric characteristics. Under these conditions we may view the electron as liberated from its parent atom, passing through successive layers of molecules being deflected through collision with them but never entering the system of the atom to disappear through absorption.

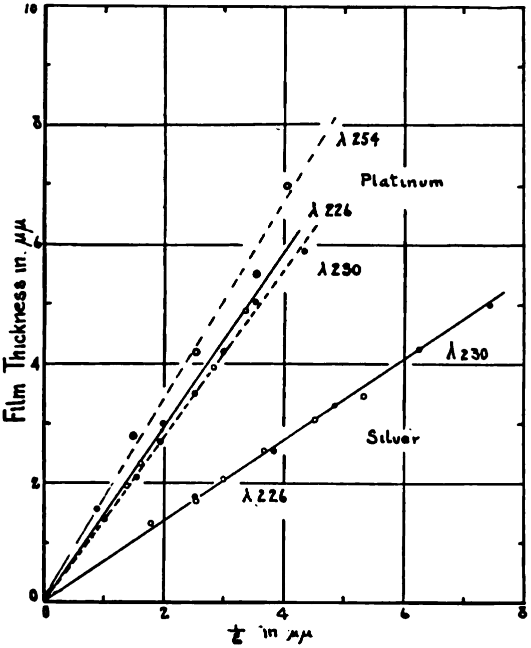

Fig. 7.

If therefore we consider two electrons, liberated with the same initial velocity, in the two metals, we will find after having passed through equal thicknesses of metal that the electron must have undergone a greater energy change per unit distance in passing through platinum than through silver. This ratio is, within experimental error, the same as the ratio of the densities of the two metals. This means that if equal masses are compared the amount of energy scattered is the same in the two cases. A similar relation exists for secondary radiation produced by Röntgen rays.[1] The above results are shown graphically in Fig. 7. In general they also seem to indicate that the greater the velocity of separation of the electron from its parent atom, the less the scattering of its energy as it passes through neighboring atoms. The observations for platinum λ 226 do not fall in line with this conclusion. This exception can however be attributed to experimental error. The results for Ag λ 254 are here omitted so as to prevent too much crowding in the figure. The results for λ 226 and 230 lie so close together that it is impossible to separate them in the graph, though the actual results from which the curve is compiled show that λ 230 lies to the left of the results for the smaller wave-length.

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[1] Barkla, Phil. Mag., 7, p. 543, 1904.

If ϵ represents the fraction of the total light energy lost through absorption and reappearing as electronic energy, in going through a sheet of metal of molecular thickness t, I_0 the intensity of the beam entering the sheet and I the intensity of the beam leaving it, then $1/\epsilon$ the intensity coming out of the sheet is given by

$$\frac{I_0 - I}{I_0} = \frac{1}{\epsilon} = kt, \tag{6}$$

where k is a constant. From which we get

$$I = I_0(1 - kt). \tag{7}$$

For larger values of t under the same circumstances it was found experimentally that

$$t \log \epsilon = \text{const.},$$

then

$$\frac{I_0 - I}{I_0} = \frac{1}{\epsilon} = e^{-c/t}, \tag{8}$$

or

$$I = I_0(1 - e^{-c/t}). \tag{9}$$

These relations do not however conform to our simple theory advanced at the beginning of the paper which involved the assumption of an exponential absorption leading to equation (4).

THEORY OF SINGLE AND COMPOUND SCATTERING.

Rutherford[1] has advanced a theory of single scattering which leads to the conclusion that the proportion I/I_0 of the rays which remain undeflected after passing through a thickness t, when t is small enough,[2] is given by the relation

$$I = I_0(1 - kt).$$

A relation deduced from the above experimental data for thicknesses less than 7 $\mu\mu$ (Fig. 6).

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[1] Rutherford, Phil. Mag., V., 21, p. 684, 1911.

[2] Richardson, Electron Theory, p. 496.

For very small thicknesses we must therefore have electrons passing through the metal film, colliding according to Rutherford's hypothesis of "single scattering" and little or no energy is contributed to the atom through which the electrons passed. From this it is seen that the relation between I/I_0 and t should be nearly linear in the initial stage, and this is found to be the case for all wave-lengths for both metals investigated.

If however the thickness penetrated by the electrons becomes greater a larger number of collisions necessarily results, and then the relation $t \log \epsilon =$ const expresses the state of affairs. It seems as if the electron might be considered as at first moving in fairly straight lines with little deflection and little or no energy exchange, but as the penetration increases the deflexions become more numerous and violent resulting in the rate of expenditure of energy increasing so rapidly that the electron seems to come to a sudden stop.

This stage of the process seems to be equivalent to what Thomson[1] describes as "compound scattering." He showed that the probability of an average deflexion of an electron on passing through a metal sheet of thickness t, if less than a given angle, is equal to $1 - e^{-c/t}$ where c is a constant for any particular substance, which under the present experimental circumstances means that the proportion I/I_0 of the electrons undeflected is given by $I = I_0(1 - e^{-c/t})$ a relation identical with the empirical equation (9).

The experimental arrangement and method therefore appears to fulfill the requirements for single scattering when the metal films are very thin (less than 7 $\mu\mu$), and compound scattering when the metallic film is thicker (Fig. 6).

In these experiments the angular opening ϕ through which the electrons escape from the film is not varied, and if the same metal be examined so that the absorbing medium is also kept the same and the velocity of the electrons varied through change in wave-length of the illumination both Rutherford's and Thomson's theory leads to the relation,

$$\left(\frac{mv^2}{t_0^{1/2}}\right. = \text{const.}, \tag{10}$$

where t_0 is the value of the thickness required to cut down the radiation to half value.

This relation can be tested with film thicknesses in which single scattering takes place. In the experimental arrangement ϕ is kept constant. Now the maximum energy of release of the electron is independent of the kind of atom and only depends on the frequency of the exciting light.

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[1] Camb. Phil. Proc., 15, p. 465, 1910. See Richardson, Electron Theory, p. 493.

If therefore we consider values of $1/\epsilon$ for equal values of t_e and λ in the two metals, we have a measure of the relative average energies with which the electrons appear after passing through equal thicknesses. This also gives us a comparison of the distances to which electrons penetrate so as to have their initial energies reduced to $1/e$ of their original value. The results obtained from the data as shown in Fig. 7 indicate that equation (10) is valid within experimental error for film thickness less than 10^{-7} cm.

An interesting and rather important change takes place after the electron penetrates further into the metal. In Fig. 6, we see an illustration of the thickness at which this occurs. We are here confronted with another illustration of the sudden change in the property of a metal when its thickness is less than 10^{-7} cm. Similar changes occur in the optical and electrical properties of these metal films in this immediate vicinity.[1]

Unfortunately not enough data were available from individual results for λ 2536, to show clearly that the smaller values of t followed the ϵt = const. relation (Fig. 6), so that the values for λ 2260 were added to show how close the results fitted the theoretical curves, here shown as solid and broken lines. The latter results were therefore shifted so that the axes of the curve coincided with the axes of the curve for values λ 2536. In this way the results show graphically just where the region of separation exists to which the above theories apply.

If we now consider thicknesses in which compound shattering exists we are limited on one side by the thickness comparable to 7×10^{-7} cm. and the maxima of the curves shown in Figs. 4 and 5. As seen from the sample data shown in Table II. the coefficient decreases less rapidly than the increase in thickness. This is to be expected, if the above theory can be applied; for the amount of scattering becomes more marked with decrease in speed and increase in thickness. This continues until a critical thickness is reached where the absorption of the number of electrons becomes predominant, finally leading to a decrease in the amount of emergence radiation.

The initial rise in the curves can therefore be divided into two distinct regions. For thicknesses less than 10^{-7} cm. the velocities of the emergence electrons must be nearly equal or so nearly that very few if any electrons appear whose energy is less than that demanded by the Einstein relation. Here little if any absorption takes place so that the electron passes through subsequent layers with slight loss of energy. From here on however the number of electrons possessing less than this maximum energy decreases, they undergo frequent and violent collision with a rapid loss in energy until some electrons seem to come to a sudden stop. Further penetration is then accompanied by loss in numbers in addition to loss in energy.

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[1] Patterson, Phil. Mag., IV., p. 663, 1902. Minor, Ann. d. Phys., 10, p. 581, 1903.

ABSORPTION OF PHOTOELECTRONS.

Our theory as outlined at the beginning of the paper indicated a maximum for values of t given by the relation

$$t_m = \frac{1}{\alpha - \epsilon} \log_e \frac{\alpha}{\epsilon}.$$

The values for t_m as picked off of the ionization curves are given in Table IV., Fig. 8. The values of t_m there indicated represent the distances to

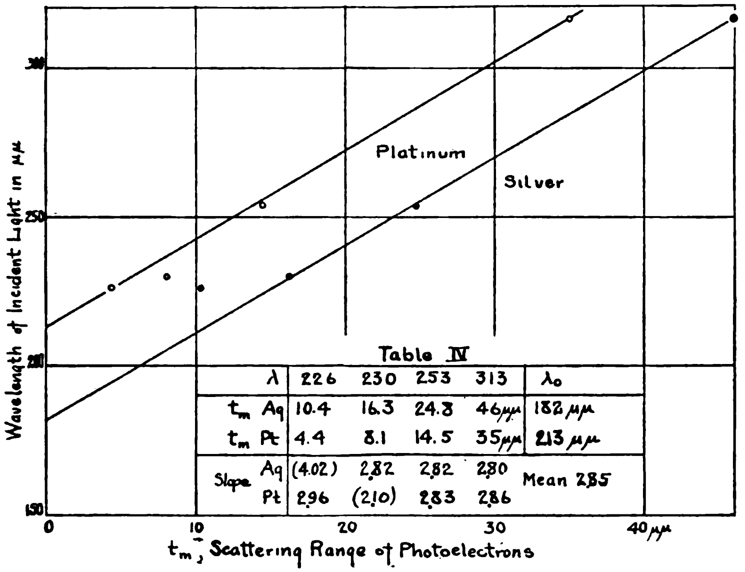

Table IV

	λ	226	230	253	313	λ₀
t_m	Ag	10.4	16.3	24.8	46μμ	182μμ
t_m	Pt	4.4	8.1	14.5	35μμ	213μμ
Slope	Ag	(4.02)	2.82	2.82	2.80	Mean 2.85
	Pt	2.96	(2.10)	2.83	2.86	

Fig. 8.

which an electron, starting from the lowest layer with initial energy $\frac{1}{2}mv^2 = h\nu$, can penetrate, before it disappears through absorption. The distance t_m is here called the scattering range. It does not measure the actual distance pursued by an electron in the form of the summation of all the zigzag paths, but the distance perpendicular to the surface of the metal. The values obtained in this way are plotted as a function of the wave-length of light causing the separation of the electron from its parent atom. The slope of the curve is the same for the two metals, the bracketed values in the table are however marked exceptions to this rule. The cause for this unusual difference cannot be accounted for through experimental error. It seems likely however that the straight line relation is perhaps a first approximation to the truth and more numerous data determined for longer and shorter wave-lengths than here investigated will determine to a more exact degree the necessary relations.

The present curves however fit the relation $\lambda - \lambda_0 = at_m$ to a marked degree. Here a is the slope of the curve and equal to 2.85, λ_0 is the value of the minimum wave-length producing electrons possessing an effective scattering range equal to zero. This would mean that λ_0 would be the minimum wave-length limit which could produce a photoelectric current under the above experimental conditions. Higher frequences might liberate electrons, but their energies would be so great that they could penetrate the next atoms in their path and completely disappear in their system. These electrons would be absorbed so promptly that we could expect a metal to possess the property of selective absorption for electrons possessing these critical velocities. The minimum wave-lengths of the incident light energy producing electrons under these conditions are 213 and 182 $\mu\mu$ for platinum and silver respectively.

TABLE V.

	Velocity $\times 10^{-7}$ Cm. per Sec.				Mean Velocity.	λ_0.
λ in $\mu\mu$.	226.	230.	253.	313.		
Silver	14.5	14.0	13.3	13.5	13.8×10^7	182 $\mu\mu$
Platinum	14.1	13.8	13.2	13.4(?)	13.6×10^7	213 $\mu\mu$
$v = \sqrt{\frac{2h\nu}{m}}$	14.05	13.95	13.25	11.92	$\times 10^7$ cm/sec	

If however the above wave-length relation is reduced to its equivalent energy form we have

$$\frac{1}{v^2} - \frac{1}{v_0^2} = bt_m, \tag{11}$$

where $b = 1.42m/hc$, assuming the relation $\frac{1}{2}mv^2 = h\nu$ to hold in the above case. We define m as the mass of the electron h as Planck's element of action and c as the velocity of light. Under these conditions v would be the velocity the electron must possess, to be able to penetrate a thickness t_m. Table V. shows the initial velocities the electrons should have, to penetrate thickness t_m, as computed from equation (11) and the velocities computed from the Einstein photoelectric equation. The contact difference of potential is here assumed equal to zero, since t_m is defined as the distance the electron travels in the metal without passing through the contact difference of potential layer. This use of the value of t_m is however not rigidly true but owing to lack of further information it is offered as an approximate solution. We see in Table V. to what extent the values for the computed velocities check the experimental values. In all cases the velocity decreasing with increased wave-length of the exciting energy.

Now $1/t_m$ may be defined as the stopping power. So that the stopping power of the metal increases as the energy of the electron increases, to eventually approach a limiting value. Comparing these values we note that the heavier the atom the greater the stopping power. If we therefore compare the distance traveled in platinum to that in silver for electrons of equal initial velocity we find that the distance traveled in silver is 1.17 as great as that in platinum when the energy in each is reduced to the same amount. Hence if we compare equal volumes of the material traversed we find that the ratio of the atomic volumes of silver to platinum is 1.13 or of the same magnitude as the relative distances traveled in the two metals. So that electrons after traversing equal masses have their initial energies reduced to the same extent.

ABSORPTION COEFFICIENTS.

After passing through a thickness t_m the photoelectric thickness curve falls off slowly at first, then nearly linear and finally more slowly again

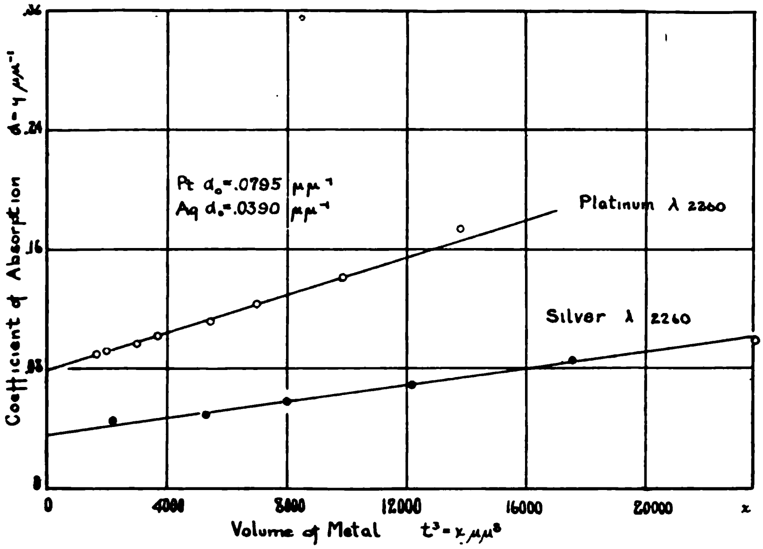

Fig. 9.

cutting the thickness axis at what appears to be a finite value. Unfortunately the latter part of the curve was very difficult to determine owing to the very minute number of electrons coming through such thick metallic layers.

The initial drop of the curve seems to indicate that there is little absorption of electron numbers as yet and that scattering is still predominant. From here on however the curve takes a very definite form. The problem is a very complex one, the steam of photoelectrons which are traveling in any given direction must suffer loss in numbers both through true absorption and scattering to such a degree as to completely disappear from the original direction and in doing so must lose energy as well. What the relative importance of these actions is in the present case is difficult to tell. A partial solution is however attempted.

The coefficients of absorption α were computed in the usual way for the absorption branch of the curve. These results were then plotted as ordinates and values of t, as abscissæ. The curve approximated nearest to a straight line when values of t^3 were used as shown in Fig. 9. Thus the relation $\alpha - \alpha_0 = mt^3$ would represent the relations to a comparative degree of accuracy limited by the experimental errors in the original observations. The values computed for α and values for m, the slope of the curves, are given for comparison, to show to what degree of accuracy the results may be valid.

TABLE VI.

Platinum, λ 226.			**Silver, λ 226.**		
α.	t.	m.	α.	t.	m.
.167 $\mu\mu^{-1}$cm^{-1}	24.0 $\mu\mu$	63.0 $\mu\mu^{-3}$cm^{-3}	.099 $\mu\mu^{-1}$cm^{-1}	29.5 $\mu\mu$	23.4 $\mu\mu^{-3}$cm^{-3}
.141	21.5	62.8	.083	26.0	25.0
.123	19.2	62.3	.070	23.0	25.4
.112	17.5	61.6	.059	20.0	25.0
.102	15.5	61.9	.050	17.5	20.5
.0972	14.5	60.0	.043	12.8	20.0
.0917	12.6	61.2			
.0907	11.8	68.0			
	Mean	62.6 $\mu\mu^{-3}$cm^{-3}		Mean	23.2 $\mu\mu^{-3}$cm^{-3}

The coefficients of absorption thus obtained are not constant, but increase very rapidly with increasing values of thickness traversed. The intercepts on the y axis are however of interest. They would represent the absorption coefficients for values of thickness commensurable to a molecular layer. These values of α_0 for platinum and silver were found to be .0795 and .0390 $\mu\mu^{-1}$ respectively for photoelectrons emitted with a velocity of 13×10^7 cm. per sec. from their parent atoms. If we divide the above values by the density of the respective metals to which they apply we arrive at the following interesting relation

$$\left(\frac{\alpha_0}{\rho}\right)_{\mathrm{Pt}} = 37{,}000\ \mathrm{cm}^2\mathrm{gm}^{-1},$$

$$\left(\frac{\alpha_0}{\rho}\right)_{\mathrm{Ag}} = 37{,}100\ \mathrm{cm}^2\mathrm{gm}^{-1},$$

showing the mass absorption to be constant for λ 226. This is the "density law" of absorption at first suggested by Lenard.[1] Crowther,[2] experimenting with β rays from uranium showed that the "density law" did not hold for these larger velocities. Lenard experimenting with heterogeneous cathode rays arrived at the following values for the metals gold and silver here given for comparison.

$$\frac{\alpha_0}{\rho} = 2880\ \mathrm{cm}^2\mathrm{gm}^{-1},$$

$$\frac{\alpha_0}{\rho} = 3070\ \mathrm{cm}^2\mathrm{gm}^{-1}.$$

Compton and Ross working with sputtered platinum obtained from photoelectric measurements the average value $\alpha/\rho = 28{,}900$. While Partzsch and Hallwachs (*l. c.*), using some data supplied by Rubens and Ladenburg, obtained the value $\alpha/\rho = 47{,}400$ for gold in the form of a leaf of about 100 $\mu\mu$ thickness.

The exigencies of the times have interrupted the completion of this subject at this time, it is hoped however that in the near future further data can be published which will support the above conclusions. The experimental part of this paper was performed at the University of Pennsylvania and I take great pleasure in acknowledging the ever generous coöperation of Prof. Goodspeed in furnishing the funds for the necessary apparatus.

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[1] Lenard, *l. c.*

[2] Crowther, Phil. Mag., 12, p. 379, 1906.

The Physics Laboratory,
State University of Iowa,
Iowa City, Iowa.

WALLACE CLEMENT SABINE.

Wallace Clement Sabine, Hollis Professor of Mathematics and Natural Philosophy at Harvard University and formerly dean of the Harvard Graduate Schools of Applied Science, Vice-President of the American Physical Society and recent member of the editorial board of the Physical Review, died at Boston on the tenth day of January, 1919, in his fifty-first year.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE PITTSBURGH MEETING, December 27–29, 1917.

THE ninety-second meeting of the American Physical Society (the nineteenth Annual meeting) was held in the Applied Science Building of the Carnegie Institute of Technology, December 27 to 29, 1917. This was a joint meeting with Section B (Physics) of the American Association for the Advancement of Science. Two sessions were held on each of the three days of the meeting, Thursday, Friday, and Saturday.

On the first two days the following program of research papers was presented. This program was in charge of the officers of the Physical Society.

The Optical Properties of Rubidium. J. B. NATHANSON.

A Preliminary Study of the Luminescence of the Uranyl Salts Under Cathode Ray Excitation. FRANCES G. WICK and LOUISE S. McDOWELL.

Note on a Phosphorescent Calcite. E. L. NICHOLS and H. L. HOWES.

The Visibility of Radiation in the Blue End of the Visible Spectrum. L. W. HARTMAN.

An Improved Form of Mercury Vapor Air Pump. CHAS. T. KNIPP.

Heat Conductivity of Cerium. C. N. WENRICH and G. G. BECKNELL.

Temperature and Heat of Fusion. J. E. SIEBEL. (By title.)

Report on the Construction of Certain Mathematical Tables. C. E. VAN ORSTRAND.

Mobilities of Ions in Vapors. KIA-LOK YEN.

The Size and Shape of the Electron. ARTHUR H. COMPTON.

The Coefficient of Emission and Absorption of Photo-electrons from Platinum and Silver. OTTO STUHLMANN, JR.

Ionization and Excitation of Radiation by Electron Impact in Nitrogen. BERGEN DAVIS and F. S. GOUCHER.

Energy in Continuous X-Ray Spectra. C. T. ULREY.

An Experimental Investigation of the Characteristic X-Ray Emission from Molybdenum and Palladium. BENJAMIN A. WOOTEN.

Characteristic X-Ray Emission as a Function of the Applied Voltage. BERGEN DAVIS.

A Standard of Sound. (Demonstration.) CHAS. T. KNIPP.

The Air Damped Vibrating System: Theoretical Calibration of the Condenser Transmitter. I. B. CRANDALL.

A Photographic Method of Measuring the Velocity of Sound Waves near the Source of Sound. ARTHUR L. FOLEY. (By title.)

The Effect of Intensity and Distance on the Velocity of Sound. ARTHUR L. FOLEY. (By title.)

The Influence of the Pressure and Time Employed in Condensing a Dental Amalgam upon its Crushing Strength, at Temperatures Between 10° and 100° C. ARTHUR W. GRAY and PARIS T. CARLISLE.

Absorption in Paraffined Paper Condensers. L. PYLE.

An Interesting Case of Resonance in an Alternating Current Circuit. H. K. DODGE.

On Electromagnetic Induction and Relative Motion, II. S. J. BARNETT.

Eddy-current and Hysteresis Losses in Iron at High Frequencies. C. NUSBAUM.

Some Energy Transformations with Oscillatory Currents. E. F. NORTHRUP.

The Effects produced upon Audion Characteristic Curves by Different Kinds of Signals (Buzzer, Electron Relay and 60-cycle A.C.) A. D. COLE.

Influence of a Series Spark on the Direct Current Corona. S. F. CROOKER. (By title.)

On the Effect of a Magnetic Field upon Cathode Rays. L. T. MORE and LOWELL M. ALEXANDER. (By title.)

The Physical Characteristics of X-Ray Fluorescent Intensifying Screens. MILLARD B. HODGSON.

Barometric Ripples. W. J. HUMPHREYS.

The Ultra-Violet and Visible Absorption Spectra of Phenolphthaleins. W. E. HOWE and K. S. GIBSON.

The Ultra-violet and Visible Absorption Spectra of Orcinolphthaleins. R. C. GIBBS, H. E. HOWE and E. P. T. TYNDALL.

The Ultra-Violet Absorption Spectra of Acetone. E. P. T. TYNDALL.

Thermal Conductivity of Metals. EDWIN H. HALL. (By title.)

A Mercury Manometer of High Sensibility. J. E. SHRADER.

A Simple Gauge for Very Low Pressures. J. E. SHRADER.

Resonance and Ionization Potentials for Electrons in Magnesium Vapor. PAUL D. FOOTE and F. L. MOHLER.

The Spectral Photoelectric Sensitivity of Molybdenite. W. W. COBLENTZ and M. B. LONG.

The program on Saturday was in charge of Section B, A.A.A.S. In the forenoon there was a symposium on "The Relationship of Physics to the War." Among the speakers were Lieut. G. P. Thomson, R.F.C. (son of Sir Joseph J. Thomson), Lieut. Giorgio Abeti of the Italian Mission to the United States, Dr. A. L. Day, Director of the Geophysical Laboratory, Washington, who gave an interesting account of the present optical-glass situation in this country and Major C. E. Mendenhall, who discussed the recent war activities of the National Research Council.

In the afternoon Professor Henry A. Bumstead delivered an address as retiring Vice-president of Section B on "Present Tendencies in Theoretical Physics" (published in Science, January 18, 1918).

At a short business session of the Physical Society the result of the mail ballot for officers of the society for 1918 was announced. They are as follows: H. A. Bumstead, president; J. S. Ames, vice-president; D. C. Miller, secretary, and G. B. Pegram, treasurer. A. L. Day and G. F. Hull are the new members of the Council. (Former elected members holding over are C. E. Mendenhall, G. W. Stewart, I. Langmuir, G. B. Pegram, G. O. Squier and H. A. Wilson.) G. K. Burgess, A. C. Lunn and A. D. Cole are the newly elected members of the Editorial Board of the PHYSICAL REVIEW. (Former members of the board holding over are F. Bedell, Managing Editor, and E. P. Lewis, W. C. Sabine, A. Trowbridge, N. E. Dorsey, Wm. Duane and O. M. Stewart.)

The reports of the treasurer and managing editor were presented, accepted, and ordered printed. (These printed reports were afterward mailed to all the membership.)

The visiting members of the Society were much indebted to Dean F. L. Bishop of the University of Pittsburgh and to Prof. H. S. Hower, of the Carnegie Institute of Technology, and to other resident members for many courtesies extended to them.

ALFRED D. COLE,
Secretary for 1917.

MINUTES OF THE BALTIMORE MEETING, DECEMBER 27 AND 28, 1918.

THE ninety-fifth meeting (the twentieth annual meeting) of the American Physical Society was held at Johns Hopkins University, in Baltimore, Maryland, on December 27 and 28, 1918, in affiliation with Section B—Physics of the American Association for the Advancement of Science. Professor Bumstead is now serving as Scientific Attaché to the American Embassy in London and his resignation as president of the society was accepted by the Council on November 30, 1918. The vice-president, J. S. Ames, thus became acting president, and he presided at the several sessions of the society and the council. The maximum attendance at the technical meetings was about one hundred, while eighty-eight members and visitors were present at the time of the business session.

On the afternoon of December 27 there were two sessions under the auspices of Section B, the presiding officer being the vice-president and chairman of the section, Major G. F. Hull. At two o'clock, p.m., the retiring vice-president and chairman W. J. Humphreys, gave an address on "Some Recent Contributions to the Physics of the Air." At five o'clock, p.m., Dr. George E. Hale gave an address before the entire Association on "The National Research Council."

The annual business meeting was held at eleven o'clock, a.m., on December 28, 1918. The revised form of the Constitution and By-Laws was unanimously adopted by letter ballot. The amendments do not alter the intent or purpose of the constitution in the old form, except in one respect; the managing editor is made a member, *ex-officio*, of the council. The amended constitution will be published in the next printed list of members.

The following officers were elected for the year 1919: For President, J. S. Ames; for Vice-President, W. C. Sabine; for Secretary, D. C. Miller; for Treasurer, G. B. Pegram; for Members of the Council (four-year term), G. K. Burgess, J. C. McLennan; for Member of the Council (one-year, unexpired term), Max Mason; for members of the Board of Editors of the PHYSICAL REVIEW, Henry Crew, L. V. King, H. S. Uhler.

Colonel Millikan explained the purposes of the Smith-Howard Bill now before Congress, authorizing Federal coöperation with the States for the promotion of engineering and industrial research. After general discussion it was unanimously voted that the American Physical Society favors federal aid and coöperation with the several States in support of research in science and engineering and in industrial research. The society favors the creation of Boards of eminent scientists and engineers within each state for the administration of the funds appropriated for all research within the state.

The reports of the treasurer and of the managing editor of the PHYSICAL REVIEW were presented at the meeting of the council. It was ordered that, after auditing, these reports be printed and sent to all members.

The secretary reported that within the year sixty-two persons had been elected to associate membership and eight elected to regular membership. Eighteen associate members had been transferred to regular membership. There was one resignation from membership. Two members had died within the year, C. C. Trowbridge, and Ernest Weibel, the latter having been killed in action in France. The total membership now is 935, there being 402 regular members.

As a matter of record it is noted that, because of conditions growing out of the war, three meetings, which, following custom, would have been held in the year 1918, were cancelled. These meetings were the usual Washington meeting of April, a special joint meeting with the American Institute of Electrical Engineers which had been arranged for October 11 and 12, and the Chicago meeting of November 30. A special meeting of the council was held in Baltimore on November 30, 1918, to transact necessary business.

At a meeting of the Council held in Baltimore on November 30, 1918, the following elections were made: *elected to regular membership*, Shoji Nishikawa; *elected to associate membership*, W. F. Angell, H. C. Arnold, J. W. Cook, F. A. Ferguson, O. F. Gish, Peter Hidnert, Samuel C. Hoffman, Frank C. Hoyt, I. L. Jones, Lewis V. Judson, Herbert Kahler, Charles B. Kazda, Otto Koppius, M. B. Long, John P. Minton, Newell C. Page, Herbert H. Palmer, Arthur R. Payne, C. E. Pierce, Anthony S. Santos, C. D. Shallenberger, Joseph Valasek, Hugo P. Wahlin, Dorothy W. Weeks, W. Wubbe.

At a meeting of the Council held in Baltimore on December 27, 1918, the following elections were made: *elected to regular membership*, M. D. Hershey,

J. C. Hunsaker, Vladimir Karapetoff, Paul D. Merica, John M. Miller, H. B. Phillips; *elected to associate membership*, R. C. Colwell, Wm. B. Creagmile, G. A. Dasney, A. E. F. Fant, E. O. Hoffman, J. J. Hopfield, F. L. Hunt, George Lewis, H. J. Lockwood, C. W. D. Parsons, D. E. Sharp, Margery Simpson, Marguerite D. Tschaler, I. H. Van Horn, F. C. Wente, C. F. Whittemore, D. T. Wilber; *transferred from associate to regular membership*, E. C. Crittenden, A. T. Jones, E. F. Kingsbury, Louise S. McDowell, Wm. F. Meggers, A. Norman Shaw, H. B. Williams.

At the morning sessions of December 27 and 28, 1918, thirty papers were presented as follows, four being read by title:

The Unique System of Units. W. W. STRONG.

A Simple Stretched Wire Dilatometer. ARTHUR W. GRAY.

Monochromatic and Neutral Tint Screens in Optical Pyrometry. W. E. FORSYTHE.

The Temperature, Pressure, and Density of the Atmosphere in the Region of Northern France. W. J. HUMPHREYS.

Refinements in Spherometry. G. W. MOFFITT.

A New Type of Hot Wire Anemometer. T. S. TAYLOR.

The Linear Thermal Expansion of Glass at High Temperatures. C. G. PETERS.

Some Characteristics of Glasses in the Annealing Range. A. Q. TOOL and J. VALASEK.

Striæ in Optical Glass. L. E. DODD and A. R. PAYNE.

Preliminary Determination of the Thermal Expansion of Molybdenum. LLOYD W. SCHAD and PETER HIDNERT.

On the Characteristics of Electrically Operated Tuning Forks. H. M. DADOURIAN. (Read by title.)

Ionization and Resonance Potentials for Electrons in Vapors of Arsenic, Rubidium, and Caesium. PAUL D. FOOTE, O. ROGNLEY and F. L. MOHLER.

Absorption Coefficient of the Penetrating Radiation. OLIVER H. GISH.

Photoelectric Sensitivity vs. Current Rectification in Molybdenite. W. W. COBLENTZ and LOUISE S. MCDOWELL.

A Device for the Automatic Registration of the α- and β-Particles and γ-Ray Pulses. ALOIS F. KOVARIK.

Note on the Distribution of Energy in the Visible Spectrum of a Cylindrical Acetylene Flame. EDW. P. HYDE, W. E. FORSYTHE and F. E. CADY.

Preliminary Note on the Luminescence of the Rare Earths. E. L. NICHOLS, D. T. WILBER and F. G. WICK.

On the Critical Absorption Frequencies of Chemical Elements of High Atomic Numbers. WILLIAM DUANE and TAKIO SHIMIZU.

Some Interesting Results of Eclipse Magnetic Observations. L. A. BAUER.

The Minimum Temperature at the Base of the Stratosphere. W. J. HUMPHREYS. (Read by title.)

Why Clouds Never Form in the Stratosphere. W. J. HUMPHREYS. (Read by title.)

Speeds in Signaling by the Use of Light. W. E. FORSYTHE.

Thermal Conductivity of Various Materials. T. S. TAYLOR.

Further Observations on the Production of Metallic Spectra by Cathode Luminescence. EDNA CARTER and ARTHUR S. KING.

Effect of Crystal Structure Upon Photoelectric Sensitivity. W. W. COBLENTZ. (Read by title.)

A Mechanically Blown Wind Instrument. A. G. WEBSTER.

The Dynamics of the Rifle Fired at the Shoulder. A. G. WEBSTER.

Interior Ballistics, by a New Gun Indicator. A. G. WEBSTER.

Residual Gases in Highly Exhausted Glass Bulbs. J. F. SHRADER.

Silvering Quartz Fibers by Cathodic Sputtering. J. F. SHRADER.

DAYTON C. MILLER,
Secretary.

THE SPECTRAL PHOTOELECTRIC SENSITIVITY OF MOLYBDENITE.[1]

BY W. W. COBLENTZ AND M. B. LONG.

THE samples of molybdenite MoS_2, examined were thin folia, 0.01 to 0.05 mm. in thickness, soldered to copper electrodes.

In this examination, which is of a preliminary nature, the sample of molybdenite was exposed to radiations of equal energy (measured with a thermopile) of different narrow spectral regions, isolated by means of screens. These bands of spectral radiation, having their maximum intensity at wave-lengths λ_m = 0.5, 0.55, 0.6, 0.75, 0.9, 2.4, 4.4 and 8.5 μ respectively, were obtained by transmission of the radiations from a Nernst glower through colored glasses, by the emission from the Bunsen gas flame and by utilizing residual radiations reflected from quartz.

The results obtained indicate that molybdenite, like selenium and stibnite, has its maximum photoelectric sensitivity just beyond the visual red, 0.85 μ; but it differs from these substances in that it is quite sensitive to infra-red rays. For example, at 2.0 μ the photoelectric sensitivity is perhaps from one third to one half that of the maximum, and at 4.4 μ (Bunsen flame which has weak emission bands at 1.8 and 2.7 μ) it is about one tenth the maximum.

The temperature coefficient of resistance of molybdenite is negative; and hence a rise in temperature produces a change in resistance which is in the same direction as that caused by the photoelectric effect. It will therefore be necessary to determine the magnitude of these two effects in order to establish a true photoelectric effect for wave-lengths greater than 4.4 μ. These various factors including reflection, etc., will be considered in the complete paper.

WASHINGTON, D. C.
December 22, 1917.

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[1] Abstract of a paper presented at the Pittsburgh meeting of the American Physical Society, December 27–29, 1917.

THE UNIQUE SYSTEM OF UNITS.[1]

BY W. W. STRONG.

SOME thirty years ago it would have almost been universally admitted by physicists and chemists that all masses were integral multiples of the masses of the atoms constituting the given mass. The masses of the atoms were universal invariants and an atomic system of units of this kind has been proposed. The effort of Prout was that of making a unique and universal atomic unit of mass that could be used wherever matter is present.

Today it is almost universally assumed that the charge of the electron is a unit of electrical charge in much the same way as the atomic unit of mass in the system of Prout. The variable mass of the electron as its velocity changes indicates that if there is an invariable element of mass it is much smaller than Prout imagined and that it cannot be larger than that quantum of mass that appears when the electron is accelerated. Let us assume that there are such invariant quanta of electrical charge and mass. If some five such invariant quantities as we may consider as applying to electrical charge, mass quanta and radiant energy (with a supposed constancy of the velocity of light) were consistent with phenomena we would be approaching a unique set of units. Let us then state some of the properties of such an ideal system of units.

1. There are in nature certain invariant atomic quantities that serve as the basis for a unique and a universal system of units that is absolute in reality. At the present time the electron (unit) is usually made to play such a rôle.

2. The science of measurement is not then based upon a complicated and an artificial process of creating and comparing standards and in defining terms such as the coincidence of two events or of comparing two lengths in relative motion but upon a unique set of natural and invariant units that are universally available.

3. If there is an ether it appears logical to assume that this set of unique units should be provided for us by the structure of the ether or at least as many of them as are necessary to fully describe all phenomena relating to the ether, the electric, magnetic and gravitational fields. Derived units such as T and L should be based on an ether structure since these are used in treating the propagation of light.

4. The unique set of units are made so by all phenomena being consistent with the assumption of their being unique and hence their use is supported by all scientific knowledge. If all electrons are alike there is no necessity of defining equal charges of electricity as is done in connection with the derivation of the electrostatic system of units. If there is an elementary quantum of mass there is no necessity to consider the matter of relative motion in the definition of mass. How can a consistent system of units be obtained by assuming the Newtonian mechanics in the definition of our units and the de-

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

scription of phenomena and then trying to build a relativity theory on another system of mechanics and at the same time using the old system of units?

5. The derived units should be uniquely determined by the fundamental or natural unique units.

6. It hardly seems possible to develop a consistent system of physics without at first determining a system of units that is universally applicable to all phenomena to be investigated.

S. I. E. M. Co.,
MECHANICSBURG, PA.

A SIMPLE STRETCHED WIRE DILATOMETER.[1]

BY ARTHUR W. GRAY.

At the request of Dr. Arthur L. Day, Optical Glass Committee, War Industries Board, the writer undertook to determine the thermal expansivity of some optical glass which had been produced under the direction of the former for naval searchlight mirrors. The decision as to whether or not the glass was suitable rested exclusively upon its thermal expansivity, a knowledge of which was desired up to about 200° C. With the dilatometer which had to be hastily constructed for the purpose, expansions were, however, measured at various temperatures up to 300° C.

The apparatus, which uses the stretched wire method of measuring linear displacements,[2] could easily be adapted for commercial testing. It differs from the precision apparatus developed by the writer at the Bureau of Standards, mainly in the substitution of an ordinary Brown and Sharpe micrometer screw for the expensive microscope comparator and reference bar.

Extending vertically downwards from the cover of the temperature bath are two tubes, the axes of which are separated by approximately the length of the specimen under investigation. At the lower end of each tube are two pointed co-axial screws for clamping the specimen from opposite sides. One of the tubes is rigidly attached to the cover, the other is hinged so as to permit free expansion of the specimen. A fine wire, of annealed tungsten, attached to the lower end of each tube extends vertically upwards through the tube and the cover for several decimeters above the latter, where it passes over a pulley and supports a weight which keeps it tight and straight. The wire bears firmly against the rounded end of the specimen and likewise against a round pin situated a decimeter or so below the pulley. By means of a micrometer screw and suitable guides this pin is movable horizontally in the plane of the two wires, so that their upper ends can be brought closer together or separated a measured distance. Just above the cover of the temperature bath is placed a horizontal bar of low and known thermal expansivity, protected as far as possible from temperature changes accompanying warming and cooling of the bath below.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

[2] Expansion measurements by this method were reported to the Physical Society at the Washington meeting in December, 1911, and at various times since then. The principle was also described in a communication to the Washington Academy of Sciences, in which the two arrangements that form the basis of the Bureau of Standards' present expansivity equipment were illustrated diagrammatically. (A. W. Gray, "New Methods for Displacement Measurements and Temperature Uniformity Applied to the Determination of Linear Expansivity," Journ. Wash. Acad. Sc., 2, 248–258, 1912.) One arrangement is used with electric furnaces, the other with liquid baths.

After the specimen is placed in the temperature bath, the upper end of each stretched wire is moved until the wire just touches the end of the bar above the cover. Contact is indicated electrically by means of an inexpensive reflecting galvanometer in series with a dry cell and a suitable resistance. As the specimen under investigation expands, it moves the wires away from the contact bar. The magnitude of the expansion is determined from the movements of the micrometer screws required to restore contact.

Only one graduated screw is actually used. An ordinary screw of moderately fine pitch moves a horizontal bar to which are rigidly attached both the pin that communicates the motion to the left-hand wire *A* and the nut of the graduated screw which moves the right-hand wire *B*. In this way a single micrometer screw is made to measure the total displacement of both wires.

The accuracy of the apparatus just described is limited mainly by the accuracy with which contact can be detected between the stretched wires and the bar above the temperature bath. The effect of errors in the measuring screw varies inversely as the ratio of the distances from the contact bar to the pins for moving the wires and to the specimen. In the apparatus actually used these pins were 600 mm. above and the specimen 150 mm. below the bar, so that the micrometer screw measured four times the expansion of the specimen. The results showed that with proper use deviations of individual screw readings from the average should not exceed 5 microns. The average deviation of 107 individual readings from the average reading for each temperature was less than 2 microns, corresponding to an average deviation of 0.5 micron in determining the expansion of the specimen.

The apparatus actually constructed as outlined above illustrates merely one particular arrangement for applying the general principles of the method. It was hastily designed and built to meet an emergency. Various details could obviously be modified. For example, two independent graduated screws for moving the stretched wires *A* and *B* would be a convenience; any sensitive detector of contact is usable; tungsten wires are not necessary, etc. Moreover, micrometer screws can be arranged to replace microscopes for use with either an air-bath (as in an electric furnace) or a liquid bath; so that the range of application is not limited to the interval between room temperature and 300° C.

Physical Research Laboratory,
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Milford, Delaware,
Nov. 29, 1918.

MONOCHROMATIC AND NEUTRAL TINT SCREENS IN OPTICAL PYROMETRY.[1]

BY W. E. FORSYTHE.

IT is very difficult to find a monochromatic or a neutral tint glass screen for optical pyrometry. Very fortunately absolutely monochromatic or absolutely neutral tint screens are not necessary for such work. For the so-called monochromatic screen what is wanted, for the most part, is a screen that is sufficiently monochromatic to enable comparisons in brightness to be made between the comparison source, or pyrometer filament for the Morse pyrometer, and the source that is being investigated.

When it is necessary to use glass absorbing screens to reduce the apparent brightness of the source studied, the main requirement is to have a screen that is nearly enough neutral tint to permit comparisons in brightness to be made. The degree to which it is necessary for the absorbing screen to have a spectral transmission independent of the wave-length, depends upon the so-called monochromatic glass used. It is quite evident that if the colored glass used is absolutely monochromatic, any absorbing glass would answer.

With two pieces (thickness about 6 mm.) of Jena red glass no. 4512 in the eyepiece of the pyrometer the total transmission, which amounted to about 1.5 per cent., of a piece of noviweld absorbing glass was measured for a particular temperature by two observers whose visibility curves are quite different and the two results agreed to within a fraction of a per cent. If a red glass is used, by total transmission is meant the ratio of the brightness observed through the red glass and the black glass to the brightness observed through the red glass alone. If the entire visible spectrum is used, it is generally very hard to make such measurements owing to the color differences involved with even the best absorbing glasses. If, however, a good red glass is used in the eyepiece, transmission measurements can easily be made.

The transmission of the absorbing glass when used with a red glass can be calculated for any black body distribution by the following formula taken from Preston's theory of light:

$$T_B = \frac{\int J_\lambda V_\lambda T'_R T'_B d\lambda}{\int J_\lambda V_\lambda T'_R d\lambda}. \tag{1}$$

$J_\lambda d\lambda$ = black body energy for interval λ to $\lambda + d\lambda$, V_λ = visibility, T_R' and T_B' = spectral transmission of red and absorbing glasses respectively. An absorbing glass that is not neutral tint is sometimes used to cut down the apparent brightness of a source when measuring the temperature with an optical pyrometer having a red glass in the eyepiece. It has been stated[2] that if an absorbing glass which is not neutral tint is used in connection with the red glass, the effective wave-length to be used is different from the effective wave-length that is to be used with a sector. In what follows it is shown that such is not the case but that the same effective wave-length is to be used in each case. Suppose that a sector with a transmission T_S were found such that the brightness observed through the sector would equal that observed through the black glass using the same red glass in both cases, that is

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

[2] Foote, Bull. Bureau of Standards, Vol. 12, p. 491.

$$\int J_\lambda V_\lambda T'_B T'_R d\lambda = \int J_\lambda V_\lambda T'_R T_S d\lambda = T_S \int J_\lambda V_\lambda T'_R d\lambda.$$

The question to be considered is what effective wave-length is to be used in calculating the temperature of the source whose brightness is thus measured. When the brightness is measured using the rotating sector the temperature T_2 is calculated from T_1 the temperature corresponding to the pyrometer reading if no sector is used by means of the formula

$$1/T_2 - 1/T_1 = \frac{\lambda_e \log T_S}{c_2 \log e}, \tag{2}$$

when T_S = transmission of sector λ_e the ordinary effective wave-length between T_1 and T_2. The effective wave-length has been defined as the wave-length such that

$$\left[\frac{J(\lambda, T_1)}{J(\lambda, T_2)}\right]_{\lambda e} = \frac{\int J(\lambda, T_1)\, V_\lambda T_R' d\lambda}{\int J(\lambda, T_2)\, V_\lambda T_R' T_S d\lambda}.$$

T_S = transmission of sector. Suppose that using the black glass screen before the pyrometer lamp the same current A through the pyrometer filament is required to cause an apparent brightness match between the same source at temperature T_2 and the pyrometer filament. This means that the sector and glass have the same transmission. The temperature T_2 can be calculated by an equation similar to equation (2).

$$1/T_2 - 1/T_1 = \frac{\lambda'_e \log T_B}{c_2 \log e}. \tag{3}$$

T_B = transmission of black glass screen and λ_e' an effective wave-length. As T_1 and T_2 are the same in each case

$$\lambda_e \log T_S = \lambda_e' \log T_B,$$

or

$$\lambda_e = \lambda_e',$$

that is, as the transmission of the black glass given by equation (1) is the same as would be obtained by comparing the transmission of the glass with a sector the same wave-length is to be used in each case.

Nela Research Laboratory,
National Lamp Works of General Electric Company,
Nela Park, Cleveland, Ohio,
December, 1918.

THE TEMPERATURE, PRESSURE AND DENSITY OF THE ATMOSPHERE IN THE REGION OF NORTHERN FRANCE.[1]

BY W. J. HUMPHREYS.

THE records of 416 sounding balloon flights made at Trappes, Uccle, Strassburg and Munich during the years 1900–1912 have been grouped in such manner as to give the average summer and winter distribution of temperature, humidity, pressure and density from the surface up to 20 kilometers above sea level. The temperatures have further been grouped according to weather conditions.

All this data is given in brief tables and shown in graphs.

A NEW TYPE OF HOT WIRE ANEMOMETER.[1]

BY T. S. TAYLOR.

IN all hot wire anemometors thus far used, the instruments have been so designed as to measure the average velocity over the area of the pipe or for a length of 10 cm. or greater at different points in the plane of cross section. None of these types is suitable for the measurement of velocity in small tubes. It was for the purpose of making these latter measurements, that the present instrument was constructed. It consists of a platinum heating wire 0.007 inch in diameter and one half to one inch long, stretched across a suitable framework, say of glass, and having a copper-constantan thermocouple made of .002 inch wire attached at its mid point. The instrument can be used satisfactorily by measuring the current that is necessary to maintain the temperature, as determined by the thermocouple, say 300° C., above the temperature of the gas in which it is placed. This method facilitates the measurements greatly, and avoids the possibility of accidentally burning out the wire. As for the other types of anemometers, it is necessary to calibrate it under the conditions under which it is to be used. This can be done by revolving it at various speeds in the gas whose velocity is to be measured at various temperatures. To do the latter it is necessary to have the revolving part of the calibration device contained in an oven whose temperature can be regulated quite accurately.

By using this method it is possible to measure quite accurately the velocity of gases at various temperatures, and has the great advantage of really measuring the velocity of gas at a point.

A second type, constructed after the usual one, has also been used in exactly the same manner as the above. It determines the average velocity over a length of from one to three millimeters. It is constructed by having two small .002-inch platinum wires attached about 2 millimeters apart near the middle of the .007-inch platinum heating wire. In calibrating and using it, the current is measured that is required to maintain a certain potential drop, say 60 millivolts between the two potential leads. A good portable potentiometer such as Leeds and Northrup's potentiometer indicator is suitable both for the temperature measurements in the first type and the potential drop in the second type. The current can be measured quite satisfactorily by use of a good ammeter of proper range. To be sure more accurate results can be obtained when the measurements of both current and potential or thermal electromotive force are made by means of a precision potentiometer.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

A decided advantage of the thermocouple type is that it enables one to determine the temperature of the gas directly, immediately before or after taking velocity measurements. This can be done also for the second type by the insertion of a separate thermocouple near the resistance wire or from the temperature coefficient of the platinum wire.

December 3, 1918.

The Linear Thermal Expansion of Glass at High Temperatures.[1]

By C. G. Peters.

THE thermal expansions of different kinds of optical glass, chemical glassware, and ordinary glass tubing, were measured by the Fizeau method. Rings cut from the samples were placed between two fused quartz interferometer plates and heated in an electric furnace from 20° to 650° C. Curves with degrees Centigrade plotted against increase in length show that the rate of expansion of each sample was nearly constant until the annealing temperature was reached. This annealing temperature varied with different glasses from 450° to 550° C. After the annealing temperature was passed the glass seemed to be in a slighty plastic condition and the rate of expansion increased by four to six times. This rapid expansion continued for about 100° when a softening point was reached. From the curves the temperature where careful annealing is required can be accurately located.

Bureau of Standards,
Washington, D. C.

Some Characteristics of Glasses in the Annealing Range.[1]

By A. Q. Tool and J. Valasek.

A STUDY has been made of the behavior of a number of glasses near their softening temperatures in order to obtain data that would serve to indicate the proper annealing process, as well as the maximum temperatures at which the various glasses should be annealed.

The " relaxation times " for the stresses were determined by two methods, the first being by the bending of a glass strip as described by Twyman and the second by observing the rate at which the double refracting power of a sample of strained glass disappeared. This relaxation time T, defined by the equation

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

$$\frac{dF}{dt} = E\frac{ds}{dt} - \frac{F}{T}$$

due to Maxwell, decreases with increasing temperature in such a way that

$$T_\theta = T_0 e^{(\theta_0 - \theta)/k}.$$

The magnitude of the constant k was found to depend on the nature of the glass. For optical glasses it was found to range from 10 to 13 for temperatures expressed in degrees Centigrade.

In the temperature range where the glass softens to such a degree that T becomes only a few tenths of an hour, it was found that heating curves, obtained by the use of a differential couple, showed a relative cooling of the glasses with respect to the neutral body. Cooling curves show a relative heating.

Measurements by C. G. Peters, of this bureau, have shown that in this same range there is an abnormal increase in the expansion coefficient. Consequently it is to be suspected that the softening of the glass, the increased expansion and this apparent cooling effect are closely related.

HIGH TEMPERATURE MEASUREMENT SECTION,
BUREAU OF STANDARDS,
WASHINGTON, D. C.

PRELIMINARY DETERMINATION OF THE THERMAL EXPANSION OF MOLYBDENUM.[1]

BY LLOYD W. SCHAD AND PETER HIDNERT.

THE thermal expansion of an exceptionally pure specimen of molybdenum was determined from −142° to +305° C.

A short description of the apparatus and of the method used in obtaining high and low temperatures is given.

The results are shown in the form of tables, from which were computed by the method of least squares, the two following empirical equations which satisfy the observations:

$$L_t = L_0 (1 + 5.15t \times 10^{-6} + 0.00570t^2 \times 10^{-6})$$

and

$$L_t = L_0 (1 + 5.01t \times 10^{-6} + 0.00138t^2 \times 10^{-6}),$$

where L_t is the length of the specimen at any temperature t within the proper range; in the first case 19° to −142° C. and in the second case 19° to +305° C. The probable error of the length computed from these equations is less than 3×10^{-6} per unit length.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918. Complete paper may be found in the Scientific Papers of the Bureau of Standards, No. 332.

Speeds in Signaling by the Use of Light.[1]

By W. E. Forsythe.

IN connection with some other work it was necessary to have some data concerning the maximum speed of light signaling as limited by the inertia of the eye and to compare this with the speeds attainable with various types of lamps.

A preliminary investigation was carried out to determine the relative lengths of dot, space and dash necessary for the greatest speed. Three observers made readings for this test and all agreed very well with the following ratio for the maximum speed:

dot: dash: space:: 1: 4: 3.

To test out the speed for a sharp cut off of the signals, using this ratio for dot, dash and space, a sector was made with which the following signals could be shown (space, of course, follows each dot or dash):

1—dot dash dot dash
2—dash dot dot dash
3—dash dot dot
4—dash dot dash dot
5—dash dash dot dot
6—dot dash dash dot
7—dot dash dash

This sector was mounted so that the observers saw the light source through a small opening. The observer was shown a series of twelve signals for each of a series of speeds beginning at a speed so low that there was no trouble in reading all the signals. This was repeated for higher and higher speeds until the observer was unable to read the signals.

To test out the speed where the heating and cooling of the different lamps was a factor, a commutator was constructed that would turn the lamps on and then off so that for any particular speed the voltage was applied to the lamp for the dot and dash the same length of time that the flashes of light were visible with the sector. Three observers made comparative readings on the signals as given by the sector and the commutator using a tungsten lamp with a ribbon filament mounted in argon. The results for each observer were about the same for either method.

As almost the same results were obtained from the lamp as from the sector this lamp, as a standard, was compared with a vacuum ribbon lamp and a gas-filled wire lamp. The lamps were so mounted as to have the same intensity in the direction of the observer stationed at a distance of 100 or 170 yards. Three observers took part in these tests and the results without exception were in favor of the gas-filled ribbon lamp.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

As all the above tests were carried out at a comparatively short distance it was thought well to make a test at a much larger distance. To do this the gas-filled ribbon lamp was mounted in a signaling unit which gave about a 5° spread and two observers made readings on signals sent as above at a distance of about 2,700 yards, both in the daytime and also at night. Assuming a breakdown at the speed at which two of the twelve signals shown were missed, one observer was able to read the signals on a clear day when the four-part signals were shown at such a speed that the time from the beginning to the end of the signal was about 1.6 seconds. This observer made about the same record at night. The other observer was able to read the signals on a clear day when the time from the beginning to the end of the signal was 2.2 seconds and at night his record was the same as that of the first observer.

A word might be said about how the signals appeared to the observer at the breakdown point. Each observer was instructed to record what he saw, not by code letter, but by actually writing down the dots and dashes. A study of these records shows that on clear days the breakdown seemed to be due to the observer's inability to see initial dots, finally seeing only the dashes. At night at the distance at which observations were made, the light was very intense and at the breakdown point the observers reported that they did not see the spaces between the light flashes. The light just seemed to flicker.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,
NELA PARK, CLEVELAND, OHIO,
December, 1918.

THERMAL CONDUCTIVITY OF VARIOUS MATERIALS.[1]

BY T. S. TAYLOR.

THE present investigation was undertaken to determine the thermal conductivity of various materials used in certain manufacturing processes. It was found that the direct method was by far the most satisfactory. This consisted of measuring the quantity of heat that passed through a given thickness of the material between two equi-temperature surfaces and the temperature drop through the sample. The hot equi-temperature surface or source consisted of a soapstone heater about 9 inches in diameter and 1/2 inch thick. It was composed of two discs each having a heating element wound in a spiral groove cut in its surface. The two discs were cemented together with faces having heating wire adjacent. Potential leads were brought out so that the potential drop could be determined for a determined area (about 4 sq. in.) surrounding the center of the heater. The cool equi-temperature surfaces were reservoirs through which water was kept circulating. The heat generated in the heater passed laterally through the samples to the cold reservoirs. The temperature drop through the samples was measured by means of small copper constant thermocouples inserted on each side of the sample. Extra turns of wire being wound around the outer edge of the heater made it possible to have the surface of the heater a very constant temperature source. Consequently from the amount of heat developed per unit area of the heater, the thickness of the sample and the temperature drop through the sample the thermal conductivity of the material is readily calculated.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

By this method the conductivity of a large number of materials has been determined. The following list contains some of the results.

Material.	Thickness, Inches.	Direction of Heat Flow.	Temp.	Cal./cm. °C./sec.	Temp. Coef.
Hard rubber	0.380	Transversely.	25–50	0.000380	.0015
White fiber	0.383	"	20–80	0.000695	.0012
Woods:					
White pine	0.519	Across grain.	20–120	0.000255	.0015
" "	0.732	Along grain.	30–80	0.000613	.0020
Maple	0.508	Across grain.	20–80	0.000434	.0006
"	0.785	Along grain.	20–80	0.001037	.0008
White oak	0.516	Across grain.	20–80	0.000455	.0018
" "	0.754	Along grain.	40–70	0.000944	.0006
Plate Glass	0.252	Transversely.	20–100	0.00195	.0018
" "	0.289	"	20–120	0.00201	.0012
Soapstone	0.715	"	70–130	0.00800	
Sheet steel .0172 inch	0.416	Transversely.	20–80	0.00158	.0025
Same painted with asphalt	0.425	"	20–80	0.00625	.0009
Wool felt dark gray	0.98	"	40–100	0.000175	.0076
Solid graphite	1.04	"	50–130	0.110	.0012
Powdered graphite:					
1. through 20- on to 40-mesh	0.476	"	40–100	0.00320	.0048
II. through 40-mesh screen	0.476	"	40–100	0.00100	.0040
III. through 100	0.476	"	40–110	0.000482	.0034
Lamp Black:					
Eagle brand, Germantown	0.476	"	40–150	0.000166	.0006
Sil-O-Cel Brick	0.977	"	30–150	0.000262	.0015

The values as recorded are given in each case for the average temperature over which the observations extended. Thus for white pine the average temperature extended from 20° C. to 120° C. The conductivity as recorded is therefore for 70° C. The temperature coefficient of thermal conductivity was determined from the observations over the range of temperature in question.

One fact that is noticeable in particular is the amount the transverse thermal conductivity of iron sheets can be increased by painting the component sheets when building up the sample.

FURTHER OBSERVATIONS ON THE PRODUCTION OF METALLIC SPECTRA BY CATHODE LUMINESCENCE.[1]

BY EDNA CARTER AND ARTHUR S. KING.

IN continuation of previous experiments, the spectra of titanium, manganese, iron, magnesium, calcium and cadmium have been examined The metal was vaporized by the impact of cathode rays in a high vacuum and the spectrum of the vapor in the cathode stream observed. The later apparatus was constructed largely of quartz in order to withstand the considerable heating of certain parts.

The experiments have shown the possibility of obtaining by this method fairly rich spectra even for metals of high melting point. Thus for titanium about 140 lines were identified, and 150 for manganese. In the detailed publication a comparison will be made with the spectra of other sources. In general, the majority of lines in the luminescence spectrum are those easily excited in the arc and furnace, but peculiarities appear which mark this as a source having special characteristics, and the action appears to be materially different for different vapors. For titanium the spectrum closely resembles that of the furnace, the enhanced lines being absent, while with cadmium, magnesium, and calcium there is a marked tendency to show enhanced lines. A relatively high intensity of lines in the ultra-violet was noted.

In the regular arrangement, the vapor whose spectrum was photographed was out of the path of the current, the anode being at one side of the chamber. When the anticathode was used as the anode, a much more intense spectrum was obtained, but for the elements tested, no decided difference in relative intensity of lines was observed. A discharge of this kind gave about 120 lines in the iron spectrum, the relative intensities among these lines showing many features of interest. Certain groups are strongly emitted, some of these being characteristic of low temperature conditions, while others require in other sources an intense excitation. In further work, the discharge conditions governing the appearance of certain classes of lines will be more fully tested.

VASSAR COLLEGE,
MOUNT WILSON SOLAR OBSERVATORY,
December, 1918.

RESIDUAL GASES IN HIGHLY EXHAUSTED GLASS BULBS.[1]

BY J. E. SHRADER.

THE investigation of high vacua has been extended by making observations on the effect of the sealing off of highly exhausted glass bulbs, and noting the subsequent change in pressure with time over an extended period.

In this work the mercury diffusion pump and the Knudsen Absolute gauge were employed. The glass ware was cleaned with potassium bichromate solution followed by thorough rinsing with hot water to remove soluble salts. Heat treatment of the bulbs and gauges in an electric oven at 500° C. for two hours sufficed to remove gases and vapors from the glass so that pressures of the order of 5×10^{-8} mm. Hg. could be obtained.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

On sealing off the bulbs and gauges by melting the glass at a constriction there is a comparatively large evolution of gases and vapors which can be partly removed by pumping during the time of sealing off, but the pressure in the bulbs is always four or five times as large as the pressure before sealing off. For several days after sealing off, there is a comparatively rapid increase in pressure, after which the change in pressure is slower and is proportional to the time.

After the exhausted bulbs had stood for about two months they were subjected to further heat treatment at successively higher temperatures and the change in pressure was noted after each heat treatment. Reheating at temperatures below 250° C. resulted in only slight increase in pressure while higher temperatures produced increasingly larger changes in pressure.

A Device for the Automatic Registration of the α- and β-Particles and γ-Ray Pulses.[1]

By Alois F. Kovarik.

THE detection of individual α- or β-particles has been in all cases dependent on the visual or audible method. These methods are quite trying on the nerves, because close attention is necessary on account of the probability law of emission and long periods of counting are essential because the method is a statistical one. The method described in this paper makes it possible to get a record of individual particles on a chronograph paper, permitting a study of it at convenience, and thus enhancing the accuracy of the statistical method. The method consists in amplifying the current to the needle-point electrode in the counting chamber by means of a three electrode vacuum tube amplifier and using this current to operate a very sensitive relay which in turn operates a local battery circuit which operates the electromagnet on a chronograph. The writer wishes to express his sincere thanks to the Western Electric Co. for the fine spirit of coöperation in making him a loan of both the sensitive relay and the amplifier.

The accuracy of the method was tested (1) for the α-particles, by checking the count by the scintillation method as in previous work, (2) for the β-particles, by determining their number by a measurement of the charge carried by the β-particles and (3) by getting a record of a large number of β-particles emitted over a long time and finding the number emitted in a given interval of time and checking up, or rather, in the case of the β-particles, proving the application of the law of probability to the emission of the β-particles. To the writer's knowledge, this is the first automatic record of any random distribution which could be readily applied to test the probability law.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

A record of the γ-rays is also obtained. The writer is not ready, at present to say that it is the γ-ray pulses which are registered although evidence seems to point that way. The law of inverse square distance (absorption in air for penetrating γ-rays neglected) is verified.

SHEFFIELD SCIENTIFIC SCHOOL,
YALE UNIVERSITY,
NEW HAVEN, CONN.,
November 23, 1918.

PHOTOELECTRIC SENSITIVITY *vs.* CURRENT RECTIFICATION IN MOLYBDENITE.[1]

BY W. W. COBLENTZ AND LOUISE S. McDOWELL.

IN the course of an investigation of the spectral photoelectric sensitivity of molybdenite[2] it was found that samples which are sensitive photoelectrically have a considerably higher electrical resistance (100 to 1,000 times greater) than samples which are not sensitive to light. Moreover, the conductivity of the insensitive samples was found to be quite independent of the direction in which the current passed (lengthwise) through the crystal.

On the other hand, samples of molybdenite which are sensitive photoelectrically were found to possess a much higher conductivity when the electric current (from a 4-volt battery) was passed in one direction than when it was passed in the opposite direction through the crystal. This difference in current leakage ("dark current") as dependent upon the direction of the current through the crystal, varied from 10 to 30 per cent., for different crystals. The photoelectric current was therefore proportionately increased by connecting the crystal into the battery circuit in the proper direction to obtain the maximum conductivity.

It was, therefore, of interest to determine whether any relation exists between photoelectric sensitivity and the rectifying action which occurs when the crystal is placed in a high frequency oscillating circuit.

For this purpose crystals (size 10 by 4 by 0.1 mm.) were selected which showed one or two small photoelectrically sensitive spots, but which were quite insensitive throughout the remainder (of the surface) of the crystal.

The rectification tests were made by means of an ordinary tuned buzzer circuit. The telephone and detector, in series, were placed in parallel with the condenser of the secondary circuit. The coupling was adjusted to produce strong oscillations in the secondary circuit. The molybdenite crystal was explored for rectification by touching the surfaces by means of a fine metal point.

The results of these tests show that the low-resistance, photoelectrically-insensitive samples of molybdenite are far more efficient rectifiers than the high-resistance, light-sensitive specimens. One such specimen, which showed no photoelectric sensitiveness whatever, as determined by a sensitive galvanometer, produced decided rectification over almost its entire surface. In fact it was difficult to find a spot which produced no rectification. On the other hand, the samples of molybdenite, having a high resistance exhibited the rectifying action only in spots, which usually did not coincide with the photoelectrically sensitive spots. These radio-electrically sensitive spots were widely scattered over the surface, but sometimes comparatively large areas were found which showed no electrical rectification, as indicated by the telephone. Only the most sensitive spots of the high resistance material produced a rectification approaching that of the specimens having a low resistance.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

[2] PHYS. REV., 11, p. 497, 1918.

In two instances, especially electrically sensitive spots were found upon the light-sensitive areas, but other, equally sensitive spots occurred in the areas which were photoelectrically insensitive.

The conclusions to be drawn are therefore that: (1) the low-resistance, photoelectrically-insensitive molybdenite exhibits much greater rectifying action than the high-resistance, photoelectrically-sensitive specimens; (2) there is no apparent relation between the electrical rectification and the photoelectric sensitivity observed in molybdenite.

BUREAU OF STANDARDS,
November 9, 1918.

ABSORPTION COEFFICIENT OF THE PENETRATING RADIATION.[1]

BY OLIVER H. GISH.

IT is stated several times in the literature that the "penetrating radiation of the atmosphere" has much greater penetrating power than the hardest γ-rays but no attempts to measure this directly have been made previously.

The following table gives values obtained indirectly from Kohlhörster's observations.

Absorption Coefficient in Air (Computed).

Altitude, m.	Absorption Coef. (μ) (Computed) (Cm.$^{-1}$).	Absorption Coef. of γ-Rays in Air Divided by μ.	Observer.
– – – –	1.10×10^{-5}	4.8	Kohlhörster
111– 686	1.53 "	3.4	Gish
3861–9071	0.772 "	6.8	"
– – – –	0.746 "	7.0	E. v. Schweidler

Using a Wulf type electroscope in the form of a vertical cylinder and adding concentric cylinders of lead at intervals the following results were obtained.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

Absorption Coefficient in Lead (Measured).

Altitude, m.	Absorption Coef. (μ) (Measured) (Cm.$^{-1}$).	Absorption Coef. γ-Rays in Lead + μ.	Place.
359	1.22	0.41	Lincoln, Nebr.
1857	0.82	0.61	Colorado Spr's, Colo.
4308	0.45	1.11	Pike's Peak

The values in the first table are less than those found for gamma-rays (6.0×10^{-5}) while those in the second table are (except for the last) greater than the coefficient of absorption of gamma-rays in lead (0.50). It should however be said that in the latter case the values obtained are too large due to the method of computing them. Both tables are however significant in that they show a decrease in the absorption coefficient with altitude. In view of this it seems probable that the "upper component" of the penetrating radiation is more penetrating than gamma-rays.

WESTINGHOUSE RESEARCH LABORATORY,
EAST PITTSBURGH, PA.

IONIZATION AND RESONANCE POTENTIALS FOR ELECTRONS IN VAPORS OF ARSENIC, RUBIDIUM, AND CÆSIUM.

BY PAUL D. FOOTE, O. ROGNLEY AND F. L. MOHLER.

THE study of electron currents in metallic vapors has been extended to arsenic, rubidium and cæsium. The resonance potential or potential of inelastic impact and ionization potential were measured by the method used in previous work.

The results with arsenic gave the resonance potential at 4.7 volts and ionization at 11.5 volts. As these values are close to the corresponding potentials for mercury of 4.9 volts and 10.3 volts the experiment was repeated with all possible precautions with similar results. A wehnelt discharge showed no visible arc with a current of 30 milliamperes but when some mercury was added to the arsenic the arc was visible with a current of .02 milliampere. In the case of metallic vapors so far studied the quantum relation $Ve = h\nu$ shows these potentials to be related to the frequencies of the first line and limit of a prominent series in the spectrum of the metal. No such relation can be found in the case of arsenic but little is known of series relations in arsenic.

The potentials found may predict a series with its first line and limit near $\lambda = 2{,}620$ and $\lambda = 1{,}070$ respectively.

In rubidium vapor the resonance potential is 1.6 volts and the ionization potential 4.1 volts, in cæsium 1.48 volts and 3.9 volts respectively. Tate and Foote[2] have shown that in the case of sodium and potassium these potentials are determined from the quantum relation $Ve = h\nu$ by the frequencies of the first line and limit of the principal series.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

[2] Phil. Mag., 36, p. 75, 1918.

These series in the case of rubidium and cæsium are widely separated doublets. Foote and Mohler[1] found in the case of thallium that the short wave-length line of the doublet determined the value of the resonance potential. The accompanying table shows that the series relations of rubidium and cæsium are analogous to sodium and potassium. The agreement is well within experimental error if we take as in the case of thallium the short wave-length line of the doublet.

Resonance and Ionization Potentials for the Alkali Metals.

Metal.	λ(Å) ($\nu = 1.5\,s - 2p$).	**Resonance Potential Volts.**		λ(Å) ($\nu = 1.5\,s$).	**Ionization Potential Volts.**	
		Computed.	**Observed.**		**Computed.**	**Observed.**
Lithium.......	6707.85	1.839	——	2299.67	5.363	——
Sodium........	5895.94 5889.97	2.092 2.094	2.12	2412.63	5.112	5.13
Potassium.....	7699.01 7664.94	1.602 1.609	1.55	2856.69	4.318	4.1
Rubidium.....	7947.64 7800.29	1.552 1.581	1.6	2968.40	4.155	4.1
Cæsium.......	8943.46 8521.12	1.379 1.447	1.48	3184.28	3.873	3.9

Bureau of Standards,
Washington, D. C.
December 9, 1918.

Note on the Distribution of Energy in the Visible Spectrum of a Cylindrical Acetylene Flame.[2]

By Edw. P. Hyde, W. E. Forsythe and F. E. Cady.

A KNOWLEDGE of the distribution of energy in the visible spectrum of an acetylene flame has become important within the last few years through the use of this flame, in cylindrical form, in investigations of the visibility of radiation. It can be shown by computation that the data on acetylene published by Coblentz form a curve in the visible spectrum which will not agree with that of a black body at any temperature to better than 7 or 8 per cent. As this would mean that no color match could be obtained and as previous experience of the authors had led to the conclusion that the energy curve of acetylene differed in shape from that of a black body only in the extreme red, a short investigation was undertaken to verify this conclusion.

Tungsten lamps whose current color-temperature relation was carefully determined in this laboratory, were sent to the Eastman Kodak Company and to the Bureau of Standards with the request that they be compared with the acetylene flame and the current for color match be found. The results gave an average value of 2360° K ± 10° K., and neither laboratory reported any difficulty in obtaining a match in color. However the Bureau of Standards reported a difference amounting to about 75° K. between the flame as given by the Eastman standard burner and that given by the "Crescent Aero" burner, the latter being higher.

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[1] Phys. Rev., 11, p. 487, 1918.

[2] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

The spectral distribution of the flame was measured by means of a spectrophotometer and a spectral-pyrometer and the results gave a curve agreeing within the limits of error with that of a black body at 2360° K. In the extreme red, beyond 0.70 μ there was indication of a higher emissivity for the acetylene. A photographic method gave results corroborating those just mentioned.

A test of the sensibility of the color-match method to show differences in the spectral energy curve, showed that if two spectral curves matched at 0.5 μ and 0.7 μ and differed by as little as 4 per cent. in the middle of the spectrum, the two light sources could not be made to match in color.

In conclusion it is recommended that the relative emission intensities of a cylindrical acetylene flame, at least for that type represented specifically by the Eastman standard burner and for the wave-length interval from 0.4 μ to 0.7 μ should be taken as identical with those of a black body at 2360° K.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,
NELA PARK, CLEVELAND, OHIO,
December, 1918.

PRELIMINARY NOTE ON THE LUMINESCENCE OF THE RARE EARTHS.[1]

BY E. L. NICHOLS, D. T. WILBER AND F. G. WICK.

THE rare earths in solid solution in CaO and certain other media exhibit fluorescence and absorption of special interest on account of the narrow bands in their spectra.

By experiments now in progress we have established the following points:

1. All bands visible when luminescence is excited by the iron spark coincide precisely with the bands of kathodo-luminescence. It is probable therefore that, as in the case of the uranyl salts, willemite, etc., the location of bands is independent of the mode of excitation.

2. When excited at the temperature of liquid air, these spectra undergo a marked narrowing of the bands and, particularly in the blue and violet, broad hazy bands are resolved into numerous line like components.

3. At low temperatures the bands usually ascribed to erbium in the blue appear to be greatly enhanced whereas the bands due to samarium, chiefly in the red, are weakened. In preparations containing both of these elements, the change in the color of fluorescence thus produced is very striking.

4. Since bands due to several elements are present in all preparations thus

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

far made, the search for series is greatly complicated. There is evidence however of the occurrence of constant frequency intervals, as in the case of all other fluorescent substances thus far examined.

On the Critical Absorption Frequencies of Chemical Elements of High Atomic Numbers.[1]

By William Duane and Takeo Shimizu.

In several papers[2] presented to the American Physical Society accounts have been given of measurements of the critical X-ray absorption frequencies of some of the chemical elements. In making these measurements an X-ray spectrometer with a calcite crystal has been employed. Before striking the crystal the X-rays passed through two narrow slits and after reflection through a third slit in front of the ionization chamber, which was wide enough to admit the entire reflected beam of X-rays into the chamber. This arrangement makes a large correction for the absorption of the X-rays by the crystal unnecessary. The critical absorption wave-lengths were obtained by drawing curves representing the ionization currents as functions of the angle made by the crystal planes with the X-ray beams (the grazing angles of incidence). If a thin sheet of a chemical element (or of one of its compounds) is placed in the path of the X-rays, a marked break in the curve appears at a certain point. The ionization current on the high frequency side of this break is much smaller than on the low frequency side, indicating a substantial increase in the absorption of the shorter X-rays by the chemical element. The formula

$$\lambda = 6.056 \sin \theta \times 10^{-8} \text{ cm.} \qquad (1)$$

gives the wave-length λ, in which the double grazing angle is obtained by measuring from the center of the sharp drop in the ionization curve on one side of the zero line of the spectrometer to that on the other side.

The papers above referred to contain the results of experiments on chemical elements up to and including cerium, with an atomic number of 58. The present paper deals with an extension of the measurements to chemical elements of atomic numbers higher than 58. We used the same apparatus as in the previous researches with the single exception that the current exciting the X-ray tube came from a high tension transformer (with a system of kenotrons and condensors, etc., to give approximately a constant difference of potential), instead of from the high tension storage battery previously employed. With this apparatus we have measured the critical absorption frequencies for a number of chemical elements up to mercury (atomic number 80). The lantern slides shown indicate that the square roots of these frequencies plotted against the atomic numbers depart from the straight line law to a still greater extent than the measurements previously recorded for chemical elements of lower atomic numbers.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society December 28, 1918.

[2] The Critical Absorption of Some of the Elements for High Frequency X-rays, F. C. Blake and William Duane, Phys. Rev., December, 1917. On the Relation between the K X-ray series and the Atomic Numbers of the Chemical Elements, William Duane and Kang-Fuh Hu, Phys. Rev., June, 1918. On the Critical Absorption and Characteristic Emission X-ray Frequencies, William Duane and Kang-Fuh Hu, Phys. Rev., June, 1918.

In one of the papers referred to it was shown that the velocity v of the electron required to produce the K characteristic X-rays, calculated from the simple formula

$$h\nu = \frac{1}{2}\frac{mv^2}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{2}$$

is a linear function of the atomic number N, it being given by the equation

$$v = 0.00678c(N - 3/2) \tag{3}$$

to within about 1/5 per cent. up to the atomic number 58. As indicated by the lantern slide shown the departure from this law for chemical elements up to mercury ($N = 80$) amounts to only 1 per cent.

We have plotted on the same diagram the results of the interesting experiments recently published by de Broglie. He used a photographic method for measuring the critical absorption wave-lengths. The correction for the absorption by the crystal in this method does not appear to be quite clear, especially as the correction must depend upon the finite size of the crystal. As a matter of fact the velocities we have calculated from his data by equation (2) fall about as far above the straight line represented by equation (3) as our values do below it.

In dealing with these chemical elements of high atomic numbers we must remember that the velocities of the electrons required to produce their characteristic X-rays exceed half the velocity of light, and that several formulas have been proposed for the electron's kinetic energy. The velocities calculated by these formulas differ from each other (in the case of mercury for instance) by several per cent. We postpone a discussion of these points, however, until the critical frequencies for radium, thorium and uranium have been obtained.

SOME INTERESTING RESULTS OF ECLIPSE MAGNETIC OBSERVATIONS.[1]

BY L. A. BAUER.

BECAUSE of the bearing of a definite detection of magnetic effects during solar eclipses upon theories of magnetic variations, the Department of Terrestrial Magnetism of the Carnegie Institution of Washington made special arrangements for magnetic and allied observations during the solar eclipse of June 8, 1918. In the program of work, there participated magnetic observatories and universities distributed over the entire zone of visibility, from China and Japan to the Atlantic seaboard. There were thus obtained magnetic data from about twenty-five stations.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

The results gained confirm the general conclusions reached from similar observations made in accordance with the author's scheme of observation during various eclipses since 1900. It may now be regarded as definitely proved that during a solar eclipse the earth's magnetic field passes through a minute oscillation, the period of which approximates that of the eclipse and the range of which, as compared with that of the solar diurnal magnetic variation, is approximately proportional to the amount of sunlight and other solar radiations cut off from the earth by the moon.

The character of the magnetic effect during the eclipse is in general the reverse of that which occurs during the sunlit hours of the day. Its main characteristic features occur from station to station not according to absolute time, nor to local time but according to eclipse time, as of course should be the case if the effect is attributable to the eclipse.

The range of the effect on the magnetic declination is about 0.1 of that of the solar-diurnal variation, or about one minute of arc for our latitudes. The maximum effect on the strength of the earth's magnetic field is about 0.03 per cent., or equivalent to the effect found by the author for a ten per cent. change in the solar radiation as revealed by changes in the solar-constant values observed at Mt. Wilson by the Smithsonion Institution.

The mathematical analysis of the eclipse magnetic variation may throw light upon the causes of other variations of the earth's magnetic field, notably of those attributed to the sun and moon. Fuller publication will be found in the Journal of Terrestrial Magnetism and Atmospheric Electricity, Vols. 23 and 24.

Why Clouds Never Form in the Stratosphere.[1]

By W. J. Humphreys.

It is obvious from the occurrence of cirrus clouds at the base of the stratosphere that at this level the atmosphere frequently is fully saturated. We should, therefore, expect the stratosphere, even if wholly free from vertical convection, to become humid through long continued diffusion. Nevertheless clouds do not form in it even under the influence of rapid drops in temperature of 10° C. or more. It must, therefore, have a humidity far below saturation. And to this condition there are at least two contributing causes.

(*a*) In the absence of vertical convection the vapor pressure must rapidly decrease, under the influence of gravity, with elevation, just as does the pressure of the atmosphere as a whole. Hence, at anything like constant temperature, the stratosphere, even if saturated at the base, must rapidly become very dry with increase of elevation. But this is not sufficient to account for the total absence of clouds at these levels.

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[1] Abstract of a paper presented (by title) at the Baltimore meeting of the American Physical Society, December 27, 1918.

(*b*) The lower portion of the stratosphere normally has a minimum temperature during the passage of each anticyclone which obviously prevents water vapor from diffusing upward to a greater density at any level than that of saturation at this low temperature. The little vapor that does diffuse a short ways upward during the few days prevalence of any milder temperatures, such as accompany cyclonic conditions, is precipitated on the passage—never long delayed—of the next anticyclone. In this way the stratosphere is kept exceedingly dry, and clear skies instead of hazy made the rule for anticyclonic weather.

THE MINIMUM TEMPERATURE AT THE BASE OF THE STRATOSPHERE.[1]

BY W. J. HUMPHREYS.

NUMEROUS records obtained by sounding balloons have shown conclusively that, on the average, the temperature at the base of the stratosphere is lower than at any other level, above or below.

As this is not in accord with a state of thermal equilibrium one naturally looks for the cause in some type of forced convection.

An analysis of the records shows that this minimum temperature is especially pronounced in anticyclonic regions, or where the barometric pressure is high. But in these regions the chief flow of the atmosphere is towards lower latitudes. Hence it lags, or loses some of its eastward velocity, and thereby interferes with the flow of the prevailing winds from the west and causes them to rise, as over a mountain barrier, to unwonted heights, and thus to cool to abnormally low temperatures.

The base of the stratosphere recovers somewhat from its minimum temperature after the passage of an anticyclone, but only to be again dynamically cooled below the point of radiation equilibrium by the next anticyclone, and so on indefinitely.,

On the average, therefore, the lower portion of the stratosphere has a mechanically enforced minimum temperature more or less below the point of radiation equilibrium.

STRIÆ IN OPTICAL GLASS.[1]

BY L. E. DODD AND A. R. PAYNE.

VARIOUS types of striæ in optical glass, classified on the basis of visible appearance, are described, and photographs shown. Certain of these types are then discussed from the standpoint of their nature and probable origin. A relationship between gas bubbles in the glass and striæ is pointed out. The application of pressure to the molten glass is advocated as a preventive, not only of bubbles, but also of those striæ associated with bubbles. Another probable source of the more numerous striæ is given as the separation of lead or its oxide, in the lead glasses. Striæ having such origin, however, are more localized, near the walls and bottom of the pot, and may be reduced to a minimum in number by proper heat treatment and stirring. It is thought that striæ in the more central part of the glass mass in the pot, which are regarded as being mostly associated with bubbles, may be largely prevented by application of pressure, and that thereby the per cent. of good glass may be considerably raised from the present amount of 20 or 25 per cent. of the total melt. The experimental application of pressure is proposed as a special research, primarily of commercial value, but indirectly of scientific value by its possibility of opening the way to the manufacture of the larger lenses free from defect.

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[1] Abstract of a paper presented (by title) at the Baltimore meeting of the American Physical Society, December 27, 1918.

Bureau of Standards,
Pittsburgh Branch.

Effect of Crystal Structure upon Photoelectric Sensitivity.[1]

By W. W. Coblentz.

In a previous investigation[2] it was observed that plates of silver sulphide which had been prepared in the laboratory and subjected to hammering, were rather insensitive photoelectrically. On the other hand, the natural mineral acanthite, Ag_2S is sensitive photoelectrically. The sample tested was a spindle-shaped rod about 8 mm. long, irregular in outline, and consisting principally of three crystals. Wishing to produce a wider receiving surface for spectral photoelectric examination, the central crystal was flattened by hammering. It was then found that this flattened crystal was quite insensitive photoelectrically, whereas, the rest of the material remained sensitive. Evidently the change in internal configuration of the crystal had some effect upon the photoelectric sensitivity. It is, therefore, proposed to observe the spectral photoelectrical sensitivity of the sample before and after mechanical manipulation.

Bureau of Standards,
Washington, D. C.,
December 27, 1918.

Silvering Quartz Fibers by Cathodic Sputtering.[1]

By J. E. Shrader.

The device by Williams[3] for silvering quartz fibers has been so modified that the measurement of the resistance of the fibers can be measured in situ.

Sputtering of fibers has been done with D.C. from generators, rectified A.C. from a transformer, and A.C. from a transformer rectified only by the sputtering device itself. The sputtering was done in hydrogen at different voltages and current densities, while the resistance was measured for varying lengths of time of sputtering.

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[1] Abstract of a paper presented (by title) at the Baltimore meeting of the American Physical Society, December 27, 1918.

[2] Bull. Bur. Standards, 14, p. 591, 1918.

[3] Phys. Rev., 4, pp. 517, December, 1914.

After silvering the resistance of the fiber has been observed to change with time and this change has been measured over a short period of time.

ON THE CHARACTERISTICS OF ELECTRICALLY OPERATED TUNING FORKS.[1]

BY H. M. DADOURIAN.

THE communication presents the results of a series of experiments performed to determine the conditions affecting the frequency of electrical tuning forks. Detailed descriptions of the experiments will be published in an early issue of THE PHYSICAL REVIEW.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
December 10, 1918.

A MECHANICALLY BLOWN WIND INSTRUMENT.[1]

BY A. G. WEBSTER.

THE principle employed is that of the squeaky faucet or fluttering safety-valve. A spring of variable tension holds the valve in place and the proper pressure causes a puff of air, which generates a sound wave in the horn, which on reflexion arrives at the valve in the proper phase to maintain vibration. The theory, combined with the author's theory of acoustical impedance, leads to a new boundary problem in partial differential equations.

THE DYNAMICS OF THE RIFLE FIRED AT THE SHOULDER.[1]

BY A. G. WEBSTER.

A RECORD is made on a smoked glass attached to a rifle in the act of being fired. From this, the acceleration of the gun may be calculated and thus the elastic properties of the shoulder determined. Experiments are made with various rifles, including the Browning Machine Gun.

INTERIOR BALLISTICS, BY A NEW GUN INDICATOR.[1]

BY A. G. WEBSTER.

AN indicator is described in which the stiffness of the spring is so great and the damping so perfect that oscillations due to the apparatus are eliminated. Results obtained regarding the burning of the powder are described.

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[1] Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

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THE

PHYSICAL REVIEW.

THE STRUCTURE OF RADIOACTIVE ELEMENTS.

By Ingo W. D. Hackh.

THE progress of knowledge in chemical and physical sciences has come mainly through the interpretation of experimental facts. Many phenomena of modern physical research can only be explained by theories of the structure of atoms. Thus a branch of "sub-atomic" physics and chemistry has been created. The majority of papers dealing with the structure of the atom deal with the elements of lighter atomic weight, while the elements of highest atomic weight which exhibit the phenomena of radioactivity seem to be neglected. Nevertheless these later elements offer a more substantial basis for theories concerning the constitution of atoms, because, by their disintegration some of the building stones of the atoms are revealed.

In further developing the periodic system of the chemical elements[1] the following scheme was devised for the series of the radioactive elements, by which the change in their relative electromotive force and valency during the different stages of disintegration is brought in harmony with the total charge of the respective substance. This scheme not only offers proof of the completeness of the uranium-radium series, but also points to a theory of "metastasic" electrons and incidentally establishes temporarily structure formula and logical symbols for the radioactive substances.

According to the current interpretation of disintegration the radioactive elements exhibiting beta-radiation, discharge an electron, and become relatively more negative by an increase in valency and corresponding change of their position in the periodic system.

Thus, *e. g.*, tetravalent uranium x', an isotope of thorium in group 4, loses an electron and becomes pentavalent uranium x'' or brevium[2] occupying a position in group 5, which is thus more electro-negative. This very short lived substance loses another electron and is transformed into the more stable hexavalent uranium 2 of group 6. Therefore with each loss of an electron (or negative charge) the valency has increased and the relative electromotive force of the substance has become more electro-negative. According to the conception of the conservation of energy the *total* negative charge of uranium 2 must be less than the total negative charge of uranium x'' or uranium x', although the electromotive force of uranium 2 is more electro-negative than its two predecessors.

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[1] J. Am. Chem. Soc., 40, 1023, 1918. Am. J. of Science, 46, p. 481, 1918.

[2] Fajans, Phys. Z., 14, 877, 1913. See footnote, Ber., 46, 3492, 1913.

TABLE I.

Structure of the Members of the Uranium-Radium Series. (Fig. 1.)

	Atomic Number, *AN.*		**Helium Nuclei,** *He.*	**Electrons.**			**Structure Formula.**
				Unstable Shell. E_i.	**Valency Shell,** E_v.	**Total,** E_m.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	92	Uranium 1..	$8 + x$	$8 + y$	6	$14 + y$	u-He_8E_{8+6}
B	90	Uranium X¹.	$7 + x$	$10 + y$	4	$14 + y$	u-He_7E_{10+4}
C	91	Brevium....	$7 + x$	$8 + y$	5	$13 + y$	u.He_7E_{8+5}
D	92	Uranium 2..	$7 + x$	$6 + y$	6	$12 + y$	u.He_7E_{6+6}
E	90	Ionium	$6 + x$	$8 + y$	4	$12 + y$	u.He_6E_{8+4}
F	88	Radium.....	$5 + x$	$10 + y$	2	$12 + y$	u.He_5E_{10+2}
G	86	Niton	$4 + x$	$12 + y$	0	$12 + y$	u.He_4E_{12+0}
H	84	Radium A...	$3 + x$	$6 + y$	6	$12 + y$	u.He_3E_{6+6}
I	82	Radium B...	$2 + x$	$8 + y$	4	$12 + y$	u.$He_2\dot{E}_{8+4}$
J	83	Radium C...	$2 + x$	$6 + y$	5	$11 + y$	u.He_2E_{6+5}
J¹	81	Radium C¹¹.	$1 + x$	$8 + y$	3	$11 + y$	u.He_1E_{8+3}
K	84	Radium C¹ .	$2 + x$	$4 + y$	6	$10 + y$	u.He_2E_{4+6}
L	82	Radium D. .	$1 + x$	$6 + y$	4	$10 + y$	u.He_1E_{6+4}
M	83	Radium E...	$1 + x$	$4 + y$	5	$9 + y$	u.He_1E_{4+5}
N	84	Radium F...	$1 + x$	$2 + y$	6	$8 + y$	u.He_1E_{2+6}
O	82	Radio-lead..	x	$4 + y$	4	$8 + y$	u E_{4+4}

Assuming that the disintegration manifesting itself as alpha-radiation is the result of a *single* atom emitting a *single* helium-nucleus (which later becomes a helium-atom) and beta-radiation the result of a *single* atom discharging a *single* electron, it becomes evident that each step in disintegration must have been accompanied simultaneously by an intra-atomic shift of electrons between the outer valency ring and the interior part of the atom. This change in the position of "metastasic" electrons can occur according to the two types of disintegration, in two directions, namely; (1) in α-radiation two valency electrons move from the outer shell to the unstable inner ring and take the place of the emitted helium nucleus, decreasing the valency thereby by two and the relative electromotive force of the resulting substance will be less negative; (2) in β-

radiation there is in addition to the emission of an electron the transfer of another electron from the unstable inner ring to the outer valency ring, thus increasing the valency and making the electromotive force more negative.

There are in the uranium-radium series 16 members, eight of which exhibit alpha-radiations, six show beta-radiations and two final end products exhibit apparently no radiation. Thus the difference from uranium 1 to radio-lead consists of eight helium-nuclei and six electrons. This difference between the structure of U and Pb has been used in the calculation of the number of helium-nuclei and electrons for the different members of the series as shown in Table I. Beginning with uranium 1 there are in the unstable ring eight helium-nuclei and eight electrons, while the valency ring contains six valency electrons. If now one helium-nucleus is thrown off, two of the valency electrons take temporarily the place of the helium-nucleus, the unstable ring of the new metabolon or atom consists of seven helium-nuclei and ten electrons, while four valency electrons are left in the outer ring (uranium x'). This new system of seven He and ten E is very unstable and in the next step of disintegration one electron is thrown off entirely, while one electron shifts to the outer ring, leaving in the unstable ring the seven helium nuclei and eight electrons (Ux$''$ or brevium), and increasing thereby the valency electrons from four to five.

Table II.

Structure of the Members of the Thorium Series.

Atomic Number, *AN*.		Helium Nuclei, *He*.	Electrons.			Structure Formula.
			Unstable Shell, E_i.	Valency Shell, E_v.	Total, E_m.	
(2)	(3)	(4)	(5)	(6)	(7)	(8)
90	Thorium........	$6 + x^1$	$8 + y^1$	4	$12 + y^1$	t.He_6E_{8+4}
88	Mesothorium 1...	$5 + x^1$	$10 + y^1$	2	$12 + y^1$	t.He_5E_{10+2}
89	Mesothorium 2...	$5 + x^1$	$8 + y^1$	3	$11 + y^1$	t.He_5E_{8+3}
90	Radio-thorium . .	$5 + x^1$	$6 + y^1$	4	$10 + y^1$	t.He_5E_{6+4}
88	Thorium X......	$4 + x^1$	$8 + y^1$	2	$10 + y^1$	t.He_4E_{8+2}
86	Thorium-emana'n	$3 + x^1$	$10 + y^1$	0	$10 + y^1$	t.He_3E_{10+0}
84	Thorium A......	$2 + x^1$	$4 + y^1$	6	$10 + y^1$	t.He_2E_{4+6}
82	Thorium B......	$1 + x^1$	$6 + y^1$	4	$10 + y^1$	t.He_1E_{6+4}
83	Thorium C......	$1 + x^1$	$4 + y^1$	5	$9 + y^1$	t.He_1E_{4+5}
81	Thorium D......	x^1	$6 + y^1$	3	$9 + y^1$	t.E_{6+3}
84	Thorium C^1......	$1 + x^1$	$2 + y^1$	6	$8 + y^1$	t.He_1E_{2+6}
82	Thorium-lead....	x^1	$4 + y^1$	4	$8 + y^1$	t.E_{4+4}

In Fig. 1 this idea is graphically illustrated and the number of helium nuclei and electrons of each member is shown. From these data are gotten the temporary structure formula shown in column 8 of Table I. In a similar way the results for the members of the thorium series are given in Table II. The sole difference between the two types of structure formula is in the value *u* and *t*. The symbol *u* indicates "lead from uranium," that is a nucleus with atomic weight 206.4 containing *x* helium nuclei and *y* electrons, while *t* indicates "lead from thorium" with atomic weight 208.2, which consists of *x′* helium nuclei and *y′* electrons. Whether or not there is a difference in the values *x* and *x′*, and *y* and *y′*, can not be definitely decided at present, for the experimental difference existing between *t* and *u* may be due to a different arrangement of the same number of helium nuclei (*x* = *x′*) and electrons (*y* = *y′*).

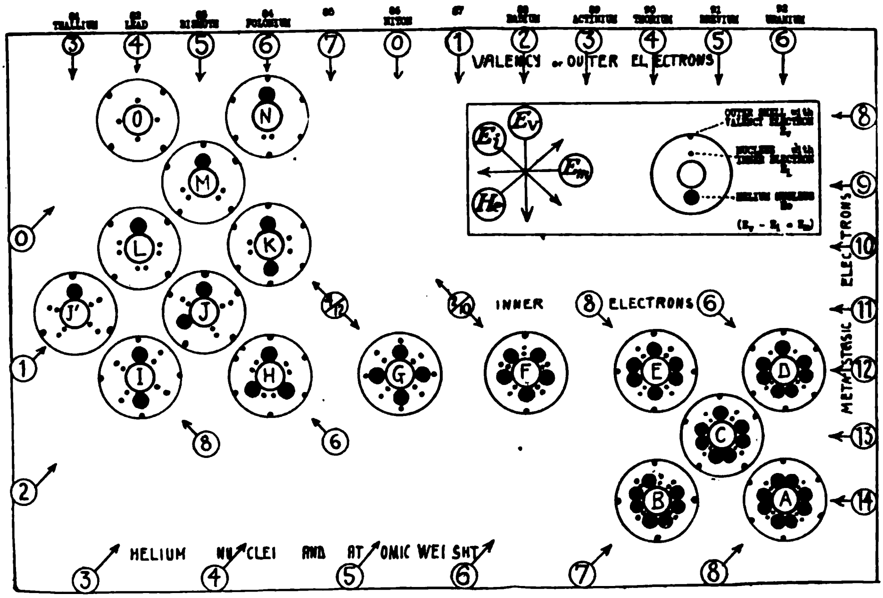


Fig. 1.

Diagram indicating the structure of the members of the uranium-radium series. The diagram is thought to be attached to the periodic system and the position of the radioactive substance in this scheme determines its structure. In any of the four axis the number of helium nuclei and electrons is the same for different members, thus the members which are located on the same southwest to northeast axis have the same atomic weight and the same number of helium nuclei (He), corresponding to the data of column 4 in Table I.; northwest to southeast axis have the same number of inner electrons (E_i), column 5 of Table I.; north to south axis belong to the same group, have the same valency (E_v) and are the isotopes of the element shown on the top of the axis (column 6 of Table I.); west to east axis have the same number of metastasic electrons (E_m), column 7 of Table I.

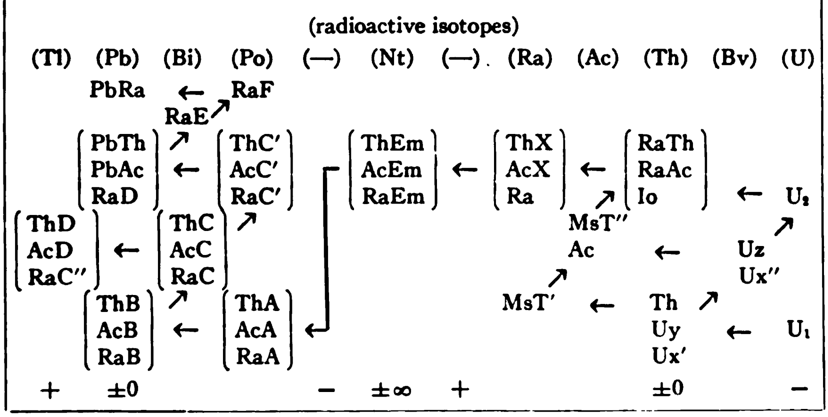
For the present the structure formula offer certain advantages preferential to the symbols employed for the radioactive substances, for they not only indicate the series (by *u* and *t*) and groups (by valency electrons), but also the number of helium nuclei and electrons. For instance the formula $u.He_2E_{6+5}$ indicates that the substance which it represents is (1) a member of the uranium series, differing from uranium 1 ($u.He_8E_{8+6}$) by 8 − 2 or six helium nuclei (corresponding to six alpha-radiations), and (8 + 6) − (6 + 5) or three electrons (= three beta-radiations), and is therefore the ninth member of the series; (2) it possesses an atomic weight of 214.4, which is the sum of u = 206.4 and He_2 = 8.0; (3) it contains (6 + 5) or eleven metastasic electrons of which five are in the outer valency shell and it belongs therefore to group five of the periodic system, and is therefore an isotope of bismuth.

The completeness of both series from uranium or thorium to lead, be-

TABLE III.

The Periodic System and the Radioactive Isotopes.

	4	5A	6A	7A	0	1A	2A	3A	4	
		(non-metals)			(inert gases)	(light-metals)				
	∓ o	−	− −	− − −	∓ ∞	+ + +	+ +	+	∓ o	
Vb	82.Pb	83.Bi	84.Po	85.—	86.Nt	87.—	88.Ra	89.Ac	90.Th	VI
IVb	50.Sn	51.Sb	52.Te	53.I	54.Xe	55.Cs	56.Ba	57.La	58.Ce	Va
IIIb	32.Ge	33.As	34.Se	35.Br	36.Kr	37.Rb	38.Sr	39.Y	40.Zr	IVa
IIb	14.Si	15.P	16.S	17.Cl	18.Ar	19.K	20.Ca	21.Sc	22.Ti	IIIa
Ib	6.C	7.N	8.O	9.F	10.Ne	11.Na	12.Mg	13.Al	14.Si	IIa
0		1.H			2.He	3.Li	4.Be	5.B	6.C	Ia
		(heavy metals)								
III′	22.Ti	23.V	24.Cr	25.Mn	26.Fe 27.Co 28.Ni	29.Cu	30.Zn	31.Ga	32.Ge	III′
IV′	40.Zr	41.Cb	42.Mo	43.—	44.Ru 45.Rh 46.Pd	47.Ag	48.Cd	49.In	50.Sn	IV′
V″	58.Ce	Pr Nd — Sa Eu Gd Tb Dy Ho Er Tm Db Yb							72.Lu	V″
V′	72.Lu	73.Ta	74.W	75.—	76.Os 77.Ir 78.Pt	79.Au	80.Hg	81.Tl	82.Pb	V′



comes evident by the periodic system because all places are occupied according to the rule that the position is changed two places to the left by alpha-radiation and one place to the right by beta-radiation. The deviation of the actinium-series is still doubtful and the structure formula of its members have therefore not been calculated.

The relationship of the radioactive elements to the other elements is shown in Table III. In this table the radioactive elements are appended to the new periodic table, the isotopes being placed vertically under each other. It also shows that the disintegration of all three series runs parallel, beginning with RaTh, RaAc and Io.

CONCLUSIONS.

1. The phenomena of disintegration enables the establishment of a partial atom-structure formula for each radioactive element. These atom-structure formulæ indicate the atomic weight, the valency or place in the periodic system and the respective position in the uranium-radium or thorium-series.

2. A theory of metastasic electrons has been developed by which (*a*) alpha-radiation or the emission of a helium-nucleus causes the change of position of two electrons from the valency ring to the interior unstable ring, while (*b*) beta-radiation or the emission of an electron causes the shift of another electron from the inner or unstable ring to the outer or valency ring.

3. This theory supports the view that the seat of disintegration is located in the next inner shell, and not in the outer or valency shell, and is caused by the shifting of two electrons.

4. The periodic table offers indications that the uranium-radium series and the thorium series down to lead are completely known and that no more members of these two series can be expected.

ON THE VIBRATIONS OF ELASTIC SHELLS PARTLY FILLED WITH LIQUID.

By Sudhansukumar Banerji.

I. Introduction.

The problem considered in this paper is chiefly of acoustical interest in relation to the theory of "musical glasses." This class of instrument consists of a series of thin-walled elastic shells whose gravest modes of vibration are tuned to form a musical seal by partially filling them with a liquid and are excited either by striking or by tangential friction on the ruins. The principal features of interest requiring elucidation are (1) the dependence of the pitch of the vibration upon the quantity of liquid contained in the vessel and (2) the mode of vibration of the liquid itself. These features are discussed in this paper for the three cases in which the elastic shell is respectively (1) a hemispherical shell, (2) a cylindrical vessel with a flat bottom and (3) a conical cup, these forms approximating more or less closely to those used in practice. The analytical expressions show that the motion of the liquid is very marked near the margin of the vessel and is almost imperceptible near the center and also at some depth inside the liquid. This feature becomes more and more marked in the case of the higher modes of vibration of the vessel. Numerical results have also been obtained and tabulated showing the theoretical relation between the quantity of liquid contained in the vessel and the vibration frequency. These show that the rapidity with which the frequency falls on addition of liquid is greatest when the vessel is nearly full, this being specially noticeable in the case of the higher modes of vibration.

The general principle of the analytical method used is similar to that adopted by Lord Rayleigh[1] in treating the two-dimensional case of a long cylinder completely filled with liquid which was studied by Auerbach.[2] This case has also been recently discussed by Nikoloi.[3] The lowering of the pitch produced by the liquid is of course due to the added inertia exactly as in the related case of the vibrations of a bar or a string immersed in a liquid which have been studied by Northway,[1] Mackenzie and Kalähue.[2]

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[1] Lord Rayleigh, Phil. Mag., XV., pp. 385–389 (1883). Scientific Papers, Vol. 2, pp. 208–211.

[2] Auerbach, Wied. Ann., 3, p. 157, 1878, and also Wied. Ann., 17, p. 964, 1882. Reference may also be made to the papers by Montigny, Bull. del' Acad. de Belg., [2], 50, 159, 1880, and by Koläcek, Wied. Ann., 7, 23, 1879, and also Sitz. math. naturw. d. Wien, 87, Abath. 2, 1883.

[3] Nilokoi, Journ. Russk. Fisik Chimicesk, 41, 5, pp. 214–227, 1909.

Musical glasses are sometimes excited by rotating them about a fixed vertical axis, the tangential friction being produced by a rubber kept in a fixed position. No attempt is made in this paper to consider this somewhat complicated case,[3] which I hope to be able to deal with on a future occasion.

2. HEMISPHERICAL CUPS.

The force which a thin sheet of matter subjected to stress opposes to extension is very great in comparison with that which it opposes to bending. From this Lord Rayleigh concluded that the middle surface of a vibrating shell remains unstretched and proposed a theory[4] of flexural vibrations of curved plates and shells in accordance with this condition. As the direct application of the Kirchhoff-Gehring method led to equations of motion and boundary conditions which were difficult to reconcile with Lord Rayleigh's theory, his theory gave rise to much discussion. Later investigations have, however, shown that any extension that may occur must be limited to a region of infinitely small area near the edge of the shell and that the greater part of the shell vibrates according to Lord Rayleigh's type.

Let the radius of the hemisphere be equal to a. Let a point whose natural coördinates are a, θ, ϕ be displaced to the position $a + u$, $\theta + v$, $\phi + w$, where u, v, w are to be treated as small.

From the condition of inextension

$$\begin{aligned}(\delta s)^2 &= a^2(\delta\theta)^2 + a^2\sin^2\theta(\delta\phi)^2\\ &= (a+u)^2(\delta\theta+\delta v)^2 + (\delta u)^2 + (a+u)^2\sin^2(\theta+v)\,(\delta\phi+\delta w)^2, \quad (1)\end{aligned}$$

Lord Rayleigh obtains the three differential equations

$$\frac{\partial v}{\partial\theta} + \frac{u}{a} = 0,$$

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[1] Northway and Mackenzie, PHYS. REV., 13, pp. 145–164, 1901.

[2] Kalähne, Ann. d. Physik, 46, 1, pp. 1–38, 1914.

[3] Reference may be made in this connection to papers by Prof. Love on "The Free and Forced Vibrations of an Elastic Spherical Shell Containing a Given Mass of Liquid," Proc. Lond. Math. Soc., Vol. XIX., where he has studied the case of a rotating spherical shell completely full of liquid, and by Prof. Bryan on "The Beats in the Vibrations of a Revolving Cylinder or Bell," Proc. Comb. Phil. Soc., Vol. VII., 1892.

[4] Lord Rayleigh, Proc. Lond. Math. Soc., Vol. XIII., p. 4, 1881. See also Proc. Roy. Soc., Vol. 45, pp. 45 and 443, 1881, Theory of Sound, Vol. I., Chap. XV, and Love's Elasticity, Chap. XXXIII.

$$\frac{\partial v}{\partial \phi} + \sin^2\theta\frac{\partial w}{\partial \theta} = 0,$$

$$\frac{u}{a} + \cot\theta\cdot v + \frac{\partial w}{\partial \phi} = 0,$$

which can be integrated in the forms

$$\frac{u}{a} = \begin{matrix}\sin\\ -\cos\end{matrix}\, m\phi[A_m(m+\cos\theta)\tan^m \tfrac{1}{2}\theta - B_m(m-\cos\theta)\cot^m \tfrac{1}{2}\theta],$$

$$\frac{v}{\sin\theta} = \begin{matrix}-\sin\\ \cos\end{matrix}\, m\phi[A_m \tan^m \tfrac{1}{2}\theta - B_m \cot^m \tfrac{1}{2}\theta],$$

$$w = \begin{matrix}\cos\\ \sin\end{matrix}\, m\phi[A_m \tan^m \tfrac{1}{2}\theta + B_m \cot^m \tfrac{1}{2}\theta],$$

A_m and B_m being arbitrary constants. These equations determine the character of the displacement of a point in the middle surface.

Since the pole $\theta = 0$ is included the constant B_m must be considered to vanish and the type of vibrations in a principal mode is expressed by the equations

$$\begin{aligned} u &= A_m\alpha(m+\cos\theta)\tan^m \tfrac{1}{2}\theta \sin m\phi,\\ v &= -A_m \sin\theta \tan^m \tfrac{1}{2}\theta \sin m\phi,\\ w &= A_m \tan^m \tfrac{1}{2}\theta \cos m\phi, \end{aligned} \tag{4}$$

in which A_m is proportional to a simple harmonic function of the time.

The potential energy of bending of the vibrating shell is given by

$$\begin{aligned} V &= \frac{8}{3}\pi\mu\frac{\tau^3}{a^2}m^2(m^2-1)^2A_m^2\int_0^{\pi/2}\tan^{2m}\frac{\theta}{2}\frac{d\theta}{\sin^3\theta}\\ &= \frac{2}{3}\pi\mu\frac{\tau^3}{a^2}(m^3-m)(2m^2-1)A_m^2, \end{aligned} \tag{5}$$

where τ = thickness of the shell and μ = rigidity.

The kinetic energy T is given by the expression

$$\begin{aligned} T &= \frac{1}{2}\pi\sigma\alpha^4\tau\left(\frac{dA_m}{dt}\right)^2\int_0^{\pi/2}\sin\theta\{2\sin^2\theta + (\cos\theta+m)^2\}\tan^{2m}\tfrac{1}{2}\theta d\theta\\ &= \frac{1}{2}\pi\sigma\alpha^4\tau\left(\frac{dA_m}{dt}\right)^2\int_1^2\frac{(2-x)^m}{2^m}[(m-1)^2+2(m+1)x-x^2]dx \qquad (6)\\ &= \frac{1}{2}\pi\sigma\alpha^4\tau f(m)\left(\frac{dA_m}{dt}\right)^2, \end{aligned}$$

where σ represents the density of the shell, and

$$f(m) = \int_1^2\frac{(2-x)^m}{x^m}[(m-1)^2+2(m+1)x-x^2]dx, \tag{7}$$

which can be evaluated for any integral value of m.

Since the types of vibrations of the shell are entirely determined by the geometry of the middle surface of the shell, it is obvious that the types can under no circumstances be altered by the presence of the liquid in the shell. The liquid gives rise to a surface traction and affects only the arbitrary constant A_m, that is to say, the amplitude and the frequency of vibration of the shell.

The motion of the liquid will depend upon a velocity potential which satisfies the equation

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{2}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Phi}{\partial\theta}+\frac{\operatorname{cosec}^2\theta}{r^2}\frac{\partial^2\Phi}{\partial\phi^2}=0. \tag{8}$$

A solution of this differential equation which will correspond to the type of vibration of the shell can be obtained by assuming Φ to be of the form

$$\Phi=\left(C_m r+\frac{D_m}{r^2}\right)\sin m\phi\cdot\Delta_\theta,$$

where Δ_θ is a function of θ only. Substituting in the differential equation we find that Δ_θ satisfies the equation

$$\frac{d^2\Delta_\theta}{d\theta^2}+\cot\theta\frac{d\Delta_\theta}{d\theta}+(2-m^2\operatorname{cosec}^2\theta)\Delta_\theta=0.$$

The general solution of this differential equation is

$$\Delta_\theta=E_m\tan^m\tfrac{1}{2}\theta(m+\cos\theta)+F_m\cot^m\tfrac{1}{2}\theta(m-\cos\theta).$$

Neglecting solutions of the type $\cot^m\frac{1}{2}\theta(m-\cos\theta)$, we see that Φ is of the form

$$\Phi=\left(C_m r+\frac{D_m}{r^2}\right)\sin m\phi\tan^m\tfrac{1}{2}\theta(m+\cos\theta), \tag{9}$$

where C_m and D_m are two arbitrary constants. Let us first take

$$\Phi=C_m\frac{r}{a}\tan^m\tfrac{1}{2}\theta(m+\cos\theta)\sin m\phi\cos pt. \tag{10}$$

The relation between C_m and A_m of (4) is readily found by equating the value of $\partial\Phi/\partial r$, when $r=a$, to $\partial u/\partial t$, both of which represent the normal velocity at the circumference. We get

$$C_m\cos pt=a^2\frac{dA_m}{dt}. \tag{11}$$

The expression (10) determines the principal mode of vibration of the liquid. The simple character of the fluid motion as determined by this expression will however be a little disturbed on account of the existence of a free surface and we shall have to add a small correction to this expression. The condition to be satisfied at the free surface is

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0, \text{ when } z = h,$$

where h denotes the depth of the liquid surface below the center of the hemisphere. We shall neglect the force of gravity, inasmuch as the period of free waves of length comparable with the diameter of the shell is much greater than that of the actual motion. The condition to be satisfied at the free surface then becomes simply

$$\Phi = 0, \quad \text{when } z = h.$$

Hence we must have

$$\Phi = C_m\frac{r}{a}\tan^m \tfrac{1}{2}\theta(m + \cos\theta)\sin m\phi\cos pt + f(r, \theta, \phi)\cos pt, \tag{12}$$

where $f(r, \theta, \phi)$ is a solution of $\Delta^2\Phi = 0$ and is such that its differential coefficient with respect to r vanishes on the spherical boundary and it has the value

$$-C_m\frac{h\sec\theta}{a}\tan^m \tfrac{1}{2}\theta(m + \cos\theta)\sin m\phi \tag{13}$$

on the free surface.

In the particular case, when the shell is completely full of liquid, the differential coefficient of $f(r,\theta,\phi)$ with respect to r vanishes on the spherical surface and $f(r, \theta, \phi)$ has the value

$$-mC_m\frac{r}{a}\sin m\phi \tag{14}$$

on the surface defined by $\theta = \pi/2$.

For the determination of the function $f(r, \theta, \phi)$, spherical harmonics of the complex degree $-\frac{1}{2} + p\sqrt{-1}$ are extremely suitable. The properties of these harmonics and their applications to some physical problems have been investigated by Hobson.[1] Solutions of Laplace's equation of the form

$$\frac{1}{\sqrt{r}}\sin(p\log Ar)\,{}^{\sin}_{\cos}\, m\phi K_p^m(\cos\theta),$$

where $K_p^m(\cos\theta)$ is a harmonic of degree $-\frac{1}{2} + p\sqrt{-1}$ and order m, and is defined by the hypergeometric series

$$K_p^m(\cos\theta) = F(m + \tfrac{1}{2} + pi, m + \tfrac{1}{2} - pi, m + 1, \sin^2\tfrac{1}{2}\theta),$$

are suitable for our present purpose. These solutions are finite and continuous for all points in the space inside the hemispherical shell (except infinitely near the origin which may be supposed excluded by surrounding it by an infinitely small sphere).

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[1] Hobson, "On a Class of Spherical Harmonics of Complex Degree with Application to Physical Problems," Trans. Camb. Phil. Soc., Vol. 14, pp. 212–236, 1889.

Let us assume

$$f(r, \theta, \phi) = \Sigma_p B_p \sqrt{\frac{a}{r}} \sin\left(p \log \frac{r}{h}\right) \frac{K_p^m(\cos\theta)}{K_p^m(\cos\alpha)} \sin m\phi, \qquad (15)$$

where $h/a = \cos\alpha$. Then

$$\frac{\partial}{\partial r} f(r, \theta, \phi) = 0,$$

when $r = a$, if the p's are the roots of the equation

$$\tan(p \log a/h) - 2p = 0,$$

and the summation in the above series extends over all the roots of this equation.

The values of the constant B_p have to be obtained from the equation

$$- Cm \frac{\cos\alpha}{\cos\theta} \tan^m \tfrac{1}{2}\theta (m + \cos\theta) = \Sigma_p B_p \sqrt{\frac{a}{h}} \frac{\sin[p \log(\sec\theta)]}{\sqrt{\sec\theta}} \frac{K_p^m(\cos\theta)}{K_p^m(\cos\alpha)},$$

which must be satisfied for all values of θ between the limits $0 < \theta < \alpha$. Approximate values of the constants B_p's can be easily obtained from this equation. In the particular case when the shell is completely full of liquid, the values of the constants B_p's can be obtained in a very simple form. Since the origin is a singular point, we exclude the point by surrounding it with a small sphere of radius ϵ, and assume that $f(r, \theta, \phi)$ vanishes on the surface of this sphere. Since in this case $\alpha = \pi/2$, we can assume

$$f(r, \theta, \phi) = \Sigma_p B_p \sqrt{\frac{a}{r}} \sin\left(p \log \frac{r}{\epsilon}\right) \frac{K_p^m(\cos\theta)}{K_p^m(0)} \sin m\phi,$$

where

$$K_p^m(0) = \frac{\sqrt{\pi}\,\Pi(m)}{\Pi(\frac{1}{2}m - \frac{1}{4} + \frac{1}{2}pi)\Pi(\frac{1}{2}m - \frac{1}{4} - \frac{1}{2}pi)}$$

$$= \frac{\sqrt{\pi}\,\Pi(m)}{\left\{\frac{(2m-1)^2 + p^2}{2^2}\right\}\left\{\frac{(2m-3)^2 + p^2}{2^2}\right\}\cdots}$$

and the summation extends for all values of p which are the roots of the equation

$$\frac{d}{da}\left[\frac{\sin[p \log(a/\epsilon)]}{\sqrt{a}}\right] = 0,$$

that is to say, the equation

$$\tan\left(p \log \frac{a}{\epsilon}\right) - 2p = 0. \qquad (16)$$

The constants B_p's have to be determined by the condition that $f(r, \theta, \phi)$ must have the value $-mC_m(r/a)\sin m\phi$ on the free surface which is given by $\theta = \pi/2$. Hence B_p's are given by

$$-mC_m\frac{r}{a} = \Sigma_p B_p\sqrt{\frac{a}{r}}\sin\left(p\log\frac{r}{\epsilon}\right).$$

Putting $r = \epsilon\cdot e^\lambda$, it is easy to see that

$$\begin{aligned}B_p &= -\frac{2(p^2+\frac{1}{4})\cdot mC_m}{p^2\log(a/\epsilon)+\frac{1}{2}[\frac{1}{2}\log(a/\epsilon)-1]}\left(\frac{\epsilon}{a}\right)^{3/2}\int_0^{\log a/\epsilon} e^{3/2\lambda}\sin p\lambda d\lambda\\ &= -\frac{8(4p^2+1)\cdot mC_m}{4p^2\log(a/\epsilon)+[\log(a/\epsilon)-2]}\left[\frac{\sin[p\log(a/\epsilon)]+p(\epsilon/a)^{3/2}}{9+4p^2}\right].\end{aligned} \quad (17)$$

To obtain an idea of the magnitude of the constant B_p, we shall obtain its value when a/ϵ is a very large quantity. It is easy to see by the method of successive approximation that the roots of the equation (16) are given by

$$\begin{aligned}p\log\frac{a}{\epsilon} = X &- \frac{1}{2X}\log\frac{a}{\epsilon} - \left(\log\frac{a}{\epsilon}\right)^2\left[\frac{1}{4}-\frac{1}{24}\log\frac{a}{\epsilon}\right]\frac{1}{X^3}\\ &- \left(\log\frac{a}{\epsilon}\right)^3\left[\frac{1}{4}-\frac{1}{12}\log\frac{a}{\epsilon}+\frac{1}{160}\left(\log\frac{a}{\epsilon}\right)^2\right]\frac{1}{X^5}\\ &- \text{etc.,}\end{aligned}$$

where $X = (s+\frac{1}{2})\pi$, s being any integer. Now, if we take $a/\epsilon = 10^6$, the roots are successively the following:

$$p_1 = \cdot 321,\ p_2 = \cdot 628,\ p_3 = \cdot 960,\ p_4 = 1.205,\ \text{etc.}$$

Hence we easily find that the constants B_{p1}, B_{p2}, B_{p3}, etc., have approximately the values $B_{p_1} = -\cdot 09\ C_m$, $B_{p_2} = -\cdot 08\ C_m$, $B_{p_3} = -\cdot 06\ C_m$, etc., from which we infer that the surface correction $f(r, \theta, \phi)$ is a small one. The principal mode of vibration of the liquid is therefore expressed by

$$\Phi = C_m\frac{r}{a}\tan^m\tfrac{1}{2}\theta(m+\cos\theta)\sin m\phi\cos pt. \quad (18)$$

If q represent the velocity of the liquid as given by this expression, we have

$$\begin{aligned}q^2 = C_m^2\frac{1}{a^2}\tan^{2m-2}\tfrac{1}{2}\theta[(m+\cos\theta)^2(\sin^2 m\phi\tan^2\tfrac{1}{2}\theta+\tfrac{1}{4}m^2\cos^2 m\phi\sec^2\tfrac{1}{2}\theta)\\ +\{\tfrac{1}{2}m(m+\cos\theta)\sec^2\tfrac{1}{2}\theta-\tan\tfrac{1}{2}\theta\sin\theta\}^2\sin^2 m\phi]\cos^2 pt.\end{aligned} \quad (19)$$

Since q is independent of r, the velocity of the liquid at any point in a given radius vector is constant. We see that the velocity varies as $\tan^{m-1}\frac{1}{2}\theta$. Hence if we move along any given meridian, the velocity increases from a zero value at the pole at first very slowly then rather abruptly to a large value at the surface, the abruptness of rise being greater the larger the quantity m, that is to say, the higher the mode of vibration of the liquid. Since the velocity of the liquid is constant along any given radius vector, we see that if we consider the motion of the liquid on the surface of a cone of semi-vertical angle θ, and trace the motion of the liquid as a whole as θ increases, the velocity remains small as θ increases and assumes a large value only at or near the surface. It is obvious therefore that in every case when the cup is not quite filled to the brim, the velocity of the liquid has a very large value near the margin of the vessel and is almost imperceptible near the center and at some depth in the liquid. In the particular case, when the shell is almost filled to the brim, the velocity of the liquid as given by this expression at a point which is near the center and also near the free surface is not small. But in this case the free surface correction $f(r, \theta, \phi)$ to the expression for the velocity potential becomes of some importance and has a sign opposite to it. Consequently the velocity of the liquid near the center always remains very small. These indications of theory are all confirmed by experiment.

To calculate the kinetic energy of the liquid, we have to integrate $\Phi \times \partial\Phi/\partial n$ over the whole boundary of the fluid. At the free surface $\Phi = 0$. We have therefore only to consider the spherical surface.

Therefore

$$
\begin{aligned}
T &= \tfrac{1}{2}\rho \int\int \Phi \frac{\partial \Phi}{\partial n} dS \\
&= \tfrac{1}{2}\alpha\rho \cos^2 pt \int_0^{2\pi} \int_0^{a} [C_m \sin m\phi \tan^m \tfrac{1}{2}\theta(m + \cos\theta) + f(r, \theta, \phi)] \\
&\quad \times C_m \sin m\phi \tan^m \tfrac{1}{2}\theta(m + \cos\theta) \sin\theta d\theta d\phi \\
&= \frac{\pi}{2} a\rho \cos^2 pt\, C_m^2 \int_0^{a} \tan^{2m} \tfrac{1}{2}\theta(m + \cos\theta)^2 \sin\theta d\theta \qquad (20) \\
&\quad + \frac{\pi}{2} a\rho \cos^2 pt\, C_m \sum_p \beta_p \frac{\sin [p \log (a/h)]}{K_p{}^m (\cos\alpha)} \\
&\qquad\qquad \times \int_0^{a} \tan^m \tfrac{1}{2}\theta(m + \cos\theta) K_p{}^m (\cos\theta) \sin\theta d\theta,
\end{aligned}
$$

ρ being the density of the liquid.

Since the liquid is supposed to be incompressible, the potential energy is zero.

The sum of the kinetic and potential energies of the solid and liquid together must be independent of the time. Thus we get

$$
\left[a^5\rho \int_0^{a} \tan^{2m} \tfrac{1}{2}\theta(m + \cos\theta)^2 \sin\theta d\theta + \alpha^5\rho K + \alpha^4\tau\sigma f(m)\right] \frac{d^2 A_m}{dt^2}
+ \frac{4}{3}\mu \frac{\tau^3}{\alpha^2}(m^3 - m)(2m^2 - 1)A_m = 0,
$$

where

$$K = \sum_p \frac{B_p}{C_m} \frac{\sin [p \log (a/h)]}{K_p{}^m (\cos \alpha)} \int_0^\alpha \tan^m \tfrac{1}{2}\theta (m + \cos \theta) K_p{}^m (\cos \theta) \sin \theta d\theta$$

$$= \sum_p \frac{B_p}{C_m} \frac{4 \sin^2 \alpha}{9 + 4p^2} \cdot \frac{\sin [p \log (a/h)]}{K_p{}^m (\cos \alpha)} \left[K_p{}^m (\cos \alpha) \frac{d}{d \cos \alpha} \{ \tan^m \tfrac{1}{2}\alpha (m + \cos \alpha) \} \right.$$

$$\left. - \tan^m \tfrac{1}{2}\alpha (m + \cos \alpha) \frac{d}{d \cos \alpha} K_p{}^m (\cos \alpha) \right].$$

If we put

$$F(\alpha, m) = \int_0^\alpha \tan^{2m} \tfrac{1}{2}\theta (m + \cos \theta)^2 \sin \theta d\theta$$

$$= \int_{1+\cos \alpha}^2 \left(\frac{2 - x}{x} \right)^m (m - 1 + x)^2 dx,$$

and if A_m varies as $\cos (pt + \epsilon)$, we get

$$[a\rho F(\alpha, m) + \tau\sigma f(m) + \alpha\rho K]p^2 = \frac{4}{3} \frac{\mu}{\alpha^3} \left(\frac{\tau}{\alpha} \right)^3 (m^3 - m)(2m^2 - 1). \quad (21)$$

This equation gives the frequency of vibration of the shell with different quantities of liquid.

The fall of pitch for the three gravest tones given by $m = 2$, $m = 3$ and $m = 4$ for a brass hemispherical shell 10 cm. in radius, 2 mm. in thickness and of density 8.6 with different quantities of liquid are shown in Table I. In Fig. 1, the frequencies have been plotted against the quantity of water in the vessel for these three modes of vibrations.

TABLE I.

α.	Quantity of Water in the Shell.	$m = 2$.			$m = 3$.			$m = 4$.		
		$F(\alpha, m)$.	$f(m)$.	p × Const.	$F(\alpha, m)$.	$f(m)$.	p × Const.	$F(\alpha, m)$.	$f(m)$.	p × Const.
90°	$\pi a^3 \times .667$	1.114	1.53	1.80	1.580	1.88	4.63	2.030	2.296	8.76
80°	$\pi a^3 \times .494$	.570	"	2.24	.641	"	6.50	.479	"	14.51
70°	$\pi a^3 \times 338$	.291	"	2.75	.125	"	9.53	.097	"	19.42
60°	$\pi a^3 \times 208$	.123	"	3.29	.032	"	10.57	.009	"	21.45
50°	$\pi a^3 \times .133$	.058	"	3.61	.008	"	10.95	.002	"	21.64
40°	$\pi a^3 \times .034$	.026	"	3.81	.003	"	11.14	.000	"	21.68
30°	$\pi a^3 \times .014$	.015	"	3.88	.001	"	11.22	.000	"	21.69
20°	$\pi a^3 \times .003$	.013	"	3.90	.000	"	11.23	.000	"	21.69
10°	$\pi a^3 \times .001$	.011	"	3.91	.000	"	11.23	.000	"	21.69
0°	0	0	"	3.93	0	"	11.24	0	"	21.69

The frequencies of a brass hemispherical shell of about the same radius and thickness loaded with different quantities of water have also been determined experimentally by a photographic method. The results showed a general agreement with the calculated values. As a shell of uniform thickness and of uniform elastic properties throughout could not be procured, and the one that was used was very much deficient in these respects, the slight discrepancy that was noticed between the calculated and the observed values of the frequency, was probably due to these defects.

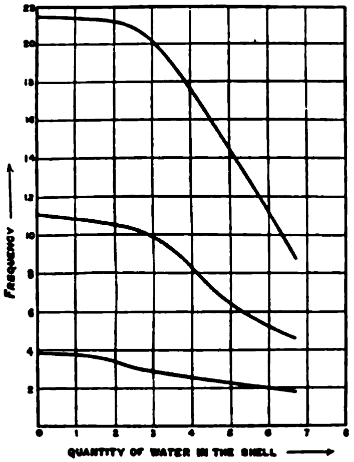

Fig. 1.

3. Cylindrical Cups.

The problem of the flexural vibrations of a cylindrical shell is considered in Lord Rayleigh's Theory of Sound, Vol. I., §235C. If the displacements at any point a, θ, z of the cylinder be δr, $a\delta\theta$, δz, then

$$\begin{aligned} \delta r &= -\,n(A_n a + B_n z)\sin n\theta, \\ a\delta\theta &= \quad (A_n a + B_n z)\cos n\theta, \\ \delta z &= -\,n^{-1}B_n a \sin n\theta. \end{aligned} \tag{22}$$

Supposing now that the cup has been formed by an inextensible disk being attached to the cylinder at $z = 0$, the displacements δr, $a\delta\theta$ must vanish for that value of z. Hence $A_n = 0$, and

$$\delta r = -\,nB_n z \sin n\theta,\ a\delta\theta = B_n z \cos n\theta,\ \delta z = -\,n^{-1}B_n a \sin n\theta, \tag{23}$$

the constant B_n is proportional to a simple harmonic function of the time, say, cos pt.

Since the displacements δr and $a\delta\theta$ are proportional to z and the displacement δz is independent of z, it is obvious, that when z is large the displacements δr and $a\delta\theta$ are also very large compared to δz, that is to say, near the free end of the shell, the displacement δz is negligible compared to δr or $a\delta\theta$. But at the bottom of the shell, the displacements δr and $a\delta\theta$ vanishes and δz remains constant. We conclude, therefore, from the law of continuity, that the disk at the bottom of the shell must have a small normal vibration. If w denote the normal displacement of the disk, it is well known that w satisfies the differential equation

$$\frac{\partial^2 w}{\partial t^2} + c^4\nabla^4 w = 0, \tag{24}$$

where

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{1}{r^2}\frac{d^2}{d\theta^2} \text{ and } C^4 = \frac{E\tau^{12}}{3\sigma(1-\epsilon^2)},$$

E being Young's modulus, σ the density, τ^1 the thickness and ϵ the Poisson's ratio.

If $w \alpha \cos(pt + e)$, then the equation becomes

$$\nabla^4 w - v^4 w = 0,$$

where $v^4 = p^2/c^4$. A solution of this differential equation is known to be

$$w = C_n J_n(vr) \sin n\theta.$$

Hence we shall take

$$w = C_n J_n(vr) \sin n\theta \cos pt. \tag{25}$$

The value of the constant C_n can be obtained from the condition that w and δz must be continuous at the boundary.

This gives

$$C_n \cos pt = -\frac{B_n}{n} \frac{a}{J_n(va)}. \tag{26}$$

We can assume that $J_n(va)$ is very large and consequently that the normal vibration of the disk is very small. The potential energy of deformation for a length l of the cylinder is

$$V = \frac{4\pi\mu\tau^3}{3a}(n^2 - 1)^2 \left[\frac{\lambda + \mu}{\lambda + 2\mu} \frac{n^2 l^2}{3a^2} + 1 \right] B_n^2. \tag{27}$$

The potential energy of vibration of the disk is given by

$$\frac{E\tau^{13}}{3(1 - \epsilon^2)} \int_0^a \int_0^{2\pi} \left[(\nabla^2 w)^2 - 2(1 - \epsilon) \left\{ \frac{\partial^2 w}{\partial n^2} \frac{\partial^2 w}{\partial y^2} \right\} - \left(\frac{\partial^2 w}{\partial n \partial y} \right)^2 \right] r d\theta dr.$$

The value of this integral can be easily obtained. But as we regard the vibration of the disk compared to that of the cylindrical surface to be very small, the value of this expression is also very small.

If the volume density be σ, we get the expression for the kinetic energy in the form

$$T = \tfrac{1}{2}\pi\sigma\tau l a [\tfrac{1}{3} l^2 (1 + n^2) + n^{-2} a^2] \left(\frac{dB_n}{dt} \right)^2 + \tfrac{1}{2}\pi\sigma\tau' \frac{a^2}{n^2 [J_n(va)]^2} \int_0^a [J_n(vr)]^2 r dr \left(\frac{dB_n}{dt} \right)^2. \tag{28}$$

If the cylinder contains frictionless incompressible fluid, the motion of the liquid will depend upon a velocity potential Φ which satisfies the equation $\nabla^2 \Phi = 0$, or in cylindrical coördinates

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0.$$

The solution of this differential equation can be written in either of the forms

$$\Phi = \alpha_n zr^n \sin n\theta \cos pt, \tag{29}$$

$$\Phi = \beta_n e^{-kz} J_n(kr) \sin n\theta \cos pt. \tag{30}$$

The boundary conditions to be satisfied by Φ are

$$\begin{aligned} &\text{(i)} \quad \frac{\partial\Phi}{\partial r} = \frac{d\delta r}{dt}, \text{ when } r = a; \\ &\text{(ii)} \quad \frac{\partial\Phi}{\partial z} = \frac{dw}{dt}, \text{ when } z = 0; \\ &\text{(iii)} \quad \Phi = 0, \text{at the free surface, } u, \text{ when } z = h. \end{aligned} \tag{31}$$

We assume that

$$\Phi = \alpha_n zr^n \sin n\theta \cos pt + \sum_k J_n(kr)[D_k \cosh kz + E_k \sinh kz] \sin n\theta \cos pt, \tag{32}$$

where the summation extends for all values of k which are the roots of the equation

$$\frac{d}{da} J_n(ka) = 0. \tag{33}$$

We at once get by condition (*i*)

$$\alpha_n \cos pt = -\frac{1}{a^{n-1}} \frac{dB_n}{dt}.$$

The condition (*ii*) gives

$$-\left[\frac{a}{n}\frac{J_n(vr)}{J_n(va)} - \frac{r^n}{a^{n-1}}\right]\frac{1}{k}\frac{dB_n}{dt} = \sum_k E_k J_n(kr) \cos pt.$$

This equation must be satisfied for all values of r between the limits $(0 < r < a)$ and will give the value of the constant E_k. Now since

$$\int_0^a r^{n+1} J_n(kr)dr = \frac{na^n}{k^2} J_n(ka),$$

$$\int_0^a J_n(kr)J_n(vr)rdr = \frac{av}{k^2 - v^2} J_n(ka)J_n'(va),$$

and

$$\int_0^a [J_n(kr)]^2 rdr = \tfrac{1}{2}a^2\left(1 - \frac{n^2}{k^2a^2}\right)[J_n(ka)]^2,$$

we get

$$-\frac{1}{k}\frac{dB_n}{dt}\left[\frac{a}{nJ_n(va)}\int_0^a J_n(kr)J_n(vr)rdr - \frac{1}{a^{n-1}}\int_0^a r^{n+1}J_n(kr)dr\right.$$

$$= E_k \cos pt \int_0^a [J_n(kr)]^2 rdr,$$

and therefore

$$E_k \cos pt = -\frac{2a}{ka}\frac{dB_n}{dt}\frac{k^2a^2}{(k^2a^2 - n^2)J_n(ka)}\left[\frac{v}{(k^2 - v^2)n}\frac{J_n'(va)}{J_n(va)} - \frac{na}{k^2a^2}\right]. \tag{35}$$

The condition (iii) gives

$$\alpha_n h r^n + \sum_k (D_k \cosh kh + E_k \sinh kh)\, J_n(kr) = 0$$

for all values of r between the limits $(0 < r < a)$.
Therefore we get

$$D_k \cosh kh + E_k \sinh kh = -\alpha_n \frac{2na^n h}{(k^2a^2 - n^2)J_n(ka)}. \qquad (36)$$

The equations (35) and (36) give the values of the constants D_k and E_k.

To calculate the kinetic energy we have to integrate $\Phi \times \partial\Phi/\partial n$ over the boundary of the shell. At the free surface $\Phi = 0$. We have therefore only to consider the cylindrical surface and the bottom. The expression can be written in the form

$$\begin{aligned} T = \tfrac{1}{2}\pi\rho \cos^2 pt \Bigg[& n\alpha_n^2 a^{2n} \frac{h^3}{3} + n\alpha_n a^n \Sigma J_n(ka) \Bigg\{ D_k\left(\frac{h \sinh kh}{k} \right. \\ & \left. - \frac{1}{k^2}\cosh kh + \frac{1}{k^2}\right) + E_k\left(\frac{h \cosh kh}{k} - \frac{1}{k^2}\sinh kh\right) \Bigg\} \\ & - \sum_k \Bigg\{ \alpha_n D_k \frac{na^n}{k^2} J_n(ka) + \tfrac{1}{2} ka^2 E_k D_k \left(1 - \frac{n^2}{k^2a^2}\right) [J_n(ka)]^2 \Bigg\} \Bigg]. \qquad (37) \end{aligned}$$

The constants E_k and D_k are very small compared to α_n. If we neglect E_k and D_k, the expression for the kinetic energy reduces to the simple form

$$T = \frac{\pi}{2} \rho n \alpha_n^2 a^{2n} \frac{h^3}{3} \cos^2 pt. \qquad (38)$$

In this case the expression for Φ reduces to the form

$$\Phi = \alpha_n z r^n \sin n\theta \cos pt. \qquad (39)$$

This expression represents the principal mode of vibration of the liquid and all the other coexistent modes are very small compared to this one. Since the expression for the velocity varies as $(r/a)^{n-1}$, the velocity is very marked near the margin of the vessel and is almost imperceptible near the center. Using the principle that the sum of the kinetic and potential energies of the solid and liquid together must be independent of the time, we easily obtain an expression for the frequency of vibrations in the most general case from the expressions for the kinetic and potential energies already given. If we neglect E_k and D_k, the frequency equation takes a very simple form. The expression in this case is

$$\begin{aligned} & [\sigma\tau l a\{\tfrac{1}{3}l^2(1 + n^2) + n^{-2}a^2\} + \tfrac{1}{3}\rho n h^3 a^2] p^2 \\ & \qquad = \frac{8\mu\tau^3 l}{3a}(n^2 - 1)^2 \left[\frac{\lambda + \mu}{\lambda + 2\mu} \frac{n^2 l^2}{3a^2} + 1 \right]. \qquad (40) \end{aligned}$$

Thus we see that the law of variation of the frequency with the height of water in the vessel can be expressed in the form

$$p^2 = \frac{1}{A + B\,(h/l)^3},$$

where A and B are two constants for the vessel.

For a glass cylinder whose dimensions are given by $l/a = 4$, $\tau/a = .02$ and which has the density $\sigma = 2.6$ and the elastic constants $\mu = 1.8$ and $\lambda = 1.53$, we easily find that the frequency p_n is given by

$$\left[.052\left\{(1 + n^2) + \frac{.1875}{n^2}\right\} + n\left(\frac{h}{l}\right)^3\right] p_n^2 = \frac{8\mu\tau^3}{3a^2l^2}(n^2 - 1)^2\,[10.4\,n^2 + 3].$$

For the three gravest tones given by $n = 2$, $n = 3$ and $n = 4$, the values of the frequencies p_2, p_3 and p_4 with different quantities of water in the cylinder are shown in Table II.

TABLE II.

h/l.	$p_2 \times$ Const.	$p_3 \times$ Const.	$p_4 \times$ Const.
0	12.47	34.44	65.63
.1	12.33	34.40	65.48
.2	12.02	33.98	64.47
.3	11.29	32.05	61.95
.4	10.15	29.45	57.79
.5	8.85	26.26	52.45
.6	7.61	22.99	46.68
.7	6.56	19.97	41.09
.8	5.60	17.34	36.05
.9	4.97	15.13	31.67
1.0	4.21	13.25	27.93

The values of the frequencies given in Table II. have been plotted in Fig. 2.

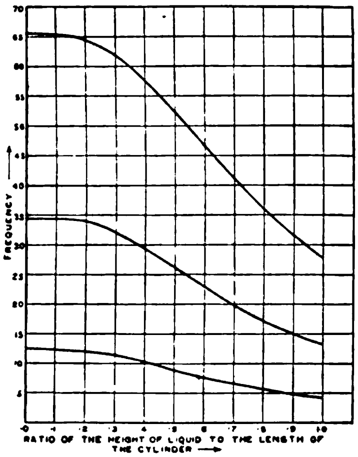

Fig. 2.

4. CONICAL CUPS.

It is shown in Lord Rayleigh's Theory of Sound in the article already referred to that if a cone for which $\rho = \tan\gamma\cdot z$, γ being the semi-vertical angle, executes flexural vibrations, the displacements $\delta\rho$, $\delta\phi$, δz at any point whose cylindrical coördinates are (ρ, ϕ, z) are given by

$$\delta\rho = n \tan\gamma\,(A_n z + B_n)\sin n\phi,$$

$$\delta\phi = (A_n + B_n z^{-1})\cos n\phi,$$

$$\delta z = \tan^2\gamma[n^{-1}B_n - n\,(A_n z + B_n)]\sin n\phi.$$

If the cone be complete up to the vertex at $z = 0$, then $B_n = 0$, so that

$$\delta\rho = n \tan\gamma \cdot A_n z \sin n\phi,$$

$$\delta\phi = A_n \cos n\phi,$$

$$\delta z = -nA_n \tan^2\gamma \cdot z \sin n\phi.$$

If the displacements in polar coördinates (r, θ, ϕ) be denoted by $\delta r, \delta\theta, \delta\phi$, we easily obtain

$$\delta\phi = A_n \cos n\phi$$

$$\delta r = \delta\rho \sin\gamma + \delta z \cos\gamma = 0,$$

$$r\delta\theta = \delta\rho\cos\gamma - \delta z \sin\gamma = nA_n \tan\gamma \cdot r \sin n\phi.$$

It is easy to see that the potential energy of deformation for a length l of the cone

$$W = \frac{4\pi}{3}\mu\tau^3 \frac{\lambda+\mu}{\lambda+2\mu} A_n^2 \sin\gamma \left[\left(-n^3\frac{\tan\gamma}{\sin^2\gamma} + n\tan\gamma + n\cot\gamma\right)^2 + \cos^2\gamma\right]\log\frac{2l}{\tau},$$

where τ = thickness.[1]

The expression for the kinetic energy of vibration of the shell can be easily obtained in the form

$$T = \frac{\pi}{8}\sigma\tau l^4 \sin^3\gamma\,[n^2\sec^2\gamma + 1]\left(\frac{dA_n}{dt}\right)^2. \tag{45}$$

If the cup contains frictionless incompressible fluid, the velocity potential of the fluid must satisfy Laplace's equation. Let us assume that the velocity potential is given by

$$\Phi = C_n r^2 \phi_n(\cos\theta)\sin n\phi \cdot \cos pt \tag{46}$$

where ϕn $(\cos\theta)$ is a function of θ only. It is easy to see by substitution in the differential equation

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{2}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial\Phi}{\partial\theta} + \frac{\operatorname{cosec}^2\theta}{r^2}\frac{\partial^2\Phi}{\partial\phi^2} = 0,$$

that ϕn $(\cos\theta)$ satisfies the equation

$$\frac{\partial^2\phi_n}{\partial\theta^2} + \cot\theta\frac{\partial\phi_n}{\partial\theta} + (6 - n^2\operatorname{cosec}^2\theta)\phi_n = 0.$$

---

[1] This expression can be readily deduced from a very general expression for the potential energy due to strain in curvilinear coördinates obtained by Prof. Love. (Vide his paper on "The Small Free Vibrations and Deformation of a Thin Elastic Shell," Phil. Trans., Vol. 179, 1888, A.) The expression has been criticized by Prof. Basset (Phil. Trans., Vol. 181, 1890, A) on the ground that Prof. Love has omitted several terms which involve the extension of the middle surface. As the inextensional vibrations only have been considered in this paper, this criticism does not affect us in any way.

A solution of this differential equation can be easily obtained in the form

$$\phi n\,(\cos\theta) = \tan^n \tfrac{1}{2}\theta\left[\,(1-n)(2-n) - 6(2-n)\cos^2\frac{\theta}{2} + 12\cos^4\frac{\theta}{2}\right].$$

The relation between C_n and A_n can be easily obtained by equating the value of $\partial\Phi/r\partial\theta$, when $\theta = \gamma$, to $d(r\delta\theta)/dt$, both of which represent the normal velocity at the boundary. We thus get

$$C_n\cos pt\frac{\partial}{\partial\gamma}\left[\tan^n\tfrac{1}{2}\gamma\left\{(1-n)(2-n) - 6(2-n)\cos^2\frac{\gamma}{2} + 12\cos^4\frac{\gamma}{2}\right\}\right]$$
$$= n\tan\gamma\frac{dA_n}{dt}. \quad (48)$$

The principal mode of vibration of the liquid will therefore be expressed by (46) except for a small correction to be introduced on account of the existence of a free surface. At the free surface the condition to be satisfied is given by

$$\Phi = 0,\ \text{when } z = h,$$

where h denotes the height of the liquid.

To satisfy this condition, we assume

$$\Phi = C_n r^2\phi_n(\cos\theta)\sin n\phi\cos pt + \sum_m D_m r^m P_m{}^n(\cos\theta)\sin n\phi\cos pt, \quad (49)$$

where the summation extends for all values of m which are the roots of the equation

$$\frac{\partial}{\partial\gamma}P_m{}^n(\cos\gamma) = 0. \quad (50)$$

The constants D_m's have to be determined by means of the equation

$$C_n(h\sec\theta)^2\phi_n(\cos\theta) + \sum_m D_m(h\sec\theta)^m P_m{}^n(\cos\theta) = 0, \quad (51)$$

which must be satisfied for all values of θ between the limits $0 < \theta < \gamma$. Approximate values of the constants D_m's can be easily obtained from this equation. To get an idea of the magnitude of the constant D_m, we shall obtain its value in the particular case when the semi-vertical angle γ of the cone is small and the height h of the liquid is large compared to the radius of the cross-section of the cone by the free surface. In this case the free surface can be taken to be practically coincident with the surface of the sphere $r = h$. The equation for determining D_m is then

$$C_n h^2\phi_n(\cos\theta) + \sum_m D_m h^m P_m{}^n(\cos\theta) = 0.$$

Now since

$$\int_{\cos\gamma}^{1} P_m{}^n(\cos\theta)P_m{}^n(\cos\theta)\sin\theta d\theta = 0,$$

m, m^1 being two different roots of the equation (50), and

$$\int_{\cos\gamma}^{1}[P_m{}^n(\cos\theta)]^2\sin\theta d\theta=\frac{1-\cos^2\gamma}{2m+1}P_m{}^n(\cos\gamma)\frac{\partial^2}{\partial m\partial\cos\gamma}P_m{}^n(\cos\gamma),$$

we easily get

$$D_m=-\frac{2m+1}{1-\cos^2\gamma}\frac{C_nh^{2-m}\int_{\cos\gamma}^{1}\phi_n(\cos\theta)P_m{}^n(\cos\theta)\sin\theta d\theta}{P_m{}^n(\cos\gamma)\dfrac{\partial^2}{\partial m\partial\cos\gamma}P_m{}^n(\cos\gamma)}.$$

Neglecting the small correction introduced by the free surface, we see that the kinetic energy of the fluid motion is

$$\frac{\pi}{2}\rho C_n{}^2\cos^2 pt\phi_n(\cos\gamma)\frac{\partial\phi_n(\cos\gamma)}{\partial\gamma}\sin\gamma\sec^5\gamma\frac{h^5}{5}.$$

Since the sum of the kinetic and potential energies of the solid and liquid together must be independent of the time, we easily obtain, on assuming that $A_n\alpha\cos pt$, the frequency equation in the form

$$\left[\frac{1}{4}\sigma\tau l^4\sin^3\gamma(n^2\sec^2\gamma+1)+\rho n^2\tan^2\gamma\sin\gamma\frac{\phi_n(\cos\gamma)}{\dfrac{\partial}{\partial\gamma}\phi_n(\cos\gamma)}\frac{H^5}{5}\right]p^2$$
$$=\frac{8}{3}\mu\tau^3\frac{\lambda+\mu}{\lambda+2\mu}A_n{}^2\sin\gamma\left[\left(-n^3\frac{\tan\gamma}{\sin^2\gamma}+n\tan\gamma+n\cot\gamma\right)^2\right.$$
$$\left.+\cos^2\gamma\right]\log\frac{2l}{\tau}.$$

where $H=h\sec\gamma$, H being the slant height of the liquid. In this case, we see that the law of variation of frequency with the height of liquid can be expressed in the form

$$p^2=\frac{1}{A+B(h/l)^5},$$

A and B being two constants for the particular shell.

The frequencies p_2, p_3 and p_4 with different quantities of water for the three gravest modes of vibrations given by $n=2$, $n=3$ and $n=4$ have been calculated from this expression for a cone of semi-vertical angle 30°, the ratio of the thickness of the sides of the cone to the slant height being equal to .02 and are shown in Table III.

The curves showing the fall of frequency for these three modes of vibrations of the cone when loaded with different quantities are plotted in Fig. 3.

TABLE III.

H/l.	$p_2 \times$ Const.	$p_3 \times$ Const.	$p_4 \times$ Const.
0	5.030	13.58	26.75
.1	5.030	13.58	26.75
.2	5.028	13.57	26.73
.3	5.008	13.54	26.69
.4	4.937	13.41	26.47
.5	4.761	13.08	25.94
.6	4.428	12.43	24.87
.7	3.942	11.64	23.13
.8	3.399	10.07	20.79
.9	2.808	8.63	18.07
1.0	2.310	7.26	15.41

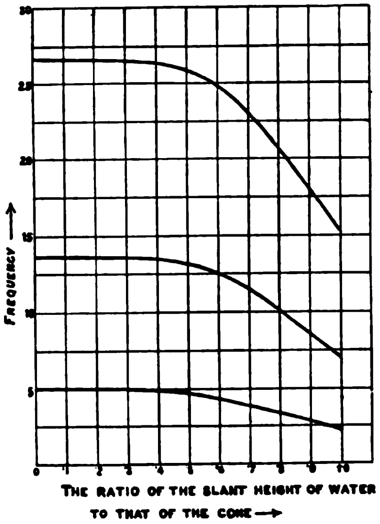

Fig. 3.

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THE RELATION OF POTENTIAL DISTRIBUTION TO HYSTERESIS EFFECT IN THE WEHNELT TUBE.

By R. A. Porter.

1. Introduction.

As a part of his extended study of the discharge in a vacuum tube having a hot oxide as cathode, Wehnelt[1] published a Braun-tube oscillograph curve showing the general character of the voltage and current values in the tube when it is connected to an alternating current generator. Later I made a large number of oscillograph records and from them a study of the relationship of voltage to current in the same form of tube, the cathode being covered with calcium oxide.[2] The curves published at that time show the characteristics for static voltage conditions and for various frequencies of alternating current as gas pressure, maximum current, temperature of cathode, and kind of gas in the tube are varied.

The curves which I obtained showed quite a variety of forms, but one of the marked features was the different current values obtained for a single anode-cathode voltage under both static and alternating current (dynamic) conditions. In general, the static and the dynamic characteristics showed a hysteresis effect, that is, larger current values for decreasing than for increasing voltages.[3] The present paper gives the results of a study of the potential distribution under which three values of current are obtained for the same anode-cathode voltage. The work was undertaken for the purpose of obtaining further knowledge of the conditions producing the hysteresis effect.

2. Apparatus.

The discharge tube used for this study was in form and dimensions almost identical with that of the tube which I used previously in studying the dynamic characteristics, it being essentially the commercial form of Wehnelt tube manufactured by Gundelach, and was obtained from him. It is spherical in shape, 14 cm. in diameter. The only modification from the former tube used consists in the addition of a movable sound, shown at *A* in Fig. 1. This sound is made of platinum wire sheathed with glass up to a point within 4 mm. of the end. The other end of the platinum wire is connected electrically to the terminal outside the tube by a coiled strip of bronze. The iron band *L*, attached to the glass tubing, allows the sound to be moved magnetically The anode, *K*, is of iron about 0.22 cm. in diameter and extends about 2.5 cm. within the tube. The end of the anode is 4.28 cm. from the plane of the cathode strip.

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[1] A. Wehnelt, Ann. d. Phys., 19, p. 138, 1906.

[2] R. A. Porter, Ann. d. Phys., 40, p. 561, 1913.

[3] See figures 4, 15, 10, 11, etc., *l. c.*

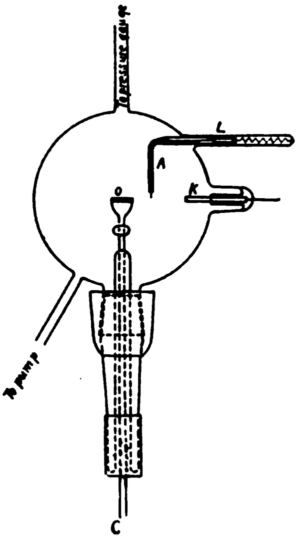

Fig. 1.

The cathode strip, *O*, is platinum, 1.5 × 0.25 × 0.0015 cm. With a heating current or 11.65 amperes, the temperature as obtained with a Wanner pyrometer was 1285° C. with no discharge current, the pressure being 0.010 mm. With a discharge current of 30 milamperes, the temperature read 20° higher. The calcium oxide coat was obtained by placing Kahlbaum c.p. calcium nitrate upon the platinum strip and heating it electrically in the air.

The negative terminal of the storage battery supplying the heating current was grounded at the base of the tube, at *C*. This junction was also the point connected to the electrometer in making measurements of the potential drop from sound to cathode. All potentials were measured finally with the electrometer. A potentiometer method described by Tate and Foote[1] was tried and found to work very well for the larger currents. However, as potentials for the non-luminous discharge had to be measured with an electrometer, it seemed best to use it for all measurements.

A resistance of 1,000 ohms was used in the anode-cathode circuit to stabilize the discharge current, which was furnished by a 300-volt d.c. generator. The applied voltage was measured with a Weston voltmeter, the discharge current, with a Weston millivoltmeter used an an ammeter.

During the observations here recorded, no trap was used to prevent mercury vapor from passing from the pump or the McLeod gauge into the discharge tube.

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[1] John T. Tate and Paul D. Foote, Jour. Washington Acad. Sci., 7, p. 482, 1917.

3. Potential Distribution and Static Characteristics.

After a number of voltage and current measurements had been made with this tube, three characteristic and readily reproducible stages in the discharge were recognizable, the non-luminous discharge and two forms of luminous discharge. Some transient effects were observed and further study may disclose the conditions under which these are obtained.

The wide difference in the characteristics of such a tube for luminous and for non-luminous discharge have already been pointed out.[1] Child[2] has made potential measurements for the luminous and for the non-luminous discharge in a cylindrical tube with CaO cathode. Lester[3] found that with a CaO cathode "there were generally three possible ranges for the current-potential curves" and that "the current broke suddenly from one range to another." A knowledge of the distribution of potential in the space between anode and cathode for the three forms of discharge observed in the Wehnelt type of tube seemed important for the explanation of the phenomenon of hysteresis as well as that of a multiple valued current for a single anode-cathode potential.

The measurements thus far made have been taken at a pressure of 0.010 mm. With this pressure and a cathode temperature in the neighborhood of 1290° C. (heating current 11.65 amperes), either a luminous or a non-luminous discharge could be obtained for all anode-cathode

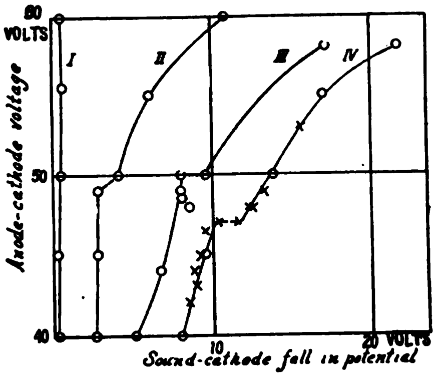

Fig. 2.

Distance of sound from cathode for curve *II*, 0.60 cm., for curve *III*, 2.40 cm.; for curve *IV*, 3.96 cm. These curves are obtained with luminous discharge. Curve *I* shows approximately the sound-cathode fall for non-luminous discharge for all three positions of the sound. Due to change in position, the sound voltage for non-luminous discharge does not vary more than 0.7 volt.

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[1] R. A. Porter, Ann. d. Phys., 40, p. 583, 1913; Richardson and Bazzoni, Phil. Mag., 32, 426, 1916.

[2] C. D. Child, Phys. Rev., 32, 492, 1911.

[3] H. Lester, Phil. Mag., 31, 549, 1916.

voltages between 40 and 60 volts. Whether the discharge is luminous or non-luminous depends on whether this region of voltage is approached from a voltage above 60 or from one below 40 volts. Curves showing the relationship between anode-cathode voltage and sound-cathode fall in potential for three positions of the sound are given in Fig. 2. The sound lay at a point on a line connecting the anode with the middle of the cathode.

The curves of Fig. 3 show more readily the relation of the sound-cathode fall to the distance from the cathode for non-luminous and for luminous discharge for the anode-cathode voltages indicated by the figures at the right of the curves. The three curves of group I. were obtained with the non-luminous discharge; curves II., III., IV. and V. with luminous. A similar set of curves has been made for a heating current of 10.65 amperes. The only striking difference is that for the smaller heating current the potentials near the anode are more noticeably lower than that midway between anode and cathode, a condition which Thomson[1] has observed in a cylindrical form of tube. The apparent irregularity in the readings in the neighborhood of 48 to 50 volts is more pronounced at the higher temperature of the cathode.

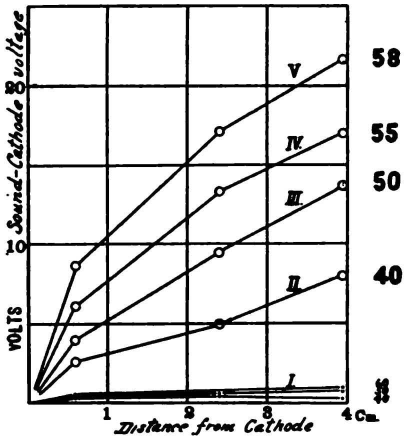

Fig. 3.

Potential distribution for non-luminous and for luminous discharge in Wehnelt tube.

Fig. 4 shows the relation between anode-cathode voltage and current for luminous discharge.[2] The relatively large change in current at an applied voltage of about 50 volts suggests a change in the mechanism of the discharge at this point, just as the wide difference in the values of current for the non-luminous and the luminous discharge for the voltage range 40 to 60 volts indicated a difference in the process of conduction for those two forms of discharge. Observation of the appearance of the glow in the tube also indicates a transition as the voltage goes through the region 48–50 volts. When the glow is present with smaller voltages, it is a faint glow extending from anode to cathode, not filling the whole space inside the tube. It is most intense in the space directly between anode and cathode, where one might expect the current density due to the pure electron (non-luminous) discharge to be greatest, and decreases in intensity and disappears in the region where the current density may be less.

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[1] J. J. Thomson, Phil. Mag., 18, 441, 1909.

[2] In the articles by Richardson and Bazzoni and by myself, Phil. Mag., 32, 428, 1916, and Ann. d. Phys., 40, 583, 1913, respectively, voltage-current curves for tubes with CaO cathodes are given for both the luminous and the non-luminous discharge.

At a voltage of 48–50 volts, however, if the applied voltage is being increased, a flaming bluish glow appears on the sides of the anode and simultaneously the anode-cathode voltage may decrease slightly and the current increase. As the voltage is raised, the anode light increases in extent and the general glow extends until it fills the whole space inside the walls of the tube; then the anode light contracts and at a higher voltage disappears leaving the whole space inside the tube almost uniformly luminous. After the anode light appears, the current increases very rapidly for small increases in applied voltage. In fact the anode-cathode voltage seemed to remain constant for a considerable range in current. When 50 volts is reached voltage and current rise together.

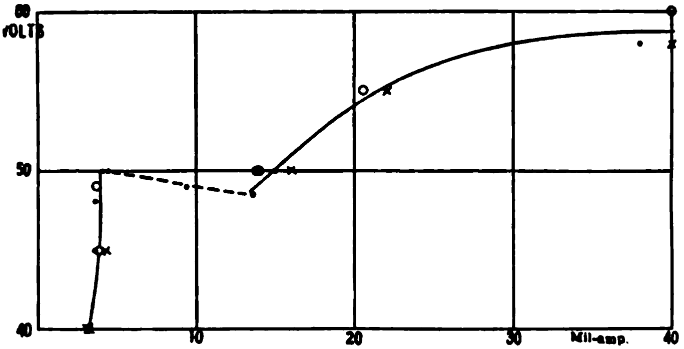

Fig. 4.

Static characteristic for luminous discharge.

As the anode-cathode voltage is decreased from 60 volts with a luminous discharge, it may be brought below 50 volts before the anode light disappears. When about 49 volts is reached the discharge changes to the form of a general glow, the anode-cathode voltage rises perhaps a volt and at the same time there is a marked decrease in current.

For one value of pressure and heating current a set of measurements has been made of the anode-cathode voltage, the current, and the potential fall in the line between anode and cathode, for the two stages in the luminous discharge. The curves of Fig. 5 show the relationship of sound-cathode fall to the total anode-cathode fall for three positions of the sound for voltages near 50 volts. Points marked with circles were obtained when the anode-light stage had been reached, those marked by dots show the sound-cathode fall when there is a general glow.

The points on curve I. at 49 volts indicate a considerable variation in the sound-cathode fall for that voltage when the anode light is present. Readings in a single set of observations, however, show regularly that there is a difference of two to three volts in this fall for the two forms of discharge with an applied voltage of 49.

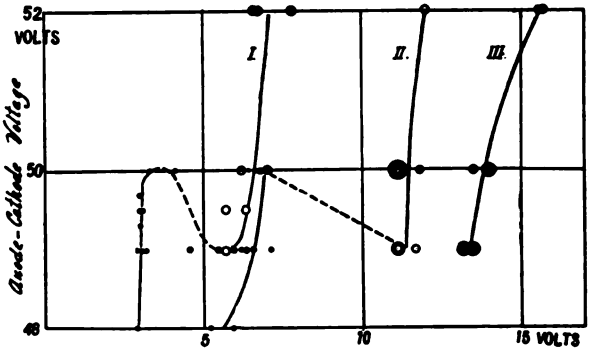

Fig. 5.

Sound—Cathode Voltage. For curves *I* and *II*, *III* the respective distances of sound from cathode were 0.62, 1.96 and 3.93 cm.

The smaller sound-cathode fall occurs when there is a general glow only partly filling the tube. This is the form of luminous discharge occurring first as the voltage is increased and may be called stage II. of the discharge, stage I. being non-luminous. When the region of 50 volts is approached from higher voltages, the blue anode light is present and is accompanied by a larger sound-cathode fall and larger currents. We may call this stage III.

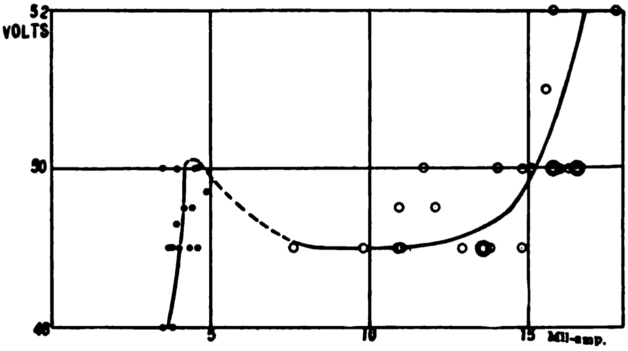

Fig. 6.

Static characteristic of Wehnelt tube in region of 48–52 volts.

The relation of current to voltage for the region 49 to 50 volts is shown in Fig. 6. The potentials existing in the tube for the three stages in the discharge are brought together in Fig. 7.

Readings at higher cathode temperatures indicate an even more sharply marked region of voltage for which the three forms of discharge already noted may occur. At higher temperatures, stages II. and III. apparently have a wider range of voltage in common and there is a larger change in current as the discharge changes from one stage to the other. Modifications of the tube now being used will be necessary in order to obtain a full set of readings at higher cathode temperatures.

When the sound was near the anode, the anode light appeared and disappeared more gradually; no sharp limit of the two forms of discharge could be distinguished. The curves for this stage in the discharge showing the relation between potential and distance from the cathode in Fig. 7 have therefore been drawn for only about half the anode-cathode distance.

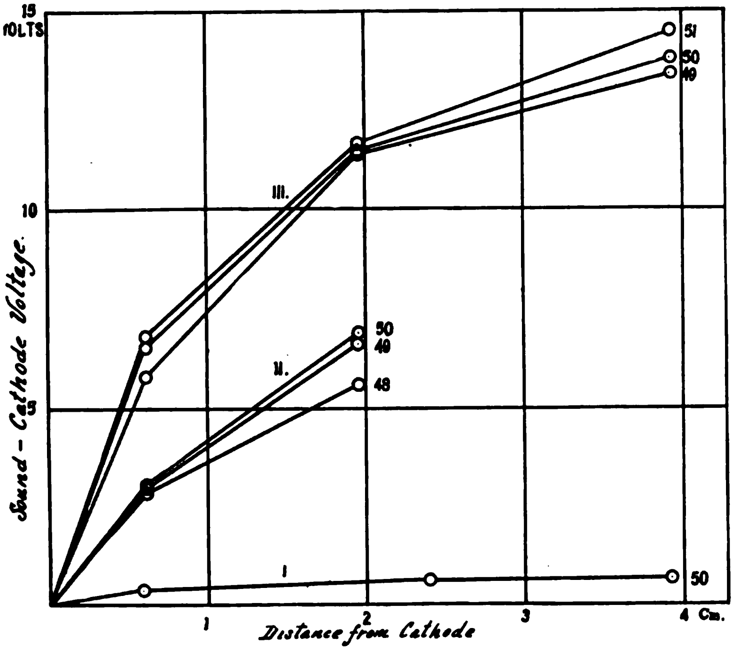

Fig. 7.

Potential distribution for three stages in discharge of Wehnelt tube. The figures at the right of the curves give the anode potentials.

From the results above given it can be seen that the following changes accompany the transition of the discharge from stage I. to stage II. and from stage II. to stage III.: (1) An increase in the cathode fall of 2 to 3 volts; (2) a small increase in the approximately uniform field in the free space between anode and cathode; (3) a decrease in the anode fall; (4) a decrease in the anode-cathode voltage; and (5) an increase in the discharge current. Each of these changes is consistent with the assumption that there is an increase in the supply of positive ions in the region of the anode, that is, a more vigorous ionization at that terminal, as the current increases.

4. THE HYSTERESIS EFFECT.

The direct observation of the discharge and the study of the dynamic characteristics and of the oscillograph records previously obtained show that during each half-cycle of the alternating current which has a suitable maximum value and takes place under the same conditions of pressure and cathode temperature as here used, all the stages of the discharge here described exist. We may conclude therefore that the widely different values of current at the beginning and at the end of a half-cycle, *i. e.*, just before the beginning and just before the end of the luminous discharge, correspond with entirely different potential distributions of the anode-cathode voltage and hence to different localizations of charges and of ionization.

When the anode light, or stage III., appears in the cycle, there is another abrupt change in the ionization and in the distribution of charge. At each of these transition stages, the old conditions tend to persist. Time and voltage seem to be required to change them.

SUMMARY.

Three stages in the discharge of a vacuum tube with CaO cathode have been identified and potential distribution curves for one pressure and heating current have been obtained for each stage. Voltage-current curves for the two stages of luminous discharge are given. The potential distribution curves lie in three groups corresponding to the three stages of the discharge. With the pressure and cathode temperature used, non-luminous discharge may exist for the voltage range 40–50 volts; another stage in the discharge may begin at 48 volts. In the region of 49 volts, therefore, any one of the three stages of the discharge may be obtained. Each of the three stages of the discharge and potential distribution tends to persist as the voltage is varied, or a hysteresis in the current values and in the potential distribution exists when the potential is cyclically varied. This is apparent for very slow cycles as well as for cycles of higher frequency.

The American Association for the Advancement of Science has granted assistance from its research funds in obtaining apparatus for this work. The author gratefully acknowledges this aid.

SYRACUSE UNIVERSITY,
SYRACUSE, N. Y.,
November 12, 1918.

ELECTRIC DISCHARGE ON THE SURFACE OF A SOLID ELECTROLYTE.[1]

By Was. Šulejkin.

The Theory of Electrolytic Safeties[2] Against Overtension.

As is well known, with the electrolysis of some solutions, the aluminum anode is covered with a coating of oxides in a very short space of time after closing the circuit. At the same time the strength of the current falls almost to zero, as the thin layer of oxides and the layer of gas enclosed in the pores offer an enormous amount of ohmic resistance. It is well known also that by increasing the tension, sometimes by hundreds of volts, the strength of the current is increased only inconsiderably, but this phenomenon remains so only until the tension has passed a certain limit, the so-called "critical tension."

On further increasing the tension, even if to only a small extent, the strength of the current suddenly and sharply rises. On the basis of this has been established the practice of applying the so-called "safety appliances against over-tension," the elements of which (practically) do not allow the current to escape to the ground from the line of transmission of electrical energy when the tension in the line is normal, but immediately and rapidly conducts discharges of electromagnetic waves to the ground, before over-tension in the protected part of the line can arise.

It is customary to say that such "safeties" cannot bear the "strain" when the critical tension is passed; the layer is broken through, and so a direct communication is set up between the fluid and the metallic anode.

But one glance at the aluminum electrodes (placed in a glass vessel to facilitate observation) suffices to show that it is not so. At a tension even considerably below the "critical," bright spots appear on the surface of the electrode (anode), here and there little bubbles of gas are given off; this all shows that here we have a case, not of static, but of kinetic equilibrium. In fact, even at low tension the layer is broken through here and there, but the breaks quickly close up; at a tension above the critical, however, such "cicatrization," for some reason, does not take place.

In order to find out this reason, I tried to use for my experiments the same "model" of aluminum electrode as I used in my investigations of the *electrolytic rectifier*[1]. Instead of an aluminum anode, a system of parallelly connected wires of 0.1 mm. was placed into the electrolyte, sealed into glass tubes (Fig. 1*a*). The part of the breaks in which the fluid comes into contact with the aluminum anode was therefore played here by the extremities of the wires placed in the fluid and projecting out of the glass about 0.5 to 0.75 mm.

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[1] Reported at the Moscow Institute of Science, May 19, 1918.

[2] Lightning arresters.

It proved that on passing the critical tension each tube immediately broke. On changing the concentration of the solution, as is known, the critical tension point for the safeties is also changed. It was found that this law is followed when the concentration is changed, and that the attainment of the critical tension coincided with the breaking of the glass tubes.

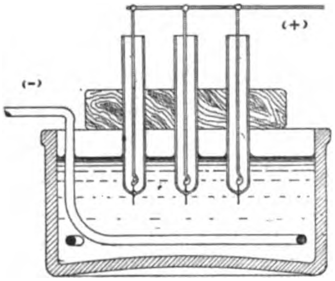

Fig. 1*a*.

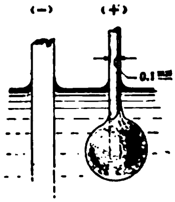

Fig. 1*b*.

The moment preceding the breaking of the glass, at the bottom end of the tube at the spot where the platinum is fused in, there is a sudden flash of blindingly bright, white light, quite different from the yellowish, weak light of the discharge between the platinum and the solution. When the glass was removed, and an anode, consisting only of fine platinum wires, was put into the solution, no change at all in the character and appearance of the discharge was observed when passing the former "critical " tension; the strength of the current did not sharply increase.

This refutes the opinion of Günt. Schultze,[2] who supposes that at the critical tension the discharge between the metal and the fluid begins to proceed with the help of quite different ions than up to the critical point (analogous to the Glaser-effect[3]). A critical point for discharge between pure metal and the solution, therefore, does not exist.

But, on passing a certain tension, you have only to touch the end of the wire-anode *under water* with some piece of glass, marble, jasper, enamel, or the like to produce at the point of contact a blinding white light. Such a flash of light appears, as may easily be observed, also at various points of the aluminum electrode (anode), when passing the critical tension.

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[1] W. Šulejkin, Archives des Sciences Physiques, I., 1918.

[2] Ann. d. Phys., 34, p. 657, 1911.

[3] H. Nernst, Ber. d. Deutsch. Chem. Ges., p. 1547, 1897.

If the end of the wire is removed to some distance from the glass (or other substance), in the interval between them an electric arc is formed, the anode of which is the wire, while the cathode is the glass (or other substance), to which, as may then be observed, particles of metal, coming away from the anode in a powdered state, are fused.

From this it is clear that the layer, hitherto an insulator, at critical tension begins to conduct the current. A necessary condition for a discharge of this sort is the near proximity of the metal and the conductor of the second kind, both being immersed in the solution and moistened by it. In aluminum safeties this condition is fulfilled; therefore, whenever at any spot the insulating layer is broken through *at a tension above the critical*, there is immediately a characteristic discharging flash at the adjacent part of the layer. At the same time the insulating layer around it is quite destroyed, in just the same way as the glass of the tube, or, for example, the lamina of the jasper. If on any part exposed in this way a fresh layer of oxides should be formed again by the action of the electrolysis, it will immediately be destroyed with a similar discharge. Meanwhile, by each initial rupture the exposure of the metal extends more and more, by which a sharp increase in the strength of the current in the electrolytic safeties is produced, when passing the critical tension.

But a complete resemblance of the work of the aluminium electrode and its "model" may be determined only by observing them under various conditions.

In order, on the one hand, to demonstrate that the critical point characteristic with electrolytic safeties depends on the peculiarities of the layer of oxides, separate parts of which, as it proves, playing a very essential part in the discharge; on the other hand, to become better acquainted with the processes which go on in the layer itself or on its surface; and, finally, to judge in some way of the movement of the ions in the close proximity of the anode, I would pass on to a description of detailed experiments.

I. CRITICAL TENSION WITH SOLUTIONS OF BI-CARBONATE OF SODA OF DIFFERENT CONCENTRATIONS, INTO WHICH AS AN ANODE IS IMMERSED AT FIRST AN ALUMINUM LAMINA AND THEN A WIRE, FUSED INTO GLASS (FIG. 1, *a*).

If we take into consideration that aluminum itself is corroded by the solution (this does not occur in the case of the Pt. wire), which introduces a certain error, it may be seen from the series of figures (see Table I.) that the character of the action of the aluminium electrode and the action at the point of contact of our experimental wire with the glass are the same. This gives us a means for determining the critical point even for solutions in which the aluminum anode is not covered with an insoluble layer of oxides (true in the majority of cases); gives us, finally, the possibility of always reproducing the phenomenon in its pure state. The advantage of this will become especially clear if we observe that, for example, with an electrolysis of sulphuric acid between aluminum electrodes the destruction of the insulating layer, *due to the corrosion of the aluminum*, takes place at a tension[1] 100 volts below the real critical tension.

TABLE I.

Concentration.	0.6.	0.3.	0.15.	0.075.	0.0375.
Critic. t. of the safety	180 volt	250	280	350	420
Cr. tens. of Pt. wire..............	164	220	270	340	430

In view of this, further experiments designed to explain the nature of the phenomenon were carried out with "models" of the aluminum electrode.

II. THE DEPENDENCE OF THE VALUE OF CRITICAL TENSION UPON THE CONCENTRATION OF THE ELECTROLYTE FOR DIFFERENT SOLUTIONS.

For experiments, solutions of the following substances have been used: (*a*) Caustic soda ($NaOH$); (*b*) sodium bicarbonate ($NaHCO_3$); (*c*) iron sulphate ($FeSO_4$); (*d*) zinc sulphate ($ZnSO_4$); (*e*) sulphuric acid (H_2SO_4); (*f*) sodium chloride ($NaCl$); (*g*) ammonium chloride (NH_4Cl); (*h*) hydrochloric acid (HCl).

Into the electrolyte as an anode were immersed fine wires (platinum or nickel) at the lower ends of which were fused drops of glass of 1 mm. diameter (Fig. 1, *b*).

At first a certain amount of soluble substance was measured out and after each observation the solution was diluted with a double quantity of water, again a double quantity, and so on. In this way a full concentration in gram-equivalents (m) to the liter was determined.

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[1] G. Schultze has called this "seemable critical" tension.

With each concentration the tension was observed at which the bright light flashed out round the part of the glass adjacent to the wire.

The values of P_{crit} were marked off in Descartes coördinates on the axis ordinates; the quantities m were marked off on the axis of abscissa. As by decreasing the concentration by half, again by half, and so on, the critical tension rises, at first slowly, afterwards very rapidly, for greater precision the points were transferred to logarithmic paper (on the abscissa axis was marked off Lgm, on the ordinate LgP_{cr}).

On doing this, a remarkable law became evident: all the characteristics of all the solutions experimented with, without exception, were represented on the logarithmic paper by straight lines.

Let us examine their peculiarities. (*a*) First of all it will be noticed from the drawing (Fig. 2) that all the straight lines are similar in appearance, being represented by the equation.

$$A \cdot LgP_{cr} + B \cdot Lgm = C, \tag{1}$$

where A, B and C are positive coefficients.
Hence:

$$LgP_{cr} = \frac{C}{A} - \frac{B}{A} Lgm = Lg\alpha - \beta \cdot Lgm \tag{2}$$

(For convenience we have introduced the symbols:

$$\frac{C}{A} = Lg\alpha \text{ and } \frac{B}{A} = \beta).$$

Therefore, it is not difficult to see, for all the solutions:

$$P_{cr} = \frac{\alpha}{m^\beta}\,. \tag{3}$$

(*b*) As the characteristics of $ZnSO_4$ and $FeSO_4$, of NaCl and NH_4Cl respectively coincide, the height of the critical tension, therefore, does not depend on composition of the cation, but depends only on the anion ($\overset{--}{SO_4}$, $\overset{-}{Cl}$, $\overset{-}{OH}$, etc.).

(*c*) In the diagram it is seen that the critical tensions of the following characteristics also coincide:

NaCl (or NH_4Cl) — to *HCl*; and $FeSO_4$ (or $ZnSO_4$) — to H_2SO_4.

But it is known that the dissociated acids (HCl, H_2SO_4) are incomparably stronger than their salts. With equal full quantities of the dissolved electrolyte ($m_{acid} = m_{salts}$) in the solution of acid there are incomparably more *dissociated* ions than in the salt solutions. Notwithstanding this, as we see, the critical tension in both cases is the same.

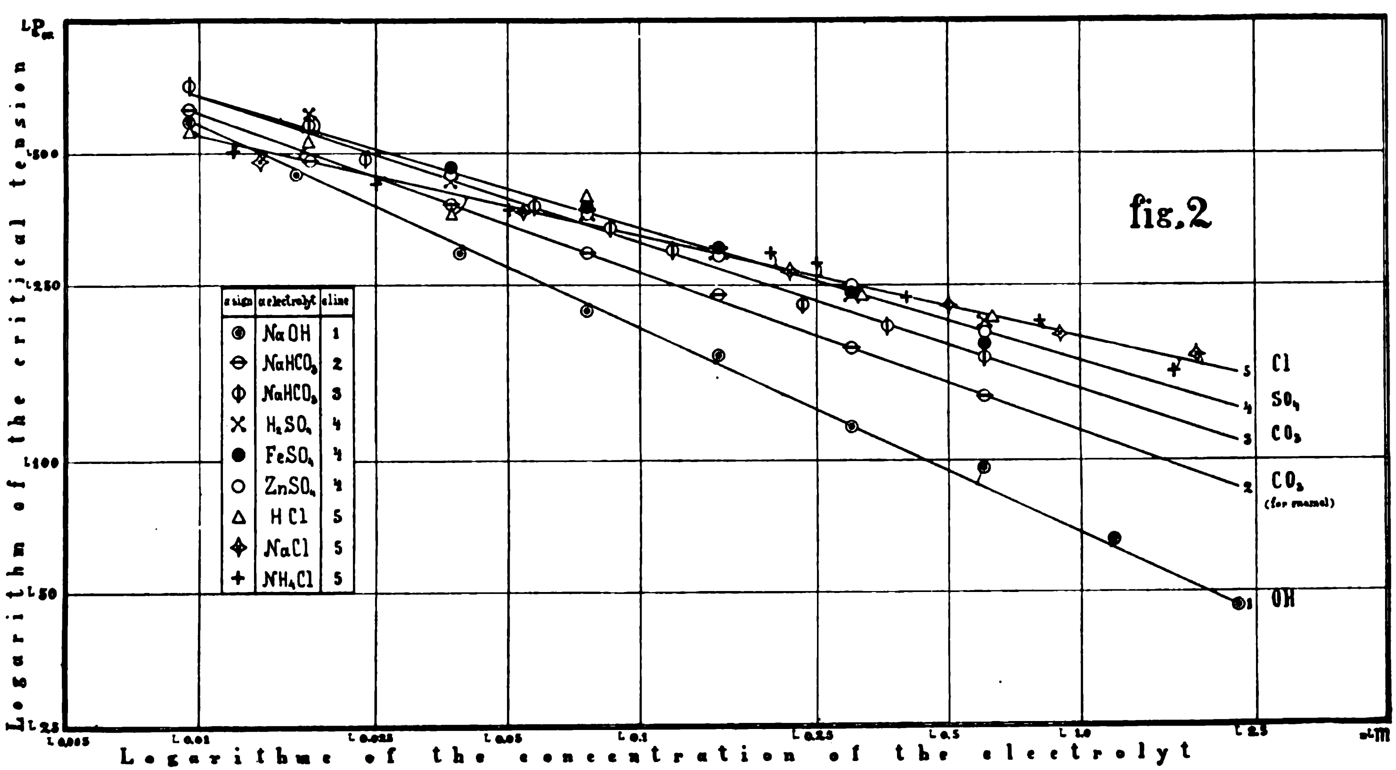
fig. 2
Logarithm of the critical tension
^LP_m
L 500
L 250
L 100
L 50
L 25
Logarithme of the concentration of the electrolyt
L 0.005
L 0.01
L 0.025
L 0.05
L 0.1
L 0.25
L 0.5
L 1.0
L 2.5
m
sign
electrolyt
line
NaOH 1
NaHCO$_3$ 2
NaHCO$_3$ 3
H$_2$SO$_4$ 4
FeSO$_4$ 4
ZnSO$_4$ 4
HCl 5
NaCl 5
NH$_4$Cl 5
Cl 5
SO$_4$ 4
CO$_3$ 3
CO$_3$ (for manual) 2
OH 1

Fig. 2.

Hence it may be concluded that the value of the critical tension does *not depend* on the number of free ions, but depends on the full amount of the dissolved electrolyte. Evidently, the processes at the anode are extremely violent; at its surface new masses of the electrolyte are continually and rapidly dissociated, depending upon the manner in which the discharge takes place.

Upon the number of free ions depends, it is true, the decrease of tension in the solution, but as the critical tension attains hundreds of volts, the decrease of tension in the solution may evidently be neglected, as compared with the decrease of tension in the close proximity of the anode.

(*d*) As regards the mutual relations of the different characteristics (for the different anions), the following peculiarities are clearly noticeable:

The stronger the acid character of the anion, the less is the inclination of the corresponding straight line on the logarithmic paper (the less is β) and, on the other hand, the higher it intersects the ordinate "$\log m = 0$" ($m = 1$) (the greater is α).

So, from the observations, the greatest α (the smallest β) corresponds to the anion $(C\overset{-}{l})$, the smallest α (the greatest β) to the anion $(O\overset{-}{H})$.

Table II.

No. of a Line.	Anion.	α.	β.
1	$(OH)^-$	68	0.460
3	$(HCO_3)^-$	141	0.336
4	$(SO_4{}^{--}$	165	0.304
5	$(Cl)^-$	186	0.230

III. The Dependence of the Height of Critical Tension on the Composition of the Substance in Contact with the Wire.

To the ends of the wires were fused drops of different sorts of glass, of enamel, of jasper, etc. The following results were obtained:

(*a*) The critical tension does not depend on the degree of fusibility or infusibility of the glass in contact with the wire, which was tested by experiments with *six* sorts of glass, from the most fusible to the most insusceptible to fusing.[1]

(*b*) A slight coloring of the glass (transparent glass) does not change the critical tension appreciably. A considerable admixture of metallic oxides (enamel), however, affects the character of the discharge: α decreases considerably, β slightly increases.

For an example the characteristic has been set down on the diagram

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[1] Not to obscure the drawing, the points obtained from these experiments are not marked on the straight line.

(N2), corresponding to the little drops of enamel (dark blue or sky blue) fused to the ends of the fine wires immersed in the solution of bicarbonate of soda ($NaHCO_3$). For this case we obtained $\alpha = 114$; $\beta = 0.34$; while for glass drops, $\alpha = 141$; $\beta = 0.336$ (N3). Similar results were obtained from the experiments with drops of jasper.

(*c*) When the wires come into contact with such a crystal as does not possess electrolytic, but metallic[1] conducting properties, our characteristic discharge is not observed.

IV. MICROPHOTOGRAPHS.

In order to be able to determine in some measure the form of the discharge in question, which is not seen by the naked eye, microphotographs were taken. As the strength of the light of the discharge, not withstanding its comparatively great intensity, is still insufficient to give a highly magnified representation in ordinary microphotographic chambers, it was necessary to take the photograph in a somewhat different way. The object was placed within the range of a small microscope (maximum linear magnifying power—60 times). The weakest object glass was used, giving a full linear magnification of 20 times. After the microscope (with external light) had been focused, the eyeglass was unscrewed, and on the diaphragm beneath it (just at the spot where the real image is formed, was placed (with red light) a small square photographic plate (1.5 cm. × 1.5 cm.). The current was turned on (for an instant), and the discharged flash was recorded on the plate, magnified, for example, 5 times. From the miniature negative thus obtained, the final enlargement was then made with the assistance of a projecting lantern.

The first photograph (Fig. 3) shows a discharge at the point of contact of the fine wire with the drop of glass fused to it and immersed in the electrolyte. The discharge encircles the wire with a hoop of light (the points of light on the surface of the drop are only gleams from the refracted and reflected light illuminating the drop).

From the drop a fine hair of glass attached to the wire was drawn upwards along the wire. On the photograph it may be seen that a discharge took place on it also.

The second (Fig. 4) photograph represents a part of the aluminum electrode at critical tension. As the layer of oxides is very thin, it is very rapidly destroyed by the discharges, which spread in rings from the points of incidental rupture. The representation, therefore, is blurred and dull.

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[1] As, for example, rock salt, as shown by A. F. Joffe.

PHYSICAL REVIEW, VOL. XII., SECOND SERIES.
March, 1919.

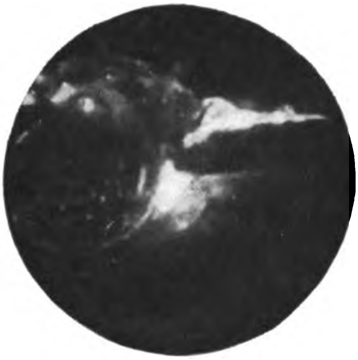

FIG. 3.

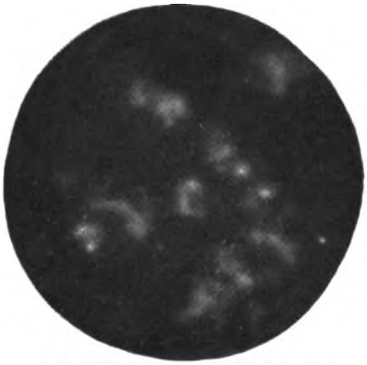

FIG. 4.

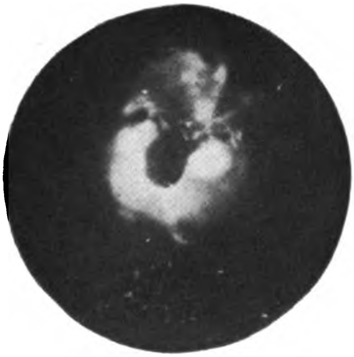

FIG. 5.

WAS. ŠULEJKIN.

Considerably clearer, however, does the discharge appear round the artificial rupture on the *third* photograph (Fig. 5). The black speck in the middle is a bared portion of the metal. In the adjacent layer of oxides (glass!) there was a flashing discharge, encircling the breach like a ring.

From the material just presented some conclusions regarding the nature of the discharge in question may be drawn. First of all, it is apparent that at the critical tension the path of the discharge changes. Namely, at a lower tension its course is "flui$\overset{-}{\text{d}}$—$\overset{+}{\text{f}}$ilm of ga$\overset{-}{\text{s}}$—$\overset{+}{\text{m}}$etal" (A), at a higher tension it is "flui$\overset{-}{\text{d}}$—$\overset{+}{\text{o}}$xide$\overset{-}{\text{s}}$—$\overset{+}{\text{m}}$etal" . . . (B). The decrease in tension in each path is so great that henceforth we shall consider it equal to the tension between the electrodes, ignoring the fall in tension in the fluid on the way from the metal cathode to the film near the anode. Combined with such a great fall in tension is the generation of a considerable amount of heat at the surface of the anode, and it is quite natural to assume that the discharge's *change of path* is caused by the heating of the layer of oxides, which reaches a "critical" degree at "critical tension." Point III_c shows that the discharge may take place only at the surface of a solid electrolyte, but not of a crystal possessing metallic conducting properties (rock salt NaCl). Consequently, an essential condition for the formation of a characteristic discharge (change of discharge from path *A* to path *B*) is the increase of conducting power in a body by heating. But this condition alone is not sufficient. In fact, at the same temperature, the mobility of ions, and consequently the conducting power, for example, with easily fusible glass is greater than with less easily fusible glass.

Still, however, we see from III_a that the height of critical tension does not depend on fusibility.

Let us see what indications the external appearance of the discharge may give. The first striking thing of all to observe is that the discharge flashes out suddenly, with a leap, no intermediate stage being noticeable; the portion of glass adjacent to the wire (or the portion of the oxide layer surrounding the point of rupture) all at once becomes heated to white heat. All this reminds us, as nothing could more clearly, of the electric arc phenomenon. This similarity is especially emphasized also by the appearance of the microphotographs *1* and *3* (Fig. 3 and Fig. 5), and the peculiarity III_b. But even by the microphotographs, taken as they are through a certain thickness of fluid, one cannot definitely be convinced that electric arcs really appeared. As these arcs, if they really exist, can arise only in the interval glas$\overset{-}{\text{s}}$—$\overset{+}{\text{m}}$etal (or oxide$\overset{-}{\text{s}}$—$\overset{+}{\text{m}}$etal), I endeavored to make an experiment of the following character:

At the ends of two platinum (or nickel) wires, placed at a distance of 0.5 to 1 cm. from one another, was fixed a "little bridge" of glass tubing, the ends of which were fused to the wires. If you connect the wires[1] to the source of a constant current with a tension of 100–200 volts and place the "little bridge" over the flame of a Bunsen burner, after some time near the wire-anode will appear bright luminous points.

The points continually move about, going out in one place and appearing in another. Through a magnifying glass it may be distinctly seen that these are miniature electric arcs, seldom reaching the length of 1 mm. If we examine the glass "bridge," after such a discharge, it will be seen that the glass around the wire-anode has acquired a spongy construction. Inside the empty spaces which have been formed move the arcs, which, as it were, "lick" the glass.

If the current is not switched off, but only the heating by the gas flame is discontinued, the arcs for some time do not go out, but the smallest movement of the air is sufficient to extinguish them, after which, without outside heating, they do not appear again.

Now, it seems, we may pass on to an explanation of the critical point of the discharge in question.

The phenomenon, most probably, proceeds thus:

Under the influence of the discharges between the fluid and the metal anode, the layer of glass adjacent to the wire is heated, its conducting power is increased, and a part of the current follows the course of $\text{flui}\overset{-}{\text{d}}$—$\overset{+}{\text{g}}\text{las}\overset{-}{\text{s}}$—$\overset{+}{\text{m}}\text{etal}$. But, as shown by Nernst,[2] in the glass near the anode a rapid evolution of gas takes place, which "polarizes" the system, separates the glass from the metal. Therefore, at a tension below "critical," the current within the glass may freely be ignored; the discharge proceeds, practically, by way of the $\text{flui}\overset{-}{\text{d}}$—$\overset{+}{\text{m}}\text{etal}$.

But if the tension is increased more and more, the edges of the glass at last become heated to such an extent that they begin to emit electrons. At this moment the arc appears between the glass and the metal. The tension, accordingly, at which the arc-path, $\text{flui}\overset{-}{\text{d}}$—$\overset{+}{\text{g}}\text{las}\overset{-}{\text{s}}$—$\overset{+}{\text{m}}\text{etal}$, is formed is also the critical tension.

For a sufficient heating of the layer of glass, let there be in its proximity, in the gas separating the fluid from the metal, a generation, under the influence of the discharges, of W joules of heat per second to the unit of surface.

The heat energy of the discharges depends on the fall of tension, P, and on the number of passing ions, q.

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[1] Essentially in accordance with a rheostat of about 100 Ω.

[2] Ann. d. Phys.

This latter quantity we cannot determine directly; neither can we judge it by the indications of an ammeter, as its pointer oscillates irregularly; but some observations of the *appearance* of the discharges between the liquid and the metallic wire give reason to *assume* that q is proportionate to P[1] and m.[2]

So we suppose:

$$q = k_1 mP, \tag{4}$$

where k_1 is a coefficient of proportionality.

The fundamental condition $W =$ const. may be set down thus:

$$W = k_2 \cdot P \cdot q = k_1 \cdot k_2 \cdot mP^2 = \text{const} \tag{5}$$

or

$$mP^2 = \text{const}$$

Hence:

$$P_{cr} = \frac{\alpha}{m^{0.5}} \tag{6}$$

where a is constant.

If we remember our characteristics of discharges, we see that, for example, the dependence of P_{crit} on m for the system glass—metal immersed in a solution NaOH is expressed thus. Instead of an exponent of 0.5, we have only $\beta = 0.46$.

Why do all the characteristics, differing from each other in quantity, present nevertheless such remarkable simplicity? Why is there always between P_{crit} and m such simple gradual dependence, notwithstanding the fact that α and β for different substances are different? Our assumption led us to the exponent $\beta = 0.5$; in its turn Fig. 2 and formula (3) perhaps, will give some indication of how *in reality* the ions of the fluid near the surface of a solid electrolyte move.

Conclusion.

1. The behavior of the aluminum anode *up to* critical tension is explained by a *kinetic*, but not a static equilibrium. In the layer of oxides new ruptures are continually being formed, while the old ones close up.

2. At tensions *above* the critical, the equilibrium is disturbed by the fact that the layer of oxides adjacent to each rupture begins to take part in the discharge, which from the path "$\overset{-}{\text{fluid}}$—$\overset{+}{\text{metal}}$" passes over to "$\overset{-}{\text{fluid}}$—$\overset{+\quad -}{\text{oxides}}$—$\overset{+}{\text{metal}}$."

As in the first case (A), on the way from fluid to metal, so also in the second (B), on the way from oxides to metal, the discharge takes place in a thin film of gas (oxygen). In the latter case the discharge has the form of an electric arc, heating the edge of the layer of oxides (cathode of the arc) to white heat, and rapidly destroying it.

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[1] It is easy to observe the increase in number of the cluster of discharges with the increase in P or m.

[2] With regard to m, see II_6.

3. Such a form of discharge may be produced also artificially. To produce it, a metal conductor (anode), touching *below the surface of the fluid* the surface of glass, marble, enamel, or other so-called solid electrolyte, must be immersed in the electrolyte.

4. The critical tension at which this discharge begins, depends on the concentration of the solution, and on the composition of the anion, the dependence between P_{crit} and m for all the solutions experimented with being expressed by the formula:

$$P_{cr} = \frac{\alpha}{m^{\beta}}. \tag{3}$$

For the dependence of α and β on the character of the anion Table II.

P_{crit} does not depend on the degree of dissociation.

5. In this article it has been possible to explain only roughly the reason for this simple dependence so clearly obtained from the experiments. For the investigation of the actual movement of the ions near the electrode, it may, perhaps, serve as a thread for persons more competent in questions of electro-chemistry.

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HIGHER TECHNICAL SCHOOL,
MOSCOW, May, 1918.

MAGNETO-STRICTION WITH SPECIAL REFERENCE TO PURE COBALT.

By Howard A. Pidgeon.

Part I. The Wiedemann Effect.[1]

The inter-relations between magnetization and mechanical strain in the ferro-magnetic metals are widely varied in their nature and some of them are extremely complex in character. Their study has formed such a wide field of investigation that no attempt will be made here to more than briefly outline some of the more salient features of the extensive literature on the subject.[2]

The Joule Effect.

In 1847 Joule[3] found that the length of a soft-iron bar was slightly increased when magnetized, and at about the same time Matteucci[4] discovered that the longitudinal magnetism in an iron rod was increased by the application of a longitudinal pull. It was later discovered by Vallari[5] that this effect is reversed in sufficiently strong magnetic fields, and is commonly known as the Vallari reversal. The reciprocal relations between magnetization and mechanical strain discovered by Joule and Matteucci were shown by the work of later investigators to extend throughout the field of magneto-striction and have been dealt with from theoretical considerations by J. J. Thomson,[6] Kirchhoff,[7] Heydweiller,[8] Gans,[9] Houstoun[10] and others, who have derived mathematical relations for these reciprocal relations, which, however, have been only very roughly verified by data,[1] due no doubt largely at least to magnetic and elastic hysteresis which play a very important role in all magneto-elastic phenomena.

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[1] This is the first of a series of articles in preparation under the general title given above.

[2] For a fuller discussion of these effects the reader is referred to the following: Poynting and Thomson, Electricity and Magnetism; Ewing, Magnetic Induction in Iron and Other Metals; Maxwell, Electricity and Magnetism, Vol. II., 3d edition, pp. 90–94; Wiedemann, Electricität, Vol. 3, p. 519. A fairly comprehensive bibliography of the subject up to that date is given at the end of an article by H. G. Dorsey, Phys. Rev., Vol. 30, p. 718, 1910.

[3] Phil. Mag., Vol. 30, pp. 76, 225, 1847.

[4] Comptes Rendus, 1847.

[5] Pogg. Ann., 1868.

[6] Thomson, Applications of Dynamics to Physics and Chemistry.

[7] Wied. Annalen, 1885, Vol. 24, p. 52.

[8] A. Heydweiller, Ann. d. Phys., Vol. 11, p. 602, 1903.

[9] R. Gans, Ann. d. Phys., Vol. 13, p. 634, 1904.

[10] Phil. Mag., Vol. 21, p.78,1911.

The first really comprehensive view of the Joule effect, as the change in length due to magnetization is called, is due to the extensive work of Shelford Bidwell[2] who worked with specimens of iron, nickel and cobalt, and also studied the change in the effect produced by the application of longitudinal stress. As a result of his work and also that of more recent investigators, especially of Honda and his co-laborers,[3] the general relations between longitudinal magnetization and strain in iron and nickel have been quite definitely established, and with considerably less certainty in cobalt.

The principal features of the work done may be summarized as follows: soft iron elongates when subjected to a magnetic field of weak or moderate strength, retracts in larger fields and in strong fields becomes shorter than its original length. The effect of pull is to increase the intensity of magnetization in weak or moderate fields, and to decrease it in strong fields. The results for steel are similar, but the initial magnetic elongation is less in general.

In the case of nickel, longitudinal magnetization is always accompanied by a decrease in length which approaches an asymptotic value in strong fields, while the effect of longitudinal tension is to diminish the magnetization for all values of the magnetic field.

In the case of cobalt the results have been far less conclusive. Bidwell[4] experimenting with a short rod of cast cobalt found a decrease in length with increasing magnetic field, followed by a retraction reaching the original length at about 750 gauss, and in still stronger fields the specimen became longer than its original length. Thus we see that the effect in cobalt was found to be the exact reverse of that in iron. This result was later confirmed by Nagaoka and Honda[5] for cast cobalt, but a similar annealed specimen was found to decrease in length gradually and did not reach a maximum shortening even in a magnetic field of 2,000 gauss. It was found as expected that the effect of longitudinal tension upon the cast cobalt was to diminish the magnetization in fields of moderate strength and to increase it in very strong fields, while longitudinal pull always diminished the intensity of magnetization in annealed cobalt.

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[1] Honda and Tereda, Phil. Mag., Vol. 14, 1907, pp. 65–107.

[2] Proc. Roy. Soc., 1886, Vol. 40, pp. 109, 257; Phil. Trans., 1888, Vol. 149, p. 205; Proc. Roy. Soc., 1890, Vol. 47, p. 469.

[3] Phil. Mag., 1898, Vol. 46, p. 261; 1900, Vol. 49, p. 329; 1902, S. 6, Vol. 4, pp. 45, 338, 459, 537; 1903, S. 6, Vol. 6, p. 392; 1905, S. 6, Vol. 10, p. 548, 642.

[4] Phil. Trans., vol. 149.

[5] Phil. Mag., Vol. 4, p. 51, 1902.

Now all of these specimens were undoubtedly of very impure cobalt, so there has remained some doubt as to what the character of the effect would be in specimens of pure cobalt, since it is known that a marked change may be produced by relatively small quantities of impurities. Bidwell gives no analysis of his specimen but makes the statement that it was soft and easily worked which indicates a considerable amount of impurity since pure cobalt is hard and brittle and not easily worked without special heat treatment. Honda,[1] however, gives an analysis of his specimens which contained from four to five per cent. of nickel, more than one per cent. of iron, and a very high carbon content of from 1.38 to 1.64 per cent.

It was shown by the work of Kelvin,[2] Ewing[3] and others that the effect of longitudinal compression upon magnetization is just the reverse of that produced by longitudinal pull, and that the effect of transverse stress upon longitudinal magnetization is just the reverse of that produced by longitudinal stress.

The work of Nagoaka and Honda[4] has shown that magnetization is also attended by a change in volume which is, in general, of a much smaller order than the change in dimensions.

THE WIEDEMANN EFFECT.

Some years previous to Joule's discovery Wiedemann[5] experimenting with iron wires made the interesting discovery that when the specimen was suspended in a very small vertical magnetic field, while at the same time an electric current flowed through the specimen, its free end was observed to twist in such a direction that to an observer looking along the specimen in the direction of the flow of current and also in the direction of the magnetic lines of force, the lines of twist were in the direction of a right-handed screw, that is the twist was positive in direction. The direction of twist was reversed upon the reversal of either the current or the longitudinal field. If the current in the specimen remained constant and the longitudinal field was gradually increased, the twist reached a maximum in fields of from 15 to 36 gauss, gradually decreased in stronger fields until in very strong fields the direction of twist changed and became negative or left handed. This is commonly known as the Wiedemann effect.

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[1] Phil. Mag., S. 6, Vol. 4, p. 48.

[2] Kelvin, Reprint of Papers, Vol. II., pp. 332–407.

[3] *Loc. cit.*, p. 202.

[4] Phil. Mag., S. 5, Vol. 46, p. 261; S. 5, Vol. 49, p. 329.

[5] Electricität, Bd. 3.

Wiedemann also found that the reciprocal relations already referred to hold here too, for if a wire in which a current was flowing was twisted it was found that longitudinal magnetization was developed, or if a specimen was subjected to a longitudinal field and then twisted circular magnetization resulted.

Investigation by Knott,[1] and Nagoaka and Honda[2] with nickel wires showed that the twist was always negative in direction and much larger than in iron.

The inability to draw cobalt into the form of wires until recently proved an almost insurmountable obstacle to the study of the Wiedemann effect in that metal. However, Honda and Shimizu[3] did succeed in making some measurements on two cobalt rods 21 cm. in length and approximately one centimeter in diameter. They found that for cast cobalt the twist in small magnetic fields was in the same direction as for nickel. It reached a maximum in a somewhat higher field and then decreased, finally reversing its direction in very strong fields. The magnitude of maximum twist was found to be approximately the same as that in similar specimens of iron. In the case of annealed cobalt the twist was very small and did not reach a maximum until a field of about 200 gauss was reached; it then gradually decreased but did not reverse even in very strong fields.

Beside the uncertainty due to the use of impure cobalt, there is still an additional uncertainty here because it was found that the curves obtained for the twist in similar rods of iron and nickel varied considerably from those obtained when wires approximately one millimeter in diameter were used. It would therefore seem especially desirable in this case to obtain data using specimens of as nearly pure cobalt as possible. Although there are many other interesting magneto-elastic effects we shall not discuss them here since the Joule and Wiedemann effects are the fundamental ones and the only ones studied in this work.

As the result of a detailed study of his own work and that of others, Kelvin was able to explain many of the magneto-elastic effects on the basis of magnetic æolotropy produced by stress. In the case of iron in moderate fields he showed that the effect of a simple pulling stress is to produce a greater permeability along than across the lines of strain, while a compressional stress has just the opposite effect. Above the Vallari reversal point the effect is reversed. Kelvin further showed that the idea of magnetic æolotropy may be employed to explain the Wiedemann effect and its reciprocal relations. In this case by the super-position of the circular field due to the current in the specimen, upon the longitudinal field, the direction of the resultant field in any given element is a diagonal lying in a plane tangent to the element and making an angle with the transverse plane, which varies from zero at the center to a maximum at the circumference. According to Kelvin's theory this must result (for moderate fields) in an elongation in the direction of the resultant field and a shortening in a direction at right angles to it. Since the change in volume is of a smaller order, there must result a shearing strain in a plane making an angle of 45 degrees with the resultant field and, consequently, this strain will have a component in a direction tangent to the element and in a plane perpendicular to the axis of the wire. Such a strain must result in a twist of the specimen. Applying similar reasoning one can predict the nature of the effect produced by stress upon circular or longitudinal magnetization.

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[1] Trans. Roy. Soc. Edin., Vol. 32, p. 193, 1883.

[2] Phil. Mag., S. 6, Vol. 4, p. 61.

[3] Phil. Mag., S. 6, Vol. 5, p. 650.

Although the conception of magnetic æolotropy is sufficient to explain many of the magneto-elastic effects qualitatively at least, after making due allowance for hysteresis effects, it does not afford a satisfactory explanation of many of the experimental facts.[1] For example, according to this theory the Wiedemann effect may be regarded as only a special case of the Joule effect, and one would accordingly expect to find a comparatively simple relation between the two. However, this is not the case, as has been pointed out by S. R. Williams.[2]

Object of This Work.

The object of this work is two-fold: first, to study the Joule and Wiedemann effects in specimens of pure cobalt wire and if possible to establish a definite relation between the two effects; second, to make a comparative study of these effects in specimens of cobalt, iron and nickel wire previously subjected to exactly the same heat treatment. As has already been indicated, previous investigation of the Wiedemann effect in cobalt has been of such a meager character and all work on cobalt with such impure specimens that it seemed highly desirable to repeat the work with pure specimens now available.

Moreover, as has been pointed out by S. R. Williams,[3] there has been a great lack of coördination in the work done in this field. Much of the experimentation has been upon specimens of whose chemical composition or previous history but little or nothing is known, and since both have a very important influence upon the magneto-elastic effects, much of the data taken has a questionable value so far as making a comparative study is concerned. It is evidently highly desirable that as many as possible of these effects be studied in the same specimens which have previously been subjected to the same heat treatment. It was with this object in view that a considerable portion of this work was undertaken. This paper will deal with the Wiedemann effect in cobalt and for comparison, also in iron and nickel; while a later paper will deal with the Joule effect and a comparative study of these and other magnetic phenomena.

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[1] See Ewing's Magnetic Induction in Iron and Other Metals, pp. 244 and 246.

[2] Phys. Rev., Vol. 32, p. 295.

[3] Phys. Rev., Vol. 34, p. 258.

SPECIMENS.

The specimens of cobalt used were obtained from the laboratory of the School of Mining in Queen's University, through the generosity of Eugene Haanel, director of the Mines Branch, under whose direction H. T. Kalmus and others made an extensive study of cobalt[1] and succeeded not only in producing very pure metal on a commercial basis but also by special heat treatment, in drawing it into wires suitable for many tests otherwise almost impossible. As will be seen from the following analysis given by the School of Mining, the specimens were comparatively pure.

Specimen *A*.

Co = 99.73
Fe = 0.14
Ni = 0.00
C = 9.00
S = 0.019
Si = 0.02

Specimens *B* and *C*.

Co = 98.71
Fe = 1.15
Ni = 0.00
C = 0.039
S = 0.012
P = 0.01
Si = 0.14

TABLE I.

No. of Specimen.	Material.	Condition.	Length in CM.	Diam. in CM.
A	Cobalt	Annealed in hydrogen.........	25.02	0.0884
B	Cobalt	Unannealed..................	24.92	0.0914
C	Cobalt	Annealed in hydrogen.........	25.06	0.0912
E	Nickel	Annealed in hydrogen.........	25.01	0.1003
F	Nickel	Unannealed..................	25.10	0.1002
H	Iron	Annealed in hydrogen.........	25.00	0.1023

The nickel specimens were from wire labelled "pure nickel" and the iron from the core of an old induction coil, so although no chemical analysis has thus far been made they are undoubtedly of fairly pure material. All of the specimens tested were approximately 25 centimeters in length and varied from 0.85 mm. to 1.00 mm. in diameter. The exact dimensions are given in Table I.

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[1] See Bulletin entitled, The Physical Properties of the Metal Cobalt, Canadian Department of Mines, Part II., by H. T. Kalmus and C. Harper.

Specimens A, C, E and H were annealed at 800 degrees centigrade for three hours, after which the temperature was very slowly reduced to that of the room, the process occupying several hours. Oxidation was prevented by keeping a stream of hydrogen flowing through the furnace. After this treatment the iron and nickel were extremely soft and flexible but the cobalt was still hard and showed a decidedly crystalline structure. Cobalt specimen B has the same composition as C but was left in the original condition. Nickel specimen F is the same as E except that it was not annealed. Data was also obtained from specimens of iron and nickel annealed by alternating current, but the results are not given as they did not differ materially from those for specimens E and H.

Apparatus.

The arrangement of apparatus is shown diagrammatically in Fig. 1. The solenoid, *S*, was attached to the plank, P_1, supported firmly between two massive brick pillars about three feet apart. The specimen, *W*, was soldered to two brass rod extensions and the whole suspended by means of the knife-edge support, *O*, from the plank, P_2, which was supported by the brick piers entirely independent of the plank, P_1. This was to guard against any possible disturbance due to mechanical displacement produced by the field in the solenoid.

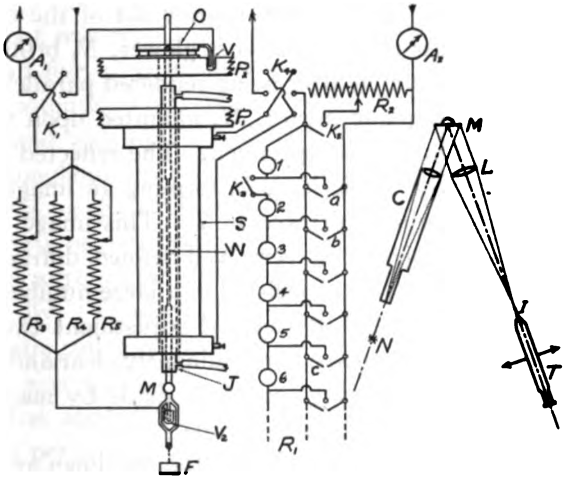

Fig. 1.

The earth's field was determined and the vertical component compensated for by an additional winding on the outside of the solenoid. As it was found in some cases that the rise in temperature of the specimen due to the heating of the solenoid produced quite an appreciable effect upon the readings, a double-walled water jacket, J, made of brass tubing was placed inside the solenoid and water from a hydrant kept flowing through it during the experiment. Electrical connection with the specimen was made by means of the mercury cups, V_1 and V_2, into which dipped copper contacts attached to the specimen. By means of a special construction illustrated in the figure, a small weight, F, could be suspended from the specimen without interfering with its freedom of motion. The weight was made just large enough to keep the wire accurately vertical in the central line of the solenoid, and to prevent the weight from vibrating it was suspended in oil at the end of a long flexible cord. The entire suspended system was enclosed to prevent the disturbing influence of currents of air in the room.

As it was desired to measure angles varying from zero to considerably more than one degree in magnitude with an accuracy of at least one second for the smaller angles, a special device was necessary for the purpose. After trying several methods, the very simple one illustrated in Fig. 1 was chosen. Lengthwise in the fairly narrow slit of the collimator, C, was mounted a fine glass fiber. A Nernst glower, N, brilliantly illuminated the slit, rays from which after being rendered parallel by the collimator lens fell upon the plane mirror, M, mounted upon the brass rod attached to the lower end of the specimen. The reflected parallel rays were focussed by an achromatic lens, L, forming an image at I, which was viewed by the traveling microscope, T. This image when viewed through the microscope showed a very well-defined diffraction pattern produced by the double slit, and consisted of a large number of alternate light and dark parallel bands. By special adjustment two or three of these lines could be made to appear predominantly clear and well defined, and upon these very good settings could be made by means of double cross-hairs in the eye-piece of the microscope.

The system was calibrated by replacing the specimen and mirror, M, with another mirror mounted in the axis of a lever approximately 65 cm. in length moved by a micrometer screw. It was found that one division of the micrometer screw of the microscope corresponded to approximately one second of twist of the specimen. By taking an average of two or three settings it was possible to set within one half division, corresponding to $0.5''$ of arc or $0.02''$ per centimeter length of specimen, for small angles. With larger deflections the image became less distinct so that the accuracy of setting was not quite as great but in all cases amply sufficient. Considerable difficulty in obtaining measurements was experienced due to vibrations in the building or produced by passing street cars, so that it was found necessary to take data at night when traffic was reduced.

A diagram of the electrical connections is shown in Fig. 1. A special form of rheostat, R_1, made of carbon lamps was selected, which although it did not have the ease of manipulation nor advantage of continuous change of resistance possessed by a liquid or slide-wire rheostat, had the very decided advantage of certainty of setting permitting readings to be easily repeated, which was very desirable in this work. The rheostat was so constructed that all the lamps could easily be placed in series, in parallel, or in various combinations of lamps in parallel with a group of lamps. For example, with switches a, b and c closed, lamp *1* was connected directly across the line in parallel with *3*, *4* and *5* in series. This arrangement made it possible to increase the current from zero to a maximum by as few or as many steps as desired. The adjustable rheostat, R_2, was connected in parallel with R_1 to obtain the higher values of current.

Current through the specimen was controlled by means of two field rheostats, R_3 and R_4, in parallel with a slide wire rheostat, R_5. The latter provided for fine adjustment necessary to keep the current constant during a run.

The current was read by means of Weston milli-voltmeters provided with one and five ampere shunts. They were frequently calibrated by means of a potentiometer, standard cell and standard resistances.

The solenoid was 38 cm. in length and hence the field was not quite uniform throughout the 25 cm. length of the specimen. From approximate dimensions the variation from the maximum, of the average field over the length of the specimen was computed. As no accurate data for the solenoid were obtainable it was necessary to determine its constant experimentally. This was done by means of a ballistic galvanometer and a ballistic coil whose dimensions were accurately known. The galvanometer was calibrated by means of a standard condenser. The constant after applying the correction indicated above was found to be 210.5 gauss per square cm. per ampere. A later determination using a mutual inductance instead of the condenser gave a result differing from that given above by only a small fraction of one per cent.

Method of Observation.

The specimen was thoroughly demagnetized before each run by gradually decreasing an alternating current flowing through the solenoid, which process was repeated several times with decreasing voltages applied. Demagnetization by less frequent reversals made by hand and also by a mechanical commutator constructed for the purpose, was also tried and as no difference was observed it was concluded that the demagnetization by A.C. was complete in most cases. (Certain exceptions will be noted later.)

Keeping the circular field constant four runs were made as follows: both longitudinal and circular fields direct; both reversed; circular field direct, longitudinal field reversed; circular field reversed, longitudinal field direct. In many cases one or more repetitions of these runs were made. When both fields were direct the relation of circular and longitudinal fields and of twist for iron, was right handed or positive according to the convention previously mentioned.

DATA AND RESULTS.

Examination of the plotted results showed that these four runs gave two distinct curves, one when both fields were direct or reversed, the other when either one was reversed. Which one of the curves was obtained evidently depended upon the direction of twist in the specimen since the agreement between the curves of each pair seems to eliminate other possibilities. This lack of agreement has been noticed by other investigators and ascribed to various causes, consequently a study of this effect was made to determine its origin.

Curves *1* and *2*, Fig. 2, show typical results for cobalt specimen A, when both fields are direct and when one is reversed respectively. Curve *3* is the average of curves *1* and *2*.

Curves *5*, *6* and *7* show the same thing for iron specimen H; and curves *9*, *10* and *11* for nickel specimen E.

It was found in the case of every specimen that after careful demagnetization the application of either the longitudinal or circular field alone produced a small twist, whose value varied with different specimens and the magnitude of the magnetizing field. The initial twist due to the circular field was, however, extremely small for most of the specimens. The maximum value of the twist produced by the longitudinal field varied from 0.18″ to 4.33″ per cm. in different specimens and, although its value varied somewhat with different runs on the same specimen, it was always in the same direction and seemed to be practically independent of the previous condition of magnetization or of demagnetization so long as the latter was reasonably thorough. In general the twist was greater in unannealed than in annealed specimens of the same material. Curves *4*, *8* and *12* show average results for these correction curves for specimens A, H and E respectively.

In the case of nickel the initial twist due to the circular field varied quite erratically with different runs both in magnitude and direction. However, if the twist, as measured from the original zero when neither field was operating, be plotted from the correction curve obtained when the longitudinal field alone was operating, as a new zero axis of twist the resulting corrected curves almost coincide. An example of the result of this operation is shown in Fig. 2 by the double row of dots, obtained

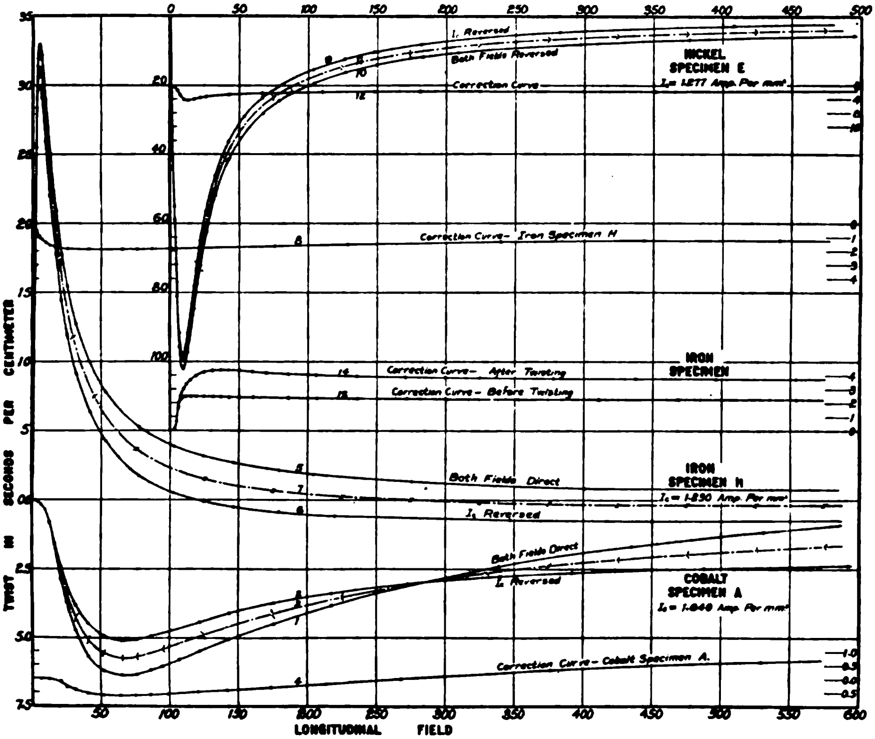

Fig. 2.

by applying curve *12* as a correction to curves *9* and *10*. It is seen the dots almost coincide with curve *11* which is the average of curves *9* and *10*. In some cases there was some divergence near the peaks of the curves but the agreement was always good for higher values of the longitudinal field.

Treating the data for iron in the same manner gave even better results, an example of which is shown by the dots almost coinciding with curve *7*, obtained by applying curve *8* as a correction to curves *5* and *6*.

In the case of cobalt, however, the best agreement was obtained by measuring the twist not from the initial zero but from that obtained after the application of the circular field.

An analysis of these rather complex results indicates that the lack of symmetry in twist is apparently due to various combinations of the three following factors: imperfect demagnetization, change in temperature of the specimen due to the heating effect of the current in it, and æolotropic structure of the specimen.

Imperfect demagnetization was in evidence in the annealed nickel specimens only, which apparently accounts for the initial twist of erratic character due to circular field. This is not at all surprising in this case since for very small values of the longitudinal field, not only is the susceptibility relatively high making perfect demagnetization difficult, but the twist is also very large, thus greatly magnifying the effect of even an extremely small amount of residual magnetism. Indeed, it may be added that this method affords an extremely delicate means of detecting residual magnetism in annealed nickel. No such effect is observed in hard drawn nickel since both the initial susceptibility and the twist are very low for small values of the longitudinal field.

The fact that this initial twist in cobalt and iron due to the circular field alone, was always in the same direction and of about the same magnitude for any given specimen seems to indicate that if residual magnetism were present it was so small that its effect could not be observed. The observed twist was evidently due to heating and æolotropy in these cases.

Twist caused by either of these two effects alone could be distinguished by the fact that æolotropy produces a true magnetic twist for which correction is necessary as in the case of iron; while the effect of heating is merely to change the zero of twist by producing a shift in the equilibrium configuration of the specimen allowing a partial adjustment of residual strains. Such was the case with cobalt. However, if both of the above effects be present, the net twist due to circular field is the algebraic sum of the two components, correction for only one of which should be made. Consequently, the application of the corrections in the manner previously described results, in general, in two parallel curves, the intercept between which is twice the variation of the correction actually made for the circular field (it was zero in the case of cobalt) from the component due to æolotropy alone.

Since the corrected curves for iron and cobalt almost coincide as shown by the curves, we conclude that the twist due to circular field in the case of the former is due almost wholly to æolotropy, and to rise of temperature in the case of the latter.

That this initial effect in cobalt is due primarily to change in temperature is further supported by the following facts. Not only did the maximum twist show a very marked increase with rise of temperature, but in one case after making a run at approximately 60 degrees centigrade and demagnetizing, a twist of approximately 4.0″ per cm. was observed on reducing the temperature to approximately 15 degrees.

Furthermore, the initial twist in cobalt seemed to increase approximately as a parabolic function of the current which one would expect if due to the heating effect.

On the other hand the effect of temperature change in iron and nickel was so small as to be almost completely masked by the other factors.

The twist produced in all specimens by the application of the longitudinal field alone is apparently due almost wholly to æolotropy caused perhaps by permanent residual strains sustained in drawing, by subsequent coiling of the wire, etc. As we have seen, such an effect may give a circular component to longitudinal magnetism.

This view is supported by the fact that annealing reduced the effect, and by the following experiment. One end of an iron specimen which

Table II.

Annealed Iron Specimen H. Current, Ic, in Specimen in Amperes per Square Millimeters.

$I_c = 0.1229$.		$I_c = 0.2951$.		$I_c = 0.615$.		$I_c = 1.230$.	
H.	*θ.*	*H.*	*θ.*	*H.*	*θ.*	*H.*	*θ.*
1.50	0.87	1.50	2.50	1.39	8.11	2.93	26.90
2.14	2.32	2.14	6.02	2.14	14.41	4.71	31.39
3.21	3.28	3.21	8.21	3.16	18.12	6.64	30.61
4.28	3.72	4.28	9.16	4.28	19.52	9.33	27.43
5.57	3.74	5.57	9.18	5.56	19.29	12.74	23.49
7.28	3.57	7.28	8.71	7.33	18.01	17.08	19.34
9.21	3.29	9.31	7.96	9.26	16.45	21.41	16.08
11.25	3.01	11.30	7.30	11.30	14.90	25.9	13.61
14.35	2.66	14.35	6.41	14.35	12.94	32.3	10.89
18.42	2.26	18.41	5.43	18.40	10.82	53.3	6.01
23.88	1.83	23.88	4.42	23.92	8.65	77.2	3.49
36.4	1.18	30.4	3.41	30.4	6.76	102.7	2.16
49.5	0.79	49.7	1.89	36.4	5.57	124.5	1.51
69.3	0.50	59.3	1.45	49.7	3.75	146.3	1.06
84.6	0.30	69.4	1.14	59.3	2.92	179.3	0.61
104.8	0.23	84.7	0.81	69.4	2.30	204.1	0.38
133.8	0.15	104.9	0.54	84.7	1.71	275.5	0.00
168.7	0.08	133.9	0.30	104.9	1.19	347	−0.20
196.6	0.07	169	0.16	134.0	0.76	401	−0.30
246.5	0.02	197	0.10	169.1	0.48	450	−0.34
332	0.00	223	0.05	197.0	0.33	501	−0.35
362.5	−.003	275	−0.02	223	0.23	552	−0.34
450.2	−0.05	334	−0.06	274	0.03	608	−0.33
		392	−0.10	334	−0.03	659	−0.31
		452	−0.13	391	−0.10	708	−0.30
				451	−0.13	800	−0.25
						854	−0.22

$I_e = 2.459.$		$I_e = 3.554.$		$I_e = 5.331$	
H.	θ.	H.	θ.	H.	θ.
2.89	24.35	2.5	18.37	2.3	12.04
4.6	33.98	4.1	27.71	4.1	20.32
6.6	37.68	5.6	34.55	5.7	28.31
9.4	37.12	7.7	38.85	7.8	35.42
12.8	33.80	10.4	39.62	10.5	39.50
17.1	29.07	14.0	37.27	14.1	40.52
21.5	24.82	17.4	33.92	17.8	39.23
26.1	21.40	21.1	30.25	21.5	36.17
32.6	17.42	26.5	25.80	27.1	31.95
42.1	13.41	34.6	20.62	35.1	26.35
53.4	10.11	44.1	16.01	44.9	20.87
77.7	6.08	54.2	12.61	66.0	13.53
103.0	3.86	65.7	9.95	88.1	8.98
125.3	2.67	73.0	8.59	106.3	6.76
147.4	1.87	83.0	6.39	126.1	4.86
179.7	1.11	106.2	4.68	154.0	3.01
205.7	0.71	126.0	3.36	190.9	1.55
282	−0.05	153.9	2.10	242.5	0.25
406	−0.59	190.7	1.09	300	−0.60
508	−0.68	242	0.24	353.5	−1.05
616	−0.59	329	−0.56	400	−1.33
715	−0.53	397	−0.92	445	−1.45
864	−0.49	490	−1.05	492	−1.45
		591	−0.99	544	−1.42
		685	−0.90	593	−1.37
		778	−0.80	637	−1.35
				700	−1.28
				779	−1.19

had been tested previously, was twisted through 360 degrees, leaving a permanent twist of 180 degrees. Subsequent tests showed that the average maximum twist was reduced and in strong fields the twist was increased in one direction and decreased in the other. The twist due to either circular or longitudinal field alone was greatly increased, the latter being almost doubled as shown by curves *13* and *14*, Fig. 2. Data for curve *13* were taken before and for curve *14* after twisting.

One peculiarity still remains unexplained. In some cases the corrected curves showed a considerable divergence in the region of maximum twist. It is perhaps due to a difference in the modulus of rigidity for direct and reversed twist, as magnetic twist must always be accompanied by actual mechanical strain since the tangential component of shear due to the magnetic fields does not vary uniformly from the center to the circumference of the wire.

At any rate it seems that one may safely conclude that the average curve for any given specimen gives very closely at least, the single curve that would be obtained if experimental conditions could be better controlled and the specimen freed from accidental eccentricities.

In this work the data and curves shown are the result of selecting data from runs giving nearly average results for twist in the two directions and averaging these results. Where runs could not be found very closely approximating the average result an actual average of the different runs is given.

The results for iron are shown in Table II. and in Fig. 3 for several values of current as indicated. In general characteristics the results agree quite well with those obtained by other observers; in fact they show a striking similarity to results obtained for steel tubes by S. R. Williams.[1] It will be observed that the values of the maximum twist increase roughly proportional to the current in the specimen at first then seemingly approach a maximum. This is what one would expect if the Wiedemann effect is a special case of the Joule effect, since the elongation in iron reaches a maximum in fairly low fields, and consequently with large circular fields the magnitude of the resultant field and its angle with the axis of the wire do not combine to form a condition so favorable for a large twist.

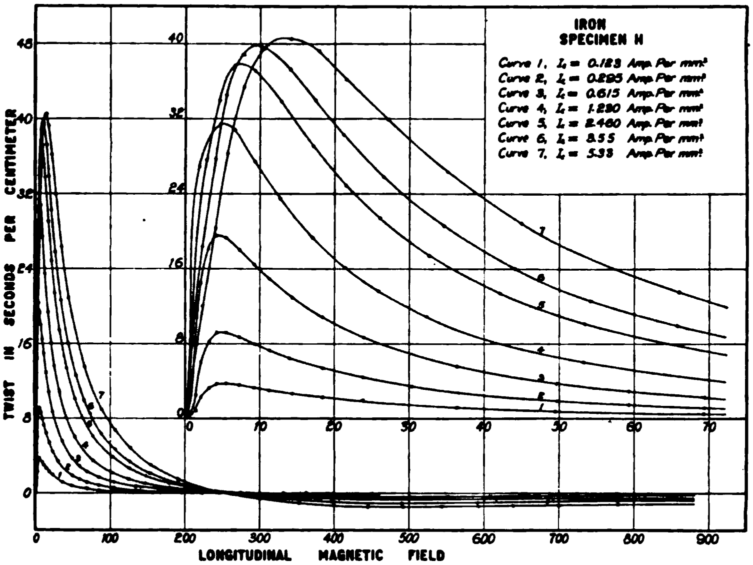

Fig. 3.

The maxima which gradually move into higher longitudinal fields with increase of circular field, are not only very sharp in this specimen but also occur at somewhat lower fields than observed by others. This is doubtless due to the extremely soft, well-annealed condition of the iron, since in another specimen evidently not so well annealed by alternating current, the maxima were broader and came in stronger fields.

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[1] *Loc. cit.*

The curves show rather striking characteristics in stronger fields. Not only do they all reverse their direction of twist at approximately the same value of longitudinal field,—a fact the reason for which is not apparent,—but in still higher fields they reach a maximum negative twist and at the highest values of field obtained, seem to be very gradually approaching the axis of zero twist. So far as the author knows this characteristic has never been previously observed. Apparently in only one other instance[1] has the Wiedemann effect been studied with values of longitudinal field above 400 or 500 gauss, and that was with rods about one centimeter in diameter whose characteristics were quite different. A further study of this feature,—for which the Joule effect offers no apparent explanation,—will be necessary to determine its cause and whether it is characteristic of all well annealed iron specimens.

The results for annealed nickel specimen E are given in Table III.

Table III.

Annealed Nickel Specimen E. Current, Ic, in Specimen in Amperes per Square Millimeters.

$I_c = 0.306$.		$I_c = 0.639$.		$I_c = 1.277$.	
H.	θ.	H.	θ.	H.	θ.
2.9	16.57	2.9	39.2	2.8	56.8
4.5	27.17	4.5	55.0	4.6	79.0
6.5	32.97	6.5	63.8	6.6	93.7
9.2	33.30	9.2	64.6	9.4	99.8
12.7	28.88	12.7	56.5	12.8	94.5
16.86	23.17	17.1	45.8	17.1	81.5
21.3	18.64	21.4	37.4	21.6	69.0
25.7	15.61	25.8	31.5	26.1	59.4
32.1	12.68	32.2	25.6	32.7	49.0
41.7	9.90	41.8	20.1	42.4	39.0
52.9	7.86	53.2	16.07	66.8	25.85
77.0	5.80	77.4	11.39	78.3	22.43
102.5	4.36	102.8	8.81	103.8	17.28
124.3	3.64	124.8	7.34	126.4	14.32
146.0	3.16	146.6	6.23	148.3	12.18
179.7	2.64	180.2	5.12	182.3	9.83
223.2	2.10	224	4.13	216.4	8.40
279	1.80	280	3.33	289.5	6.30
349	1.38	406	2.32	392	4.69
405	1.18	508	1.86	503	3.66
507	1.00	619	1.54	620	3.03
619	0.84	721	1.36	720	2.59
718	0.80	867	1.15	866	2.19
867	0.66				

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[1] Honda and Shimizu, Phil. Mag., Vol. 5, S. 6, p. 650.

$I_t = 2.554$.		$I_t = 3.691$.		$I_t = 5.536$.	
H.	*θ.*	*H.*	*θ.*	*H.*	*θ.*
2.7	62.1	2.9	62.9	2.9	56.6
4.5	90.3	4.6	93.4	4.50	88.8
6.4	113.2	6.5	119.5	6.4	116.9
8.9	130.7	9.3	142.4	9.1	146.7
12.4	137.2	12.7	155.4	12.5	170.5
16.5	132.3	17.0	157.7	16.3	186.2
20.8	121.4	21.5	149.7	21.2	189.9
25.1	109.7	25.9	139.2	25.6	186.3
31.3	94.8	32.3	123.5	32.0	174.8
40.5	77.8	41.8	104.0	41.3	154.8
51.5	63.9	53.1	86.4	52.4	132.4
65.4	51.9	67.1	71.2	76.3	98.0
75.0	46.1	77.2	62.8	101.1	75.7
102.0	34.6	102.4	48.2	122.8	63.0
120.8	29.56	124.3	40.1	144.9	53.8
142.0	25.09	146.1	33.9	174.6	44.39
174.1	20.51	179.6	27.7	220.5	35.30
206.47	17.31	222.7	22.40	275.4	28.35
271	13.16	280	17.77	344	22.72
338	10.54	348	14.28	396	19.60
392	9.06	403	12.36	495	15.67
489	7.28	502	9.86	598	13.07
593	6.03	607	8.29	694	11.40
688	5.22	701	7.08	833	9.40
824	4.33	838	5.95		

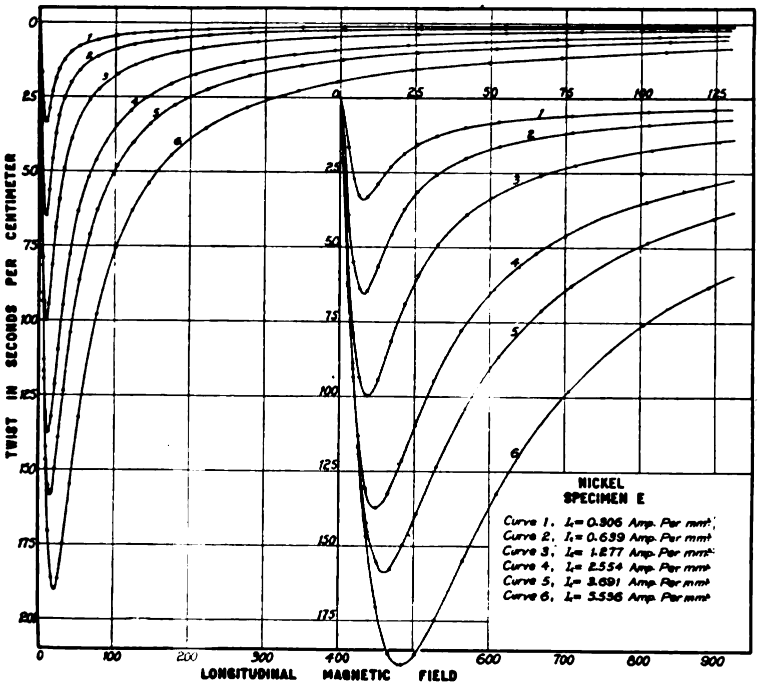

Fig. 4.

TABLE IV.

Unannealed Nickel Specimen F. Current, Ic, in Specimen in Amperes per Square Millimeters.

$I_c = 0.641$.		$I_c = 1.281$.		$I_c = 2.563$.		$I_c = 3.704$.	
H.	θ.	*H.*	θ.	*H.*	θ.	*H.*	θ.
2.7	0.02	2.6	0.02	2.5	0.09	2.7	0.14
4.5	0.05	4.3	0.06	4.3	0.17	4.5	0.235
6.4	0.065	6.3	0.09	6.2	0.24	6.4	0.36
12.6	0.14	8.6	0.15	8.6	0.36	9.1	0.51
21.4	0.32	11.8	0.21	11.8	0.49	16.9	1.10
32.2	0.73	15.7	0.34	19.9	1.06	25.8	2.74
41.8	1.08	19.9	0.50	30.1	2.66	41.7	6.54
53.2	1.44	29.9	1.18	38.9	4.19	53.3	8.30
66.2	1.74	38.7	1.98	49.4	5.50	65.9	9.66
77.3	1.92	49.2	2.68	61.4	6.54	77.2	10.62
88.2	2.03	61.2	3.23	71.9	7.21	102.5	11.63
102.6	2.12	82.0	3.83	82.3	7.68	117.6	11.96
115.8	2.16	95.2	4.06	95.9	8.08	124.2	12.04
124.5	2.17	101.5	4.14	101.7	8.24	136.4	12.07
136.9	2.19	109.0	4.22	109.3	8.37	146.0	12.06
146.5	2.19	115.0	4.26	115.9	8.45	163.9	11.96
164.2	2.17	126.4	4.29	127.4	8.54	179.2	11.78
179.9	2.13	135.1	4.30	136.0	8.56	204.0	11.46
204.5	2.06	151.0	4.28	152.0	8.52	222.2	11.16
223	2.01	165.1	4.22	166.4	8.44	278	10.26
256	1.92	187.6	4.14	188.6	8.26	347	9.18
348	1.65	203.5	4.06	205.5	8.08	400	8.49
404	1.52	232	3.90	237	7.72	450	7.91
505	1.32	252	3.80	257.5	7.48	499	7.40
613	1.16	285	3.60	295	7.07	555	6.90
711	1.04	314	3.42	321	6.80	603	6.51
785	0.98	363	3.18	373	6.28	655	6.12
851	0.92	407	2.96	417	5.88	695	5.84
		455	2.78	464	5.50	760	5.44
		498	2.60	508	5.18	825	5.06
		550	2.44	565	4.82		
		595	2.31	609	4.56		
		640	2.19	660	4.28		
		682	2.08	695	4.12		
		682	2.08	777	3.78		
		765	1.90				

and Fig. 4. They agree remarkably well with results obtained by other observers. It will be observed that for nickel the twist is opposite in direction to that in iron, does not reverse its direction and the maximum value is more than four and one half times as great as in iron. The maximum twist in iron for the highest value of current used is approximately 40.5″ per cm., while for a slightly higher value of current in nickel it is 190″ per cm. As in iron the peaks advance at first slowly, then more

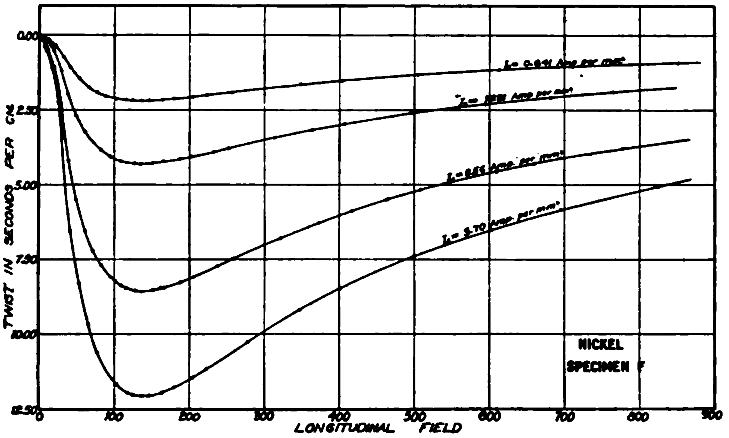

Fig. 5.

Table V.

Annealed Cobalt Specimen A. Current, Ic, in Specimen in Amperes per Square Millimeters.

$I_c = 0.393$.		$I_c = 0.820$.		$I_c = 1.640$.	
H.	*θ.*	*H.*	*θ.*	*H.*	*θ.*
4.3	0.01	4.5	0.06	2.7	0.04
6.4	0.02	6.4	0.10	4.5	0.08
9.0	0.04	9.1	0.22	6.5	0.17
12.6	0.18	12.6	0.41	9.2	0.34
16.9	0.37	16.9	0.83	12.6	0.82
21.2	0.59	21.2	1.29	16.9	1.62
25.6	0.76	25.5	1.69	21.2	2.63
32.0	0.97	31.9	2.13	25.6	3.48
41.6	1.19	41.3	2.55	32.0	4.27
53.0	1.31	52.5	2.84	41.4	5.09
65.9	1.33	65.0	2.94	52.9	5.57
74.5	1.31	76.1	2.92	65.3	5.72
87.2	1.29	86.7	2.86	76.4	5.69
101.5	1.22	100.3	2.77	87.0	5.55
123.1	1.14	121.8	2.58	101.0	5.35
144.7	1.05	143.0	2.39	122.4	4.96
177.5	0.95	175.6	2.18	143.8	4.57
201.9	0.87	216.5	1.91	176.0	4.11
275	0.65	272.5	1.66	217.5	3.62
341	0.55	339	1.41	273.0	3.10
394	0.49	391	1.30	340	2.59
490	0.42	486	1.10	392	2.30
594	0.33	533	1.01	488	1.95
686	0.26	679	0.85	592	1.64
819	0 20	811	0.75	685	1.42
				825	1.13

$I_c = 3.279$.		$I_c = 4.734$.		$I_c = 7.102$.	
H.	θ.	H.	θ.	H.	θ.
2.7	0.10	2.5	0.24	2.5	0.72
4.6	0.20	4.3	0.52	4.3	1.48
6.5	0.43	6.4	0.98	6.4	2.66
9.1	0.95	8.9	1.97	12.5	7.77
12.7	2.03	12.5	3.79	20.1	15.45
16.9	3.92	16.9	6.41	25.5	18.99
21.2	5.80	21.2	9.07	31.8	22.58
25.6	7.25	25.6	11.27	41.2	25.83
32.1	9.06	32.0	13.77	52.3	27.48
41.6	10.63	41.6	15.96	64.8	27.69
52.9	11.55	53.0	17.10	75.8	27.17
65.8	11.83	65.7	17.38	86.5	26.40
77.6	11.70	76.6	17.14	100.5	25.19
87.1	11.46	87.4	16.70	122.1	23.25
95.5	11.19	101.7	16.00	143.8	21.45
101.5	11.00	123.4	14.84	176.4	18.98
123.3	10.20	145.0	13.67	200.5	17.47
144.5	9.45	178.1	12.18	275	13.94
177.5	8.48	202.8	11.26	341	11.76
219.8	7.40	276	9.04	393	10.45
276	6.36	344	7.66	489	8.61
342	5.37	399	6.82	590	7.30
396	4.77	498	5.66	680	6.37
494	4.00	603	4.83	807	5.42
596	3.40	695	4.24		
687	3.00	827	3.57	8.9	4.78
817	2.55			16.8	11.93

rapidly into higher longitudinal fields with increasing circular field. However, the peaks of the curves seem to approach a maximum value of twist much less rapidly than in the case of iron. Here again the sharply defined maxima occurring in comparatively low fields are an indication of good annealing. A specimen of the same wire not so well annealed by alternating current gave curves (not shown) having flatter maxima of less magnitude and occurring in higher longitudinal fields as in the case of iron.

This is very strikingly illustrated by the results for specimen F, shown in Table IV. and Fig. 5, which was a hard drawn nickel wire exactly the same as specimen E except that it was not annealed. Here the broad flat maxima, not only occur in very much higher fields, but have only about one thirteenth the magnitude of the former. It clearly shows the importance in work on magneto-striction of knowing the previous history of the specimen, especially the heat treatment.

In Table V. and Fig. 6 are shown the results for cobalt specimen, A, and in Table VI. and Fig. 7 for the less pure specimen, C. A comparison

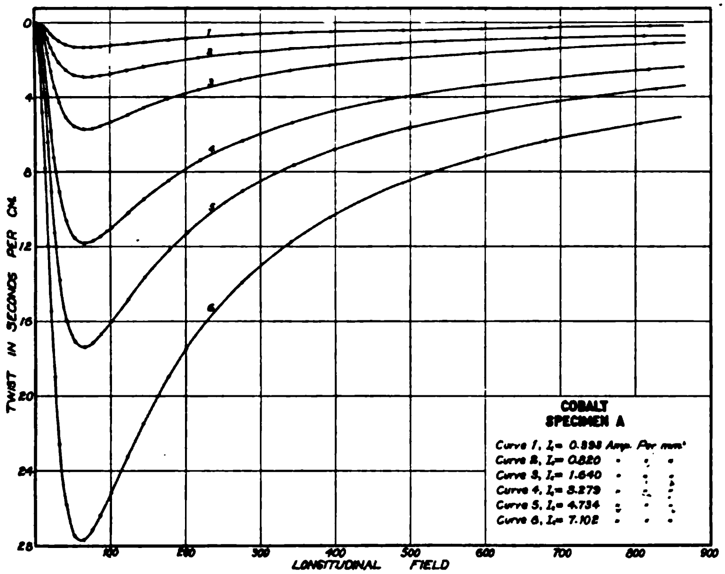

Fig. 6.

Table VI.

Annealed Cobalt Specimen C. Current, Ic, in Specimen in Amperes per Square Millimeters.

$I_c = 0.308$.		$I_c = 0.771$.		$I_c = 1.542$.	
H.	*θ.*	*H.*	*θ.*	*H.*	*θ.*
2.7	0.02	2.7	0.04	2.7	0.04
4.3	0.03	4.3	0.06	4.3	0.10
6.4	0.05	6.4	0.09	6.4	0.17
9.0	0.06	9.0	0.16	9.0	0.31
12.5	0.12	12.54	0.32	12.5	0.65
16.8	0.35	16.90	0.92	16.8	1.92
21.1	0.75	21.2	1.90	21.2	4.01
25.5	1.13	25.5	2.83	25.5	5.78
31.8	1.51	31.8	3.77	31.8	7.61
41.2	1.77	41.2	4.40	41.3	8.92
52.3	1.84	52.4	4.62	52.6	9.22
65.0	1.80	65.1	4.52	65.3	9.02
76.1	1.74	76.3	4.34	76.4	8.63
86.9	1.62	87.0	4.12	87.2	8.21
100.9	1.54	101.2	3.83	101.2	7.66
122.6	1.37	123.1	3.43	123.1	6.84
144.2	1.24	144.7	3.06	144.7	6.14
177.2	1.07	177.5	2.62	177.7	5.28
201.5	0.94	202.1	2.37	202.2	4.74
275.7	0.73	276.2	1.83	276.2	3.60
343.6	0.57	345	1.47	346	2.92
399	0.49	400	1.27	399	2.54
499	0.41	500	1.01	498	2.03
608	0.32	608	0.84	608	1.65
705	0.28	704	0.71	703	1.41
852	0.23	850	0.59	847	1.17

$I_c = 3.083.$		$I_c = 4.451.$		$I_c = 6.677.$	
H.	*θ.*	*H.*	*θ.*	*H.*	*θ.*
2.7	0.12	2.7	0.27	2.7	1.20
4.5	0.23	4.5	0.53	4.5	2.52
6.4	0.42	6.4	0.94	6.4	4.66
9.1	0.80	9.1	1.83	9.08	9.00
12.6	1.78	12.6	4.23	13.5	16.32
16.9	5.00	16.96	10.20	16.9	26.55
21.2	9.33	21.2	16.58	21.2	35.60
25.6	12.93	25.6	21.40	25.5	42.25
31.9	16.68	32.0	25.85	31.85	48.42
41 5	18.53	41.6	28.69	41.3	51.88
52.7	19.07	52.9	29.13	52.4	51.68
65.5	18.57	65.6	28.08	65.1	49.10
76.5	17.72	76.5	26.67	76.3	46.10
97.3	16.81	87.2	25.25	87.0	43.18
101.6	15.62	101.7	23.32	101.1	39.40
123.2	13.93	123.3	20.73	122.9	34.49
144.7	12.46	144.8	18.41	144.4	30.15
177.8	10.64	177.9	15.65	177.1	25.23
210.8	9.27	202.2	14.01	201.6	22.34
276	7.25	276.1	10.51	275.8	16.28
341	5.89	344	8.49	343	12.83
397	5.06	398	7.29	398	10.92
497	4.01	496	5.78	498	8.46
602	3.31	601	4.73	604	6.78
695	2.82	694	4.01	696	5.71
832	2.32	828	3.28	833	4.60

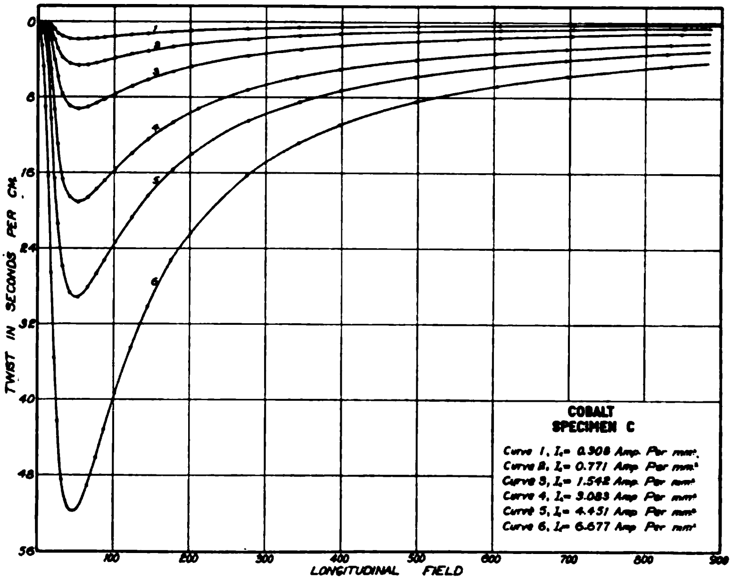

Fig 7.

of the curves shows that the maximum twist for the purer specimen is much less than for the other and occurs at higher values of the longitudinal field. This is not surprising when we consider that the purer cobalt is more highly crystalline and consequently exhibits a greater magnetic hardness and a correspondingly lower susceptibility; properties which as we have seen in the case of nickel, produce exactly this sort of an effect.

These results for pure cobalt differ very materially from those obtained by Honda,[1] for not only is there no reversal of twist in strong fields such as he found for a cast cobalt rod, but the curves show very little resemblance to those he obtained for an annealed cobalt rod. The difference is doubtless due to the difference in purity of cobalt.

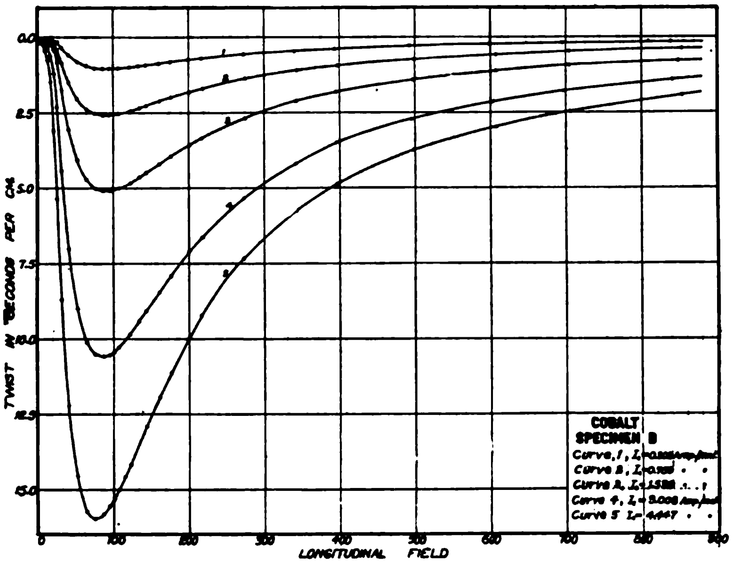

Fig. 8.

Examination of the curves also shows that there is a very marked similarity between the results for cobalt and nickel, making due allowance for the difference in the magnitude of twist in the two cases. In fact, there is actually less difference between the curves for annealed specimens of cobalt and nickel than between those for annealed and unannealed nickel. (It will be shown in a later publication that this similarity holds also for the Joule effect.) This suggests the possibility that, if a more satisfactory method of annealing cobalt could be found reducing it to a less marked crystalline condition or at least to a finer crystalline texture, the results might compare even more favorably with those for nickel. Such a suggestion receives some support from the results for cobalt specimen B, given in Table VII. and Fig. 8. This specimen differs from specimen C, only in not being subjected to further annealing.

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[1] Honda and Shimiza, Phil. Mag., Vol. 5, S. 6, p. 650.

Here again we see the marked effect of annealing and that the result corresponds exactly with that found in iron and nickel. Indeed it may be stated as a general characteristic of all three metals that, the more thorough the annealing the greater is the magnitude of the maximum twist, the sharper the peaks of the curves, and the lower the values of the longitudinal field in which they occur. Now this is exactly the sort of effect that annealing has upon the susceptibility. So, qualitatively

TABLE VII.

Unannealed Cobalt Specimen B. Current, Ic, in Specimen in Amperes per Square Millimeters

$I_c = 0.308$.		$I_c = 0.769$.		$I_c = 1.539$.	
H.	θ.	H.	θ.	H.	θ.
4.3	0.008	2.6.	0.01	2.6	0.02
9.0	0.02	4.4	0.03	4.5	0.04
16.6	0.04	6.4	0.04	6.4	0.06
20.9	0.07	9.0	0.05	9.0	0.08
25.4	0.15	12.4	0.06	12.5	0.13
31.7	0.34	16.7	0.11	16.7	0.21
40.8	0.58	21.0	0.20	21.1	0.40
52.0	0.80	25.4	0.40	25.5	0.82
64.5	0.94	31.8	0.84	31.8	1.74
75.7	1.02	40.97	1.49	40.9	3.05
86.2	1.03	52.1	2.04	52.2	4.07
94.5	1.04	64.7	2 37	64.7	4.71
100.2	1.02	75.9	2.52	76.1	5.01
106.8	1.02	86.5	2.57	86.6	5.09
121.7	0.98	94.8	2.57	95.0	5.08
133.7	0.95	100.5	2.55	100.6	5.05
143.0	0.92	107.1	2.52	107.1	5.00
160.4	0.86	122.1	2.44	122.2	4.84
175.5	0.81	134.1	2.34	134.1	4.64
199.7	0.74	143.7	2.26	145.6	4.49
216.4	0.69	161.2	2.11	161.0	4.20
272.5	0.56	176.2	1.98	176.5	3.95
340	0.45	200.6	1.81	200.6	3.58
393	0.39	218.7	1.69	217.9	3.34
493	0.28	274.5	1.36	275	2.69
599	0.24	343	1.10	342	2.12
695	0.20	397	0.93	395	1.82
839	0.16	498	0.73	497	1.41
		607	0.58	606	1.12
		704	0.47	704	0.93
		853	0.38	849	0.78

$I_c = 3.008$.		$I_c = 4.447$.		$I_c = 3.008$.		$I_c = 4.447$.	
H.	θ.	H.	θ.	H.	θ.	H.	θ.
2.6	0.05	2.6	0.10	106.9	10.27	107.1	15.00
4.4	0.08	4.3	0.18	121.8	9.83	115.3	14.57
6.4	0.13	6.4	0.26	133.8	9.42	122.2	14.17
9.00	0.22	9.0	0.44	143.2	9.07	143.7	12.90
12.4	0.35	12.5	0.75	160.7	8.45	161.2	11.93
16.8	0.61	16.7	1.46	176.0	7.90	176.5	11.12
21.0	1.20	21.0	3.08	200.2	7.11	200.9	9.96
25.5	2.30	25.4	5.35	217.9	6.64	218.7	9.22
31.7	4.42	31.7	8.68	273.6	5.34	275	7.34
40.9	7.02	40.97	12.21	342	4.20	343	5.75
52.1	8.99	52.1	14.55	396	3.50	397	4.87
64.7	10.11	64.7	15.73	495	2.72	498	3.74
75.7	10.50	76.0	15.98	602	2.15	608	2.97
86.5	10.59	86.6	15.82	697	1.79	705	2.46
94.6	10.54	95.0	15.55	841	1.40	854	1.93
100.4	10.42	100.7	15.31				

at least, there seems to be a close connection between the susceptibility and the Wiedemann effect. However, the latter is so complex that the relation is far from being a simple one. It is hoped that further study may bring out more definite relations.

The results for iron specimen H, nickel specimen E, and cobalt specimen C are presented in a somewhat different way in Figs. 9, 10 and 11 respectively, where for several values of the longitudinal field, values of current in the specimen are plotted as abscissæ and the corresponding twist as ordinates. In the case of nickel and cobalt it is seen that for values of the longitudinal field very large compared with the circular

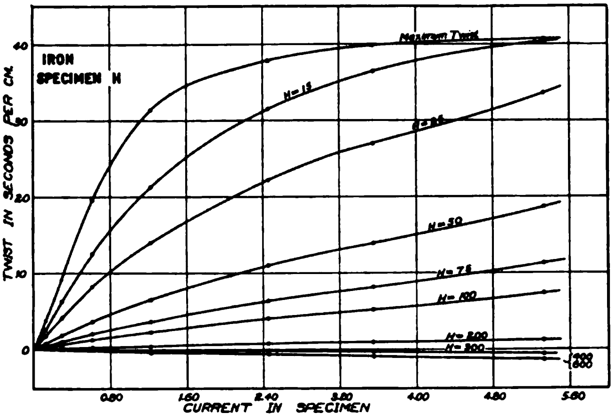

Fig. 9.

field, the twist is approximately proportional to the latter. This is exactly what would be expected if the Wiedemann effect be a modification of the Joule effect. For, let P, Fig. 12, be any particle in a cylindrical element of the wire. The approximate net effect of combined longitudinal and circular magnetization is to displace the particle through the distance λ, in the direction PP' of the resultant magnetic field, making the angle θ with the length of the element in the direction of the axis of the wire. The twist, θ, is given by,

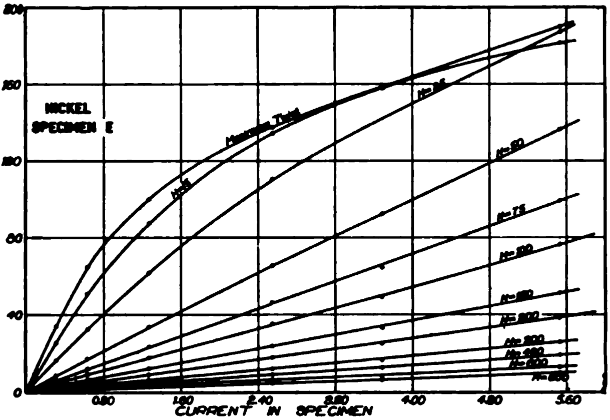

Fig. 10.

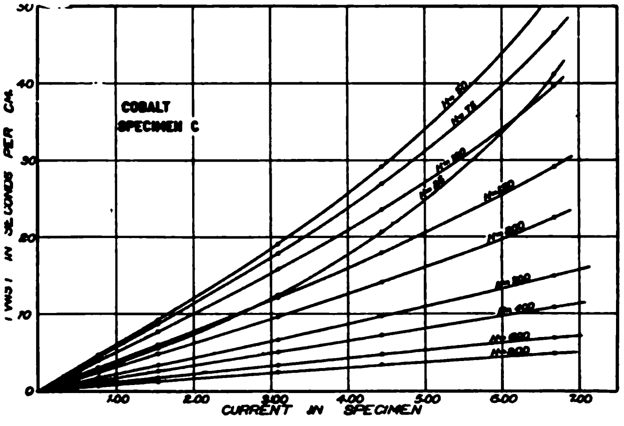

Fig. 11.

$$\theta = \frac{\lambda}{r} \sin \theta$$

$$= \frac{\lambda}{r} \frac{H_c}{\sqrt{H^2 + H_c^2}}$$

where H_c is the circular field, H the longitudinal field and r the distance of the particle from the axis. If H is very large compared to H_c then

$$\theta = \frac{\lambda}{rH} H_c.$$

Previous study of the Joule effect has shown that λ in nickel, reaches practically a constant value in moderately strong magnetic fields. From the equation it is seen that for constant values of H, θ is very nearly a linear function of H_c. This result, derived only for high values of magnetic field in a cylindrical element, at least gives an indication of what may be expected in a wire. For any given value of H_c the twist varies inversely as H, if H is very large, and consequently approaches zero as a limit as H is increased indefinitely. The curves indicate that this is perhaps the case for nickel and cobalt.

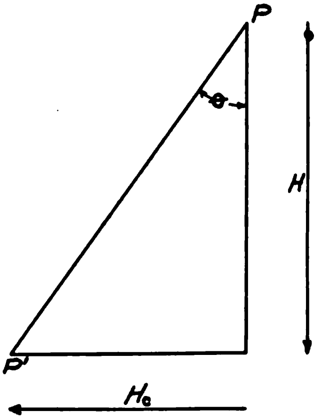

Fig. 12.

These results correspond quite well with what one might predict from the molecular theory of magnetism; for in strong longitudinal fields which magnetize the material almost to saturation, the molecular magnets are almost all aligned in the direction of the field so that the only effect of the circular field is to produce a deflection of these molecular magnets proportional to the deflecting field with a corresponding deformation of the magnetized material.

Cobalt becomes saturated only in much stronger fields than nickel and consequently the agreement is not so good.

In this case as in that of all the magneto-elastic effects in iron, the results are very complex and any attempt to explain them without further study is fruitless.

Any analysis of magneto-elastic effects is made much more difficult by the marked hysteresis occurring in all of them. In Figs. 13, 14 and 15 are shown hysteresis curves for iron, nickel and cobalt respectively, obtained by keeping the circular field constant and cyclically varying the longitudinal field. The comparatively large area inclosed in the hysteresis loop for cobalt indicates its rather extreme magnetic hardness and low susceptibility, which has been previously mentioned.

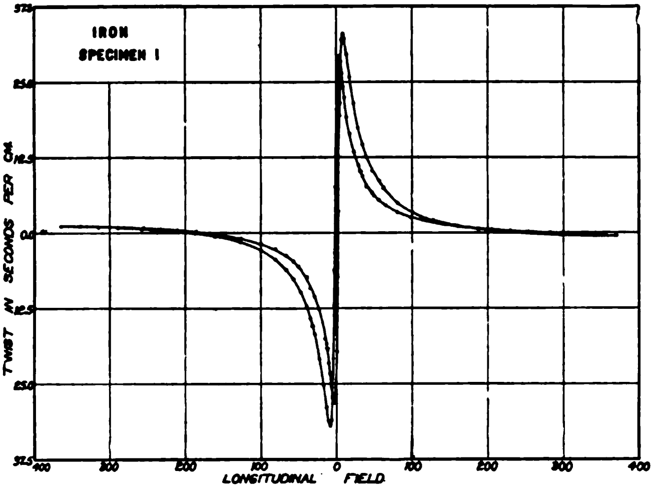

Fig. 13.

Some of the features of the Wiedemann effect will be still further discussed in a later publication in connection with the Joule effect and other magnetic phenomena.

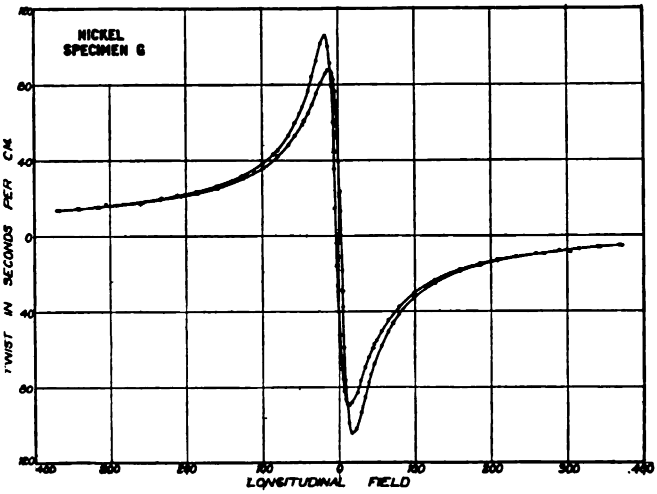

Fig. 14.

Summary.

Experimental curves are shown for the Wiedemann effect in specimens of pure cobalt wire, and for the purpose of comparison, results are also shown for specimens of iron and nickel subjected to the same heat treatment. A comparison of the results is made and certain features of the Wiedemann effect discussed.

The results for pure cobalt differ materially from those obtained by other observers working with impure specimens, and show a close similarity to the results for nickel, the twist, however being much less.

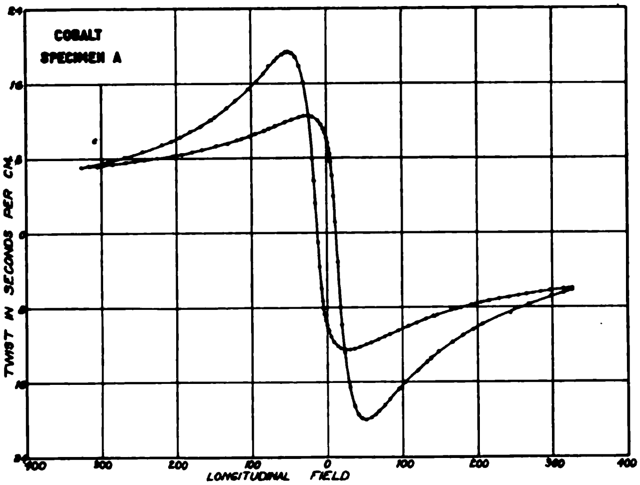

Fig. 15.

Certain eccentricities of twist observed in all specimens are studied and an explanation given which seems to account for nearly all of the observed facts.

Hysteresis curves for iron, nickel and cobalt are shown.

Experimental results for the Joule effect and other magnetic phenomena will be presented in a later publication.

The writer wishes to express his thanks to various members of the department staff for helpful suggestions given, and especially to Professor E. L. Nichols through whom the specimens were obtained and under whose direction the work was done.

Physical Laboratory,
Cornell University,
September, 1918.

ON THE MECHANICAL AND ELECTRODYNAMICAL PROPERTIES OF THE ELECTRON: ADDENDUM.

BY MEGH NAD SAHA.

IT appears that the relation

$$-\mu = -\frac{m_0c^2}{e} = \frac{f_{12}}{\Omega_{12}} = \frac{f_{23}}{\Omega_{23}} = \frac{f_{31}}{\Omega_{31}} = \frac{f_{14}}{\Omega_{14}} = \frac{f_{24}}{\Omega_{24}} = \frac{f_{34}}{\Omega_{34}},$$

which was given in my paper[1] "On the Mechanical and Electrodynamical Properties of the Electron" and which was obtained from the determinant

$$\begin{vmatrix} & f_{12}+\mu\Omega_{12} & f_{13}+\mu\Omega_{13} & f_{14}+\mu\Omega_{14} \\ f_{21}+\mu\Omega_{21} & & f_{23}+\mu\Omega_{23} & f_{24}+\mu\Omega_{24} \\ f_{31}+\mu\Omega_{31} & f_{32}+\mu\Omega_{32} & & f_{34}+\mu\Omega_{34} \\ f_{41}+\mu\Omega_{41} & f_{42}+\mu\Omega_{42} & f_{43}+\mu\Omega_{43} & \end{vmatrix} = 0$$

or

$$(f_{12}+\mu\Omega_{12})(f_{34}+\mu\Omega_{34}) + (f_{23}+\mu\Omega_{23})(f_{14}+\mu\Omega_{14}) + (f_{31}+\mu\Omega_{31})(f_{24}+\mu\Omega_{24}) = 0,$$

cannot hold. For, from the principle of least action written in Minkowski's form,

$$\delta\int m_0c^2ds - \int \delta W\cdot ds = 0,$$

we obtain the relation

$$\int\int[(f_{12}+\mu\Omega_{12})(dx\delta y - \delta x dy)] + 5 \text{ other similar terms} = 0.$$

But it is not possible to equate to zero the coefficients of the six-components $(dx\delta y - \delta x dy)$ of the area-six-vector $(dSx\delta s)$ as was done in that paper, for though $(\delta x, \delta y, \delta z, \delta l)$ represent an arbitrary displacement (dx, dy, dz, dl), is not so, but represent the actual displacements. We have, therefore, to collect the coefficients of $(\delta x, \delta y, \delta z, \delta l)$ and put them separately equal to zero. In this way we obtain

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[*] PHYSICAL REVIEW, January, 1919.

$$-\frac{m_0c^2}{e}=\frac{f_{12}w_2+f_{13}\omega_3+f_{14}\omega_4}{\Omega_{12}\omega_2+\Omega_{13}\omega_3+\Omega_{14}\omega_4}=\frac{f_{21}\omega_1+f_{23}\omega_3+f_{24}\omega_4}{\Omega_{21}\omega_1+\Omega_{23}\omega_3+\Omega_{24}\omega_4}$$

$$=\frac{f_{31}\omega_1+f_{32}\omega_2+f_{34}\omega_4}{\Omega_{31}\omega_1+\Omega_{32}\omega_2+\Omega_{34}\omega_4}=\frac{f_{41}\omega_1+f_{42}\omega_2+f_{43}\omega_3}{\Omega_{41}\omega_1+\Omega_{42}\omega_2+\Omega_{43}\omega_3}, \qquad (A)$$

which are simply another form of the Minkowskian equations

$$m_0c^2\frac{d^2x}{ds^2}=X,\quad m_0c^2\frac{d^2y}{ds^2}=Y,\quad m_0c^2\frac{d^2z}{ds^2}=Z,\quad m_0c^2\frac{d^2l}{ds^2}=L \qquad (A')$$

for

$$\frac{d^2x}{ds^2}=-(\omega_2\Omega_{12}+\omega_3\Omega_{13}+\omega_4\Omega_{14}),\quad \text{etc.}$$

The form (A) as it involves the acceleration six-vector may for certain purposes prove more convenient than the form (A').

NEW BOOKS.

The Theory of the Relativity of Motion. By RICHARD C. TOLMAN. Berkeley: University of California Press, 1917. Pp. ix + 225.

This is an introduction to Einstein's physical theory of relativity and its developments, of a scope about like that of Cunningham's book.

The first chapter gives a historical review, a mere sketch but enough to hint at the development of the physical setting from Galileo and Newton to Michelson and Morley. Chapters two to six give an elementary account of the new kinematics and dynamics of a particle together with the optical matters of aberration, Doppler effect, and Fresnel convection. The material here is about that of Einstein's pioneer papers with one important and undoubtedly wise change, in that the introduction of the concept of mass in connection with phenomena of impact gives direct access to dynamics without requiring reference to particles electrically charged, though the field of a moving charge and the relations of mass and energy are reached as corollaries. This earlier portion of the book, nearly half, uses only elementary mathematical notions.

The rest of the volume is naturally less elementary. Chapters seven to eleven deal with dynamics of systems, some aspects of thermodynamics and statistical mechanics, and elastic bodies. In each case the principle of least action and Lagrange's equations have a fundamental place, in the last case with a frank avowal of the questions left open because of the lack of unique determination of the kinetic potential. Chapter twelve outlines the theory of the electromagnetic field, including brief reference to moving dielectrics. Here the whole development is based on an assumed form for the electromagnetic kinetic potential, giving an elegance of treatment that one would not like to miss. It may be asked however whether it would not have been more appealing to most readers to exhibit first, as Einstein did, the invariance of the Maxwell-Lorentz equations, and then pass over to the variational deduction, giving an order of thought more like that used for the dynamics. The final chapter is an outline of vector analysis in four dimensions, with a few examples of its use in formulating the theory previously given.

The book deserves hearty commendation as an interesting introduction to a subject often considered difficult of access, and will win the reader's thanks after he has pruned away a few obvious blemishes. Some passing remarks are distinctly irritating; for example, the reference to physical "model-making," one of the glories of physics, especially in the nineteenth century, as a "debauch." The reviewer was once reminded however by a biological friend that irritation and stimulation are not sharply distinct.

A. C. L.

Second Series *April, 1919* *Vol. XIII., No. 4*

THE

PHYSICAL REVIEW.

ON THE IRREGULARITIES OF MOTION OF THE FOUCAULT PENDULUM.

BY A. C. LONGDEN.

SYNOPSIS. In a brief historical statement, emphasis is put upon the current opinion that a Foucault pendulum must be very long and very heavy in order to be successful. It is then shown that the elliptical motion so common in Foucault pendulum experiments is not due to insufficient length or weight, or even to atmospheric disturbances, but simply and solely to unequal freedom of motion in different directions. The disturbing influences are within the pendulum itself. Different forms of support and suspension are discussed, and methods of eliminating the disturbing influences are suggested. What the author describes as a double roller support is shown to be superior to any other form of support thus far proposed. With this support and a perfectly annealed double bifilar suspension, a 2½-meter pendulum weighing a single kilogram succeeds perfectly.

THE pendulum experiment of Léon Foucault, by means of which he demonstrated the axial rotation of the earth, is held in reverence by physicists and astronomers, not simply as one of the successful experiments of the nineteenth century, but as one of the brilliant experiments of all time.

Foucault began his work on this problem with a pendulum not more than 2 meters long, but finding the results unsatisfactory, he increased the length to 11 meters, and finally, under the great dome of the Pantheon, to 67 meters. Taking advantage of the height of the great cathedral towers of Europe, the experiment was more or less successfully repeated at Cologne, Reims, Amiens, St. Jacques, and elsewhere, where towers of great height could be made available.

Many attempts have been made to reduce this classic experiment to lecture room dimensions, but by common consent the short wire and small mass have been condemned as unsatisfactory. More than a score of well-known physicists and astronomers are on record as affirming that the Foucault pendulum must be very long and very heavy in order to give satisfactory results—some say, to prevent cross vibrations or to keep the pendulum from falling into an elliptical path. Many insist that it must be set in motion with great care and protected from external disturbances, which, it is said, develop irregularities in its motion. In a recent article in a scientific journal on "A Laboratory Method of Demonstrating the Earth's Rotation," it is stated that the Foucault pendulum method is inapplicable in many laboratories because there is no convenient place to hang a sufficiently long and heavy pendulum.

We shall probably never be quite certain of the exact conditions under which Foucault's celebrated experiment was performed. Our sources of information, in addition to the very brief records of the French Academy of Sciences,[1] are the papers published by Mme. Foucault,[2] ten years after Foucault's death, and the contributions of Lissajous[3] and Flammarion,[4] published still later.

Most of the important information concerning the pendulum of the Pantheon in its final form, is found in one of the papers in Mme. Foucault's "Recuile," entitled "Explications sur l'experience relative au mouvement de la terre." This paper was without date and had not been published by Foucault himself, though he is said to have published a similar paper in le Journal des Debats in March, 1851.

That the work was rushed through in great haste is indicated by Flammarion's statement that it was undertaken in January, 1851, and Mme. Foucault's statement that it was presented to the French Academy of Sciences, February 3 of the same year.

The records are not in perfect agreement concerning the dimensions of the pendulum in its final form, but the evidence is abundant that it consisted of a mass of 28 kilograms, suspended by a steel wire about 67 meters long, strained somewhat beyond its elastic limit. The fact that the wire was greatly strained is important later on.

A few years ago I set up a 60-foot Foucault pendulum consisting of an iron ball weighing about 8 kilograms, suspended by a No. 18 brass wire, from a tolerably good though by no means perfect double knife-edge support. The ball was provided with a spike at the bottom to enable the observer to determine its position, as it moved back and forth over a chart provided with a circular scale calibrated in degrees.

The performances of this pendulum were not very satisfactory and I attributed its partial failure to the presence of air currents in the shaft. However, after trying the pendulum a number of times, I found that its eccentric behavior was repeated over and over again in much the same way. I could not convince myself that such *regular* "irregularities" were accidental, and I was quite sure they were not due to the uncertain shifting of air currents.

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[1] Compte Rendu, t. XXXII., p. 135.

[2] Recuile des Travaux Scientifiques de Léon Foucault. Publié par Madame Veuve Foucault, Sa Mère. Paris, 1878.

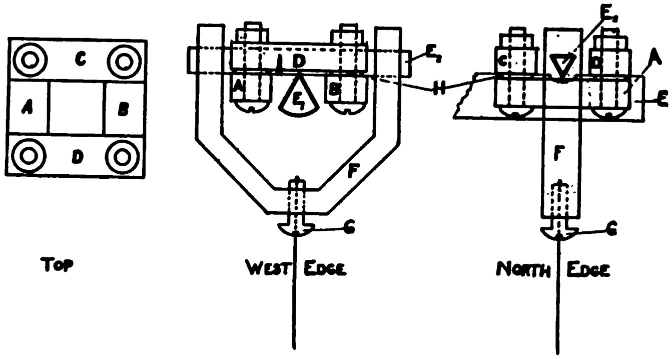
[3] Notice Historique sur la vie et les travaux de Léon Foucault.

[4] Notice scientifique sur la pendule du Panthéon.

For the purpose of making a closer study of the Foucault pendulum and its behavior, I moved this pendulum to the lecture room, suspending the 8 kilogram ball from a more carefully constructed double knife-edge support attached to one of the ceiling beams over the lecture table. The pendulum was now a little more than 2½ meters long. It was started swinging by tying the ball back and burning the thread, in the usual way. It was allowed to swing through an arc of about 10°.

A few preliminary experiments gave me reason to believe that none of the seemingly irregular movements of the pendulum were in any way chance or accidental, but that they could all be produced and reproduced under perfectly definite conditions.

The double knife-edge support used is shown in detail in Figs. 1 and 2.



Figs. 1 and 2.

E_1 and E_2 are the knife-edges, *ABCD*, the rocker, consisting of hardened and polished steel plates bolted together at the corners, and *F*, a brass hanger, carrying the wire in a machine screw, *G*, which facilitates substituting one wire for another.

The upper knife-edge could be raised or lowered by varying the vertical distance between the plates of which the rocker was made. This might be done by means of micrometer screws, but in these experiments it was done by inserting thin metal plates between the upper and lower halves of the rocker, as at *H*, in Figs. 1 and 2.

With this support the behavior was that of a Blackburn pendulum with slightly different periods in two directions at right angles to each other. Since the elliptical motion of a Blackburn pendulum depends upon the ratio of these two periods, it ought to be possible to adjust the distance between the planes of the double knife-edge support in such a way as to get any desired amount of eccentricity, or to eliminate it altogether.

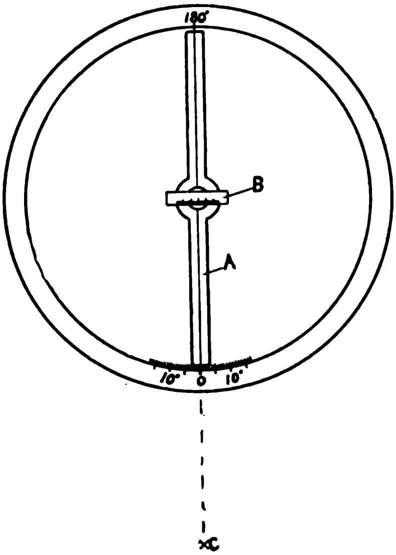


Fig. 3.

In one of the preliminary experiments with the double knife-edge support adjusted at random, the pendulum started on its elliptical path, rotating clockwise as viewed from above; and I noticed that as the minor axis of the ellipse increased in length, the shift of the plane of vibration of the pendulum, as marked by the position of the major axis of the ellipse, increased at an abnormal rate. After verifying this fact by repeated observations, I undertook to investigate the relation of the length of the minor axis of the ellipse, to the position of the major axis.

I placed a movable index, *A*, Fig. 3, upon the circular scale over which the pendulum was swinging and mounted a scale of millimeters, *B*, at right angles to it at the center of the circle. The index consisted of a light wooden bar with a screw running through it at the center, and with a fine black line running lengthwise along the bar from zero to 180° on the scale.

With a pendulum provided with a sharp spike travelling close to the black line on the movable index, and with a suitable lens placed as at

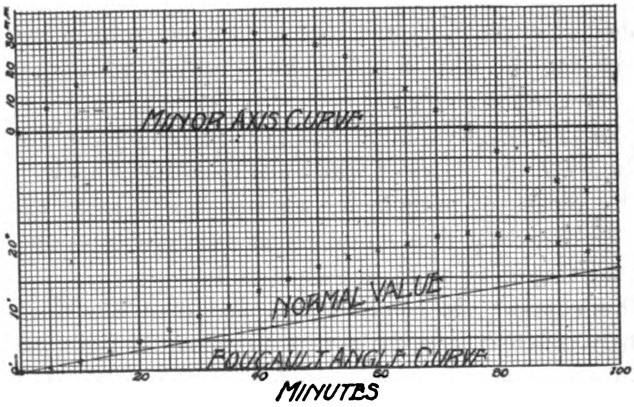


Fig. 4.

C, in Fig. 3, it is always possible to read the length of the minor axis of the ellipse to single millimeters and in many cases to approximate tenths of millimeters; while the position of the major axis can be read to within $\frac{1}{10}$° with ease and certainty, or to $\frac{1}{100}$° approximately.

Starting the pendulum swinging from north to south, with an amplitude of 20 centimeters (an arc of about 9°), I read the length of the minor axis of the ellipse, and the angle between the major axis and the meridian every 5 minutes for an hour and three quarters. The results are tabulated below and plotted in the curves shown in Fig. 4. In the column marked "Direction of Rotation," clockwise or direct is marked "*D*," and counterclockwise or reverse is marked "*R*." In the plot, the minor axis curve is above and the curve representing the position of the major axis in angular displacement from the meridian, and which we shall hereafter call the Foucault angle curve, is below. The length of the minor axis is in millimeters and the Foucault angle is in degrees, on the *Y* axis. Both are in minutes on the *X* axis. The full curves are not actually drawn, because the points plotted are so close together that it was not thought necessary to connect them.

Time in Minutes.	Minor Axis in Mm.	Direction of Rotation.	Foucault Angle in Degrees.
0	0		0
5	8.0	D	.65
10	15.5	"	1.8
15	21.0	"	3.1
20	26.2	"	5.0
25	30.0	"	6.8
30	32.0	"	9.0
35	32.5	"	10.5
40	32.0	"	13.0
45	30.5	"	14.9
50	28.0	"	16.9
55	24.0	"	18.5
60	19.2	"	19.85
65	13.0	"	20.8
70	6.0	"	21.8
75	0		22.2
80	8.0	R	22.1
85	14.0	"	21.6
90	18.0	"	20.3
95	21.5	"	18.9
100	24.0	"	17.7

The minor axis curve is certainly a close approach to a simple sine curve. It probably would be a perfect sine curve if it were not for the fact that the amplitude of the pendulum is diminishing all the time and this affects the length of the minor axis of the ellipse as well as the major axis.

Note that the length of the minor axis increased to 32.5 millimeters in 35 minutes and then diminished to zero at the end of 75 minutes, when it crossed the *X* axis of the plot and the pendulum began to rotate counterclockwise. Note also that while the pendulum was rotating clockwise, the Foucault angle increased abnormally; but that when the pendulum began to rotate counterclockwise, the Foucault angle began to diminish.

The calculated value of the Foucault angle for the latitude of Galesburg is 9.83° per hour. The straight line in the Foucault angle plot, running from the origin to the 9.83° point at the 60-minute line, represents this value. It will be noted that the plotted curve is much above this line most of the time, but that it reaches its greatest height and starts down again when the minor axis curve falls below the *X* axis of the plot, that is, when the pendulum begins to rotate counterclockwise.

At the end of this series of observations the pendulum was accidentally thrown out of adjustment. After readjusting, it was started swinging in the usual way, but this time it started rotating counterclockwise, and continued rotating in that direction during the entire series of observations which followed. That is, it did not complete a half cycle and reverse its direction of rotation as it did in the preceding series.

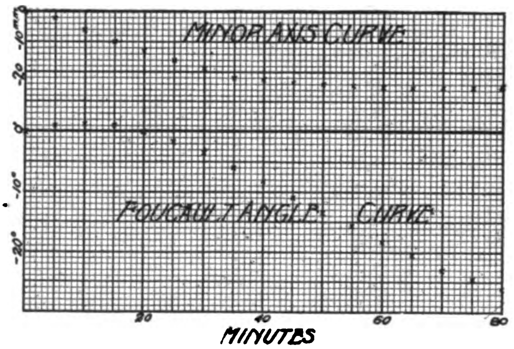


Fig. 5.

The curves for this series are shown in Fig. 5.

Our first glance at these curves failed to reveal the fact that they had anything in common with those in Fig. 4, but further inspection shows that they are alike in the fact that they both show that counterclockwise rotation of the pendulum tends to reverse the direction in which its plane of vibration is shifting.

However, I had in Fig. 4 a part of a cycle in which the rotation of the pendulum was mostly clockwise, and in Fig. 5 a part of a cycle in which the rotation was entirely counterclockwise.

It then occurred to me to shorten the cycle, so that I might have an opportunity to compare the minor axis curve with the Foucault angle curve through several complete cycles, before the amplitude of the pendulum became very greatly reduced. This was easily accomplished by readjusting the distance between the planes of the double knife-edge support, so as to change the length of the pendulum in one direction without changing it in the other.

After adjusting the support for about 3 or 4 cycles per hour, I made the series of observations represented in the curves shown in Fig. 6.

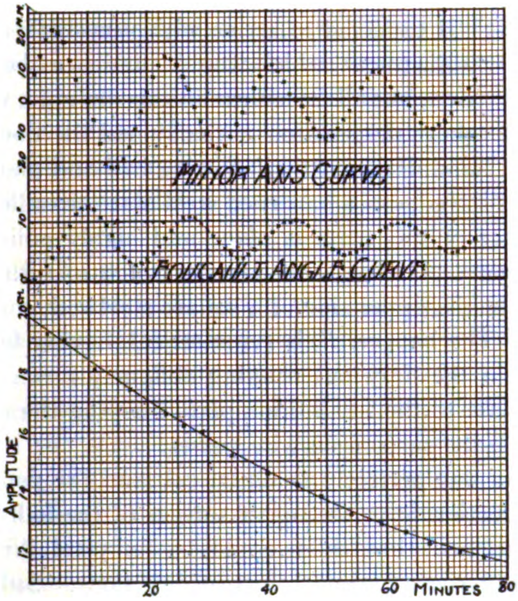


Fig. 6.

In this series, both readings were made *every minute* for an hour and a quarter. Also occasional readings of the amplitude of vibration of the pendulum, along the major axis of the ellipse, for the purpose of determining the decrement.

These curves show in a most striking manner, the relation of the elliptical motion of the pendulum to the shift in its plane of vibration. In every case, when the direction of rotation changes (when the minor axis curve changes sign) the direction of the shift in the plane of vibration also changes. Expressing it in another way, whenever the rotation of the pendulum is clockwise, the Foucault angle is increasing, and whenever the rotation is counterclockwise, the Foucault angle is decreasing. There is a slight time lag as the Foucault angle curve rises, and the reverse as it falls, because the net effect plotted is the minor axis effect plus or minus the real Foucault effect.

At this point I thought my troubles were over, for it now seemed that in order to get a normal Foucault angle, it would only be necessary to adjust the double knife-edge support for equal lengths of the pendulum in both directions. Then there would be no elliptical motion, and consequently no shifting of the plane of vibration except that due to the rotation of the earth on its axis,—the normal Foucault angle.

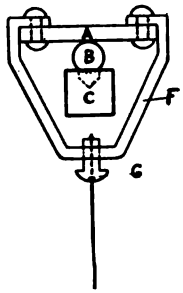


Fig. 7.

I expected to arrive with very little difficulty at an adjustment which would eliminate the elliptical motion, but all my efforts in this direction were futile. In sheer desperation I abandoned the double knife-edge and designed the following ball and plane support:

In Fig. 7, *A*, is the edge of a hardened steel plate 30 millimeters square. The lower surface of this plate is plane and highly polished. It rests at its center upon a polished steel ball, *B*, which in turn rests in a depression drilled in the upper side of a square steel rod, the end of which is shown at *C*. The hanger, *F*, is symmetrical, having four arms (only two are shown in the drawing) running obliquely down from the four corners of the plate to the small brass plate carrying the machine screw, *G*, through which the suspension wire passes.

As this hanger was perfectly symmetrical, and the pendulum free to swing in any direction from the upper surface of the ball as its point of suspension, it seemed that there could be no possibility of elliptical motion due to difference of length in two directions. Still the elliptical motion persisted.

In the first experiment with the ball and plane support, the minor axis of the elliptical path of the pendulum would increase to a maximum, the rotation being in one direction, then the axis would shorten to zero and rotation would begin in the opposite direction. These changes in the length of the minor axis of the ellipse, accompanied by corresponding changes in the direction of rotation of the pendulum, followed each other with great regularity.

This elliptical path seemed to be a splendid example of a resultant of two simple harmonic motions at right angles to each other and differing in period by a small amount which could be easily calculated, or which could be determined experimentally. Since I had eliminated the possibility of the pendulum having two lengths in different directions, by the use of the ball and plane support, I wrongly attributed the elliptical motion to unevenness in the wire, although numerous measurements failed to reveal any great lack of roundness and there was nothing to indicate imperfections of any kind. I could not see that it would have made any difference if the wire had not been round or even if it had been more flexible in one direction than in the other, for with the ball and plane support, the pendulum was perfectly free to swing in any direction and the wire was not expected to bend at all. It was expected to swing with perfect freedom from the point of contact between the ball and plane. The correctness of this assumption was demonstrated later by substituting a flat steel ribbon for a round wire.

Nevertheless the pendulum behaved as if it had two periods. Furthermore it *always started* rotating clockwise, *never* counterclockwise. This fact remained a puzzle until another wire was substituted for the wire which I had been using. The two wires were cut from the same piece, and were certainly as nearly alike as two wires could well be, and yet the pendulum now *invariably* started rotating *counterclockwise—never clockwise.*

I immediately hit upon the curl in the wire as the cause of the elliptical motion. The wire was brass spring wire, and had been lying in a coil about 25 centimeters in diameter, until the "set" in it was very pronounced.

After starting the pendulum a number of times with the new wire and getting always the same results, *counterclockwise* motion at the start, I rotated the *wire* 180 degrees on its own axis, without disturbing either the pendulum ball or the support. The wire may be rotated by turning the screw *G*, Fig. 7, without disturbing the hanger, and a similar arrangement at the bottom enables us to rotate the wire without rotating the ball. Now, the pendulum started rotating *clockwise.*

In a series of experiments which followed, for the purpose of studying the effects of the curl in the wire, I decided not to use the ball and plane support on account of its tendency to rotate at the upper surface of the ball.

The ball and plane support has no advantage over a perfectly constructed double knife-edge support. It even has some disadvantages; but it at least proved valuable in detecting some of the defects in the wire, which perhaps would not have been detected otherwise.

In order to eliminate the effects due to the rotation of the hanger in the ball and plane support, and also the possibility of two periods, resulting from lack of perfect adjustment of the double knife-edge support, and yet to retain a rotary adjustment of the wire, I designed the following torsion head plate support.

A, in Fig. 8, is a cast iron cylinder, 30 millimeters long and having an internal diameter of 30 millimeters. It is bored true on the inside and faced at the ends. *B*, *B*, are steel rods, 13 millimeters in diameter, screwed into shoulders on opposite sides of the cylinder. These rods are held in strong clamps attached to the ceiling beam. *C* is a brass head plate 10 millimeters thick, and turned in the lathe to fit into the cylinder as shown. A circular scale, not shown in the drawing, is cut upon the edge of the head plate, and a reference mark is placed on the side of the cylinder, so that when the head plate, carrying the wire, is rotated, its position can be read on the scale on its edge. *D* is a set screw, to hold the head plate firmly in any desired position. There are three of these set screws, placed 120 degrees apart, in order to prevent any slight tendency to wabble.

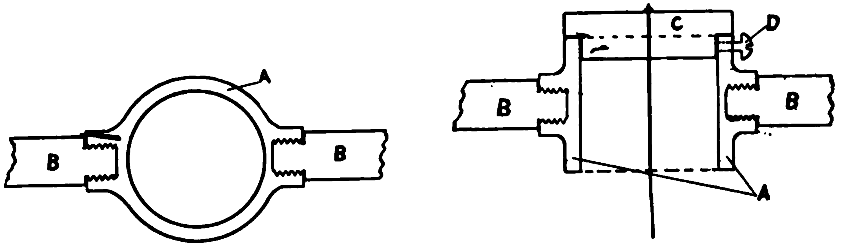


Fig. 8.

As the wire must bend at the lower edge of the head plate when the pendulum swings, it is important that there should be no looseness at this point, and particularly that there should be no greater freedom of motion in one direction than in another.

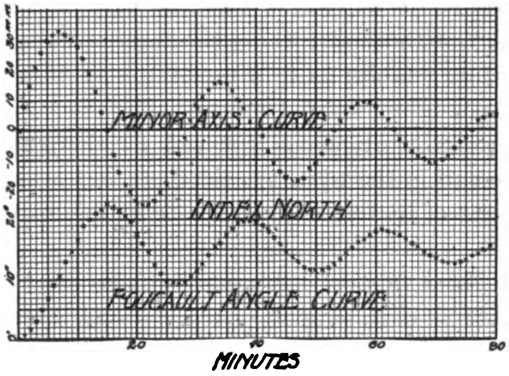


Fig. 9.

To realize these conditions, the hole in the head plate was drilled from the upper side and the plate was faced on the lower side afterwards, leaving the edges of the hole perfectly sharp at the circumference of the wire. The hole was made with a drill a size smaller than the wire. The plate was then heated and put in place on the wire by the "shrinking on" process. Finally, the wire was bent over and soldered at the upper surface of the plate, to prevent its being pulled out when the weight was placed upon it.

It is also important that the ceiling beam or other structural element to which this support is clamped, should be as rigid as possible, for it is obvious that any lack of rigidity in the support would be likely to offer greater freedom of motion in one direction than in another.

The experiments with this form of support were conducted with a steel piano wire a trifle more than a millimeter in diameter, as a suspension. The wire had been lying in a closely bound coil, 10 centimeters in diameter, and when opened up, the coil expanded to a diameter of 19 centimeters.

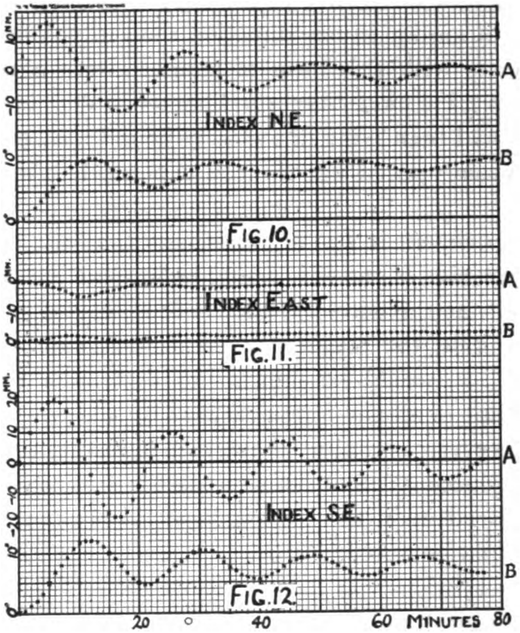
In the first experiment it was my intention to adjust the torsion head so that the convex side of the wire would be in some particular direction, —say north; but, as the wire, 2½ meters long, curled several times between its extremities, the position of the convex side was different at different points. It would be difficult to determine the direction in which the resultant of all the forces exerted by the wire acted upon the pendulum. However, assuming that such a resultant acted in some direction and produced a definite effect upon the motion of the pendulum, an index was placed upon the torsion head, so that any position arbitrarily chosen could be noted.

A series of observations was then made, on the position of the plane of vibration of the pendulum, and the length of the minor axis of its elliptical path, with the index on the torsion head pointing north. The results are shown in Fig. 9. The wire was then rotated 45° clockwise (index northeast), and another series of observations made. The results are shown in Fig. 10. In Fig. 10 and the succeeding figures, the minor axis curve is marked *A*, and the Foucault angle curve, *B*. Other observations were made with the index in other positions as shown in the curves down to and including Fig. 16.

A simple inspection of these curves in pairs leaves no room for doubt about the correctness of our inference from the curves in Fig. 6. In every case, the dependence of the Foucault angle curve upon the minor axis curve is obvious. Whatever the *cause* of the elliptical motion, the *effect* is a perfectly definite shift in the plane of vibration. In the present case, the elliptical motion was due to elastic reactions in a highly tempered steel wire.

To produce a perfect Foucault pendulum the disturbing effect dependent upon the double knife-edge is easily controlled by adjustment. The effect due to the elastic reactions within the wire, ought to be eliminated by ***annealing the wire.*** A number of more or less successful attempts in this direction were made.

In one experiment with the torsion head plate support I used a rather poorly annealed iron wire. The observations on this wire are shown in Fig. 17. I could not believe my observations had been so careless as the kinks in these curves indicated, but I repeated the series about sixteen hours later, the pendulum having remained undisturbed during the interval. This time I got the results indicated in Fig. 18.

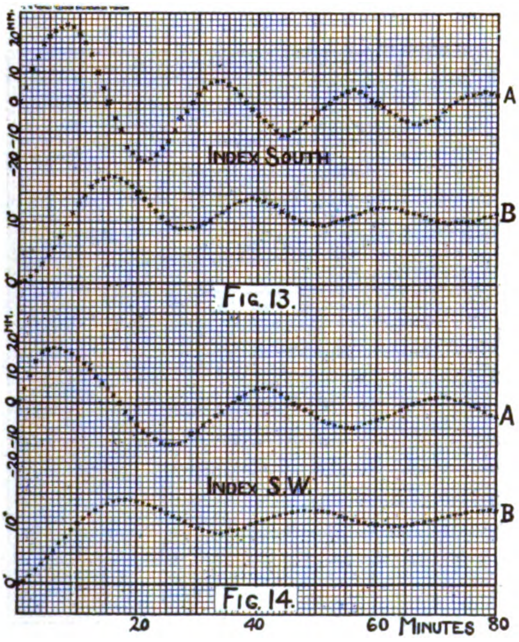


Figs. 10, 11, 12.

The curves in Fig. 18 show the same irregularities as those in Fig. 17, but in a less pronounced degree. The wire was gradually straightening out, under the influence of the weight which was upon it. The irregularities are less conspicuous in the Foucault angle curves than in the minor axis curves. In fact they are not perceptible in the Foucault angle curve in Fig. 18. I have drawn a smooth curve through both the Foucault angle curves from the 39 minute point to the 82 minute point. This hits practically every point on the Foucault angle curve in Fig. 18, but shows perceptible variations from it in Fig. 17, at about 58, 72, and 78 minutes.

I then ran a Bunsen burner flame up and down the entire length of the wire several times as it hung with the 8 kilogram weight upon it; after which, a third series of observations resulted as shown in Fig. 19. The kinks are entirely smoothed out of both curves. Evidently I had finished annealing the wire. If this wire had not been required to bend at the point of suspension, that is, if it had been used with a double knife-edge support instead of a torsion head plate support, it might have been a good Foucault pendulum after it was perfectly annealed.

To illustrate the effect of continued bending upon very soft wire I cite the following case: The 8 kilogram ball was suspended by a No. 18



Figs. 13, 14.

(B. & S. gauge) soft copper wire capable of sustaining a steady pull of about 30 kilograms. The upper end of this wire was soldered into a No. 60 hole (drill gauge size) in a brass torsion head plate. This pendulum was started swinging with an amplitude of 20 centimeters and ran with no perceptible tendency toward elliptical motion and with no perceptible variation from the true Foucault angle, but the wire became brittle and broke off close up to the plate after running about five minutes.

Any wire which is required to bend at the point of suspension must not only be perfectly annealed and perfectly round and perfectly homogeneous in all respects, but it must also be free from wear at the point of suspension and it must not be allowed to become brittle at the point where it bends. There are so many chances of violating one or more of these requirements that we may hardly expect success with a torsion head plate support.

The double knife-edge support is ideal in some respects but it has one defect which must be looked upon as serious. The knife-edges must be *perfectly sharp* and the planes upon which they rest must be so *hard* that the knife-edges will not cut them *in any degree whatever.* These are conditions which can not be fully realized, and if realized, could not be maintained. Any wear on the knife-edges converts them into cylindrical bearings, and any wear on the planes upon which they rest converts them into cylindrical troughs, so that after a time we have a cylinder rolling inside of a larger cylinder instead of a knife-edge resting upon a plane. The axis of suspension of a pendulum with a knife-edge support is at the surface of the plane upon which the knife-edge rests; but if a cylindrical roller be substituted for the knife-edge, the axis of suspension of the pendulum is no longer at the surface of the plane but at the center of the roller. We have therefore changed the length of the pendulum by an amount equal to the radius of the cylindrical roller. As the knife-edges wear, the effective length of the pendulum is

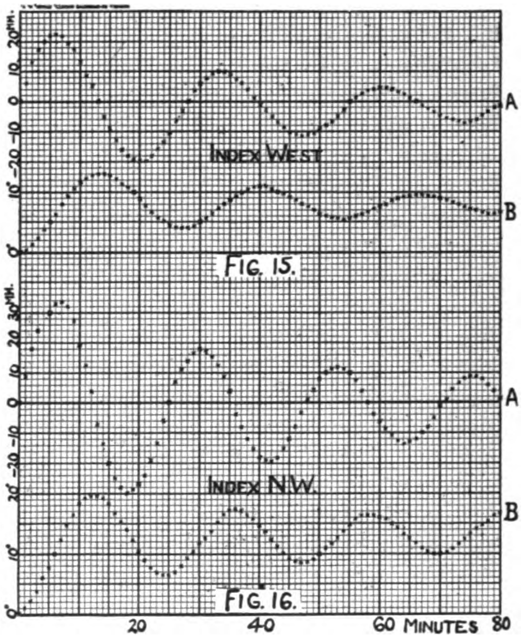
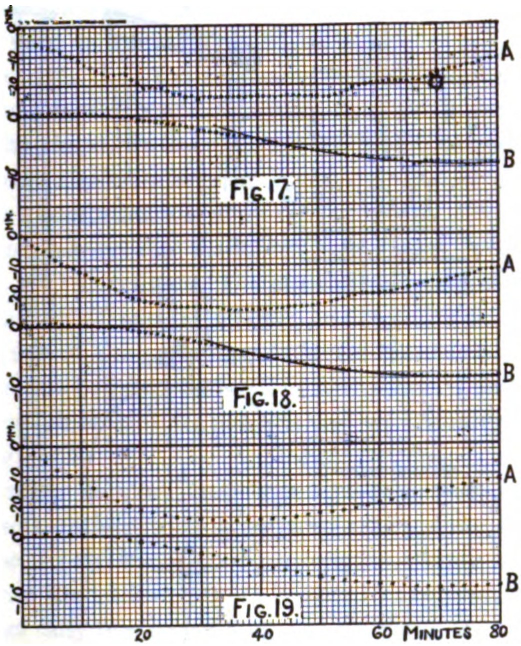


Fig. 15, 16.

increased in one direction and diminished in the other and our Foucault pendulum is gradually converted into a Blackburn pendulum.

Of course the wear can be compensated for by readjusting the distance between the planes, or in some cases by means of a counterpoise above the support; but these adjustments are really somewhat difficult to make, and as the wear continues the pendulum is never in a perfectly satisfactory condition.

The most satisfactory support which I have been able to construct is a double roller support, in which hard polished steel rollers of considerable size have been deliberately substituted for knife-edges. The wear is thus reduced to an inappreciable minimum, the radius of curvature is perfectly definite and constant and the adjustment when once made is permanent.



Figs. 17, 18, 19.

This support is simple and easily constructed and possesses a number of features which ought to recommend it. The details of construction are shown in Fig. 20.

A is a hard steel plate, 25 millimeters square and 3 millimeters thick, with shallow grooves in the upper and lower surfaces at right angles to each other as shown at *B*. The bearing surfaces, *C*, and the hardened steel rollers, *D* and *E*, which are in contact with them, are highly polished.

A double bifilar suspension, *F*, is used, consisting of soft copper wire small enough to be easily stretched when the weight of the ball, *L*, rests upon it. No. 30 (B. & S. gauge) is a suitable size for a 1 kilogram ball. The wire is simply looped over the ends of the upper roller, and ***under*** the ends of a short crossbar, *K*, at the top of the ball, *L*. Shallow grooves may be cut near the ends of the roller and crossbar, to keep the wire from slipping. If the wires are not quite equal in length, they may be equalized by carefully administering a gentle pressure directly downward upon the ball, while the upper roller is held rigidly in a horizontal position. The lower roller, *D*, acts as fixed support and should be rigidly held in its position by means of a strong clamp of some kind. If *D* is mounted in an east and west position, when the pendulum is swinging east and west, the upper roller, *E*, rolls on the plate, *A*. When the pendulum is swinging north and south, the plate, *A*, rocks upon the lower roller, *D*. *H* is a counterpoise to compensate for the difference of length of the pendulum in the two directions. This counterpoise may be run up or down on the threaded rod, *M*. (Threads not shown in the drawing.) When *H* is properly adjusted, the period of the pendulum is the same in both directions. There is then no elliptical motion and no

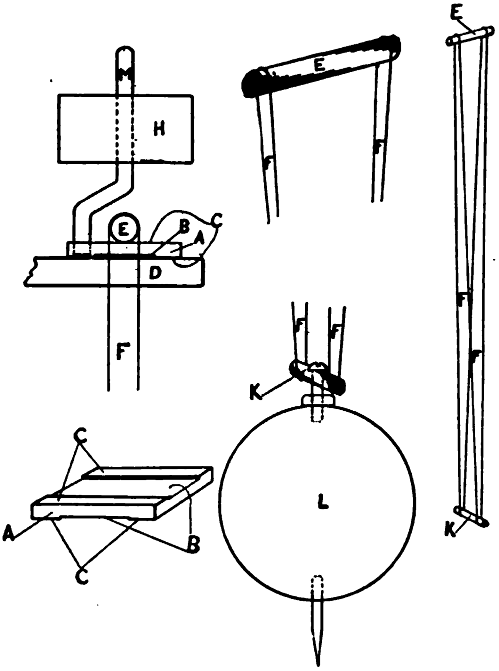


Fig. 20.

shift in the plane of vibration of the pendulum, except that due to the rotation of the earth upon its axis.

In the apparatus which I have just described, the rollers are 6.35 millimeters in diameter and the counterpoise weighs 140 grams. The counterpoise is provided with a circular scale to facilitate fine adjustment. Of course the adjustment is affected not only by the mass and position of the counterpoise, but also by the thickness of the rollers and the thickness of the plate. It is also true that changing the angle between the rollers will disturb the adjustment. For this reason, great care should be exercised in making other adjustments, not to shift the position of the upper roller or the plate. Within small limits, canting the upper roller might be used as a method of adjusting.

It has been shown that the tendency of the pendulum to run in an elliptical path may be the result of any one or more of a variety of causes, but whatever the cause, we may always develop a counter tendency by raising or lowering the counterpoise in the device now under consideration, and perfect compensation is always possible. Of course, the final adjustment must always be made by experiment, and although the process may be a little tedious, it is not difficult, and when once made, the results are all that could be desired.

If the amplitude of the pendulum is large, there is a tendency of the rollers to slip on the polished surfaces of the plate. I find an amplitude of 20 centimeters (40 centimeter path) is very satisfactory for a pendulum 2½ meters long.

The double roller support used in connection with a double bifilar suspension of soft copper wire is so satisfactory that I do not find it necessary to use a pendulum having a length of more than 2½ meters or a mass of more than 1 kilogram. In fact I have been quite successful with a pendulum only one meter long and weighing only 240 grams, but of course there is no need of going to such an extreme as this, except to demonstrate that it can be done. A pendulum weighing a kilogram, and swinging over the lecture table, from a rigid beam in the ceiling of the lecture room, is ideal.

Foucault unquestionably succeeded in minimizing his difficulties by using a very long and very heavy pendulum, but his results may not have been as exact as some of us have supposed. I find nothing in any of the original papers or early comments upon them, to indicate a high degree of accuracy. Indeed the only statements bearing upon this point indicate that the results of the experiment were only approximately correct. For example: "Quoique ces oscillations diminuent d'amplitude assez rapidement, au bout de cinq à six heures elles sont encore assez

grandes pour permettre d'observer la déviation que est alors de 60 à 70 degrés." Or, "La montre à la main, ou voit que, à Paris, la déviation est du un degré en cinq minutes."

Such statements can not be thought of as representing precise measurements. They are good enough to leave no doubt in regard to the general proposition that the earth rotates on its axis, but they are not by any means exact. If success depends upon great length and great mass, Foucault's results ought to have been very exact. The fact that they were not, indicates that there was something else to be reckoned with. I have already referred to the fact that his wire was strained beyond its elastic limit, and I have no doubt this condition of the wire contributed to his success quite as much as its great length. Of course the great mass of the ball was responsible for the strain upon the wire, but if the wire had been *annealed*, the great mass would not have been necessary.

To use a pendulum of very great mass in order to prevent atmospheric disturbances is also quite unnecessary. Air currents, if forcible enough, do disturb the pendulum slightly, but such disturbances never "develop large irregularities" in the motion of the pendulum, in spite of certain ingenious theories intended to explain how they do it. In fact they never *develop* any irregularities at all. They always die out gradually, and at the same rate as the decrement in the principal motion of the pendulum along the major axis of the ellipse, when the amplitude has been reduced to the same value. Would you dare present any other view of the case to your students if you were discussing the second law of motion?

How unimportant slight atmospheric disturbances are, may be indicated by the fact that I have succeeded perfectly with a 1 kilogram pendulum in an ordinary lecture room with all the doors and windows open (2 doors and 7 windows) and a good breeze flowing through the room.

The majestic oscillations of the Pendulum of the Pantheon, as Flammarion fittingly describes them, must have produced a profound impression upon those who had the good fortune to observe them; but it is not possible to repeat the experiment in every lecture room on such a magnificent scale. To reduce the experiment to convenient lecture room dimensions, ought to greatly extend its field of usefulness.

KNOX COLLEGE,
GALESBURG, ILL.,
October, 1918.

ON THE DIFFRACTION-FIGURES DUE TO AN ELLIPTIC APERTURE.

By C. V. Raman.

Synopsis. Photographs are shown of the diffraction pattern obtained near the focus of a converging pencil of rays from an elliptic aperture. The transiton from the Fresnel to the Fraunhofer class of diffraction figure is traced and attention drawn to the geometric law to which the pattern conforms, namely, that the brightest part of the diffraction pattern outside the elliptic cross-section of the beam lies within the geometric evolute of this cross-section and is bounded by it.

It is well known that the diffraction pattern at the focus of a convergent pencil of rays limited by an elliptic aperture is made up of a central spot of light surrounded by alternate dark and bright rings which are all elliptical in shape, the direction of the major axis of the rings being the same as that of the minor axis of the aperture and *vice versa*.[1] The distribution of luminosity in the focal plane may be readily deduced from the known results for the case of the circular aperture by a simple transformation of coördinates. The theory of the phenomena observed in *ultra-focal* planes is, however, considerably more complicated, and so far as the present writer is aware, has never been fully worked out for the case of elliptic apertures. Recently, while working in collaboration with Mr. R. S. Deoras, the writer made some observations on the diffraction of light by elliptic apertures for convergent and also for divergent pencils, and has noticed that the configuration of the fringes presents some rather striking geometrical features, particularly in the cases in which the eccentricity of the elliptic aperture is considerable. It is thought that a brief account of the observations, and the reproductions of some of the photographs of the diffraction-figures secured in the course of the work (Figs. 2 to 11) may be of interest to the readers of the Physical Review.

Figs. 2 to 9 represent the gradual transition from the Fresnel to the Fraunhofer class as the focus of the convergent pencil is approached. In all these cases, the aperture limiting the pencil was of considerable eccentricity and had its major axis vertical. The interesting feature to which the writer wishes to draw attention is the following simple geometrical law to which the diffraction-pattern is found to conform. In

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[1] Airy's Tract on the Undulatory Theory of Optics, Section 86.

Fig. 1 the heavy lines represent the elliptic cross-section of the geometric pencil of rays by the plane of observation, and also the geometric evolute of this cross-section. [The ratio of the major to the minor axis of the ellipse is taken to be greater than $\sqrt{2}$, so that two of the cusps of the evolute lie outside the cross-section.] Observation shows that the brightest part of the diffraction-pattern outside the elliptic cross-section lies within the evolute and is, in fact, bounded by it. The complete figure of the evolute may be observed visually or even photographed with sufficiently long exposures. Beyond the two cusps of the evolute, a horizontal brush or extension may also be seen on each side, as shown in Fig. 1. The evolute boundary is clearly seen in Figs. 3, 4 and 6, though in all these cases, the size of the photographic plate and the exposure were insufficient to record the whole of it.

Obviously, as the plane of observation approaches the focus of the convergent pencil, the cross-section of the pencil and the evolute both contract and ultimately reduce to a point. The horizontal brushes extending from the two cusps of the evolute, however, persist (vide Figs. 7 and 8) and break up into beads on both sides which, when the focal plane is further approached, ultimately join up and form the elliptic rings observed in the Fraunhofer pattern. Altogether, the case presents unusually interesting features in the transition from the Fresnel to the Fraunhofer class of diffraction-figure.

Fig. 10 represents the elliptic rings at the focus, for the case in which the eccentricity is small. Fig. 11 represents the rings observed in the focal plane when a *circular* reflecting surface held very obliquely forms the diffracting aperture. Some degree of asymmetry is observable in this figure.

The writer hopes to be able to present a fuller study and mathematical treatment in due course.

210 BOWBAZAAR STREET,
CALCUTTA, INDIA,
October 20, 1918.

PHYSICAL REVIEW, VOL. VIII., SECOND SERIES,
April, 1919.

PLATE I.
To face page 260.

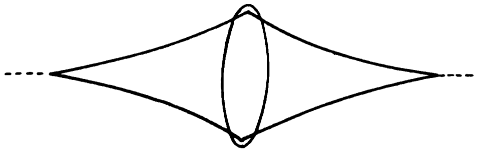

FIG. 1.

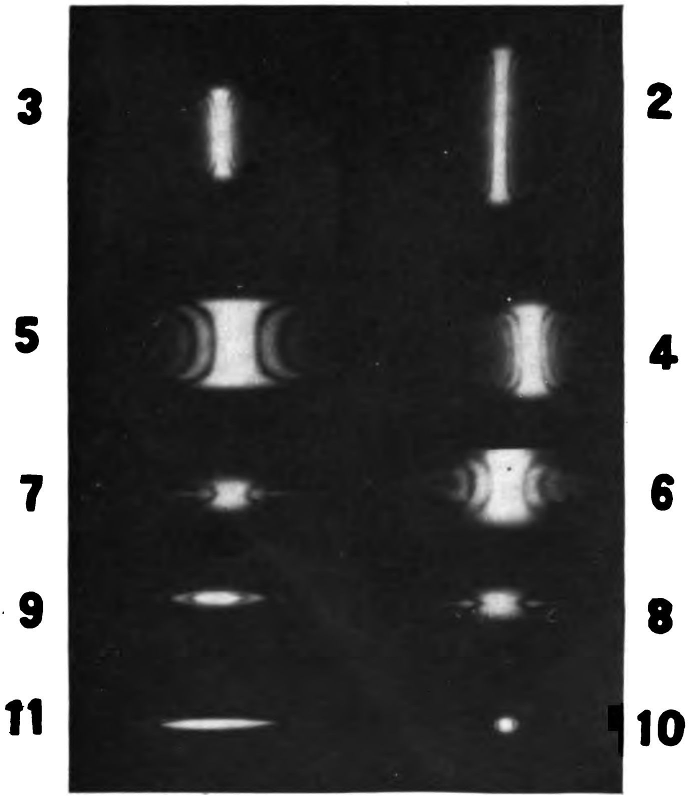

FIG. 2.

C. V. RAMAN.

REFINEMENTS IN SPHEROMETRY.[1]

By G. W. Moffitt.

SYNOPSIS. *Mechanical Methods.*—The shortcomings of the ring spherometer are discussed and a new type of contact ring with selected ball bearings for contact members, giving greater precision and reliability, is described. A modified instrument having but one contact is also described. It has certain advantages in some cases.

Optical Methods.—The auto-collimating method, as often used, is limited in its precision because of the depth of focus of the optical system. The improvement described consists in placing the lens on a nodal slide and adjusting until lateral motion of the image in the field of view is no longer noticeable when the slide is rotated through an angle. In the caustic curve methods use is made of the aberrations introduced on oblique reflection from a spherical surface. By measuring the distance from the primary to the secondary image and multiplying by a factor depending upon the arrangement of the apparatus the radius is determined with high precision for concave surfaces or for convex surfaces of short radius. This method is especially valuable when the element of surface is small.

Summary.—Surface measured with precision by the various methods described are as follows: (1) Mechanical Methods: Concave and convex surfaces of short and medium radius, the precision decreasing rapidly as the radius becomes larger. (2) Auto-collimating Method: Concave and convex surfaces of short or medium radius up to the limit of the slide on the turntable, the precision being high and practically independent of the radius. (3) Caustic Curve Methods: Concave surfaces of any radius to a high degree of precision, even when the element of surface available is too small to be measured by other methods. Convex surfaces of short and medium radius to a degree of precision decreasing with increase of radius.

IMPROVEMENTS in methods for the measurement of spherical lens surfaces do not seem to have kept pace with the development of processes for shaping the surfaces. To verify the execution of a formula, or to ascertain the constructional data of a given lens system without destroying it, requires a precise determination of the thicknesses, radii, and focal lengths of the individual elements of which the system is composed. One can measure the thicknesses with an ordinary micrometer gauge. The focal lengths and color curves may be found by the use of a good optical bench equipped with a nodal slide. But with methods now in common use it is usually difficult, often impossible, to determine the radii with a satisfactory degree of precision. I have been led, therefore, to a study of existing methods in spherometry, and in consequence, make a few suggestions for their improvement. Some methods are also given in this paper which seem to possess elements of novelty and a limit of precision surpassing that of those now in use.

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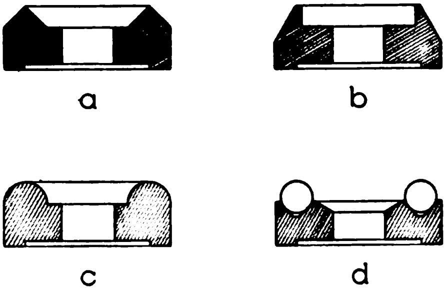
[1] Communication No. 78 from the Research Laboratory of the Eastman Kodak Company.

I. MECHANICAL METHODS.

The Ring Spherometer.

Those who may have attempted to verify the curvatures of a lens system are familiar with the shortcomings of spherometers even of the best ring type, such as that illustrated in Fig. 1.

A vertical section through the axis of a theoretically ideal spherometer ring is shown in Fig. 2*a*. But for mechanical reasons such a ring cannot be made. The cross-section usually employed is that shown in Fig. 2*b*.



Figs. 2*a*, 2*b*, 2*c*, 2*d*.

There are really two diameters, one for convex surfaces, the other for concave. But the corners are always more or less rounded and irregular, making it practically impossible to determine the true value of the diameter of the contact circle because it varies in an unknown manner with the radius being measured. Being made of soft metal to prevent injury to the lens surfaces the ring is easily damaged unless the edge be especially protected against rough usage. Even if carefully handled the ring will soon show indentations and bends under the microscope and these defects introduce further uncertainty in the constants of the ring as there is no way of telling to what extent they affect a given determination.

The Ball Ring Spherometer.

The question naturally arises whether it is possible to give the spherometer ring such form that the size and position of the contact circle may be accurately stated in every case.

One answer to the question would be to make the vertical section of the ring circular in form as in Fig. 2*c*. The formula for reducing the readings would be of the same general type as before, but the ring would be of sturdy form not easily damaged, and not detrimental to the surfaces measured upon it. On the other hand the mechanical difficulties standing in the way of making such a ring of truly circular form and cross-section make its realization hardly possible. One can, however, accomplish the same end by the use of three or four selected ball bearings whose sphericity and uniformity of size fall within the required limits of exactness, conditions which may be easily met.

A V-shaped circular groove may be turned in a disk of metal whose base fits the spherometer as does the usual type of ring (Fig. *2d*). After the ring has been ground until true the balls may be laid in the groove with the disk in place and the instrument is ready for use. It may be desirable to fasten the balls in position in the groove by some means which will not distort the upper part of the balls.

With a ring of this type the determination of the constants involves nothing but readings that can be made with a high degree of precision. In the measurement of the radius of the ball ring there is no guessing as to the point on which to set the reading microscope—no setting on ill-defined edges whose position seems to change with every change in lighting. The personal error in bringing the sharply defined image of a ball tangent to the cross wire may be easily determined by comparing the results for the diameter of a ball with the results of direct micrometer gauge measurements. But by taking the distance from one side of a ball to the same side of one on the other end of the diameter of the ring the personal equation vanishes in the determination of the diameter of the circle through the center of the balls.

For the ordinary ring spherometer the equation is

$$R = \frac{d^2}{2h} + \frac{h}{2},$$

where R is the radius to be determined, d the radius of the contact circle —different for concave and for convex surfaces—and h is the sagitta of the arc of radius R subtended by the diameter of the contact circle.

For the ball-bearing contact ring the equation becomes

$$R = \frac{d^2}{2h} + \frac{h}{2} \pm r,$$

where r is the radius of the ball bearings and the double sign is to be interpreted as meaning two separate cases in which the negative sign is used for convex and the positive sign for concave surfaces. Here d represents the radius of the circle passing through the centers of the balls.

For either type of ring the error in a determination may be written

$$\Delta R = \frac{d}{h}\Delta d + \left(\frac{1}{2} - \frac{d^2}{2h^2}\right)\Delta h \pm \Delta r,$$

for concave and convex surfaces respectively. For the ordinary type of ring the last term becomes zero. Other sources of instrumental error have been considered, but their effect on the results with any well made instrument are small compared to the error from the sources already considered. Among those considered were the errors due to lack of centering of the ring with respect to the center contact, obliquity of the plane of the ring with respect to the direction of motion of the center contact, and form of the center contact. All these sources of error would be negligible in any instrument constructed with a reasonable amount of care.

The relative merits of the two rings may be compared by assuming an average case and determining the maximum error to be expected in a determination, obtaining the maximum error by summing the numerical values of all the terms on the right-hand side of the above equation. Take, for example, the case where $d = 10$ mm., $h = 1$ mm. For the ordinary ring d may be determined to 0.01 mm., or 0.02 mm. Assume that the settings for h can be read to 0.0005 mm. Then the error in the determination of h would probably be about 0.001 mm. In the ball ring instrument the error in the determinations of d and r need not exceed 0.001 mm. and the error in the determination of h would be as in the first case about 0.001 mm. On the basis of these assumptions the maximum error to be expected in the determination of the radius of average magnitude would be about 0.15 mm. for the ordinary ring and about 0.06 mm. for the ball-bearing ring.

The gain in precision is not all that could be desired, but it is only one of the advantages realized by this modification of the instrument. When it is remembered that the error has been reduced to less than half its former value, and that the ball-bearing ring will maintain its constants for a long time and even stand considerable rough usage without injury, the gain is seen to be considerable.

The error equation shows that for a given value of R the error will be smaller the smaller d/h becomes, or for a given lens the larger the value of d. But this is limited by the aperture of the lens while h is limited by the radius of the lens face. For a given contact circle the size of the ball bearings used does not affect the precision of the measurements. Therefore it is possible to use the balls most easily obtained or for other reasons most suitable for the purpose. If the lenses to be measured are small the balls must be small in order that a ball ring of small diameter may be constructed.

While it may be best to fasten the balls rigidly in place a very serviceable ring may be made by simply placing the loose balls in the groove if care be taken to see that balls and groove are well cleaned. One set of balls and a disk with several concentric grooves will serve for practically all lenses whose curvatures are adapted to measurement on the spherometer.

The Single Contact Sliding Stage Spherometer.

A modification of the ball-bearing ring spherometer which is excellent for nearly all measurements made with a spherometer is what I have called the single contact slide spherometer. The details of this instrument are shown in Fig. 3. The troublesome contact ring has been entirely dispensed with and the center contact rod is inverted so that the contact member is at the lower end. In the base of the instrument is a sliding stage actuated by a micrometer screw capable of a range or motion about as great as the aperture of the largest lens to be measured on the instrument. On the stage is mounted the lens carriage which should have a short screw-actuated cross slide which need not be calibrated in any way, as it is used only to bring the center of curvature of the lens face into line with the axis of the contact shaft. The center contact of this instrument must be of truly spherical form. A carefully selected ball bearing of small diameter serves very well. As a means of reading the instrument the microscope method is entirely satisfactory.

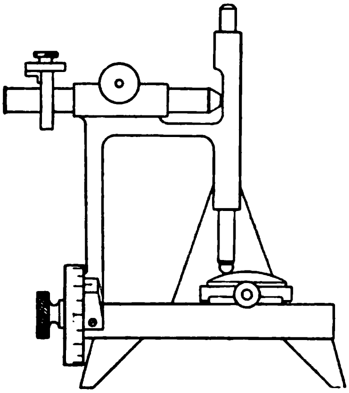


Fig. 3.

To use the instrument place the lens on the carriage and clamp in place, paying little attention to centering or leveling. Adjust the cross-slide until the contact rod reading in the microscope is a maximum—or minimum. After this has been done any movement of the main slide will cause the contact to move along the circumference of a great circle of the lens face. The main slide is then set so that the contact is at some point near the edge of the lens and slide and microscope read. The slide is then moved until the microscope reading becomes a maximum or a minimum and slide and microscope readings again taken. Then the slide is an additional distance equal to that already moved and the readings taken again. The radius may be computed by means of the ball-bearing ring spherometer formula. The last reading made was not essential but served as a check on those already taken. On the other hand a number of distributed readings may be taken and several determinations of the radius made from one set of readings.

The advantages of this type of instrument are:

1. In construction it is much easier to select one ball of precise sphericity than it is to select three or four balls of the same size and of sufficiently precise sphericity.

2. There are no instrumental constants to be determined, or rather, the constants are determined in the making of the readings.

3. The instrument affords a means of studying the deviation of a surface from the spherical form.

4. Will measure the radius of curvature when the area of the lens is too small to be determined on the ring spherometer.

Against these advantages may be placed the following disadvantages:

1. Not as rapid as the ring-type instrument.

2. Side motion of the contact shaft must be reduced to a minimum.

II. OPTICAL METHODS.

Auto-Collimating Methods.

A number of variations of the auto-collimating methods have been proposed from time to time. Of these the one shown in Fig. 4 is perhaps the most satisfactory. A short distance in front of the objective of a simple reading telescope is placed a lens similar to the objective of the telescope itself but in a reversed position. Between the two lenses is placed a partly silvered mirror which may be adjusted to reflect the light from a distant small source through the front lens onto the face of the lens to be measured which is carried in a holder at *L*. This arrangement gives a better field of view than can be had by the use of an auto-collimating eyepiece. It may be shown that the illumination at the eyepiece will be a maximum when the product of the transmission and the reflection of the mirror is a maximum. If the point of convergence of the light pencil and the center of curvature of the lens face coincide a part of the light will be reflected back through the telescope and an image of the source formed at the cross-wires. The lens is carried on a slide fitted with scale and vernier by means of which the displacement of the lens between settings may be determined.

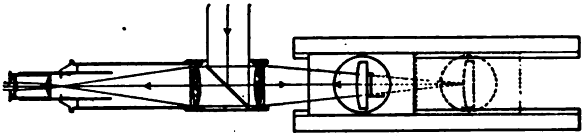


Fig. 4.

In use the slide is first adjusted so that fine dust particles on the face of the lens are in sharp focus. The slide is then moved until the source is sharply imaged on the cross-wires. The displacement of the slide from one reading to the next is equal to the radius of curvature of the lens face.

Several difficulties are encountered in the application of this method. The presence of spherical aberration in the lenses of the instrument may cause considerable uncertainty in the settings. The spherical aberration may be reduced to a minimum by correct design of the lenses, but even then the depth of focus naturally inherent in an optical system of this kind renders precise focusing practically impossible. With this in mind I have modified the method of determining the correct setting, and have investigated the precision obtainable with what may be called the

Auto-Collimating Nodal Slide Method.

The lens to be measured is mounted on a nodal slide arrangement similar to that used for determining the position of the gauss points of a lens (see Fig. 5). In other respects the apparatus is the same as before except that the scale and vernier for measuring the displacement between settings are now on the turntable of the nodal slide whose support is stationary. The method may be applied to a T testing bench by placing the telescope on a carriage mounted on one of the cross-tracks of the bench and using the regular turntable.

In use the apparatus is first adjusted as well as may be by focusing. Then the turntable is rotated back and forth through an angle limited by the aperture of the lens and further adjustment made until there is no side shift of the image with respect to the cross-wires during the rotation. When this adjustment has been completed the center of curvature of the lens face coincides with the axis of rotation of the turntable. To find the reading when the face of the lens is tangent to the axis of rotation of the turntable the carriage is moved until fine dust on the surface of the lens is in focus. The final adjustment is made by rotating the turntable back and forth and further adjusting the slide until there is no relative motion of image and cross-wires at the center of the field. If this setting cannot be made with sufficient accuracy by means of the telescope, because of its low magnification, it may be made with the regular high-power microscope on the main track of the bench. The distance through which the slide has been moved is, as in the preceding case, equal to the radius of curvature of the lens face.

The error in a determination of a radius by this method will be due almost entirely to the error in the setting for the center of curvature provided the setting on the axis of rotation has been made with a high power microscope. To evaluate this error assume that the center of curvature, C, fails to coincide with the axis of rotation of the turntable by a small distance e (Fig. 6). Let R be the true radius of curvature of the lens face and A the maximum angle through which the turntable may be rotated before the image disappears. Since e will always be small in comparison to R the position of the image of O formed by reflection from the lens face may be assumed to be at a distance $2e$ from O. Then if x represent the lateral shift for a rotation through an angle A

$$x = 4e \sin\frac{A}{2}, \quad \text{or} \quad e = \frac{x}{4}\csc\frac{A}{2}.$$

If the two lenses of the apparatus be of the same focal length x is also the lateral shift of the image with respect to the cross-wires. Assuming that a shift of 0.02 mm. may be detected and that a rotation of 30° is possible the smallest error in a setting which could be detected would be about 0.02 mm. Since the lens surface may be placed very accurately tangent to the axis of rotation of the turntable by means of a high-power microscope the above value may be taken as a fair estimate of the precision attainable with this method for any surface measurable on the apparatus. It is interesting to note that the magnitude of the radius has little effect on the precision for all surfaces measurable on the instrument.

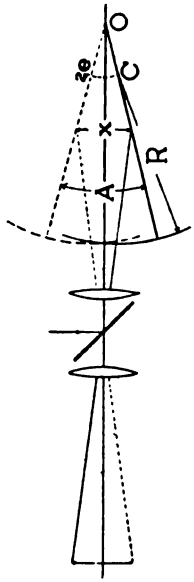


Fig. 6.

Caustic Curve Methods.

Unlike the auto-collimating methods just described where spherical aberration in the instrument itself entered as a disturbing factor, these methods are based upon the aberrations introduced by oblique reflection from a spherical surface. I have devised several variations of the general method to suit different sets of apparatus from the elaborate lens testing bench to a simple travelling microscope, pivot stand, and distant source, but the one whose description follows seems best suited to the purpose as it involves only measurements of distances. Caustic curve methods are especially valuable in cases where the element of lens surface is too small to be measured by other methods. In favorable cases the precision seems to be limited more by the errors of adjustment of the apparatus than by those of making the readings.

Fig. 7 shows the general arrangement. The light is assumed to be coming from the left and to be reflected from the lens face whose center of curvature is at C. The continuation of the spherical surface is represented by the dotted line. FN is the caustic curve of reflection in this case. It is a portion of a two-arched epicycloid. The axis of the reading microscope is perpendicular to the incident light. Lens, light, and microscope are so arranged that the chief incident ray is reflected along the axis of the microscope. Now if the microscope be moved along its axis the two astigmatic images formed by the reflection from the spherical surface will be seen one after the other. The primary image is perpendicular to the plane of the paper at PP', while the secondary image lies in the plane of the paper and parallel to the incident rays at SS', that is, the secondary image is located along the principal axis of the concave mirror of which the lens surface forms an element. The measurement consists in finding the distance between the primary and the secondary images. It will be seen that this distance multiplied by the proper constant, which comes out to be $2\sqrt{2}$, gives directly the radius sought. It may also be easily shown from the properties of the curves that the distance from the primary image to the lens surface, measured along the axis of the microscope produced, is equal to the distance between the primary and the secondary images. This valuable check relation is one of the advantages realized by choosing the perpendicular relation between chief incident and reflected rays. If a moderate magnification be chosen, and there seems to be little gain in precision by pushing the magnification beyond a reasonable limit, one is able to focus on the lens face and so measure both these distances.

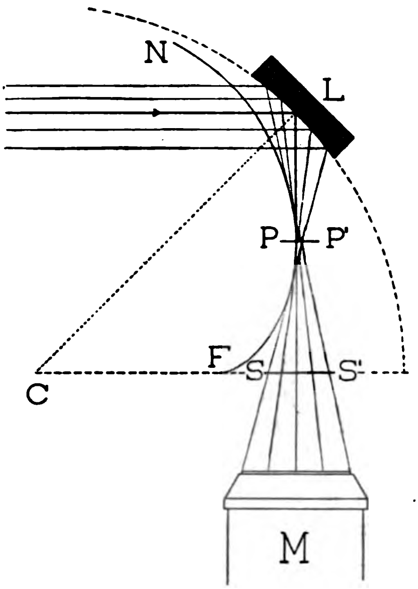


Fig. 7.

The procedure in making a determination by means of this method would be about as follows: Having placed the lens in rough position the secondary image is thrown into the field of the microscope and examined. If the image rises or falls in the field of view of the instrument as the sharp focus position is passed it is an indication that the center of curvature of the lens face does not lie in the plane defined by the axis of the microscope and the chief incident ray. The lens must be adjusted by tipping it in its holder until the secondary image does not rise or fall in the field as the working plane of the microscope is moved through the position of sharp focus, but simply diminishes in width and fuzziness, passes through sharp focus, and widens again. As will be seen later the microscope should be raised or lowered until the sharp image falls across the middle of the field. When so adjusted the center of curvature of the lens face lies in the plane defined by the incident and reflected chief ray. The microscope is next placed so as to receive the primary image. This may require a bodily movement of the lens carriage toward or away from the source if a simple rotation does not suffice. The focusing of the primary is not at all the finding of the best image of a sharp line. The line is never sharp on one side. The case is illustrated in Fig. 7, in which the image sought is in the plane *PP′*. The image will be quite sharp and bright on the edge touching the caustic and will gradually fade off toward the other edge which will be limited by the aperture of the microscope. As the microscope is moved either way from the position of correct focus it is evident that the image will move across the field of view, the sharp bright edge leading. Therefore as the true position of focus is approached and passed the image moves into the field of view with its faint side leading, then, as the true position is passed, it reverses its motion and the bright edge leads the procession back. The true position of the focus is not easy to locate precisely by a simple estimate of the position at which the reversal of motion occurs for at this position the lateral movement of the image for a given movement of the microscope is small. But at a short distance to either side of the point of reversal the motion is easily detected. By making a reading when the bright edge, or the first fringe if diffraction fringes show, is on a certain mark in the micrometer, and then moving the microscope until the true position has been passed and the edge, or bright line, is again on the chosen mark the mean of the two readings will give a good determination of the setting. The adjustment of the apparatus should be so made that the true focus on the primary image occurs when the bright edge of the image is in the center of the field of view. The microscope is then moved back and the reading for the secondary taken. The focusing in this case is simply the determination of the sharpest focus of a well-defined straight line which should be in focus all the way across the field since the secondary, being parallel to the incident light, lies parallel to the working plane of the microscope. The microscope is then moved up and focused on fine dust on the surface of the lens. As the lens surface is at an angle to the working plane of the microscope only a part of the field will be in focus at a time. The proper vertical part of the field to focus sharply is that in which the bright edge of the primary

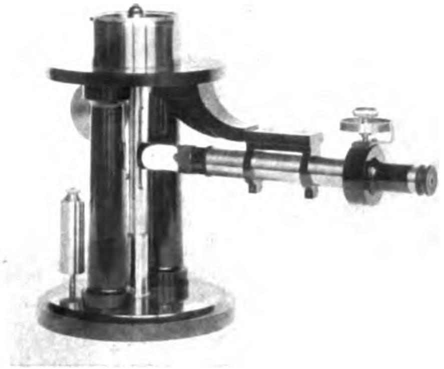


Fig. 1.

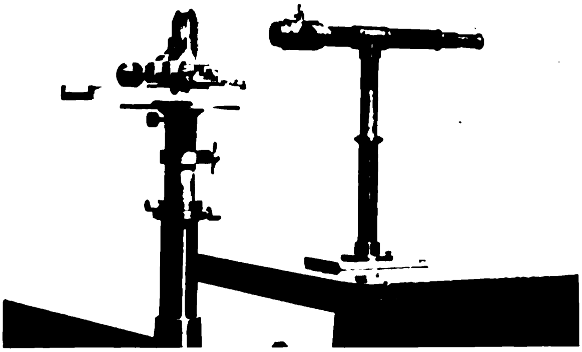


Fig. 5.

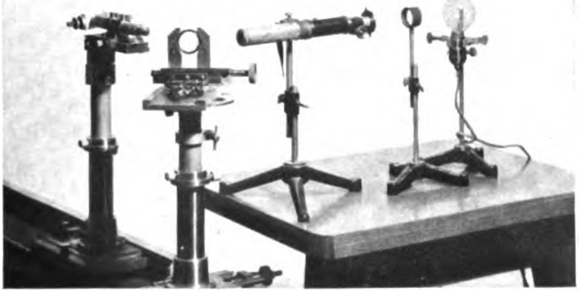


Fig. 8.

G. W. MOFFITT.

image was focused. Since the lens is curved in a vertical plane as well as in the horizontal care is also required in choosing the horizontal portion of the field on which to focus. Evidently the correct portion is that in which the secondary was focused when the setting was made on it. If, as above noted, the center of the field was used then the focusing on the lens face should be made for the center of the field also.

The radius may be computed from the measured distances between the primary and the secondary images by simply multiplying by $2\sqrt{2}$, using the distance from the lens to the primary as a check on the work. Or the reading on the primary may be disregarded, using only the readings on the lens face and on the secondary and multiplying the difference by $\sqrt{2}$. Care must be taken, however, that the setting on the lens face is focused in the proper part of the field and this can only be determined by a careful observation of the primary focus. The work of making the settings is, therefore, not reduced. Various factors affect the precision obtainable with the two methods and the one giving the best results in any case can be determined only by trial.

On concave faces where high magnification may be used results of high precision may be obtained. A radius of 100 mm. may be determined to within a few hundredths of a millimeter. On convex faces where the images are virtual and lie at a considerable distance back of the surface the working distance of the microscope must be almost as great as the radius to be determined. The precision of focusing would be considerably reduced. For such surfaces the auto-collimating method previously described would probably be preferable.

Several other adaptations of the caustic curve method have been tried. One in particular in which the angle between incident and reflected chief rays was so chosen that the distance between the primary and the secondary images was equal to the radius sought. But errors introduced by a small error in the adjustment of the angle were much greater than in the right-angled case. This, and other limitations seemed to indicate that the right-angled position would give better results and at the same time afford the check readings on the face of the lens. Fig. 8 shows the arrangement of the apparatus used in the tests on caustic curve methods.

By the use of the methods described above one may determine the radius of curvature of a polished spherical surface to a degree of precision usually quite satisfactory for the needs of optical design. In the case of convex surfaces of long radius the precision is sometimes not all that could be desired.

Rochester, N. Y.,
September, 1918.

ON THE AUTOMATIC REGISTRATION OF α-PARTICLES, β-PARTICLES AND γ-RAY AND X-RAY PULSES.

BY ALOIS F. KOVARIK.

SYNOPSIS. A device is described which makes it possible for α-particles, β-particles and γ-ray and X-rays pulses to make records automatically on chronograph paper, which records can be conveniently and accurately studied at leisure. Historically, the method developed from Rutherford's method of counting α-particles and the last previously described development was by the writer in which he used a telephone receiver whose diaphragm was actuated by the current to a needle electrode, after amplification by means of a 3-electrode vacuum tube. In the present method the amplified current actuates a sensitive relay which in turn operates the chronograph pen by means of a local circuit. The accuracy of the records was checked up for the α-particles by the scintillation method of counting and for the β-particles by determining the charge carried by the β-particles. Records of γ-rays and X-rays are similar to those of the α- and β-particles and some arguments are given supporting the idea that the records represent individual pulses or short trains of pulses. A record of β-particles taken over a 7-hour period was studied and conclusion deduced that the variations in the number of β-particles emitted in a given time interval follow the probability law.

INTRODUCTION.

THE method of counting individual α-particles by the scintillations they produce on a fluorescent screen is well established. The α-particles have also been counted by the method of ionization by collision at low pressures, devised by Rutherford[1] in connection with his experiments on the charge of an α-particle. Rutherford and Geiger[2] modified this method, later, by using a spherical electrode in helium at a pressure of about one third atmosphere instead of the wire electrode used at very low pressure in air. In this case they substituted a string electrometer for the quadrant electrometer which was a marked improvement. Later, Geiger[3] used a sharp needle inside of the chamber and was able to detect at atmospheric pressure not only α-particles entering the chamber but β-particles also. This method has been used successfully in a number of researches by Kovarik and McKeehan,[4] Chadwick,[5] and

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[1] Rutherford, E., and Geiger, H., Proc. Roy. Soc., A 81, 162, 1908.

[2] Geiger, H., and Rutherford, E., Phil. Mag., S6, 24, 618, 1912.

[3] Geiger, H., Verh. D. Phys. Ges., 15, 534, 1913.

[4] Kovarik, Alois F., and McKeehan, L. W., Phys. Zs., 15, 434, 1914; PHYS. REV., N. S., 6, 426, 1915; PHYS. REV., N. S., 8, 574, 1916.

[5] Chadwick, J., Verh. D. Phys. Ges., 16, 383, 1914.

Vennes.[1] The writer[2] has used various modifications of the method mainly in the observing instruments, Zeleny's electroscope, C. T. R. Wilson's electroscope, a telephone receiver, and a ballistic galvanometer had been used with success for this purpose. These visual or audible methods of counting are quite trying on the nerves because close attention is necessary on account of the probability law of emission of the particles; and long periods of counting are essential because the method is a statistical one. A self-recording device would therefore be an obvious improvement. Rutherford and Geiger,[3] by moving a photographic film past the illuminated fiber of a string-electrometer, obtained a photograph of the fiber showing deflection of the fiber caused by the discharges between an electrode and the surrounding chamber when ionization by collision took place in the chamber upon the entrance of individual α-particles into the chamber. This has been the only method described thus far for obtaining a record of α-particles.

The method described in this paper makes it possible to get a record on a chronograph paper, of individual α-particles or β-particles or of γ-ray pulses (?), which may then be studied at one's convenience and the count made with a greater accuracy than is obtainable with the visual or audible methods. The accuracy of the method was tested both for α-particles and β-particles. The applicability of the law of probability to the emission of β-particles was proved and some experiments were made with γ-rays.

The Self-Registration Method.

In the visual method the counting chamber is connected to a fairly high positive voltage (about 2,000 volts) and the needle electrode is connected to earth through a very high resistance (ink on paper) and also to the fiber of the string electrometer or to the gold leaves of the other electrometers above mentioned. In the audible method a telephone receiver is placed between the needle and earth, if the discharge current in the counting chamber is sufficiently large to actuate the diaphragm of the telephone receiver; or else, it is placed in the plate-filament circuit of an audion amplifier, the needle electrode being connected directly to the grid or to earth through the primary of an induction coil, the secondary coil being connected to the grid and filament. In the auto-registration method, the current between the needle electrode and the walls of the counting chamber is amplified by a three electrode vacuum tube amplifier and this current is used to operate a very sensitive relay which in turn operates a local battery circuit actuating the pen on a chronograph. Fig. 1 shows diagrammatically the apparatus used and the necessary connections. *B* is a high potential battery (about 2,000 volts) whose positive pole is connected to the counting chamber, *W*, the other pole being earthed. If *W* is made negative a much higher voltage is necessary for any given point electrode. *E* is the needle point electrode which is connected through a high resistance, H.R., to earth. The high resistance consisted of India ink on paper and was

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[1] Vennes, H. J., Am. J. Sc., L. 4, 44, 69, 1917.

[2] Kovarik, Alois F., Phys. Rev., S. 2, 9, 567, 1917.

[3] Rutherford and Geiger, *loc. cit.*

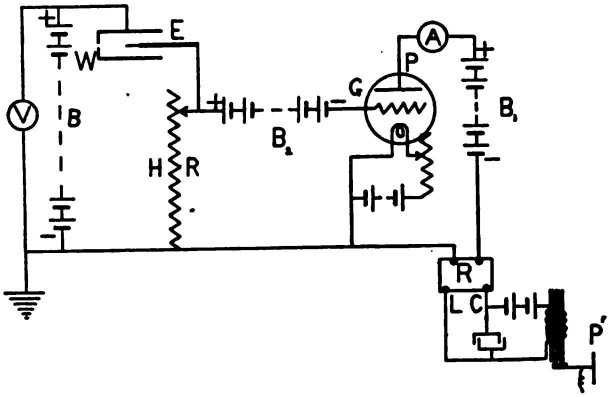

Fig. 1.

adjustable to different values. The needle electrode is also connected to the positive pole of a battery, B_2 (about 80 volts) the negative pole of which is connected to the grid of the audion amplifier. B_1 is another battery (about 160 volts) whose positive pole was connected to the plate of the amplifier and the negative pole to earth through the sensitive relay, *R*. The relay operated a local circuit, L.C., which actuated the chronograph pen, *P′*. As indicated in the figure another battery was used to heat the filament. A static voltmeter, *V*, indicated the voltage on the counting chamber, and a milammeter, *A*, registered the plate-filament current. The battery B_2 on the grid was used to oppose the battery B_1 on the plate and when properly adjusted only a negligible current would flow in the plate-filament circuit. When a discharge takes place in the counting chamber, the needle electrode becomes positively charged and because of the high resistance between it and earth, the grid comes momentarily to a higher potential and an increased electron current flows to the plate *P* which, passing through the relay *R* causes the local battery circuit to be closed and thus makes the pen register the discharge. The high resistance regulates the duration and partly the amount of the momentary potential on the grid. The sensitive relay operated on a current of a fraction of a milampere. Fig. 2 shows a photograph of a portion of a chronograph record automatically registered by β-particles entering the counting chamber at the rate of 160 per minute; and also in the lower portion of the natural discharges which occurred at the rate of about 8 per minute. The natural discharges are caused by the α-particles from the emanation in the air, by radiations from the material of the counting chamber,[1] by penetrating γ radiations from the earth and other sources, and are also perhaps partly due to imperfect needle points.

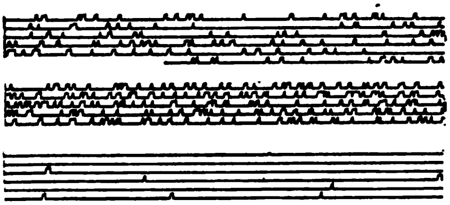

Fig. 2.

The needle point electrodes require special attention in preparation. Ordinary points may cause continuous discharge in the intense electric field. The needle electrode for our purposes must respond only when ionization is produced by radiations. The writer has experimented on many kinds of electrodes: pointed platinum wires, platinum wires with minute globules at the end produced by heating the end to melting temperature, steel points of various shapes and sizes, tungsten filaments, etc. Steel needles, ground to quite fine cylindrical ends and these treated in a flame (to slight redness) so as to form a smooth and uniform surface, have been found to serve best. The tungsten filament can easily be made into a point of any shape by heating (oxidizing) in a flame and these points serve quite well in a dry atmosphere and can easily be renewed; but humidity seems to affect the surface readily and destroy the requisite properties. A point electrode of lasting qualities has not yet been discovered. Some point electrodes may be used for more than 100 hours continuously while others break down after an hour's use.

TESTS OF THE METHOD.

The accuracy of the record is necessarily a function of the mechanical contrivances used. The sensitive relay used was tested and found to respond 100 times per second when the intervals were uniform but the electromagnet used to operate the pen was less rapid in its action. Consequently, particles following each other in a very short interval of time may not be recorded. The apparatus as used was found to give accurate records up to 400 per minute. Particles entering simultaneously would give one record, but the number of these as well as those following each other in a very short space of time becomes negligibly small for a rate of 100 or 200 per minute. These facts were obtained by varying the distance, in vacuo, between the source and the window of the counting chamber and finding that the number of α- or β-particles per minute which reached the window varied inversely as the square of the distance; also, by varying the size of the window in which case the number varied directly with the area. In the latter case the largest area was six times the smallest area.

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[1] It may be noted that a counting chamber of lead gives so large a number of natural discharges due to rays from Radium *DEF* in the lead as to make it useless for this purpose.

The counts for α-particles had been made on various polonium plates and the check was made by counting the scintillations on a good zinc sulphide screen and also by the ionization method at low pressures used by Geiger,[1] which method is, however, based on the scintillation method. Checks were always good and some have already been referred to in previous papers. Working with the β-particles in previous researches, Kovarik and McKeehan assumed that all β-particles were counted when the point gave an accurate check for the α-particles and when the number of β-particles counted remained constant with further increase of the potential on the counting chamber. This procedure has been found correct by the following experiments.

A check on the count of the β-particles from a given source was made by determining the total number of β-particles by measuring the charge carried by them and assuming each to carry one elementary charge. The source used was a radium DEF deposit on aluminum of very small area and only the more penetrating rays were used by placing the source between two aluminum plates. Radium DEF was used because the γ-ray effect is small. The plate so employed was flat and consequently the radiations through the aluminum envelope were not absorbed equally in all directions. To get the count of the total number of particles emitted per second by the plate it was necessary to count the β-particles in various directions and integrating. The source was placed axially in a brass cylinder lined with thick paper to reduce reflection. At one end of this vessel was placed a steel or brass plate having a window covered with aluminum of smallest thickness (0.0003 cm.) capable of withstanding atmospheric pressure. The counting chamber was placed against this window, being insulated from it and the brass cylinder.

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[1] Geiger, H., Proc. Roy. Soc., A 82, 486, 1909.

The brass vessel was exhausted to a pressure of less than 2 mm. of Hg in every case to eliminate absorption and scattering by air which is very important, the effect being about 50 per cent. when the source is 10 cm. from the counting chamber, for 1 atmosphere of air. The counting chamber was at atmospheric pressure. The source was set 10 cm. from the window and its plane could be rotated about an axis perpendicular to the axis of the cylinder and could thus be set for any angular position for which the count was to be made. The source was set at various angular positions from 0° to 360° but since the envelope was of uniform thickness on both sides the results for corresponding angular positions were averaged, and a curve drawn for positions within a quadrant. This curve is shown in Fig. 3, where the ordinate for 0°, *i. e.*, rays coming normally through the envelope, is taken as 100 and represents relatively the count of β-particles in a unit time through the window of the counting chamber. With the aid of this curve, the actual number counted at 0°, the area of the window, and the distance of source from the window one can determine the number of β-particles emitted into the various zones about the source as center. This integration can be made with sufficient accuracy by dividing the space into 3 or 4 zones. If 3 zones are considered with the source as center and the window of the counting chamber at 10 cm. from the source considered as the pole, a being the area (3.95×10^{-3} cm.^{2}) of the window and N (1.23 per second) the number of β-particles counted at 0°, the total number of β-particles per sec. is found to be

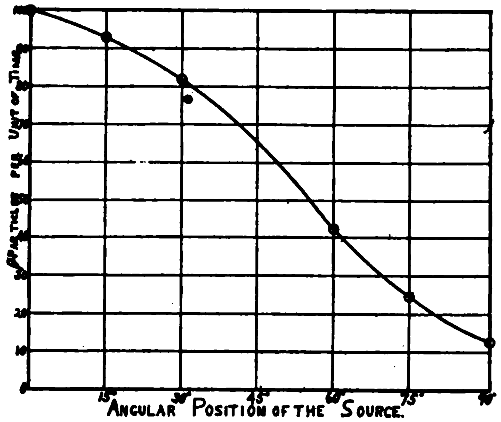

Fig. 3.

$$n = 2\frac{N}{\sec}\cdot\frac{1}{a}[X\cdot P_2 + Y\cdot P_2 + Z\cdot P_3]$$

$$= 2\left\{\frac{N}{\sec}\cdot\frac{1}{a}\cdot \pi r^2[0.268P_1 + 0.732P_2 + 1.P_3]\right\}$$

where X, Y, Z are the areas of the zones (for a hemisphere) and P_1, P_2, P_3 are the average values of the ratio of the number of β-particles per sec. for the various zones to the number counted at 0°. These ratios were obtained graphically from the accurate curve (Fig. 3). A small correction had to be made for the γ rays of radium D and the natural discharges. This was done by obtaining records when a steel or brass plate without a window, but otherwise similar to the one used in front of the counting chamber, was substituted. The total number of β-particles per sec. for the specimen used came out 1.89×10^5.

The source was then placed inside of, but insulated from a metal vessel lined with thick aluminum completely enclosing the source except for a small hole for the electrode holding the source and the whole was then placed inside another vessel which could be evacuated to 10^{-4} mm. Hg. The vessel surrounding the source absorbed the β-particles and the rate at which it charged up was measured by an electrometer, the source being alternately connected to positive and negative potential varying from 0 to 500 volts. The delta ray effects from the metal of the source and the enclosing vessel were found different, as recorded by Moseley[1] in his experiments, but were not so marked in these experiments as in his, owing to the small γ-ray effect of radium D. A magnetic field used similarly as by Moseley showed less marked influence on the results. Therefore, the mean of the rate of charging up of the electrometer with alternately positive and negative electrostatic fields between the source and the charge absorbing vessel, was used. Determinations were also made of the negative charge lost by the source and this checked with the charge absorbed by the surrounding vessel. The greatest difference between the various determinations made was 3 per cent. Assuming the electronic charge to be 4.77×10^{-11} e.s.u., the total number of β-particles by the charge method came to 1.92×10^5 β-particles per second. The number by the self recording method was 1.89×10^5 β-particles per second. The check is much closer than expected and warrants the writer to say that the β-particles are accurately recorded. It is to be noted the β-particles counted were the more penetrating rays of radium E ($\mu = 43.3$ cm^{-1} Al). Experiments were also performed with the very soft rays of radium D and E and the check was far from satisfactory and the results indicated that there are some soft rays which do not ionize but which nevertheless carry a negative charge. This particular point is being investigated.

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[1] Moseley, H. G. J., Proc. Roy. Soc., A 87, 230, 1912.

Emission of β-Particles Follows the Probability Law.

Since the record obtained on the chronograph is made by the β-particles themselves, such a record if made over a sufficiently long time would give the law of emission. A similar study was made by Rutherford and Geiger[1] of the law of emission of α-particles in which case the observer had to record the scintillations observed. In the present case the β-particles themselves made the record. A record was obtained of 37,266 β-particles in 430.6 minutes or a little over seven hours. The record was divided into intervals of 0.2 minutes and the number of β-particles in each of the 2,153 intervals was counted. The smallest number per interval was 7 and the largest number 31. The numbers of intervals containing 7, 8, · · ·, 31, were tabulated and plotted as ordinates against the number per interval as abscissas. To show that the experimental results follow a probability curve, the number of intervals which should contain any given number of particles was calculated from the total number of intervals and the average number of β-particles per interval assuming a probability distribution and using the formula of Bateman.[2] If x = average number per interval, and P = the probability that n enter per interval, then,

$$P = \frac{x^n}{n!} e^{-x}$$

from which the number of intervals containing n β-particles readily follows. Plotting these values against n we get the curve shown in Fig. 4. The observed values are plotted as circles. It will be noticed that these points follow the curve fairly closely, falling a little under at the low and high values of n and coming somewhat above at the mean

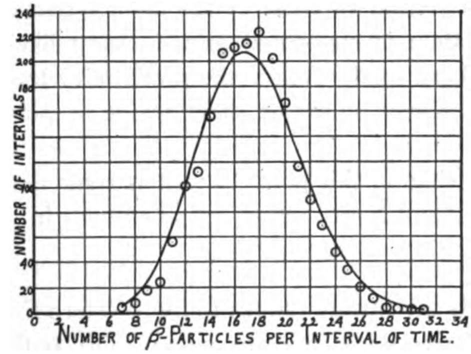

Fig. 4.

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[1] Rutherford, E., and Geiger, H., Phil. Mag., S. 6, 20, 698, 1910.

[2] Rutherford, E., and Geiger, H., Phil. Mag., S. 6, 20, 698, 1910.

values. It may be concluded that the variations in the number of β-particles emitted in a given interval of time follow the probability law.

γ-RAYS AND X-RAYS.

If the counting chamber is made without a window and of material to permit no α- or β-particles to enter from the outside, and some radium is brought into the neighborhood, discharges between the point and the chamber take place and records are obtained which are similar to those obtained in the case of the α- or β-particles. The same is true when X-rays are passed through the chamber. The number of these discharges decreases with increase of distance of the source and for the γ-rays it was shown to follow the law of inverse squares, the distance being changed from 10 feet to a couple of inches. If lead screens are interposed between the source of γ-rays or X-rays and the counting chamber, the numbers are decreased. The effect is therefore directly connected with the energy of the γ-ray and X-ray pulses. It seems likely that the β-particles produced on the inside of the counting chamber from the metal (mainly) and also from the air, are directly responsible for the point discharges. Whether a pulse produces one or a large number of such β-particles seems immaterial for causing the point discharge provided that these β-particles ionize. To all intents and purposes the β-particles produced by the pulses will act at the same time and therefore one discharge will result per pulse or train of pulses. Since the records appear similar to those of the α- or β-particles, in so far as distribution with time is concerned and do very likely (although no calculations have been made) follow the probability law, it seems likely that the γ-ray or X-ray pulses or short trains of pulses, are actually registered.

Experiments bearing on this point as well as some unsettled questions regarding the γ-ray and X-ray pulses are at present in progress and will be reported later.

I desire to express my sincere thanks to the Western Electric Company, or the special sensitive relay and the audion bulb, both of which they kindly loaned me for my experiments.

I also wish to acknowledge the assistance of Mr. Walter B. Lang in some of the observations.

SHEFFIELD SCIENTIFIC SCHOOL,
YALE UNIVERSITY,
NEW HAVEN, CONN.,
January 25, 1919.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NINETY-SIXTH MEETING.

THE ninety-sixth meeting of the American Physical Society was held in Fayerweather Hall of Columbia University, New York, on February 28 and March 1, 1919.

A regular meeting of the Council was held on February 28, five members being present, and President Ames presiding. Professor Theodore Lyman, of Harvard University, was elected vice-president of the society for the unexpired term caused by the death of Professor Wallace C. Sabine. The following were recommended to the National Research Council as representatives of the American Physical Society in the Division of Physical Sciences: H. A. Bumstead, Wm. Duane, Irving Langmuir, Earnest Merritt, R. A. Millikan, and E. B. Wilson. The following were appointed representatives of the American Physical Society in the International Astronomical Union: J. S. Ames, Henry Crew, and Theodore Lyman.

Elections to membership were made as follows: to *regular membership*, J. A. Fleming, Carl Hering, Paul W. Merril, Henry Norris Russell; to *associate membership*, Carl Darnell, J. P. Delaney, Carl F. Eyring, Joseph Foster, Clifford G. Formd, John G. Homan, Horton W. Hall, G. A. Johnstone, Edwin W. Kelly, C. N. Moore, A. M. Mahon, B. A. Orndorff, K. G. Szlupas, Paul B. Taylor, Phillips Thomas; *transferred from associate to regular membership*, S. J. Mauchly, Peter I. Wold.

Sessions for the reading of papers were held in the morning and in the afternoon of Saturday, March 1, 1919, at which twenty-seven papers were presented, six being read by title. The program was as follows:

Water Vapor Pressure in Units of Force. ALEXANDER MCADIE. (Read by title.)

The Thermal Expansion of Living Tree Trunks. C. C. TROWBRIDGE (deceased) and MABEL WEIL.

Pyrometer Absorption Glasses. PAUL D. FOOTE, C. O. FAIRCHILD and F. L. MOHLER. (Read by title.)

The Function of Phase Difference in the Binaural Location of Pure Tones. R. V. L. HARTLEY.

Temperature Coefficient of Resistance of Molybdenum. E. C. BLOM.

Measurement of Small Alternating Currents by Means of a Rayleigh Resonator. F. R. WATSON. (Read by title.)

An Addition to the Theory of the Quadrant Electrometer. A. H. COMPTON and K. T. COMPTON.

Permanent-Contact Crystal Detectors. LOUISE S. MCDOWELL.

Are the Frequencies in the Series of X-rays the Highest Frequencies Characteristic of Chemical Elements? WILLIAM DUANE and TAKEO SHIMIZU.

The Spectral Photoelectric Sensitivity of Silver Sulphide. W. W. COBLENTZ and H. KAHLER. (Read by title.)

Some Photoelectric Lecture Experiments. JACOB KUNZ. (Read by title.)

The Crystal Structure of Carborundum. A. W. HULL.

A Possible Relation between the Resonance Potential and Specific Inductive Capacity of a Metallic Vapor and its Properties in the Solid State. K. T. COMPTON.

Collisions of Slow Moving Electrons with Nitrogen Molecules. S. KARRER. (Read by title.)

The Ionizing Potentials of Argon, Neon and Helium. H. C. RENTSCHLER.

Note on the Use of the Hot Cathode Vacuum Tube for the Measurement of Voltage. LYNDE P. WHEELER.

An Interferential Method for Measuring the Expansion of very small Samples. C. G. PETERS and IRWIN G. PRIEST.

On the Measurement of the Detection Coefficient of Thermionic Vacuum Tubes. H. J. VAN DER BIJL.

The Properties of the Electron as Derived from the Chemical Properties of the Elements. IRVING LANGMUIR.

The Law of Absorption of High Frequency Radiation. ARTHUR H. COMPTON.

Some Physical Properties of Dental Materials. C. G. PETERS and W. H. SOUDER.

The Origin of the General Radiation Spectrum of X-Rays. DAVID L. WEBSTER.

Note on Bomb Trajectories. EDWIN B. WILSON.

On the Relation Between the *K* Series and the *L* Series of X-Rays. WILLIAM DUANE and TAKEO SHIMIZU.

A New Formula for the Spectral Distribution of Energy from a Complete Radiator. IRWIN G. PRIEST.

Transmission of a Quartz Grating Replica. R. S. WETZEL and WALTER SCHWANHAUSER.

On the Measurement of the True Barometric Pressure in a Rapidly Moving Current of Air. J. G. COFFIN.

DAYTON C. MILLER,
Secretary.

Thermionic Amplifier.[1]

By H. J. Van der Bijl.

The current voltage characteristic of the vacuum-tube amplifier manufactured by the Western Electric Company can be represented by the equation

$$I = \alpha(\gamma\, E_B + E_C + \epsilon)^2, \tag{1}$$

where I is the current in the filament-plate circuit, E_B the potential difference between filament and plate, E_C the potential difference between filament and grid, and ϵ a small quantity which depends on the contact potential difference between filament and grid, the potential drop in the filament, etc.

The equation is obtained as follows: Suppose the grid and filament be electrically connected and a potential difference E_B be applied between them and the plate, then the field at a point near the filament is the same as if a potential difference $E_B + \epsilon$ be applied between the filament and a plane coincident with the grid. If a potential difference E_C be also applied between the filament and the grid the effective voltage is the sum, namely $\gamma\, E_B + E_C + \epsilon$. The exponent 2 has been found empirically.

With the help of this equation the amplification equations of the tube can be obtained. It shows that the relation between plate current and grid voltage is of the same form as the relation between plate current and plate voltage. If an alternating voltage $e \sin pt$ be applied between grid and filament the current is

$$I = \alpha(\gamma\, E_B + E_C + \epsilon + e \sin \mathrm{pt})^2. \tag{2}$$

The admittance of the tube K is given by

$$\frac{1}{2\kappa}\int_0^{2\pi} \frac{\partial I}{\partial E_B}\, dt = 2\alpha\gamma\,(\gamma\, E_B + E_C + \epsilon).$$

The impedance is $1/K$, and this can be expressed as

$$R_0 = \frac{E_B + \mu_0\,(E_C + \epsilon)}{2I}, \tag{3}$$

where $\mu_0 = 1/\gamma$ and is the maximum voltage amplification obtainable from the tube.

If a resistance R be placed in the filament plate circuit, the voltage E_B between filament and plate does not remain constant, but is given by

$$E_B = E - RI, \tag{4}$$

where E is the constant voltage of the battery in the plate circuit. The characteristic equation then becomes

$$I = \alpha\,[\gamma(E - RI) + E_C + \epsilon + e \sin \mathrm{pt}]^2. \tag{5}$$

It is seen from this that the external resistance has the effect of straightening out the characteristic. For a perfect amplifier the characteristic must be straight, otherwise distortion is produced, because in virtue of the curvature of the characteristic an increase in the input voltage (filament grid voltage) produces a greater increase in current that the current decrease produced by an equivalent decrease in the input voltage. It has been found that if the external resistance is equal to or greater than the impedance of the tube itself, the characteristic is sufficiently straight to reduce distortion to a negligible quantity.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, April 27, 1918.

Considering now the case in which the circuit contains an external resistance R, it is seen that a variation in the current in R due to a variation in the grid voltage causes the plate voltage also to be variable, that is,

$$I = f\,(E_B,\ E_C)$$

and hence

$$\frac{dI}{dE_C} = \frac{\partial I}{\partial E_B}\,\frac{dE_B}{dE_C} + \frac{\partial I}{\partial E_C}\,. \tag{6}$$

Referring to equations (1) and (4) this gives

$$\frac{dI}{dE_C} = 2\alpha\,(\gamma\ E_B + E_C + \epsilon)\left(\gamma\,\frac{d(E - RI)}{dE_C}\right),$$

i. e.,

$$\frac{dI}{dE_C} = \frac{2\alpha\,(\gamma\ E_B + E_C + \epsilon)}{1 + 2d\gamma R(\gamma E_B + E_C + \epsilon)}\,.$$

Multiplying by R and putting $\gamma = 1/\mu_0$ we obtain

$$R\,\frac{dI}{dE_C} = \frac{\mu_0 K}{R + \dfrac{E_B + \mu_0(E_C + \epsilon)}{2I}}\,. \tag{7}$$

Now RdI is the voltage change set up in the resistance R and dE_C is the change in the input voltage, so that equation (7) gives the voltage amplification μ which with the help of (3) can be written

$$\mu = \frac{\mu_0 R}{R + R_0}\,. \tag{8}$$

From this it readily follows that if R_i be the input impedance of the tube, the power amplification is given by

$$\eta = \frac{\mu^2\,R\,R_i}{(R + R_0)^2} \tag{9}$$

and the current amplification by

$$\xi = \frac{\mu_0 R_i}{R + R_0}\,. \tag{10}$$

These equations have been tested and found to hold with sufficient accuracy to serve for the computation of circuits involving thermionic amplifiers.

It must be remarked that the characteristic equation used here is not sufficiently accurate to apply in the case in which the tube is used as a radio-detector. The detecting action of the tube depends on the second derivative of the characteristic, so that when using the tube as a detector a more accurate equation for the characteristic is used.

Research Laboratory,
Western Electric Co.,
New York.

WATER VAPOR PRESSURE IN UNITS OF FORCE.[1]

BY ALEXANDER MCADIE.

THERE are many instructors in physics and chemistry who are not at the present time, for good and sufficient reasons, fully conversant with the units used by aërographers.

Probably no one feature of the war, not even the use of gas, nor the success of the submersible, stands out so sharply as the development of the airship. Whether we be advocates of the plane or the blimp, representing in general heavier-than-air and lighter-than-air craft, we all recognize that a new and vastly important field of engineering has been opened up in aëronautics, and a new and promising field of investigation for the physicist in studies of the structure of the atmosphere, more briefly called aërography.

With the first use of the sounding balloon it became apparent that the Fahrenheit scale would have to be scrapped and that the Centigrade scale was likewise unserviceable. The reasons are self-evident. And so the temperatures in all upper air work are given on the Absolute scale. The form used however is not the most convenient, and we have been using with success at Blue Hill for several years a new form which has been called the Kelvin kilograde and which has been described elsewhere The zero is the absolute zero and the melting point of ice under standard pressure is marked 1000. No degree signs are used. The values used later are in these units.

In upper air work it also became evident that inches and millimeters as units for the expression of atmospheric pressure were, like the temperature units, out-of-date; and so the aërographic services of many countries, especially Great Britain, France and Italy have been expressing pressure in units of force. This usage is spreading rapidly.

So far as I can ascertain credit for the suggestion and first use of an absolute unit to express pressure must be given to Professor Theodore Richards. In the classic paper on "New Method of Determining Compressibility" (Carnegie Inst., 1903), the *bar* is clearly defined as the pressure of a dyne per square centimeter. The pressure of an atmosphere is expressed as one million bars or megabar. For convenience this is written 1,000 kilobars. The pressure of the atmosphere at a height of 1 km. is approximately 900 kb., at an elevation of 10 km. the pressure is about 300 kb., and at the top of the atmosphere or as high as we may hope to explore for some time to come, say 50 km., the pressure is as low as 5 kb.

The use of these units of force has been extended at Blue Hill to measurements of water vapor pressure. There appears to be no difficulty in using the new units and the formulæ below.

European aërographic services use the following formulæ when the cover of the wet bulb is not frozen, and it may be noted here that in high level observations the temperature is generally so low that the wet bulb is covered with ice. No satisfactory solution has yet been reached.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

With wind velocity 0 to 0.5 m/s (meter per second)

$$p_s = p_e - .0012\, P(t - f_1)\ (1 + f'/_{610})$$

with wind velocity 1 to 1.5 m/s

$$p_s = p_e - .0008\, P(t - t_1)\ (1 + t'/_{610})$$

with wind velocity exceeding 2.5 m/s

$$p_s = p_e - .000656\, P(t - t_1)\ (1 + t'/_{610})$$

When the water is frozen, the numerical coefficients .001060, .000706 .000579, are used, and 610 in the last factor changed to 689.

We prefer to express the relation in the following form

$$p_s = p_e - PK\ (t - t')$$

in which p_s is the pressure of the water vapor at the temperature of the dew-point or saturation, p_e the pressure at the temperature of evaporation or wet-bulb, P the pressure of the atmosphere, K a constant which like most constants varies somewhat (with ventilation of the bulb and probably other conditions). All pressures are given in the same unit, kilobar, and all temperatures in same unit, Kelvin kilograde. The value of K within ordinary limits is .00179. For quick determination of vapor pressure, absolute humidity and relative humidity, the value of PK may be taken as .18.

The wind effect is an uncertain quantity. All the tables of relative humidity so laboriously compiled by various meteorological services are invalidated by the omission of statement of temperature and wind values. Reference is made to papers by the author: "New Units in Aërology," PHYSICAL REVIEW, N.S., Vol. VI., No. 6, December, 1915; "Comparative Accuracy of Whirled Psychrometer," etc., PHYS. REV., February, 1918.

THE THERMAL EXPANSION OF LIVING TREE TRUNKS.[1]

BY C. C. TROWBRIDGE (DECEASED) AND MABEL WEIL.

THE transverse coefficient of expansion of living tree trunks has been found to be extremely large below 0° C. and very much smaller, *i. e.*, of about the order of the coefficient of expansion of dead wood, above that temperature. In the longitudinal direction a small *expansion* with *rise* of temperature was noted above the freezing point while an equally small *expansion* with *fall* of temperature was measured below that point. When, however, the temperature fell to about −18° C., a slight *contraction* with *fall* of temperature was observed. This behavior indicates the presence of two processes superposed, one a thermal coefficient of expansion, the other a physiological or cellular expansion. A lag of the transverse expansions and contractions behind temperature changes, also points to the same conclusion.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

PYROMETER ABSORPTION GLASSES.[1]

BY PAUL D. FOOTE, C. O. FAIRCHILD AND F. L. MOHLER.

THE proper methods of using absorption glasses and the choice of a suitable absorption glass for extending the high temperature scale are questions of considerable importance. The calibration of an absorption glass is given by the formula

$$1/\theta - 1/S = A,$$

where θ is the absolute temperature of a black body, S the absolute observed temperature with the absorption glass in place and A is in general a function of θ.

A glass is described for which A is a constant. Various methods of determining A are given. The use of two absorption glasses together is discussed and several theorems are developed in regard to effective wave-lengths of colored glasses, effective wave-length when sighted on non-black radiators, etc. A synopsis of the paper will appear in the Journal of the Optical Society and the complete paper will be published in the Bulletin of the Bureau of Standards.

THE MEASUREMENT OF SMALL ALTERNATING CURRENTS BY MEANS OF A RAYLEIGH RESONATOR.[1]

BY F. R. WATSON.

THE resonator consisted of a brass cylinder closed at one end by a telephone receiver and open at the other end through a small tube in which a Rayleigh disc was suspended by a quartz fiber. Alternating currents sent through the telephone receiver set up vibrations of the air in the cylinder that caused the disc to rotate, the amount of rotation being noted by the deflection of a beam of light reflected from the disc. A 110-volt, 60-cycle alternating current gave readable deflections for currents of the magnitude of 1×10^{-8} amperes. Later experiments indicated a greater sensitivity.

Various modifications of the apparatus were tested. A double resonator was made of two resonators similar to the one described connected by a tube with the Rayleigh disc in the center of the connecting tube. This arrangement allowed the use of two telephones and, being closed from the outside air, eliminated almost entirely the effect of extraneous sounds. An ordinary Bell telephone arrangement of a transmitter, dry cell, induction coil and receiver gave deflections for speech and musical sounds. Sensitiveness was increased by tuning the resonators and the electrical circuit to the source of sound. The frequencies of the currents ranged from 60 to 1,200 per second.

UNIVERSITY OF ILLITOIS,
December 1, 1918.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

AN ADDITION TO THE THEORY OF THE QUADRANT ELECTROMETER.[1]

BY A. H. COMPTON AND K. T. COMPTON.

THE ordinary theory of the quadrant electrometer is extended to take account of a tilt of the needle about its long axis, a vertical displacement of one pair of quadrants with respect to the other by an amount δ and a vertical displacement of the needle from its position of symmetry by an amount p. Under these conditions, if the slope of the tilted needle is s, the torque due to electrical forces is

$$T = K_1 V(v_2 - v_1) + V^2\left(K_3 \frac{Rs\delta}{h^2}\theta + K_4 + K_5 \frac{\delta p}{h^2}\right), \tag{1}$$

neglecting terms of second and higher order of small quantities. Here V is the potential of the needle, v_1 and v_2 are the potentials of the two pairs of quadrants, h is the height of the quadrants, R is the radius of the needle and θ is the deflection.

The terms

$$\left(K_4 + K_5 \frac{\delta p}{h^2}\right) V^2$$

cause the deflection usually observed when the needle is charged. By suitably adjusting δ or p these terms may be made to cancel. Then the remaining terms are balanced against the torque $K_2\theta$ from the suspension, giving

$$K_1 V(gv_2 - v_1) = \left(K_2 - K_3 \frac{Rs\delta}{h^2} V^2\right)\theta; \tag{2}$$

whence the sensitiveness is

$$S = \frac{\theta}{v_2 - v_1} = \frac{K_1 V}{K_2 - K_3 \dfrac{Rs\delta}{h^2} V^2}. \tag{3}$$

By suitably adjusting s, δ and V, the magnitude and sign of the second term in the denominator may be varied within wide limits permitting a range of sensitiveness, theoretically from zero to infinity, to be obtained with any given suspension.

A model of an electrometer designed to take advantage of these considerations is demonstrated.

PERMANENT—CONTACT CRYSTAL DETECTORS.[1]

BY LOUISE S. MCDOWELL.

THE following investigation of possible methods of securing a permanent-contact crystal detector was initiated by a request from the Signal Corps to the Bureau of Standards to investigate a method proposed by Second Lieutenant Henri Lauer. His proposal to secure a permanent multiple-contact crystal detector by alternating-current electrolysis was based upon the hypothesis that, when immersed in an electrolyte, the rcetifying action of the crystal would permit current to flow in only one direction through the sensitive areas and that upon these areas alone there should be a deposit of the metal since the flow of current in both directions through the insensitive areas would prevent deposition. To the deposits upon the sensitive areas permanent contact could then be made by direct-current electrolysis or otherwise.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Audibility tests made upon two or more detectors in parallel and upon single crystals with contacts in parallel to two or more sensitive areas indicated that no gain in loudness of signal was to be expected from the use of multiple contacts. Crystals were further divided into three classes according to the direction of flow of the rectified current. Class 1, in which the direction of the current was usually from point to crystal, included molybdenite, zincite, chalco-pyrites, iron pyrites, and bornite. Class 2, in which the direction of current flow varied from one sensitive point to another on the same crystal, included carborundum, approximately fifty per cent. of the specimens of silicon tested and one or two specimens of galena and molybdenite. Class 3, in which the direction of the current was usually from crystal to point included galena and fifty per cent. of the silicon tested.

Alternating-current electrolysis, with currents of 60 cycles and of radio frequency, was tried with crystals of class 1. A few adherent deposits of copper were obtained, but results indicated that these were due to accidental causes, not to rectification by the crystal since (1) only a few of the sensitive spots were coated, (2) there were many insensitive deposits, (3) lack of perfect adhesion between deposit and crystal increased the sensitiveness when contact was made to the deposit.

With galena, of class 3, an attempt was made to secure a metallic deposit upon the sensitive spots by cathode sputtering, using an alternating discharge and making the crystal one electrode. Results indicated that under these conditions no rectification occurs of a nature to permit deposition upon the sensitive areas of the crystal.

Further attempts to obtain a fixed, sensitive contact by the use of direct-current electrolysis, by soldering, and by welding proved unsuccessful with the more sensitive crystals such as galena and silicon although partially successful with molybdenite and carborundum. The results point to the necessity of a high resistance at the contact and suggest that with the more sensitive crystals the only satisfactory permanent-contact detector will be one with a mechanically maintained, high resistance contact.

Bureau of Standards,
Washington, D. C.

Are the Frequencies in the K Series of X-rays the Highest Frequencies Characteristic of Chemical Elements?[1]

By William Duane and Takeo Shimizu.

A NUMBER of investigators have found experimental evidence, which they interpreted as indicating the existence of radiation or absorption characteristic of a chemical element, and of higher frequency than those of its *K* series of X-rays. On the other hand curves have been published showing the emission of X-rays by chemical elements such as rhodium and molybdenum, which include rays in the general X-radiation up to frequencies 50 per cent. and 75 per cent. greater than those of the *K* radiation and these curves furnish no evidence of the existance of characteristic rays of higher frequency than the *K* absorption frequency.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

In this paper we wish to report to the society the results of experiments on the X-rays emitted by an aluminium target. The wave-lengths of the X-rays that we have examined lie between $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. This is the portion of the X-ray spectrum usually obtained from an ordinary X-ray tube excited by a difference of potential of 71,200 volts. Such a difference of potential does not produce X-rays shorter than $.1820 \times 10^{-8}$ cm. and rays longer than about 1.26×10^{-8} cm. are almost completely absorbed by the glass walls of the tube. In order to investigate such long waves, specially constructed tubes must be employed.

As the shortest wave-length in the *K* series of aluminium is 7.98×10^{-8} cm., the range of frequencies in our experiments extends from 6.34 to 43.8 times the highest frequency in this *K* series.

In making the measurements we used the apparatus described in the PHYSICAL REVIEW for December, 1917, p. 624. This consisted of an X-ray spectrometer with a calcite crystal, the beams of X-ray being defined by two slits in lead blocks, which lay between the X-ray tube and the spectrometer. The current through the tube came from a high tension transformer connected to a system of kenotrons and condensers, which produced,approximately a constant voltage. During part of the work the difference of potential amounted to 53,200 volts and during the rest of it, to 71,200 volts. The X-ray tube (kindly furnished us by Dr. W. D. Collidge) was similar to the ordinary Coolidge tube, except that an aluminium target replaced the usual tungsten target.

The curves obtained by platting the ionization currents against the grazing angles of incidence are, within the errors of measurement, smooth curves without such prominences as indicate characteristic rays, except at the following points:

(*a*) Near the angles corresponding to $\lambda = .622 \times 10^{-8}$ cm. and $\lambda = .709 \times 10^{-8}$ appears a pair of well defined prominences, indicating characteristic radiation amounting to 2 per cent. and 5 per cent. of the general X-radiation. These are undoubtedly due to the characteristic *K* radiation of molbydenum, the wave-lengths of the β and α lines of which are $.641 \times 10^{-8}$ cm. and $.716 \times 10^{-8}$ cm. respectively, as given by Wooton.[1]

(*b*) There are also prominences near the angles corresponding to $\lambda = .975 \times 10^{-8}$ cm. $\lambda = 1.18 \times 10^{-8}$ cm. These lie very close to the β and α lines in the *L* series of lead. According to Siegbahn and Friman's tables the wave-lengths of the strong β and α lines in the *L* series of lead are $\lambda = .981 \times 10^{-8}$ cm., and $\lambda = 1.17 \times 10^{-8}$ cm.

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[1] PHYSICAL REVIEW, January, 1918.

Part of the Coolidge cathode in the X-ray tube consists of metallic molybdenum so that the presence of the molybdenum lines may be accounted for either as coming from a thin layer of metallic molybdenum deposited on the target during the construction of the tube, or else as the result of secondary radiation from the molybdenum in the cathode producing tertiary radiation from the target.

The lead lines undoubtedly are due to secondary radiation from the lead blocks containing the slits through which the X-rays passed before reaching the spectrometer.

If the voltage applied to an X-ray tube lies considerably above that required to produce the characteristic rays of its target (as is the case here with molybdenum and lead), the characteristic radiation is many times more intense than the general radiation in its neighborhood. Further molybdenum and lead are far more efficient radiators than aluminium is (their atomic numbers being much higher, 47 and 82 respectively instead of 13 for aluminium).

In addition to these prominences there appears a sharp break in the ionization curve at an angle corresponding to $\lambda = .3727 \times X^{-8}$ cm. which is the characteristic ionization wave-length of iodine. This break is due to the fact that methyl-iodide was used as the gas in the ionization chamber. To make sure that no radiation characteristic of aluminium occurs near this wave-length, we removed the methyl-iodide, and refilled the ionization chamber with ethyl-bromide. Under these conditions no prominence nor break appears in the curve near $\lambda = .3727 \times 10^{-8}$ cm.

We conclude, therefore, that aluminium possesses no characteristic radiation within the range of wave-lengths examined.

The Spectral Photo-electric Sensitivity of Silver Sulphide.[1]

By W. W. Coblentz and H. Kahler.

Both the natural mineral acanthite and a laboratory preparation by Mr. G. W. Vinal, were examined. The latter material which was hammered into a thin plate, was found insensitive photo-electrically, at room temperature, but at −157° C. a sharp maximum of photo-electrical sensitivity was observed for radiations of wave-length $\lambda = 1.05\,\mu$.

At room temperatures the natural crystalline mineral differs from other photoelectrically sensitive substances studied in that the photo-electric response becomes fatigued and after an exposure of 2 to 3 seconds to light, the positive resistance change (galvanometer deflection) begins to be effective. For unlimited exposure, the galvanometer deflection returns to about one fourth of its maximum deflection. On removing the light stimulus, the galvanometer gives a negative deflection, which in the course of a few minutes returns to the original zero scale reading. In other words, the change in resistance of the crystal when exposed to radiation, is first negative then positive, the resultant change being negative and roughly one fourth the original change. At low temperatures, −158° C., this polarization phenomenon disappears, and the response to radiation is the same as that of other substances, *e. g.*, selenium and molybdenite.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

At room temperature, acanthite has a maximum of photo-electrical sensitivity for radiations of wave-length $\lambda = 1.35\,\mu$ with a region of high sensitivity at 0.8 to 1 μ.

At −158° C. the sensitivity curve is quite symmetrical with a maximum at 1.2 μ. Mechanical working of the crystal (hammering it into a thin plate) has a marked effect upon this maximum which, in one sample, occurred at 1.13 μ to 1.23 μ, depending upon the amount of hammering the crystal had undergone.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
February 5, 1919.

THE CRYSTAL STRUCTURE OF CARBORUNDUM.[1]

BY A. W. HULL.

IN the December number of the Journal of the American Chemical Society Burdick and Owen have described experiments on the crystal structure of carborundum. They conclude that the structure is like that of diamond, with half of the carbon atoms replaced by silicon, with this exception, that the carbon atoms are not in the centers of the tetrahedra formed by the silicon atoms, but are displaced toward the apices of the tetrahedra, corresponding to a lack of symmetry in the carbon atom for which we have no other evidence. Their conclusions are based entirely upon the relative intensities of the X-ray reflections, and their calculations are made in exactly the same manner as those of the Braggs on calcite and similar crystals. It is assumed that the scattering electrons of each atom are all concentrated at the center of the atom, and that some undefined cause produces a "normal" fall of intensity in successive orders represented by the numbers 100, 20, 7 and 4. The variations from this normal fall of intensity are the basis of the conclusions drawn regarding the positions of the atoms.

Inasmuch as the assumptions on which this analysis is based are certainly not valid, it seemed worth while to investigate what variations in intensity might be expected from a *perfect* diamond lattice, composed of atoms having stationary electrons in cubical arrangement, such as have been found to account for the intensities of the reflection observed in diamond and crystal silicon respectively. The only assumptions made are that the electrons are fairly stationary in the positions specified, and that they scatter X-rays independently according to classical electromagnetic theory.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

In order to make the conclusions as definite and free from arbitrary assumptions as possible, the C atom is assumed to have *exactly the same size and shape* as in diamond, namely, two electrons very near the nucleus, and four electrons at the opposite (tetrahedral) corners of a cube at distances of .77 aengstroms from the nucleus; and the silicon atom to be *exactly as in crystal silicon*, except that its four outer electrons are drawn in about seven per cent. It has two electrons very near the nucleus, eight at the corners of a cube at distances of .15 aengstroms from the nucleus, and the remaining four at opposite (tetrahedral) corners of a larger cube at distances of 1.12 aengstroms from the center. The atoms are assumed to be all similarly oriented, with the axes of their cubes parallel to the crystalographic axis of the carborundum crystal (assumed cubic). The calculations are made in a manner exactly analogous to that of the Braggs, applying Darwin's formula to *single electrons*.

The following table gives the intensities calculated in this way for the different reflections observed by Burdick and Owen.

The agreement with the experimental values is as good as the data warrants, and is fully as good as in Burdick and Owen's calculations. When more exact experimental data is available it will probably be found that the arrangement of electrons in exact cubes as assumed above is not correct, but that these cubes are distorted, at least slightly, by the forces of attraction holding the atoms together.

The facts which it is desired to emphasize here are (1), that the assumption of simple cubical atoms, that is, atoms composed of a nucleus surrounded by stationary electrons in cubical arrangement, accounts completely for the observed variation in the intensity of the reflections of different orders in the case of "normal" planes. (2) that in the case of carborundum this "shape factor" is capable of explaining completely the observed *variations* from the "normal" intensity, without assuming any lack of symmetry in the arrangement of the atoms.

The structure of carborundum may therefore be considered, as far as our present information goes, to be a lattice *exactly* like that of diamond, in which half the carbon atoms, that is, those belonging to one of the two face-centered cubic lattices, are replaced by silicon. Each carbon atom is surrounded by two electrons very near its center, plus 8 equidistant electrons, four of its own and one each from the four surrounding silicon atoms along the four "chemical bonds" joining these silicon atoms to the carbon. These eight electrons form an exact cube of side .89 Å. around the carbon atom. Each silicon atom has two electrons very near its center, surrounded by eight electrons in a cube of side .17 aengstroms, surrounded in turn by the four above mentioned electrons

Reflecting Plane.	Order of Reflection.	Intensity of Lines.			
		Observed.		Calculated.	
		Crystal I.	Crystal II.	Hull.	Burdick and Owen.
100	1	100	100	100	100
	2	126	470	47	72
	3	57	196	130	17
$\bar{1}10$	1	100	100	100	100
	2	61	38	77	20
	3	22	7	29.2	7
111	1	100	100	100	100
	2	35	55	18	31
	3	19	38	16	19
	4	13	23	3.2	4
	5	3	—	8	2
	6	1	—	3.5	2
$11\bar{1}$	1	100	100	100	100
	2	35	37	18	7
	3	44	57	16	3
	4	18	14	3.2	5
$2\bar{1}1$	1	100	100	100	100
	2	23	27	38	20
$11\bar{3}$	1	100	100	100	100
	2	12	12	18	14
$31\bar{1}$	1	100	100	100	100
	2	21	21	18	31
	3	—	8	10	19
210	1	100	100	100	100
	2	10	11	14	13

in tetrahedral arrangement, on the lines joining it to the four nearest carbon atoms. The distance of these outer four electrons from the center of the silicon atom is 1.12 aengstroms. This is slightly less than their distance, 1.20 aengstroms, from the centers of silicon atoms in crystal silicon. In other words, the atoms of carbon and silicon are about 4 per cent. closer to each other in carborundum than would be obtained by packing together carbon and silicon atoms of the exact size and shape as found in diamond and crystal silicon. This closer approach should correspond to the chemical stability of carborundum as compared with diamond and silicon. For simplicity, it has been assumed in the above calculations that the carbon atom maintained its exact size, the shrinkage being attributed entirely to the silicon electrons as being the more mobile.

February 14, 1919.

A POSSIBLE RELATION BETWEEN THE RESONANCE POTENTIAL AND SPECIFIC INDUCTIVE CAPACITY OF A METALLIC VAPOR AND ITS PROPERTIES IN THE SOLID STATE.[1]

BY K. T. COMPTON.

RECENT discoveries have shown that the long wave-length limit of photoelectric sensitiveness of a metal is greater in the solid than in the vapor state. In other words, the energy required to liberate an electron from an atom of the vapor is greater than that required to liberate an electron from a metal. However the work required to extract an electron from the metal is strikingly similar to that required to produce the "single line" spectrum of the vapor. A table is given showing the degree to which these quantities are identical, from which it appears that in the case of a majority of the metals the discrepancies fall within the limits of experimental error. Theoretical considerations indicate that exact coincidence would not be expected even if the relation suggested were true.

If this relation is true, it is suggestive of interesting speculations regarding the condition of the electrons in a metal. It would appear that the atoms retain practically the same structure as in the vapor state as far as the equilibrium orbits lying within that orbit involved in the production of the single lined spectrum are concerned, but that an electron outside this orbit becomes a "free electron" in that it is not subject to appreciable attractions from the parent atom. Thus the work required to extract an electron from a metal is very nearly that required to withdraw it from the "single line" orbit to the one next outside. These considerations are highly speculative at present, though it would not be surprising if an attempt to apply the Bohr theory of the atom to solids would yield fruitful results.

Attention is also called to the fact the argument used by the writer to derive the ionizing potential of a gas in terms of the specific inductive capacity of the gas[2] applies equally well to the resonance potential, applying to whichever has the lower value. Recent published and unpublished data on ionizing and resonance potentials and absorption spectra are shown to support the validity of the argument as a first approximation.

PRINCETON UNIVERSITY.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

[2] PHYS. REV., 8, p. 412, 1916.

THE LAW OF ABSORPTION OF HIGH FREQUENCY RADIATION.[1]

BY ARTHUR H. COMPTON.

OWENS' law for the absorption of X-rays, that

$$\mu = k_1\lambda^3,$$

where μ is the mass-absorption coefficient, λ is the wave-length and k a constant, has been modified by Barkla to account for the scattering which becomes important at very high frequencies. In his expression,

$$\mu = k_2\lambda^3 + \sigma_0$$

the mass-scattering coefficient σ_0 was originally considered as a constant, having the value 0.2, calculated by J. J. Thomson on the usual electron theory. Barkla and White have recently shown, however, that this quantity must diminish at short wave-lengths, but were unable to suggest any reason for the decrease.

If the electron is taken as a ring of electricity with a radius comparable with the wave-length of very hard X-rays and V-rays, the mass-scattering coefficient should vary according to the expression recently proposed by the writer,

$$\sigma = \sigma_0\left(1 - 29.61\frac{a^2}{\lambda^2} + 524.2\frac{a^4}{\lambda^4} - 5396\frac{a^6}{\lambda^3} + \cdots\right), \tag{1}$$

where σ_0 is the mass scattering coefficient as calculated by Thomson, and a is the radius of the electronic ring. At the same time the true or fluorescent absorption will depend upon the magnitude of the action of the incident radiation upon the electron. This will diminish for short wave-lengths, since in this case different parts of a wave may work against each other in trying to pry an electron loose from its fixed position. The energy absorbed in displacing an electron may be shown to be proportional to $\sqrt{\sigma}$. The mass absorption coefficient may therefore be represented by

$$\mu = k_3\sqrt{\sigma}\lambda^3 + \sigma, \tag{2}$$

where σ is the function of a/λ represented by equation (1).

This expression is compared with the experimental values for the absorption coefficient of aluminium as given by Hull and Rice, Williams, and Bragg and Pierce, and is found to be satisfactory, especially for the short wave-lengths where the formulas of Owen and Barkla fail.

The value of *the radius of the electron* necessary to give best agreement with the experimental values *is* 1.85×10^{-10} *cm.* Unless there is some consistent error in the measurements of Hull and Rice, this value is determined within a probable error of about 5 per cent.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

COLLISIONS OF SLOW MOVING ELECTRONS WITH NITROGEN MOLECULES.[1]

By S. Karrer.

THE collisions of slow-moving electrons with nitrogen molecules has been studied by projecting electrons having known velocities into a retarding electric field between two parallel plate electrodes. The variation of the current between the electrodes with the distance between them indicates that the collisions are inelastic, if the velocity of the electrons is below the resonance velocity and the number of collisions per second is not too great. The nature of the collisions seems to be somewhat influenced by the rapidity with which they follow each other.

No traces of ionization of nitrogen were observed with electron velocities as high as ten volts, a result in accord with the recent report of Davis and Goucher.

A plane equipotential source of electrons was used.

Physical Laboratory,
University of Illinois.

THE IONIZING POTENTIALS OF ARGON, NEON AND HELIUM.[1]

By H. C. Rentschler.

THE object of the investigation was to measure the ionizing potentials of these gases and to determine whether they show the two types of inelastic impact between an electron and an atom of the gas, as found in the case of the metallic vapors. The first type of inelastic impact occurs when the colliding electron produces a displacement of an electron of the atom without detaching the electron from the atom known as the "resonance potential." The second type of inelastic impact occurs when the colliding electron produces ionization.

The test for "resonance potential" was made by the method first used by Tate in his work on mercury vapor.

The ionizing potentials were measured by Lenard's method, and also by measuring the current between a hot tungsten filament and a surrounding cylinder as a function of the potential between the cylinder and filament.

Argon was found to have a resonance potential of about 12 volts and an ionizing potential of about 17 volts.

Neon shows no resonance potential. The ionizing potential was found to be about 21 volts instead of 16 as determined by Franck and Hertz. This difference is probably due to the greater purity of the neon used in the present experiment.

Helium shows no "resonance potential." The ionizing potential was found to be about 27 volts instead of 20 volts as found by Franck and Hertz, Powlow, and Bazzoni. This value is in close agreement with Bohr's theory and also with the value calculated by the quantum theory from the limit of the helium spectrum measured by Richardson and Bazzoni.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Experiments are described showing the probable cause for the difference between the present and former values.

RESEARCH LABORATORY,
WESTINGHOUSE LAMP COMPANY.

NOTE ON THE USE OF THE HOT CATHODE VACUUM TUBE FOR THE MEASUREMENT OF VOLTAGE.[1]

BY LYNDE P. WHEELER.

1. *Methods Employed.*—Two general schemes of connection have been tried. In one the voltage to be measured is applied between the grid and the filament of a three element tube and the change thus produced in the plate current observed. In the other, the voltage is introduced between the plate and filament of a two element tube and the resulting change in plate current measured. A third method (for higher voltages) in which the voltage is applied between the plate and filament of a three element tube and the change in grid current measured, is still under investigation.

2. *Results Obtained.*—Using the first of the above methods it has been found possible to devise a compact portable apparatus which serves satisfactorily for the measurement of low voltages (up to 50 volts) in A.C. circuits of any frequency. Such voltmeters are in every day use in the high frequency laboratory of the Sloane Laboratory at Yale University. By the second method somewhat higher voltages can be measured, and it is hoped by means of the third to extend the range still further. Only the results obtained by the first method are discussed in this note. These instruments have been found to hold their calibration well, are not too complicated in their manipulation to be put into the hands of electrical engineering seniors, and if properly used their indications can be made independent of wave form as well as frequency.

3. *Details of Operation.*—Only the smaller sizes of the high vacuum tube used for sending and receiving in radio have been tried. The best plate and grid potentials to use vary with the characteristic of the particular tube. They are determined (*a*) by the condition that the grid current must be kept very small in order not to draw appreciable energy from the circuit in which the voltage is being measured; (*b*) by the sensibility of the instrument used to measure the plate current; (*c*) by whether it is desired to make the scale of the instrument uniform when calibrated in volts. The first condition necessitates in general a negative, zero or at most a small positive grid potential. A considerable range in the choice of plate potential in no way affects the usefulness of the contrivance (provided the plate current does not surpass that at the upper bend of the characteristic) and enables the second and third conditions to be met. Any device for flattening out the characteristic which does not introduce inductance in the palte circuit will aid in meeting the third condition.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

If care is taken to keep the variations of plate current within the limits of the straight portion of the characteristic of the tube, the calibrated scale is linear. Also since under these conditions there is no distortion, the impressed voltage wave form will be faithfully reproduced.

The calibration must be carried out on a low frequency A.C. circuit. For measurements up to five volts, using tubes such as the V T-1 or V T-11, it has been found satisfactory to work with the grid at zero and the plate at one and a half volts in connection with a microammeter having a full scale reading of 120. With these values of the grid and plate potentials the scale is not quite uniform, nor are the readings independent of wave form. The compensating advantages are simplicity and that owing to the rectifying action of the tube when so used no thermocouple is needed.

4. *Limitations.*—These are of two kinds. The first is that imposed by the necessity of using two auxiliary batteries. This restricts the portability of the contrivance and of course it is essential that each battery should yield a strictly constant current. The second lies in the variability of the tubes commercially available. Each tube requires individual adjustment, although once a good tube has been selected and the working potentials properly adjusted its indications seem to be reliable for the best part of the life of the filament. Until the manufacture of tubes has been better standardized however, this variability is a distinct handicap to the use of these methods of measuring voltage.

AN INTERFERENTIAL METHOD FOR MEASURING THE EXPANSION OF VERY SMALL SAMPLES.[1]

By C. G. Peters and Irwin G. Priest.

THIS paper describes an interferential method for measuring accurately the thermal expansion of samples even much smaller than can be used in the Fizeau interferential method, which requires *three* similar pins, usually about ten millimeters long or a homogeneous ring about forty millimeters in diameter and usually ten millimeters long in the direction of the measured expansion. Data are presented relating to *single* samples about five millimeters in cross section and ten millimeters in the direction of the measured expansion. Even smaller samples may be used if necessary.

The essential novelty of the method lies in the fact that measurements are made on the change in width instead of the displacement of interference fringes. For a fuller explanation of this, the complete paper must be consulted.

The new method yields results of the same accuracy as the Fizeau method while it has the following advantages over the latter:

1. Only one small pin is needed for a sample.

2. Contact errors are decreased and the error due to creeping of the cover plates is eliminated.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

3. The computations are simplified by having the standard and sample nearly equal in length.

4. The trouble of counting the fringes while the temperature is changing is eliminated entirely.

5. The correction for change of refractive index with the density of the air is eliminated without evacuating the container.

6. The method is especially advantageous in determining small differential expansions, for the difference in expansion of small samples may be obtained directly by carrying them together through the same temperature cycle.

The complete paper will be published as a Scientific Paper of the Bureau of Standards.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
February 14, 1919.

THE PROPERTIES OF THE ELECTRON AS DERIVED FROM THE CHEMICAL PROPERTIES OF THE ELEMENTS.[1]

BY IRVING LANGMUIR.

IN a paper soon to appear in the Journal of the American Chemical Society a theory of the arrangement of electrons in atoms and molecules will be given. This theory explains the periodic properties of all the elements including those of the eighth group and the rare earths. It also meets with success in explaining the magnetic properties of the elements. It leads to a simple theory of chemical valence applying equally well to polar and to non polar substances. In the case of organic compounds the results are identical with those of the ordinary valence theory, but with oxygen, nitrogen, chlorine, sulphur and phosphorus compounds the new theory applies as well as to organic compounds, while the ordinary valence theory fails completely. The so-called physical properties such as boiling points, freezing points, electric conductivity, etc., are accounted for as easily as the chemical properties. The arrangement of electrons even in molecules of nitrogen, carbon monoxide, nitric oxide and cyanogen is worked out and explains the properties of these remarkable substances.

The theory is an extension of Lewis' theory of the "cubical atom."[2]

The very satisfactory agreement between this theory and the enormous number of experimental data of organic and inorganic chemistry gives great importance to the postulates underlying the theory for these represent the conditions which must apparenly be fulfilled if the properties of matter are to be explained. Thus far the physicist has studied the electron from a rather narrow field of observation. The chemical data now gives much additional information about the electron, some of which seems inconsistent with conclusions resulting from a study of the problem by physical methods.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

[2] Jour. Amer. Chem. Soc. *38*, 762, 1916.

The theory is based on the following postulates, given here in rather incomplete form.

1. The electrons in atoms are either stationary or rotate, revolve or oscillate about certain definite positions in the atom. In the most stable atoms, namely those of the inert gases, the electrons are arranged in pairs symmetrically placed with respect to a plane called the equatorial plane. No electrons lie in this plane. There is an axis of symmetry (polar axis), perpendicular to his plane, through which four secondary planes of symmetry pass forming angles of 45° with each other. The atoms as a whole thus have the symmetry of a tetragonal crystal.

2. The electrons in any given atom are distributed through a series of concentric (nearly) spherical shells, all of equal thickness. Thus the mean radii of the shells form an arithmetic series 1, 2, 3, 4, and the effective areas are in the ratios $1 : 2^2 : 3^2 : 4^2$.

3. Each shell is divided into cellular spaces or cells occupying equal areas in their respective shells and distributed over the surface, of the shells according to the symmetry required by postulate 1. The first shell thus contains 2 cells, the second 8, the third 18, and the fourth 32.

4. Each of the cells in the first shell can contain only one electron, but each other cell can contain either one or two electrons. All the inner shells must have their full quotas of electrons before the outside shell can contain any. No cell in the outside layer can contain two electrons until all the other cells in this layer contain at least one.

The inert gases are those in which all the cells in the outside shell have equal numbers of electrons. Thus according to the first four postulates helium has two electrons, neon has ten, argon 18, krypton 36, xenon 54, and niton 86. All atoms with an atomic number greater than that of helium (2) have as their first shell a pair of electrons close to the nucleus. The line connecting the two electrons establishes the polar axis for the atom. Neon has in its second shell eight electrons, four in each hemisphere (*i. e.*, above and below the equatorial plane), arranged symmetrically about the polar axis. The eight electrons are thus nearly at the corners of a cube. In argon there are eight more electrons in the second shell. In all elements of higher atomic number the second shell is like that in argon. Krypton has in its third shell nine electrons in each hemisphere symmetrically placed with respect to the polar axis and to the four electrons in the second shell. The ninth electron in each hemisphere goes into the polar axis. Xenon is like krypton except that it has twice as many electrons in its third shell. Niton has sixteen electrons in each hemisphere of its fourth shell. These are easily placed symmetrically with respect to the polar axis and the eight underlying electrons.

5. Two electrons in the same cell do not repel nor attract one another with strong forces. This probably means that there is a magnetic attraction (Parson's Magneton Theory) which nearly counteracts the electrostatic repulsion.

6. When the number of electrons in the outside layer is small, the arrange-

ment of the electrons is determined by the (magnetic?) attraction of the underlying electrons. But when the number of electrons increases, especially when the layer is nearly complete, the electrostatic repulsion of the underlying electrons and of those in the outside shell becomes predominant.

7. This postulate refers to the properties of atoms and need not be considered here.

8. The stable and symmetrical arrangements of electrons corresponding to the inert gases are characterized by strong internal and weak external fields of force. The smaller the atomic number, the weaker the external field.

9. The most stable arrangement of electrons is that of the pair in the helium atom. A stable pair may also be held by: (*a*) a single hydrogen nucleus; (*b*) two hydrogen nuclei; (*c*) a hydrogen nucleus and the kernel of another atom; (*d*) two atomic kernels (very rare).

10. The next most stable arrangement of electrons is the *octet*, that is, a group of eight electrons like that in the second shell of the neon atom. Any atom with atomic number less than eighteen, and which has more than three electrons in its outside layer tends to take up enough electrons to complete its *octet*.

11. Two octets may hold one, two, or sometimes three pairs of electrons in common. One octet may share one, two, three or four pairs of its electrons with one, two, three or four other octets. One or more pairs of electrons in an octet may be shared by the corresponding number of hydrogen nuclei. No electron can be shared by more than two octets.

The results obtained by the use of these postulates are so striking that one may safely reason that the results establish the fundamental correctness of the postulates.

How can these results be reconciled with Bohr's theory and with our usual conception of the electron? It is too early to answer. Bohr's stationary states and the cellular structure postulated above have many points of similarity. It seems that the electron must be regarded as a complex structure which undergoes a series of discontinuous changes while it is being bound by the nucleus or kernel of an atom. There seems to be strong evidence that an electron can exert magnetic attractions on other electrons in the atom even when not revolving about the nucleus of the atom.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

SOME PHYSICAL PROPERTIES OF DENTAL MATERIALS.[1]

BY C. G. PETERS AND W. H. SOUDER.

IN carrying out an investigation of dental amalgams, the following properties are being considered: thermal expansivity, dimensional changes, and crushing strengths.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Thermal Expansion Coefficients.—In determining the thermal expansion coefficients, a differential interference method, was adopted as the most accurate and convenient.

The thermal expansion of different sections of teeth, immersed in a solution, determined over the range 20 to 70 degrees centigrade were approximately 6.5×10^{-6} per degree centigrade for the root and 10×10^{-6} for the enamel.

The coefficient of a specimen of synthetic porcelain tested under similar conditions was about 8×10^{-6}.

Similar tests on amalgams showed coefficients from 24 to 28×10^{-6}. The critical point near 75° centigrade, discovered by other observers while making different tests, was confirmed.

This differential expansion between tooth structure and amalgam, amounting to .2 micron per centimeter per degree centigrade, is thought to be worthy of serious consideration, by dental clinicians.

Dimensional Changes During Amalgamation.—Samples of different amalgams were packed in a mold of definite shape such that these would fit the interferometer and their changes measured exactly as in the previous case. These samples were placed in position in a few minutes after amalgamation and observations taken over a period of one to three days.

Curves plotted to show change in length with time show an initial contraction which lasts about 20 minutes and is thought to be due to the specimens returning to the equilibrium temperature. This contraction is followed by an expansion of from 4 to 8 microns per centimeter at the end of 3 hours. Additional changes with time are relatively small.

Flow Under Continuous Pressure.—Subjecting specimens of amalgam to a constant pressure of about 3,000 pounds per square inch (beginning 48 hours after amalgamation) caused a flow in all cases. In most instances this was less than 2 per cent. at the end of five days. In some cases it caused a change of 10 per cent. or caused a rupture of the material.

It is thought that the rate of application of pressure modifies the crushing strength quite materially.

THE ORIGIN OF THE GENERAL RADIATION SPECTRUM OF X-RAYS.[1]

BY DAVID L. WEBSTER.

AT a meeting of this society, four years ago, Duane and Hunt[2] announced the experimental proof that the spectrum of X-rays contained no frequencies above the value whose quantum is the energy of a cathode ray; and I pointed out[3] the fact that this law meant that the old pulse theory could not hold, but that the general radiation, just as much as the characteristic, must consist of long trains of continuous waves. Since then the evidence for the wave train theory has been strengthened by a later analysis showing that for an infinitely thin anticathode, not only the derivatives of the intensity, but the intensity itself, would be discontinuous at the limit of the spectrum.

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[1] Abstract of a paper presented at the New York meeting of the American Physcial Society, March 1, 1919.

[2] W. Duane and F. H. Hunt, Phys. Rev., *6*, 166, 1915.

[3] D. L. Webster, Phys. Rev., *6*, 56, 1915.

The question suggested by this is, What is the nature of the oscillators producing these trains of waves? Are they cathode rays or are they something fixed in the atom? If they are cathode rays, we may expect them to show the Doppler effect, and with it also, according to the electromagnetic theory, we may expect an unsymmetrical distribution of intensity of X-rays as though the X-rays were thrown forward by the cathode rays. Such a combination of effects was indeed found by Kaye,[1] in 1909, using very thin targets of Al, Cu, Au and Pt. In every case he found more rays thrown forward than back, and in most cases the forward rays were more penetrating, showing the higher frequency that one would expect from the Doppler effect. With thick targets of carbon, the same sort of asymmetry of intensity was found by Stark.[2] and Kirschbaum,[3] but they could not be sure of any difference in frequency. And in thick targets of Pb investigated by Ham,[4] silver by Miller[5] and Pt by Kirschbaum,[6] no asymmetry could be detected. From this we should infer that the asymmetry was a property of light elements or thin layers of heavier ones, but not of thick layers of heavy ones. Before drawing any conclusions we should therefore try to see why the effect is limited in this way.

One obvious reason for expecting less asymmetry from heavy elements is the presence of characteristic rays, which, for various reasons, are likely to be uniformly distributed. But this is not enough. Another significant fact is that none of the experiments on the limit of the spectrum have shown any Doppler effect there, although with the high speeds of cathode rays such an effect might be expected even at angles not much different from a right angle from the cathode rays. This indicates that the highest frequencies emitted are emitted by electrons that have already lost all their velocity before emitting, although they were necessarily the fastest ones present at all when they struck the atoms that caused the emission. If so, the electrons that emit while in motion are therefore those that emit lower frequencies than the highest they are capable of, or in other words the lower frequencies will only show asymmetry when produced by electrons whose energy is initially much greater than the quanta for those frequencies. With aluminum, which gives no errors from characteristic rays, exactly this effect was found by Kaye, since the softer components of a hard beam showed more asymmetry than the harder ones. This explains at once why thick targets show less asymmetry than thin ones. They contain cathode rays of low velocity, and a large proportion of their low frequency X-rays are due to these cathode rays. And it also explains why heavy elements show less asymmetry than light ones if we may assume that their atomic electrons are bound more firmly and are less easy for the cathode rays to dislodge and move in their own direction, and their more powerful nuclei will cause greater scattering of the moving electrons.

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[1] G. W. C. Kaye, Proc. Camb. Phil. Soc., *15*, 269, 1909.

[2] J. Stark, Phys. Zeitschr., *10*, 902, 1909.

[3] H. Kirschbaum, Ann. der Phys., *46*, 85, 1915.

[4] W. R. Ham, PHYS. REV., *30*, 96, 1910.

[5] F. C. Miller, Franklin Inst. Journal, *171*, 457, 1910.

[6] Loc. cit.

This brings the experiments into line with each other, but leaves us with two very startling conclusions. First, that electrons can radiate continuous wave trains while in rapid forward motion; and second, that when an electron radiates all its energy, it stops first and radiates after stopping. If the electron is in motion, and moves the diameter of an atom while radiating a wave train, where does it get the elastic force under which it vibrates? How can it carry its elastic restoring force and whole vibrating mechanism with it for such a distance? And if an electron does radiate all its energy, as at the high frequency limit of the spectrum, and must stop first, before radiating, where is its energy after its forward motion has stopped but before it radiates? These questions seem very hard to answer on the hypothesis that the only properties of the electron are those of a small uniformly charged sphere.

But they are easily answered by the theory of heat radiation that I developed some time ago on the basis of a modified form of the Parson magneton.[1] For in this theory the electron is a thin conducting ring and the emitting oscillation is that of the electricity on the ring, and this oscillation can occur without the help of elastic forces from outside, as it must when the electron is moving. The magneton also provides for the disposal of the energy between the stoppage of the cathode ray and the radiation of the X-rays, transforming it into energy of the oscillation. And it explains why, if all the energy of translation of the cathode ray has gone into the electric oscillation it will be radiated from a stationary center, but if only part of it goes into the oscillation, the translational motion will continue during the oscillation and cause the observed asymmetry.

Massachusetts Institute of Technology,
Cambridge, Mass.,
March 1, 1919.

Note on Bomb Trajectories.[2]

By Edwin B. Wilson.

THE trajectory in vacuo would be a parabola of which the equation is $y = gx^2/2u_0$, if u_0 be the initial velocity (supposed horizontal) of the bomb. It is possible to make a simple approximation to the path when the resistance of the air is taken into account.

Namely:

$$y = \frac{gx^2}{2u_0^2} + \frac{g^2x^3}{3u_0^2U^2},$$

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[1] A. L. Parson, Smithsonian Miscellaneous Collections, *65*, *11*, 1, 1915. D. L. Webster, Proc. Amer. Acad., *50*, 131, 1915.

[2] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

where U is the limiting velocity of the bomb and where it must be assumed that the length of the path of the trajectory is not over one or two thousand feet. The equation may be solved for x in terms of y, or rather in terms of the square root of y, with the result

$$x = \sqrt{\frac{2y}{g}}\,u_0 - \frac{2}{3}\frac{u_0^2 y}{U^2}.$$

The first term in this expression represents the horizontal travel of the bomb in vacuo during a drop of amount y and the second term represents the distance that the bomb is retarded relative to the parabolic path by the action of the air supposed to resist according to the ordinary law of the square of the velocity. The approximation is good only when $\sqrt{2yg}/3\cdot u_0/U^2$ is small. If the limiting velocity of the bomb is 900 feet per second and the initial velocity is 100 feet per second, the correction remains small for very considerable lengths of drop y—in fact, for lengths much greater than appear valid for approximations previously introduced in the work.

ON THE RELATION BETWEEN THE *K* SERIES AND THE *L* SERIES OF X-RAYS.

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SOME years ago Kossel called attention to the fact that the difference between the frequencies of the β and α lines in the K series of X-rays characteristic of a chemical element very nearly equals the frequency of one of the strong lines in the L series of that element. Since then the relation between the two series of X-rays has been the subject of considerable discussion, especially with regard to its bearing on the modern theories of the structure of the atom and of the mechanism that produces characteristic line spectra. Kossel's equation cannot represent a general relation between the lines in the K series and those in the L series, however, for the L series contains many more lines than the K series contains.

In the researches described in this paper we have endeavored to measure the absorption and emission frequencies characteristic of tungsten in both the K and the L series, using the same calcite X-ray spectrometer throughout the work.

In the experiments on emission lines the X-rays came from the tungsten target of an ordinary Coolidge tube, but in most of the experiments on absorption we replaced this tube with one containing a molybdenum target, and inserted a thin layer of potassium tungstate between the tube and the spectrometer.

In each experiment the current exciting the X-ray tube came from a high tension transformer with a system of condensers and kenotrons attached to it for producing approximately a constant difference of potential.

In order to avoid errors due to the penetration of the X-rays into the crystal

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

we placed the two narrow slits that define the X-ray beam between the X-ray tube and the spectrometer.

The curves shown in the lantern slides represent the ionization currents as functions of the readings of one of the verniers attached to the crystal table.

To avoid making an accurate determination of the zero point directly readings were taken on both sides of the zero line of the spectrometer.

The curves on the first slide represent the K series of lines in both the first and the second order spectra. To produce the K series in tungsten requires a difference of potential greater than 69,200 volts applied to the X-ray tube, and in order that the peaks on the curves that represent characteristic lines should be well marked we employed a difference of potential of 100,000 volts.

The curves show the following characteristics:

(*a*) At the angles corresponding to $\lambda = .1785 \times 10^{-8}$ cm. (calculated from the grazing angle of incidence θ by the formula

$$n\lambda = 6.056 \times 10^{-8} \times \sin \theta)$$

appear sharp drops in the curves. This, then, is the characteristic absorption wave-length of tungsten in the K series.

(*b*) At the angles corresponding to $\lambda = .2087 \times 10^{-8}$ cm. and $\lambda = .2134 \times 10^{-8}$ cm. occur two sharp peaks completely separated from each other. These are the wave lengths of the two emission lines α_1 and α_2 in the K series. The peak corresponding to α_2 does not appear to be quite symmetrical. It is slightly broader on the long-wave length side than on the short, suggesting that there may be a weak line α_3 very close to α_2, with a wave-length of about $.214 \times 10^{-8}$ cm. There is also a well defined peak at the angle corresponding to $\lambda = .1842 \times 10^{-8}$ cm. (the β line in the K series), and a slight elevation close to the absorption drop (a γ line in the K series).

In the neighborhood of the peaks readings were taken 15″ of arc apart; and, since the double grazing angle is about 4°, an error of 15″ in estimating the point of a peak would produce an error of 1/10 per cent. in the value of λ. If the same error were made in corresponding peaks on the two sides, an error 1/5 per cent. would result.

The curves representing experiments with a molybdenum target and a tungsten absorbing screen show marked breaks at $\lambda = .1785 \times 10^{-8}$ cm., $\lambda = 1.230 \times 10^{-8}$ cm. (strong) and $\lambda = 1.08 \times 10^{-8}$ cm. (medium) and a less well defined break at $\lambda = 1.025 \times 10^{-8}$ cm. The first is the characteristic K absorption wave-length, and the other three are the characteristic L absorption wave-lengths of tungsten.

The following table contains the frequencies of vibration calculated from the wave-lengths obtained from the curves.

Emission and Absorption Frequencies of Tungsten $\times 10^{-19}$. (Probable error ± 0.003.)

K Absorption Frequencies.

First order tungsten target	1.680
First order molybdenum target	1.679
Second order molybdenum target	1.681
Average ν	= 1.680

L Absorption Frequencies.

Strong $\nu_1 = .2438$
Medium $\nu_2 = .2773$
Weak $\nu_3 = .2922$

K Emission Frequencies.

	β Medium
1st order	1.628

	α_1 Strong	α_2 Medium	α_3 Weak
1st order	1.435	1.405	1.40?
2d order	1.437	1.405	1.40?
Average	1.436	1.405	1.40?

Relation between K Series and L Series.

Absorption	Emission
$\nu - \nu_1 = 1.436$	$\nu_{\alpha_1} = 1.436$
$\nu - \nu_2 = 1.403$	$\nu_{\alpha_2} = 1.405$
$\nu - \nu_3 = 1.388$	$\nu_{\alpha_3} = 1.40?$

It appears from the last five lines in the table that *the difference between the K absorption frequency and one of the L absorption frequencies equals one of the α emission frequencies in the K series* to within the limits of error of the measurements. Further, if we subtract the strong L absorption frequency ν_1 from the K absorption frequency ν we get the frequency of the strong α_1 line, if we subtract the medium L absorption frequency ν_2 from ν we get the frequency of the medium K emission line α_2 and, if we use the weak L absorption frequency we get the third α line in the K series, α_3.

The curve representing experiments with a tungsten target shows about 12 lines in the L series between $\lambda = 1.2 \times 10^{-8}$ cm. and $\lambda = 1.50 \times 10^{-8}$ cm. Six of these lines can be accounted for by an extension of the above law, that an emission frequency is the difference between two absorption frequencies, if we assume the existence of two M absorption wave-lengths between $\lambda = 6 \times 10^{-8}$ cm. and $\lambda = 7.5 \times 10^{-8}$ cm. Such M absorption wave-lengths would account, also, for the presence of the β line in the K emission series.

If the above law proves to be of general applicability, characteristic absorption frequencies similar to those of X-rays should be sought for in other parts of the spectrum (the visible and ultra-violet, etc.). Characteristic X-ray absorption frequencies are quite different from the absorption lines that obey Kirchoff's law.

It is unnecessary to lay further stress upon the importance of this absorption-emission relation in speculations on the structure of matter and on the mechanism of the absorption and emission of radiation.

TEMPERATURE COEFFICIENT OF RESISTENCE OF MOLYBDENUM.[1]

By E. C. Blom.

THESE experiments were conducted to determine the temperature coefficient of resistance of molybdenum over a greater range of temperature than had previously been recorded.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Some 10 mil (0.254 mm.) drawn molybdenum wire was made into lamps. The filament length of one group of these was 150 mm. while that of the other was 120 mm. With everything else the same, this gave a filament length difference of 30 mm., which was used for all resistance calculations.

For the temperature from 0° C. to 30° C. the lamps were immersed in a water-bath until constant conditions were reached. The resistance was then measured with a Wheatstone Bridge. The temperature was measured with a calibrated thermometer. Over the range 800° C. to 2000° C. the temperature was measured with both brightness and color match optical pyrometers. Direct current was used to light the lamps and the resistance was determined by measuring volts and amperes with laboratory standard instruments. For the interval 30° C. to 300° C., the bulbs were removed, and the filaments immersed in a paraffine bath where again the temperature was measured with a calibrated thermometer and the resistance measured with the bridge. The interval 300° C. to 800° C. was not covered but the results up to 300° C. and beyond 800° C. ran so evenly that it seems plausible to assume that the curve which fits those temperatures already covered will fit the others also.

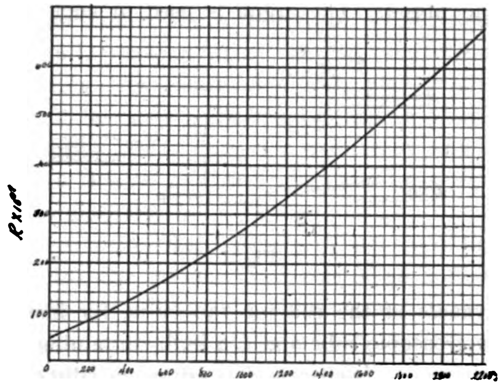

Fig. 1.

The accompanying curve gives a summary of the results obtained. The following empirical equation was found to yield the correct values:

$$R.10^7 = 44 + 0.177t + 0.000053t^2,$$

where t is in °C and R in ohms per cm.[3]

Physical Laboratory,
Westinghouse Lamp Co.,
Bloomfield, N. J.,
February 11, 1919.

SOME PHOTOELECTRIC LECTURE EXPERIMENTS

BY JACOB KUNZ.

IN a short article on the photoelectric relay in the Electrical World, Vol. 66, p. 934, October, 1915, the writer has indicated some simple experiments with the photoelectric cell and the three electrode photoelectric relay. Three more experiments will here be indicated by which some properties of the photoelectric cell can be demonstrated.

The tube *A* represents a photoelectric cell containing potassium sensitized with hydrogen and argon. The argon is used at such a pressure that the maximum sensitiveness of the cell for 130 volts, for instance, is surpassed, *B* represents a spectroscopic tube and *C* a bulb containing charcoal. Now for a given tube the photoelectric current depends essentially on three independent variables: the intensity of light, the potential difference, and the gas pressure. Let us start with a certain light intensity, for instance, with an incandescent lamp of 16 candlepower at a distance of two meters, with a certain pressure and a potential difference of 80 volts. We shall find in the galvanometer a certain deflection which will increase with increasing potential difference, first slowly, later very rapidly until the gas begins to glow. This point should not be reached with the ordinary cells, because the sensitiveness of potassium cells as a rule decreases when the glow occurs.

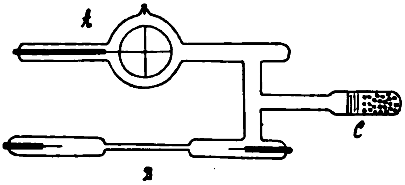

Fig. 1.

In a second experiment let us keep the potential difference, say 110 volts. and the gas pressure constant, and increase the intensity of light. The current is proportional to the illumination. Many photoelectric cells have been constructed in which this proportionality does not hold. The current may either increase more slowly or more quickly than the illumination. If the current approaches a saturation value with increasing illumination, it is apparently due to volume charges; and if it increases more rapidly than the illumination it is due to the fact that the gas approaches the glowing stage, which depends on the pressure, potential difference and illumination, like the photoelectric current itself.

In a third experiment let us keep the illumination and the potential difference constant and vary the pressure by putting the charcoal into liquid air. As the argon is absorbed the photoelectric current will at first increase, reach a maximum, then decrease and become very small when all the argon is absorbed.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

The absorption of the argon can be seen in the changes of the light in the tube *B*. When the liquid air is withdrawn, argon is emitted, the photoelectric current passes through the same maximum as before and falls off to the initial small value (Stoletow's law).

The point at which the gas of the photoelectric cell becomes luminous depends on the potential difference, the intensity of illumination and the gas pressure. If for a certain pressure and potential difference we approach the source of light, suddenly the gas begins to glow, and of course continues to glow when the light is withdrawn. This beautiful experiment is interesting insofar as it shows that an increase of the number of electrons emitted from the metal leads to such an increase in ionization that luminous discharge occurs in which the current rises enormously over the value attained without glow. The photoelectric cell becomes a fairly good source of cold light. In such experiments the sensitive layer changes considerably and it is better to use two different tubes for this set of experiments and for the previous set, which does not lead to glow discharge.

If we place a rotating sector disc between the source of light and the photoelectric current, amplify the current by means of an audion as shown by E. Pike in a recent article in the Physical Review, and use a telephone instead of a galvanometer in the secondary of the audion, then we can hear the source of light, even a very weak source, quite distinctly. This principle may be used in an instrument by which a blind man may hear the letters of a printed page.

University of Illinois,
Urbana, Illinois.
January 11, 1919.

On the Measurement of the Detection Coefficient of Thermionic Vacuum Tubes.[1]

By H. J. Van der Bijl.

The relation between current and voltage in thermionic tubes of the audion type, can be represented by

$$I = f\left(\frac{1}{\mu}\Sigma E_B + \Sigma E_c\right), \tag{1}$$

where ΣE_B and ΣE_c are the potentials of the anode and grid with respect to the filament. The function f is non-linear and hence the device can be used as a radio detector. If, for example, a varying e.m.f. e, be impressed between filament and grid, the varying current i in the circuit connecting anode and filament can be represented by a simple convergent power series:

$$i = a_1e + a_2e^2 + a_3e^3 + \cdots \tag{2}$$

In order to properly design tubes it is necessary to know the relation between the coefficients a_1, $a_2 \cdots$ and the structural parameters of the tube. This requires a knowledge of the form of the function f. When using the tube as an amplifier f can be given a simple quadratic form.[1] This is, however, not sufficiently accurate for purposes of radio detection which is determined by the second derivative of the characteristic, *i. e.*, by the coefficient a_2 of the above series. The problem we want to consider here is the direct determination of a_2, which may be called the detection coefficient. The difficulty of this determination lies in the necessity for measuring extremely small alternating currents,—ranging from 10^{-3} to 10^{-6} amp. The use of hot wire meters is therefore entirely out of the question. Hence the method shown in the accompanying diagram was used.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

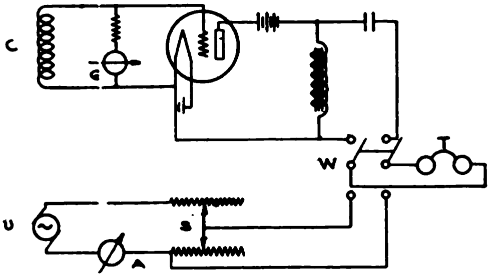

Fig. 1.

The input voltage impressed on the tube at C ranged from a few hundredths to a few tenths of a volt, and was measured with the Duddell galvanometer G. It was of the form

$$e = A \sin pt\,(1 + \sin qt) \tag{3}$$

i. e., it was a voltage of high frequency $p/2\pi$ completely modulated by a voltage of frequency $q/2\pi$ lying within the audible range. If (3) be inserted in the second term of (2) the resulting current will be seen to have an audible component of frequency $q/2\pi$ which can be heard in the telephone receiver T. In order to measure this current the generator U is used to give a current of the same frequency as that in the receiver and which is strong enough to be measured with a thermo-couple and microammeter A. In fact, the same generator supplied the current used to modulate the high frequency input. This current is then attenuated by means of the shunt S until the intensity of the note in the receiver is the same for both positions of the switch W. Knowing the value of the shunt S the current i in the receiver can be computed from i_1 as measured by A. This shunt is so designed that for all its adjustments the impedance into which the generator U works is constant, so that i_1 remains constant.

For convenience in representing the results we can put

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[1] H. J. Van der Bijl, PHYS. REV., 12, p. 171, 1918.

$$d = \log \frac{i_1}{i}, \tag{4}$$

where d can be previously determined for any combination of shunt and series resistances of the shunt S. Since the audible current i in the output of the detector is given by

$$i = a_2 e^2 \tag{5}$$

we get

$$d = -2 \log e + (\log i_1 - \log a_2). \tag{6}$$

The accuracy with which this relation holds is shown in the diagram, the circles and crosses indicating measurements made by two different observers.

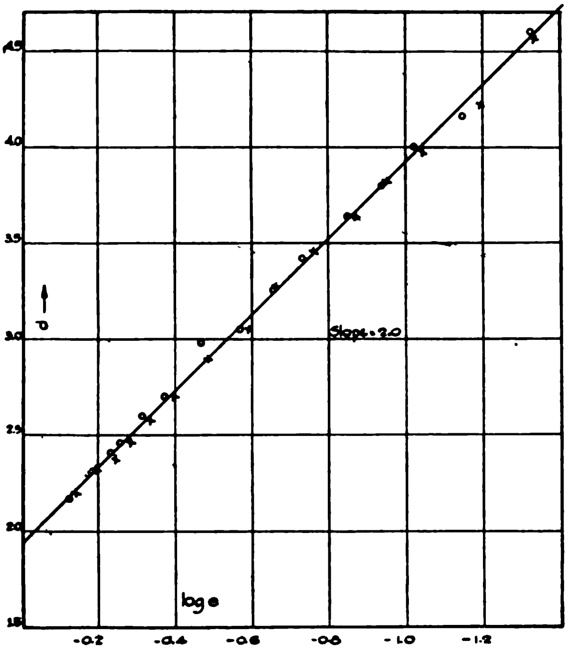

Fig. 2.

Since i_1 is large enough to be measured with the thermo-couple and microammeter A the detection coefficient a_2 can be obtained from the intercept of the line.

The detector used in these experiments was a standard Western Electric tube of the type that was designed specially for aëroplane radio telephone service. Its detecting efficiency had not changed to any noticeable extent over a period of more than seven months, during which time it was in frequent use. In these experiments the operating voltages were not adjusted to give the best efficiency.

From these results the detection coefficient was found to be 36×10^{-6} amp./(volt)^2, the resistance of the receiver being 6,400 ohms. This corresponds to a power dissipation in the receiver of 8.3×10^{-6} watt/(volt)^4. An idea of this amount of power in the receiver can be obtained by noting that the minimum power dissipation necessary in this receiver to give a sound that just barely enables one to distinguish signals is about 3×10^{-12} watt.

RESEARCH LABORATORY OF THE WESTERN ELECTRIC CO., INC.,
NEW YORK CITY.
February, 1919.

A NEW FORMULA FOR THE SPECTRAL DISTRIBUTION OF ENERGY FROM A COMPLETE RADIATOR.[1]

BY IRWIN G. PRIEST.

THE radiant energy from a complete radiator ("black body") may be expressed as a function of temperature and wavelength by the following empiric formula which does not appear to have been noticed heretofore:

$$E_\lambda = D_1 T^5 e^{-D_2[A^{-\frac{1}{2}}-(\lambda T)^{-\frac{1}{2}}]^2} \tag{1}$$

where

$E_\lambda \equiv$ energy of wave-length λ in the same sense as in the Planck equation.
D_1 and D_2 are empiric constants.
$T \equiv$ absolute temperature in degrees Kelvin.
$A \equiv$ Wien displacement constant in micron degrees. ($A \equiv \lambda_m T$, where $\lambda_m \equiv$ wave-length of maximum energy.)
$\lambda \equiv$ wave-length in microns.

From equation 1, we have for relative energy at any temperature

$$E_{\lambda r} \equiv \frac{E_\lambda}{E_m} = e^{-D_2[A^{-\frac{1}{2}}-(\lambda T)^{-\frac{1}{2}}]^2} \tag{2}$$

where $E_m \equiv$ maximum energy, of wave-length λ_m.

This formula was originally obtained as an empiric approximation to the Planck formula by graphic transformations of the latter. It was later found, for $D_2 = 4481$ and $A = 2940$, that this formula represents some of the best of Coblentz' experimental distribution data *better* than the Planck formula can represent them for any value of A and the corresponding value of C_2 ($C_2 = 4.965A$). This conclusion may be verified by reference to the accompanying table.

The proposed equation includes all of the following previously recognized radiation laws, each of which may be derived from it:

1. The Wien displacement law.

The special case of Wien's displacement law ($\lambda_{\max} T = A$) may be inferred from inspection of the equation (1 or 2) for condition of maximum E_λ, viz: $A^{-1/2} - (\lambda T)^{-1/2} =$ zero.

Further, the following equation may be derived from equation 2:

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Priest: A New Formula for Spectral Distribution, etc.

λT ↓	Values of $\frac{E_\lambda}{E_m}$ as a Function of λT						Residuals			
	Computed by Formulas					Exp. Data×	Planck for A=2890 minus Exp. Data	Proposed Equation for A=2890 minus Exp. Data	Planck for A=2940 minus Exp. Data	Proposed Equation for A=2940 minus Exp. Data
	For A=2890			For A=2940						
	Wien $C_2=14450$	Planck $C_2=14350$	Proposed Equation* Priest, Dec.	Planck $C_2=14597$	Proposed Equation* Priest, Dec.	Coblentz B.S. Sci. Pap. 284, Fig. 2 T=1596.5 K				
Column No.	(I)	(II)	(III)	(IV)	(V)	(VI)	(II)-(VI)	(III)-(VI)	(IV)-(VI)	(V)-(VI)
500	0.27×10^{-6}	0.32×10^{-6}	0.87×10^{-6}	← Note →	0.72×10^{-6}					
800	$.13\times10^{-2}$	$.14\times10^{-2}$	$.18\times10^{-2}$		$.16\times10^{-2}$					
942		$.94\times10^{-2}$	$.11\times10^{-1}$	0.78×10^{-2}	0.96×10^{-2}	0.15×10^{-1}	−0.006	−0.004	−0.007	−0.005
1000	$.16\times10^{-1}$	$.17\times10^{-1}$	$.19\times10^{-1}$							
1102		$.39\times10^{-1}$	$.42\times10^{-1}$	$.34\times10^{-1}$	$.38\times10^{-1}$	$.35\times10^{-1}$	+0.004	+0.007	−0.001	+0.003
1213		$.80\times10^{-1}$	$.82\times10^{-1}$	$.71\times10^{-1}$	$.77\times10^{-1}$	$.58\times10^{-1}$	+0.022	+0.024	+0.013	+0.019
1325		.1391	.1420	.1254	.1330	.110	+0.029	+0.032	+0.015	+0.023
1485		.2525	.2558	.2334	.2408	.233	+0.019	+0.023	+0.000	+0.008
1500	.2582	.2646	.2678							
1676		.4151	.4166	.3904	.3973	.395	+0.020	+0.022	−0.005	+0.002
1788		.5135	.5150	.4877	.4929	.495	+0.019	+0.020	−0.007	−0.002
1900		.6079	.6096	.5827	.5863	.581	+0.027	+0.029	+0.002	+0.005
2000	.6810	.6871	.6859							
2044		.7183	.7179	.6959	.6963	.700	+0.018	+0.018	−0.004	−0.004
2155		.7924	.7926	.7700	.7722	.797	−0.005	−0.004	−0.027	−0.025
λ<λm ↑ 2427		.9238	.9240	.9111	.9094	.929	−0.005	−0.005	−0.018	−0.020
2714		.9903	.9900	.9858	.9848	.994	−0.004	−0.004	−0.008	−0.008
Max. → λ>λm ↓ 2986		.9972	.9975	1.000	.9995	.998	−0.001	−0.000	+0.002	+0.002
3000	.9964	.9960	.9966							
3257		.9665	.9675	0.9769	.9753	.963	+0.003	+0.005	+0.014	+0.012
3512		.9180	.9167	.9322	.9296	.940	−0.022	−0.023	−0.008	−0.010
3624		.8916	.8900	.9078	.9053	.911	−0.019	−0.021	−0.003	−0.006
3848		.8368	.8332	.8553	.8509	.864	−0.027	−0.031	−0.009	−0.013
4000	.7887	.7987	.7933							
4071		.7788	.7730	.7976	.7929	.771	+0.008	+0.002	+0.027	+0.022
5000	.5323	.5515			.5635					
5843		.3950	.3806	.4117	.3999	.397	−0.002	−0.016	+0.015	+0.003
6000	.3465	.3712	.3566							
6003		.3712	.3570	.3868	.3751	.374	−0.003	−0.017	+0.013	+0.001
6434		.3133	.2979	.3274	.3146	.313	+0.000	−0.015	+0.014	+0.002
7089		.2439	.2288	.2558	.2437	.244	−0.000	−0.015	+0.012	−0.000
7408		.2165	.2021	.2275	.2155	.218	−0.002	−0.016	+0.010	−0.002
7685		.1949	.1807	.2047	.1926	.197	−0.002	−0.016	+0.008	−0.004
7983		.1756	.1615	.1847	.1731	.179	−0.003	−0.017	+0.006	−0.006
8000	.1501	.1746	.1607							
8238		.1606	.1466	.1690	.1574	.160	+0.001	−0.013	+0.009	−0.003
8493		.1469	.1340	.1546	.1437	.147	−0.000	−0.013	+0.008	−0.003
8765		.1349	.1212	.1412	.1305	.132	+0.002	−0.011	+0.009	−0.002
9930		$.92\times10^{-1}$	$.81\times10^{-1}$	$.97\times10^{-1}$	$.88\times10^{-1}$	$.86\times10^{-1}$	+0.006	−0.005	+0.011	+0.002
10000	$.71\times10^{-1}$	$.90\times10^{-1}$	$.80\times10^{-1}$		← Note Agreement →		↑ Note Systematic High Values for λ < λm ↓			↑ Note Small Residuals and Distribution of Signs c(III)-(II) ↓
20000	$.45\times10^{-2}$	$.86\times10^{-2}$	$.69\times10^{-2}$	Note Agreement of ↔ Cols. II, III, V. cf. Low Values in I	$.77\times10^{-2}$					
30000	$.76\times10^{-3}$	$.19\times10^{-2}$	$.15\times10^{-2}$							
40000	$.20\times10^{-3}$	$.65\times10^{-3}$	$.54\times10^{-3}$		$.63\times10^{-3}$					
50000	$.72\times10^{-4}$	$.28\times10^{-3}$	$.25\times10^{-3}$							
60000	$.30\times10^{-4}$	$.14\times10^{-3}$	$.13\times10^{-3}$		$.15\times10^{-3}$					
70000	$.15\times10^{-4}$	$.75\times10^{-4}$	$.74\times10^{-4}$							
80000	$.76\times10^{-5}$	$.45\times10^{-4}$	$.51\times10^{-4}$		$.60\times10^{-4}$					
90000	$.43\times10^{-5}$	$.28\times10^{-4}$	$.35\times10^{-4}$							
[illegible]	[illegible]	[illegible]	[illegible]		[illegible]					

Notes to Column Headings III, V, VI.	Means →		(II)-(VI)	(III)-(VI)	(IV)-(VI)	(V)-(VI)
	for λ < λm	Arith.	+0.015	+0.016	+0.009	+0.010
		Algeb.	+0.012	+0.013	−0.004	−0.0004
	for λ > λm	Arith.	+0.006	+0.014	+0.011	+0.005
		Algeb.	−0.004	−0.013	+0.008	−0.0003
	for all values of λ	Arith.	+0.010	+0.015	+0.010	+0.0075
		Algeb.	+0.003	−0.002	+0.003	−0.0003

*Proposed Equation

$$\frac{E_\lambda}{E_m}=e^{-4481\left[A^{-1}-(\lambda T)^{-1}\right]^2}$$

×Omitting points affected by atmospheric absorption.

Fig. 1.

$$\lambda T = \left[\frac{1}{A^{-\frac{1}{3}} \pm \sqrt{\frac{\log_e E_{\lambda r}}{-D_2}}} \right]^3 \tag{3}$$

which is in accord with the general case of Wien's law.

2. The law that the maximum energy is proportional to the fifth power of the absolute temperature.

This may be seen by inspection of the equation for the condition

$$\lambda = \lambda_m = \frac{A}{T}.$$

3. The empiric Stefan law that total energy (integral with respect to wave-length) is proportional to the fourth power of absolute temperature.

This follows from the fact that the definite integral between any wave-length limits for equal values of $E_{\lambda r} > 0.33 \times 10^{-9}$ is proportional to the fourth power of temperature.

From the experimental point of view, therefore, the proposed equation accords with the fourth power law just as well as the Planck equation does.

However, the theoretical Stefan-Boltzmann law that the definite integral with respect to wave-length between *zero* and *infinity* is proportional to the fourth power of absolute temperature can *not* be derived from this equation, because E_λ/E_m has a finite although very small value (0.33×10^{-9}) for $\lambda T =$ infinity.

Another interesting property of the proposed equation is the fact that it has precisely the form of the well known equation of the "probability curve." If $E_{\lambda r}$ (equation 2) be plotted as ordinate against $\sqrt[3]{1/\lambda}$ or $\sqrt[3]{300/\lambda}$ (cube root of frequency) as abscissa, the resulting curve is strictly symmetrical about the maximum ordinate. It is possible that this is the mathematical essence of what may be sometime recognized as a theoretical law of radiation.

If the general validity of this equation as representative of experimental data be admitted, the law of symmetry just mentioned provides a very simple method of determining the wave-length of maximum energy from an isothermal curve.

Graphically, the intersection of the axis of symmetry with the abscissa scale gives $\sqrt[3]{1/\lambda_m}$ from which λ_m may be found directly. Or, by algebraic solution,

$$\lambda_m = \frac{8}{\left(\sqrt[3]{\frac{1}{\lambda_1}} + \sqrt[3]{\frac{1}{\lambda_2}} \right)^3}$$

where λ_1 and λ_2 are any wave-lengths for equal relative energy.

The departures of the proposed formula from the corresponding formulas of Wien and Planck for various values of λT may be seen in the accompanying table. It will be noticed:

1. Throughout the region best known by experimental investigation, it does not depart much from the Planck formula, although it approximates closer to the experimental data.

2. The curve of the proposed equation intersects the Planck curve at a point in the "Reststrallung" region (λT = about 60,000).

For numerical evaluation this formula is more convenient than the Planck formula.

The author has no theoretical basis for proposing this equation; but recalling that the Planck equation was first are *empiric* relation to which theory was later *forced* to conform, he has thought that it might be worthy of notice and perhaps of some theoretical consideration by others.

P.S. April 18, 1919.—A second abstract giving a revised value of the constant D^2 on the basis of isochromatic data, will appear in the Proceedings of the Physical Society, Washington Meeting, April 25–26, 1919.

National Bureau of Standards,
February 25, 1919.

On the Measurement of the True Barometric Pressure in a Rapidly Moving Current of Air.[1]

By J. G. Coffin.

THE accuracy of the measurement of barometric pressure in rapidly moving airplanes is open to serious question.

If a body of any shape is moving rapidly through the air, there are regions on the surface of it which receive an aërodynamic pressure greater than the normal static or barometric pressure and there are also regions under smaller pressure than normal.

For a sphere, for example, the pressure variation is shown in Fig. 1, where radial distances outside the circle represent pressures in excess of normal and radial distances inside represent pressures less than normal. The radius, therefore, represents true barometric pressure, but not necessarily to scale.

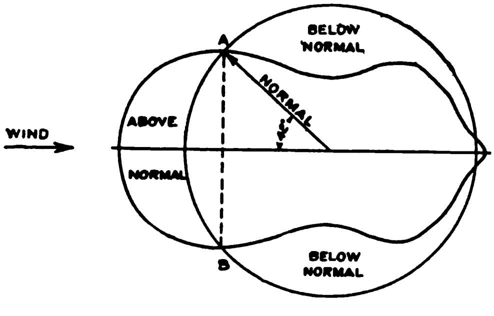

Fig 1

It is seen that there is an excess pressure on the nose where the air strikes the sphere directly and that at points on a circle of which AB is the trace there exists exactly normal pressure. At points aft of AB the pressure is below normal and on the sides reaches values as much or more under normal than the values over normal at the nose. Any form of barometric instrument must necessarily be open to the air whose pressure it is to measure, hence the position of the opening or openings is of importance.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

For example, at 60 miles per hour the total possible error may be as much as corresponds to an altitude of 240 feet and at 120 miles per hour, a usual speed at the present time, it can be nearly 1,000 feet!

This difficulty cannot be met by placing the barometer in any other receptacle or in particular, inside the fusilage; as the outside container must also be open to the air and as before the instrument will measure the total pressure obtaining at this opening.

In order to overcome the difficulty Dr. Zahm proposed, some years ago, to surround the barometer with a streamline body with a small opening at a point corresponding to a point on the circle *AB* on the sphere; that is at the point of zero aërodynamic pressure.

As is seen from Fig. 1, the rate change of aërodynamic pressure due to slight displacements along the surface is a maximum and any slight change in the air flow would cause large variations in the registered pressure. There could also be a large constant error due to initial uncertainty in the knowledge of the actual direction of air flow where the body is attached.

It occured to the writer that these objections could be overcome by placing the barograph inside of a small sphere which is fully exposed to the airstream and connecting its hermetically tight case to a small opening in the sphere.[1] If this sphere be now rotated by clockwork or other means at a uniform angular speed the opening will be carried around in a horizontal great circle and the barograph will register all the pressures, of course, passing through normal pressure two (or more) times per revolution.

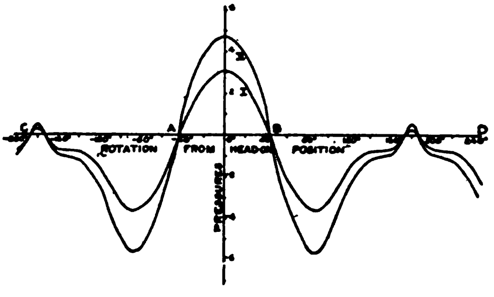

Fig II

The trace registered will be similar to Fig. 2.

From experiments made in the Curtiss Wind Tunnels at Garden City it is found that the zero points *A* and *B* remain fixed at all speeds experimented with. On the other hand, the change in speed produces an enlargement of the pressure scale and the pressures follow exactly the same law as if registered with a Pitot tube of standard design.

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[1] The barograph can, of course, also be connected to this opening by means of an air tight tube and be placed in any convenient location for observation.

It is a simple matter to calibrate such an instrument and draw the line *CD*, representing true barometric pressure, so that this troublesome problem is completely solved.

As a by-product of this idea we obtain the following advantages: If a barograph of this type is placed on an airplane and the trace examined on its return from a flight, this trace will show the true barometric pressure at every instant during the flight, the amplitude of the variations in the trace, will show at every instant the speed of the airplane and as equal periods are marked off on the graph by the rotations of the sphere, the slope of the graph will show the rate of climb at every instant so that this single trace contains a complete record of the performance of the airplane.

RESEARCH DEPARTMENT,
CURTISS ENGINEERING CORPORATION,
GARDEN CITY, L. I.

THE FUNCTION OF PHASE DIFFERENCE IN THE BINAURAL LOCATION OF PURE TONES.[1]

BY R. V. L. HARTLEY.

THE location of a source emitting a pure tone is a binaural process depending upon differences in the intensity and phase of the sounds at the two ears. Considerable experimental work has been done in which the intensities were kept equal and only the phases varied. A good review of this has been given by Stewart.[1]

If the observed location effects are the result of the listener's subconsciously perceiving the phase difference and associating it with past experiences in which he has encountered the same phase difference when the position of the source was known; it should be possible to trace a correlation between these experimental sound images and the positions of sources which would cause the phase differences used in producing the images.

Using a method due to Stokes,[2] Stewart[3] and, more recently, T. C. Fry of the Western Electric Co. have made calculations, from which, on the assumption that the head is a rigid sphere with the ears on a horizontal great circle, it is possible to plot the phase difference, P, against the lateral angular displacement of the source, for various distances and frequencies of the source. We have then to compare the direction corresponding to a particular phase difference, as given by the curves, with the direction of the image obtained experimentally with that phase difference.

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[1] Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

[2] G. W. Stewart, PHYS. REV., June, 1917, p. 502.

[3] Lord Rayleigh, Theory of Sound, Ch. 17.

[4] G. W. Stewart, PHYS. REV., 1914, 4, p. 252.

The agreement revealed by this comparison is striking. The curves account for the rotation of the image, with continuously varying *P*, that is observed at frequencies below about 600 cycles, including its motion toward the ear, followed by a range of more or less uncertainty, just before *P* reaches 180°. They also indicate that the value of *P* for maximum lateral displacement increases with the frequency as Bowlker[1] has observed. This value reaches 180° for about 600 cycles. Above that frequency, the curves indicate the appearance of a second image on the opposite side. At first this is present only for values of *P* near 180°, but with increasing frequency, it extends to smaller values, finally covering the whole cycle. A third image then appears near $P = 0$, and so on for still higher frequencies. This accounts for the generally observed falling off in the definiteness of the image with increasing frequency above about 600 cycles. Bowlker however was able to distinguish as many as three simultaneous images, and his description of them shows an agreement with those deduced from the theoretical curves which is remarkable considering the assumptions involved.

This general correlation constitutes strong evidence in favor of the theory of direct perception of phase in sound location.

RESEARCH LABORATORY OF THE
WESTERN ELECTRIC COMPANY, INC.

[1] T. J. Bowlker, Phil. Mag., (6), 15, p. 318, 1908.

Second Series *May, 1919* *Vol. XIII., No. 5*

THE

PHYSICAL REVIEW.

AN OPTICAL LEVER MANOMETER.

BY J. E. SHRADER AND H. M. RYDER.

SYNOPSIS.—To meet the demand for a gauge which will measure vapor pressures in the range between that which can be measured by the Knudsen Gage (.001 mm.) and the ordinary mercury manometer, a new type of gage has been devised. This consists of a rather large mercury manometer in one arm of which a small optical lever is so adjusted that it measures small changes in level due to differences of pressure. The range of the instrument for accurate work is from .001 mm. to 3 or 4 mm. The instrument can also be used as a differential gage with the same accuracy. In use the changes in pressure can be observed by the deflection of a beam of light and may be recorded by a photographic device. Curves are given from a photographic record of the action of the gage during the release of CO_2 and water vapor which had been frozen out in a trap with liquid air. Attention is called to the rapidity with which this gage responds to changes of pressure.

THE McLeod gage has long served as a means of measuring pressures of several millimeters or mercury down to the order of 10^{-5} mm. There are two fundamental cases, however, to which this gage is not applicable in this range, that is, when vapors are to be measured, and where there is a rapidly changing pressure to be recorded. In the first of these cases the McLeod gage becomes useless, due to the fact that Boyle's Law is not applicable to vapors. In the second case the McLeod gage, in its most favorable form, may be much too slow moving, and has no recording feature.

At the present time, when many pumps are on the market which enable more or less leaky systems to be maintained at low pressures, it has become desirable to determine accurately vapor pressure of many substances at pressures below those usually recorded. One of the writers[1] has reported a method of obtaining low vapor pressures accurately, which in a simplified and refined form is capable of operation down to the lower limits of the McLeod gage, but which is applicable to only a certain class of substances, namely those which can be distilled within range of temperatures available in the laboratory for such work, and which is not suitable for rapid determinations. The ability to observe rapidly changing pressures, or to record them, would mean, too, the facilitating of a number of operations in the laboratory or shop.

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[1] Ryder, Journal Franklin Institute, Vol. 186, No. 1, July, 1918.

One of the writers[1] has given a preliminary report of a gage designed to provide for the particular requirements suggested in the foregoing. It is the object of this paper to describe the essential details of this gage, and to give a few examples of its actual performance.

The principle of this gage is shown in Fig. 1. A mercury U-tube manometer is formed in the usual manner, except that the surfaces of the mercury are so arranged as to be of relatively large area. Above one of the surfaces, within the tube, is arranged an optical lever, as shown in the figure. This lever is supported by two knife edges, *a–a*, which rest on loops of wire, which in turn are sealed into the glass walls of the tube, a glass bead *b*, fused to the end of the lever arm acts as a float on the mercury surface, and in this way transmits the motion of the mercury surface to the lever arm. A mirror *M* attached at the position shown acts in the usual manner to reflect a beam of light from the lamp *c* to the scale *d*, if the gage is to be arranged as an indicating instrument. If the gage is to be used for recording variations in pressure, the scale may be replaced by a photographic device such as is used in oscillographic work.

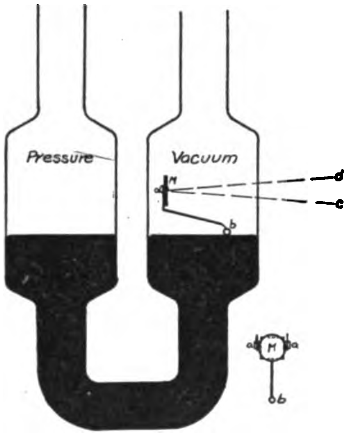

Fig. 1.
Optical Lever Manometer.

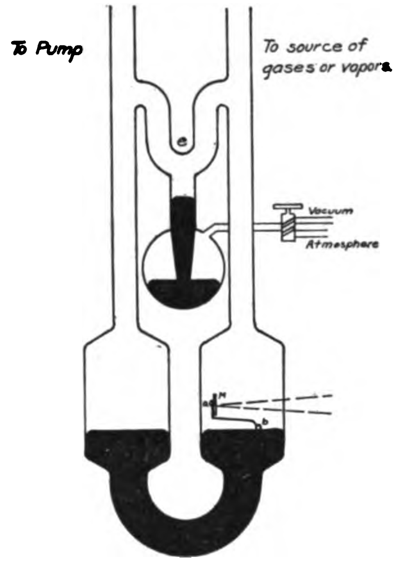

Fig. 2.
System for Manometer.

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[1] Shrader, Pittsburgh Meeting, American Physical Society, December, 1917.

Fig. 2 shows a convenient method for connecting this gage in a system. The cross connection *e* provides an easy means of evacuating the whole system with one pump located as shown. With this stop-cock or mercury cutoff open, a zero reading can be easily obtained, after which this connection may be closed and the gases or vapors introduced for measurement. This system provides also for the measurement of small variations in pressure, with an original pressure of any desired value, this value in no way affecting the absolute sensibility of the gage.

The gage as shown is a primary instrument, since the pressure corresponding to any deflection on the scale may be calculated from dimensions which may be readily determined. The lowest pressure which may be observed with the gage is that which gives the minimum appreciable movement of the mercury, and has been determined by experience to be approximately one thousandth of a millimeter of mercury in a properly designed gage. The upper limit, is, of course, determined by the dimensions of the gage, and is easily made to overlap the lower accurate readings of an ordinary U-tube manometer. The only source of error which need be considered is that due to the adhesion of the mercury to the walls of the tube, the deflection being therefore less than the actual value. If the mercury is very clean, and the glass in good condition a large mercury surface reduces this error to a negligible quantity. Mercury surfaces of from 5 cm. to 10 cm. diameter are in use. Some gages

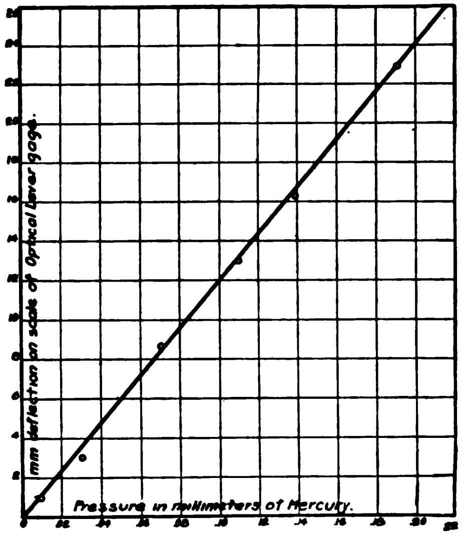

Fig. 3.

Calibration curve for an Optical Lever Gage.

have had a window of plane glass at the point of transmission through the tube of the beams of light, to prevent any possibility of a refraction of the beams due to the curvature of the glass.

Fig. 3 shows a check curve plotted between scale readings of this "optical lever" gage as ordinate and the actual pressures, as measured by a McLeod gage (hydrogen being the gas used). It will be seen that with this gage, up to the limit of this curve, .2 mm., a straight line relation exists. Above this point, the line is slightly curved, due to the flat scale used, and to the angular movement of the lever arm. This curve shows, too, the variations from the true value which may be expected from consecutive readings. The relative variation of this curve from its calculated position depends principally on the accuracy of the calculation. A convenient method of obtaining accurately the lever arm ratio consists in mounting the lever in a vessel of known cross sectional area with the float resting on mercury, in its normal position. When small known amounts of mercury are added, the movement of the beam of light on a properly placed scale may be observed, and the lever arm ratio calculated.

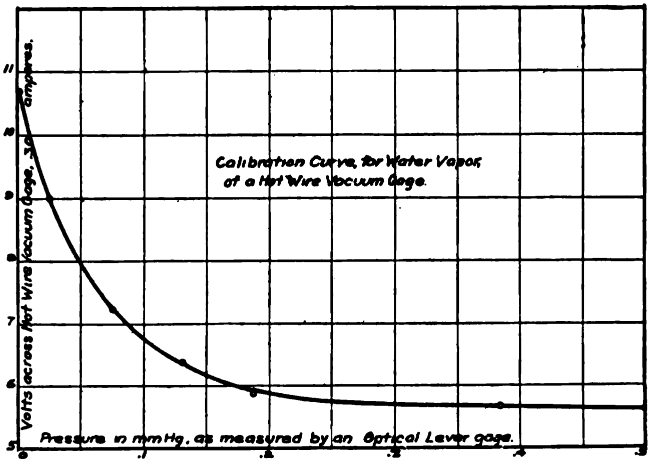

Fig. 4.

Use of Optical Lever Gage as a Primary Instrument.

By means of this gage as a primary instrument the use of the hot wire gage[1] may be extended to the measurement of vapor pressures. Fig. 4 is a calibration curve for the hot wire gage for water vapor, the pressure being determined by the "optical lever" gage.

Fig. 5 is a reproduction of a graphic chart obtained by using the

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[1] Pirani, Verh. I. Deutsch Phys. Gesell., 24, p. 686, 1906.

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PLATE I.
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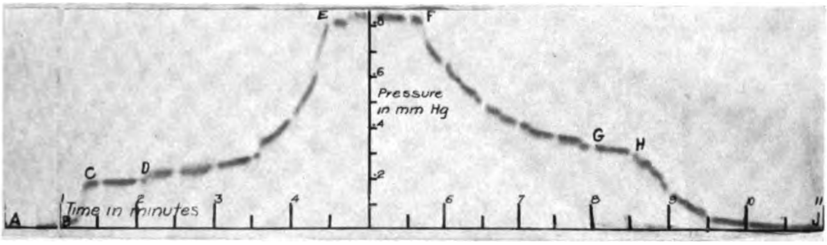

Fig. 5.

J. E. SHRADER AND H. M. RYDER.

"Optical lever gage." The apparatus used was that shown in Fig. 2 with the addition of a tube arranged for immersing in liquid air, connected to the right leg of the U-tube. At the point *A* on the chart, all gases have been removed from the system except some water vapor and CO_2 which are held frozen in the tube immersed in liquid air. With the cross connection closed, the liquid air is removed. At *B* active vaporizing starts. At *C* the CO_2 has been released, at *D* water vapor is coming off, and at *E* this vaporization is complete. At *F* solid CO_2 is applied and the water starts to freeze out again, this freezing out becoming complete at *G*. At *H* the solid CO_2 is replaced by liquid air and freezing out of the CO_2 begins. This is completed at *J*, the gage being thus brought back to zero reading. This chart is shown merely to give an idea of the possibilities of the "optical lever" gage in this direction. Many problems of diffusion, gas evolution, etc., immediately present themselves for solution with the aid of this device.

Some idea of the maximum rapidity of pressure variation which the optical lever gage will record may be gained from Fig. 6 which shows a record of the freezing out of water vapor by means of liquid air. The oscillations are due to the original sudden pressure drop, and have a period of slightly more than two seconds (approximately 1,000 grams of mercury in the gage). The design of the gage may be such that this period may be materially reduced.

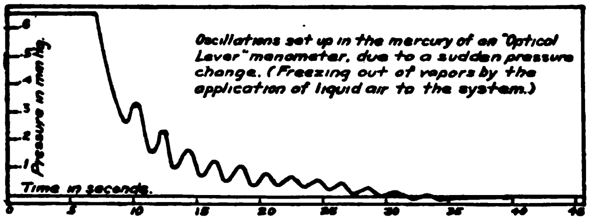

Fig. 6.

To summarize, a primary vacuum gage, consisting of the optical lever principle applied to a mercury U-tube manometer has been devised for indicating or recording pressures, including pressure variations, of vapors as well as gases, the pressure range being from several millimeters of mercury to approximately one thousandth of a millimeter. All errors of any consequence may be eliminated. By the use of this gage with the addition of a standard photographic device, charts may be obtained showing accurately pressure changes due to such phenomena as vaporization, freezing, diffusion, etc.

Westinghouse Research Laboratory,
East Pittsburgh, Pa.,
January, 1919.

THE LOGARITHMIC LAW CONNECTING ATOMIC NUMBER AND FREQUENCY DIFFERENCES IN SPECTRAL SERIES.

BY GLADYS A. ANSLOW.

SYNOPSIS.—In continuation of a previous note by Anslow and Howell graphic and algebraic relations are derived between the logarithms of the atomic numbers and of the ν, $\nu_1 + \nu_2$, ν_1 and ν_2 constant frequency differences occurring in the spectral series of alternate members of the same chemical family. Two systems of four approximately straight lines approaching parallelism resulted when a family possessed both doublet and triplet series. The equations derived are of the form $\nu = (N/mnk)^A$, where N is the atomic number of the element; A is a constant depending upon the system; n is an integer, the same for both systems in the same family; k is a constant, 0.080; and m takes on successive integral values for the ν, $\nu_1 + \nu_2$, and ν_1 equations. The agreement between the calculated and observed values of frequency differences indicates that the logarithmic lines should curve upwards for elements of high atomic number. The equations derived and the fact that an approximate linear relation seems to exist between the convergence frequencies of members of the same system of series and their atomic numbers do not seem to follow from the Bohr-Sommerfeld theory of atomic structure as so far developed and indicate that some adjustments to the theory are necessary if it is to be extended to predict optical series.

IN an earlier note Mrs. Janet Howell Clark and the author[1] called attention to an extension of the modified law of Runge and Precht[2] concerning the frequency differences of the doublet and triplet spectral series in a given family of the Mendelejeff chemical series. Ives and Stuhlmann[3] had found by a graphical method that a linear relation exists between the logarithms of the atomic numbers of the elements of one family and the logarithms of their doublet separations. Plotting the logarithms of the atomic numbers of the members of the second chemical family against the logarithms of the sum of the constant frequency differences of the triplets in their spectra, we found that linear relations exist between alternate members of the family, the points for magnesium, zinc, cadmium and mercury lying on one line, and those for calcium, strontium, barium and radium on another.

Bell[4] has recently published a treatise on the relation of atomic number to the doublet and triplet separations in the different chemical families. Testing the relation $\nu = A(N - N_0)^2$, where ν stands for the frequency difference, N for the atomic number, and A and N_0 are constants, he finds that such an equation approximately predicts the doublet and triplet separations of the different elements if we assume two such equations for each family which branch in the case of the first and second families at potassium and magnesium respectively. He also derives an equation for the logarithmic relation between the same quantities, putting the equation in the form, $\log \nu = p \log N + q$, where p and q are constants, but discards such a relation as not an essential improvement

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[1] Anslow and Howell, Proceedings of the National Academy of Sciences; III., p. 409, 1917.

[2] Runge and Precht, Philosophical Magazine; V., p. 476, 1903.

[3] Ives and Stuhlmann, PHYS. REV., V., p. 368, 1915.

[4] Bell, Philosophical Magazine, XXXVI., p. 337, 1918.

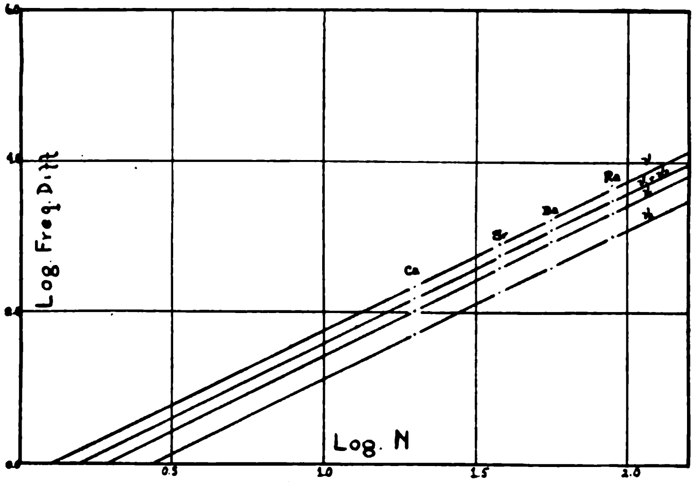

Fig. 1.

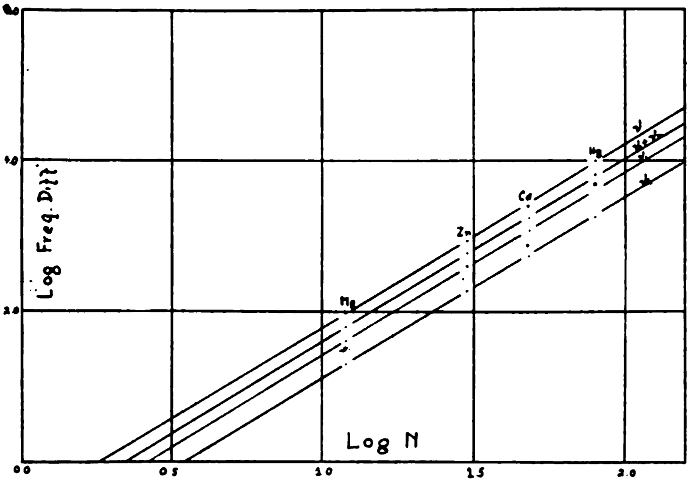

Fig. 2.

upon the relation $\nu = A(N - N_0)^2$. He also states that "the logarithmic method fails to show graphically the branching relation of the columns."

However, the plotting of the logarithmic equations does show the existence of two equations for each family as is evident from the diagrams in the article by Mrs. Clark and myself, where we attempted to make this fact clear. It happens that I had calculated the equations of the logarithmic lines for the different families where series are known and since the form in which I stated my final results is slightly different from Mr. Bell's, it shows some interesting relationships.

Since the spectral series of the elements in the second chemical family have been the most fully worked out for any family, I will consider it first. I have plotted the logarithms of the doublet separation, ν, and of the triplet separations ν_1, ν_2, and $\nu_1 + \nu_2$ against the logarithms of the atomic numbers of the elements and have obtained lines which pass through Ca, Sr, Ba and Ra (Fig. 1), and through Mg, Zn, Cd and Hg (Fig. 2). The striking feature of the result is that the lines for each system are approximately parallel. If the two systems are plotted on the same diagram, corresponding lines will intersect at points slightly below those for magnesium. This fact is shown for the ν lines in Fig. 3. The equations of the lines were first put into the form,

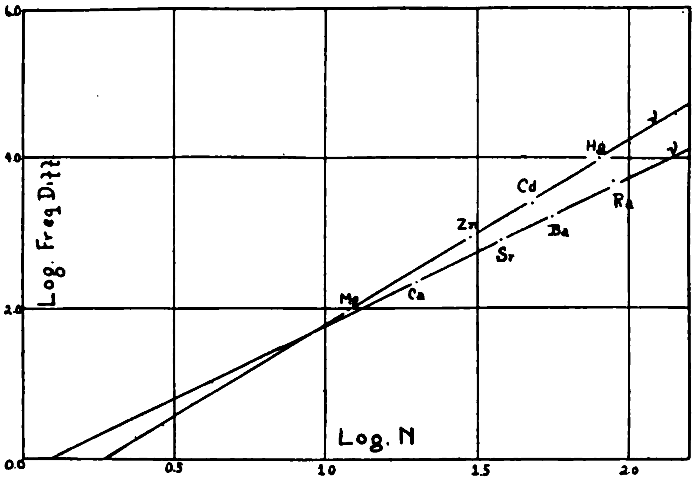

Fig. 3.

$$\log \nu = A(\log N - C)$$

and this equation then solved into $\nu = (N/B)^A$, where B = antilog. C. The constants A and B which appear in the mean equations of the lines are tabulated below.

The values for the frequency differences are taken, for the sake of comparison, the same as those taken by Bell and given by Dunz.[1] The values for the radium triplets are those given in our earlier paper. It should be noticed that the triplet differences 2050 and 832 found by Hicks[2] fit almost as well into the equations derived. Values marked * were not used in the calculation of the formulæ.

TABLE I.

Element.	N.	ν Obs.	ν Calc.	ν Calc.—ν Obs.	$\nu_1+\nu_2$ Obs.	$\nu_1+\nu_2$ Calc.	$\nu_1+\nu_2$ Calc. $-\nu_1+\nu_2$ Obs.
Ca........	20	222.9	225.6	+ 2.7	158.10	159.8	+ 1.7
Sr.........	38	801.3	795.8	− 5.5	581.49	578.0	+ 3.5
Ba........	56	1690.5	1704.6	+ 14.1	1248.7	1255.6	+ 6.9
Ra........	88	4858.0*	4140.9	−717.1	3052.7*	3103.9	+ 51.2

$A = 1.964$
$B = 1.267 = 4 \times .317$

$A = 2.002$
$B = 1.586 = 5 \times .317$

Element.	N.	ν_1 Obs.	ν_1 Calc.	ν_1 Calc. $-\nu_1$ Obs.	ν_2 Obs.	ν_2 Calc.	ν_2 Calc. $-\nu_2$ Obs.
Ca........	20	105.99	105.5	− 0.5	52.11	52.1	+ 0.0
Sr.........	38	394.44	394.7	+ 0.3	187.05	187.0	+ 0.0
Ba........	56	878.4	875.6	− 2.8	370.3*	408.4	+ 38.1
Ra........	88	2016.6*	2216.6	+200.0	1036.1*	1005.8	− 30.3

$A = 2.055$
$B = 2.072 = 6 \times .345$

$A = 1.994$
$B = 2.747$

Element.	N.	ν Obs.	ν Calc.	ν Calc. $-\nu$ Obs.	$\nu_1+\nu_2$ Obs.	$\nu_1+\nu_2$ Calc.	$\nu_1+\nu_2$ Calc. $-\nu_1+\nu_2$ Obs.
Mg........	12	92.0	91.4	− 0.6	60.86	61.0	+ 0.1
Zn........	30	872.4	872.6	+ 0.2	578.79	578.1	− 0.8
Cd........	48	2484.1*	2776.4	+312.3	1712.91*	1831.7	+118.8
Hg........	80	9835.06	9766.2	− 68.9	6397.50	6412.2	+ 14.7

$A = 2.462$
$B = 1.917 = 6 \times .319$

$A = 2.454$
$B = 2.247 = 7 \times .321$

Element.	N.	ν_1 Obs.	ν_1 Calc.	ν_1 Calc. $-\nu_1$ Obs.	ν_2 Obs.	ν_2 Calc.	ν_2 Calc. $-\nu_2$ Obs.
Mg........	12	40.95	40.9	+ 0.0	19.89	19.9	+ 0.0
Zn........	30	388.91	388.9	+ 0.0	189.78	189.8	+ 0.0
Cd........	48	1171.05*	1234.1	+ 63.0	541.86*	604.2	+ 63.3
Hg........	80	4630.31*	4328.4	−301.9	1767.19*	2127.0	+359.8

$A = 2.457$
$B = 2.648 = 8 \times .331$

$A = 2.462$
$B = 3.560$

The equations derived show that A is approximately constant for each system, as is to be expected since the plotted lines approach parallelism. Moreover, the constants B for the ν, $\nu_1 + \nu_2$ and ν_1 lines of both systems are successive integral multiples of the same factor, 0.32. The agreement between the observed and calculated values for the separations is fairly good for elements of low atomic number, but poor for those elements which lie beyond the rare earths in the chemical series. The differences between the values found for cadmium seem to indicate that either the series used are not related to those with which they are commonly supposed to be similar to in other elements, or the structure of the atom of this element is not exactly similar to that of the other elements in the same group, so that it acts as if its atomic number were less than that which has been assigned to it. Attention is also called to the fact that much better values could be calculated for the ν_1 separations of radium and mercury if the values of B were more nearly equal to $6 \times .320$ and $8 \times .320$, respectively.

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[1] Dunz, Bearbeitung unserer Kentnisse von der Serien, Tubingen, 1911.

[2] Hicks, Philosophical Transactions, A, CCXIL., 33.

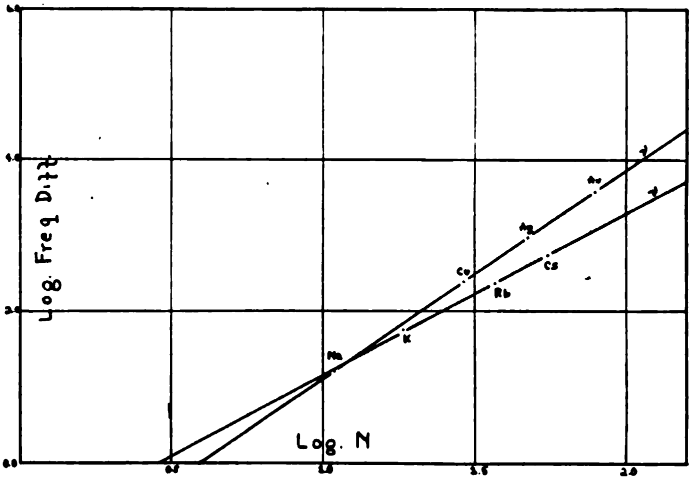

Fig. 4.

Since alternate elements are related in this way, it should be possible to predict frequency differences in elements where they are not yet known. The lines through Ca, Sr, Ba and Ra predict the separations $\nu = 9.56$; $\nu_1 + \nu_2 = 6.95$; $\nu_1 = 3.91$; and $\nu_2 = 2.17$ for beryllium. The spectrum shows the differences 8.9, 6.8, 4.1 and 2.5 between lines which occur in pairs, but not in triplets. A magnetic study of the lines in question has been made by Popow[1] proving that they belong to doublet and triplet series. Possibly the simplicity of the atom prevents the appearance of the three lines in the spectrum at once, but permits doublets with the separations corresponding to the triplet separations.

The results for Group I are plotted in Fig. 4 and the equations derived are shown in Table II. In this group only the doublet series have been discovered with the exception of copper.

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[1] Popow, Verhandlungen der Schweizerischen Naturforschenden Gesellschaft, Sept., 1913.

Table II.

Element.	N.	ν Obs.	ν Calc.	ν Calc. $-\nu$ Obs.	Element.	N.	ν Obs.	ν Calc.	ν Calc. $-\nu$ Obs.
Na......	11	17.21	17.21	+0.0	Li......	3	0.34*	1.09	+0.7
Cu......	29	248.13*	245.4	−2.7	K......	19	57.90	57.1	−0.8
Ag......	48	920.56	922.0	+1.4	Rb.....	37	237.71	239.0	+1.3
Au......	79	3817.20	3826.7	+9.5	Cs.....	55	564.10	559.7	−4.4
$A = 2.741$ $B = 3.895 = 8 \times .486$					$A = 2.147$ $B = 2.887 = 6 \times .481$				

As in Group 2 two values appear for A, and B is seen to be an integral multiple of the constant, 0.483. If the behavior of the light emitting electrons of the two families is similar, we should expect the appearance of triplet series in this group, and the equations which should predict the values of $\nu_1 + \nu_2$ and ν_1 are for Na, Cu, Ag and Au

$$\nu_1 + \nu_2 = \left(\frac{N}{9 \times .483}\right)^{2.741}$$

and

$$\nu_1 = \left(\frac{N}{10 \times .483}\right)^{2.741};$$

and for Li, K, Rb and Cs

$$\nu_1 + \nu_2 = \left(\frac{N}{7 \times .483}\right)^{2.147}$$

and

$$\nu_1 = \left(\frac{N}{8 \times .483}\right)^{2.147}.$$

Two systems of triplets have been discovered in the spectrum of copper by Rydberg[1] with the separations $\nu_1 + \nu_2 = 129.50 + 50.58 = 180.08$; and $680.10 + 212.21 = 892.41$. Assuming that A should have the same value for these triplets as for the doublets the values derived for B are

$$\begin{array}{lll} B^{\nu_1+\nu_2}, & 4.324 = 9 \times .481; & 2.432 = 10 \times .243 \\ B^{\nu_1}, & 4.918 = 10 \times .491; & 2.685 = 11 \times .244 \\ B^{\nu_2}, & 6.728 & 4.102 \end{array}$$

The agreement between the predicted values for B and those derived from the triplets with the smaller frequency differences is striking, the other system of triplets being, apparently, a secondary one with B a multiple of a constant half as large as for the other. These results support the hypothesis that the graphs for the ν, $\nu_1 + \nu_2$, ν_1, and ν_2 separations in Figs. 1 and 2 are parallel straight lines.

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[1] Rydberg, Astrophysical Journal, VI., p. 239, 1897.

If the latter three lines are parallel, the following equations can be stated for the triplets in any element

$$\nu_1 + \nu_2 = \left(\frac{N}{B_{\nu_1+\nu_2}}\right)^A \tag{1}$$

$$\nu_1 = \left(\frac{N}{B_{\nu_1}}\right)^A \tag{2}$$

$$\nu_2 = \left(\frac{N}{B_{\nu_2}}\right)^A \tag{3}$$

and subtracting (2) from (1)

$$\nu_2 = (N)^A\left[\left(\frac{1}{B_{\nu_1+\nu_2}}\right)^A - \left(\frac{1}{B_{\nu_1}}\right)^A\right] \tag{4}$$

which leads to the following relation between the constants,

$$\left[\left(\frac{1}{B_{\nu_1+\nu_2}}\right)^A - \left(\frac{1}{B_{\nu_1}}\right)^A\right]^{1/A} = \frac{1}{B_{\nu_2}}. \tag{5}$$

I have tested this result for the triplets considered. The ν_1 value for A for Ca, Sr, Ba, and Ra is so different from those for $\nu_1 + \nu_2$ and ν_1 that I have calculated second values for B from the Ca, Sr and Ba ν_1 separations, and from the Ca and Sr ν_2 separations, assuming $A = 2.002$, which is that derived for the $\nu_1 + \nu_2$ separations. The results are placed below those taken from Table I.

TABLE III.

System.	$B_{\nu_1+\nu_2}$.	B_{ν_1}.	$[(B_{\nu_1+\nu_2}^{-1})^A - (B_{\nu_1}^{-1})^A]^{-1/A}$.	B_{ν_2}.	$d(B_\nu)$.
Ca, Sr, Ba, Ra....	1.586	2.054	2.494	2.746	0.252
Ca, Sr, Ba, Ra....		1.922	2.800	2.780	0.020
Mg, Zn, Cd, Hg...	2.247	2.648	3.543	3.560	0.017
Cu..............	4.324	4.918	6.728	6.930	0.202
Cu..............	2.432	2.685	4.102	4.108	0.006

The values in the last column give the differences between the two calculated values of B_{ν_2}, and are so small that the hypothesis of parallelism between the series separations is strengthened.

Triplets appear again in Group VI., and have been discovered in the spectra of O, S and Se (Fig. 5). If the alternate members in this group are spectroscopically related those in O should not be related to those in

S and Se, but to the undiscovered triplets in Cr, Mo, W and U. From the separations $\nu_1 + \nu_2 = 18.15 + 11.13 = 29.16$ in S, and $\nu_1 + \nu_2 = 103.7 + 44.07 = 148.48$ in Se, the following equation is derived.

$$\nu_1 + \nu_2 = \left(\frac{N}{3.356}\right)^{2.160} = \left(\frac{N}{7 \times .479}\right)^{2.160}.$$

The similarity between this equation and the one predicted for the undiscovered triplets of Li, K, Rb and Cs is apparent. If they are identical ν_1 and ν_2 should be given by parallel equations with $B = 8 \times .479$, and

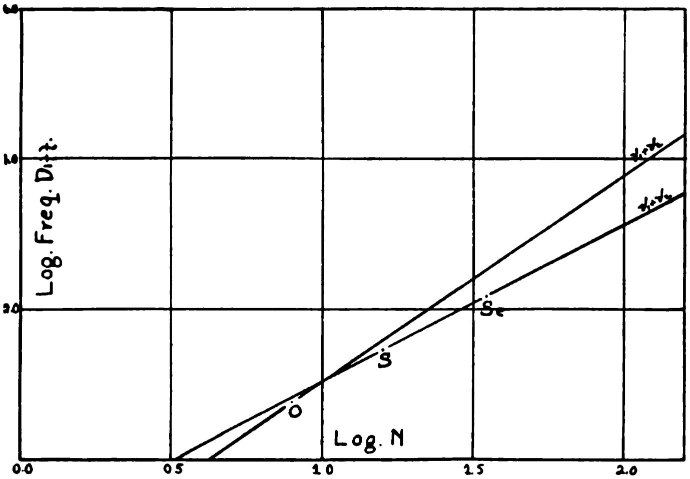

Fig. 5.

$$B = \left[\left\{\left(\frac{1}{7}\right)^{2.160} - \left(\frac{1}{8}\right)^{2.160}\right\}\left(\frac{1}{.479}\right)^{2.160}\right]^{-1/2.160}.$$

These values give

$$\nu_1 + \nu_2 = 21.12 + 7.27 = 28.39 \text{ for S,}$$

and $111.63 + 37.04 = 148.67$ for Se. Better agreement would have been obtained if the values for B predicted for Li, K, Rb and Cs had been used.

The analogy suggests that the triplets in O may be given by the equations derived from the triplets in copper. Using these equations we obtain $\nu_1 + \nu_2 = 5.43$; $\nu_1 = 3.80$; and $\nu_2 = 1.61$. The observed differences as given by Runge and Paschen[1] are $3.70 + 2.08 = 5.78$.

The only other group where series of the first type have been worked out, with the exception of those in manganese, is in the third, where doublets occur. The graph is shown in Fig. 6. The values of the constants derived from the frequency differences which are known are shown in Table IV.

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[1] Runge and Paschen, Annalen der Physik, LXI., p. 641, 1897; Astrophysical Journal, VIII., p. 70, 1898.

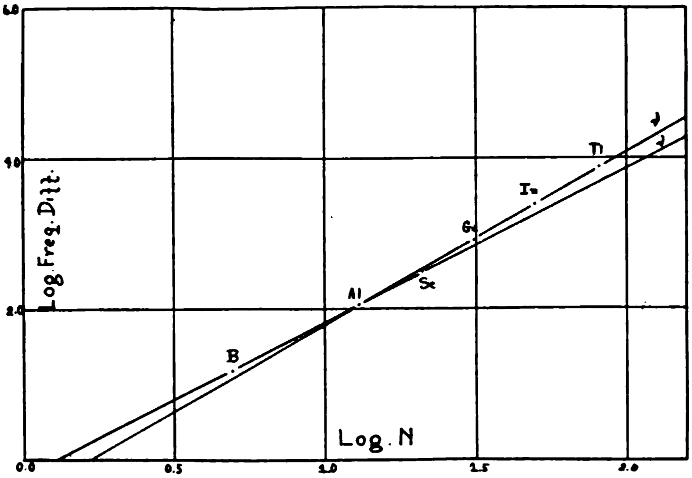

Fig. 6.

TABLE IV.

Element.	N.	ν Obs.	ν Calc.	ν Calc. − ν Obs.
Al	13	112.07	110.9	− 1.2
Ga	31	823.6	829.5	+ 5.9
In	49	2212.63	2397.1	+184.5
Tl	81	7792.45	7753.5	− 39.0
B	5	15.4	12.09	− 3.4

$A = 2.318$
$B = 1.706 = 7 \times .246$

It is seen that the value of the doublet separation in boron lies above the line connecting the points determined by the other elements which are alternate members in the family. If another line should determine the frequency differences for the other elements B should lie on it, and it should intersect the known line in the neighborhood of the point determined by Al. Hicks[1] has found a doublet difference for scandium of 320. Combining this value with that known for boron we find that $A = 2.166$, and $B = 1.433 = 6 \times .239$.

DISCUSSION.

The results stated show that the logarithmic method of obtaining the relation between atomic number and frequency differences in spectral series give comparatively good agreement between the observed and calculated values for the constant frequency differences for elements of low atomic number, although the agreement is not within the limit of probable error of the mean of the observed differences, the probable error of the observed value being usually not more than 0.5 cm^{-1}. For elements of high atomic weight the observed values are greater than the calculated, as should be expected if the mass of the electron varies with its velocity. The logarithmic line should, therefore, curve upwards for elements of high atomic weight. Attention is also called to the fact that the calculated values are all too high for the elements which just precede the rare earths in the chemical table.

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[1] Hicks, Philosophical Transactions, A, CCXIII., p. 408.

The importance of the logarithmic line does not lie so much in an attempt to obtain good agreement between observed and calculated values as in the connections which are brought out between different types of series in the same and different chemical families. The equations derived show that the values of B are successive integral multiples of the same constant for the ν, $\nu_1 + \nu_2$, and ν_1 lines in an element. This seems to mean that the frequency differences between the longer wave-length line in triplets and each of the other two are more important than the differences between adjacent lines. The equations give the same value for A[1] for all separations in the same system, and the constants B seem to be integral multiples of the same constant, 0.080, in all systems. The variation of the value of A seems to indicate that spectral series are of several types, just as there are several types of X-ray spectra. In general, the types vary with the valency of the element, but it is possible for the same type to occur in elements with different valencies as is done in groups I. and VI.

The form of the equation derived does not seem to be predicted from the equation for the frequency of lines as developed by Sommerfeld[2] for X-rays, from which we should expect an equation of the type of Bell's to be approximately true. Sommerfeld's equation, moreover, indicates that the convergence frequency of a series should be given by

$$n_0 = \left(\frac{N-k}{p}\right)^2\left[1 + \frac{a}{4}\left(\frac{N-k}{p}\right)^2 + \frac{a}{8}\left(\frac{N-k}{p}\right)^4 \cdots, \right]$$

where a, k and p are constants, a being a small quantity. Therefore, the convergence frequencies of series of the same type should be approximately proportional to $(N - N_0)$.[2] That this is not the case is seen from an inspection of Fig. 7, where the convergence frequencies of the series with constant frequency differences have been plotted. The values used were obtained from Dunz, and may also be found in Kayser's Handbuch der Spectroscopie, Vol. II. The graph shows that an approximate linear relation exists between the convergence frequencies and

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[1] An inspection of the values for A shows common differences of approximately 0.15, the value 2.60 being the only absent member in such an arithmetic series. It is a curious coincidence that the frequency differences between the α and β lines in the K system of X-ray series may be predicted by the equation $\nu = (N/.065)^{2.18}$.

[2] Sommerfeld, Annalen der Physik, LI., 1916.

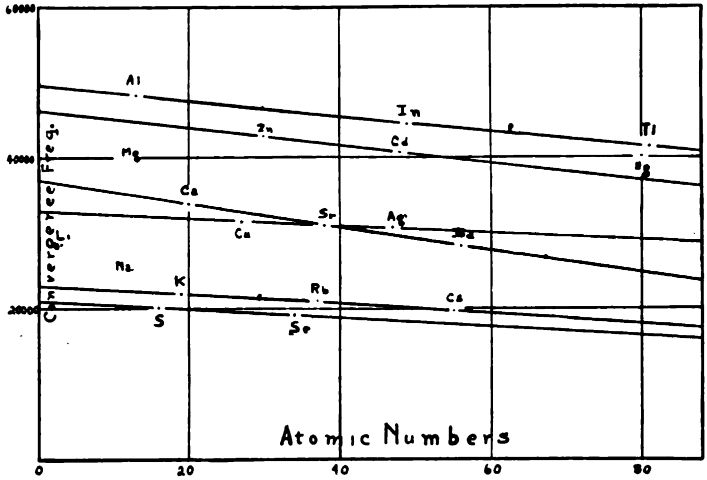

Fig. 7.

atomic numbers of K, Rb and Cs; Ca, Sr and Ba; and Al, In and Tl, which elements I have shown to be related in spectral type. Moreover, a line through the points for S and Se is approximately parallel to that through K, Rb and Cs, a fact which is in support of the hypothesis that the series in these elements is predicted by the same equation.

It would seem that an adequate atomic theory should explain the relation just stated between atomic number and convergence frequency, and the physical reason for the approximate relation

$$\nu = \left(\frac{N}{mnk}\right)^A,$$

where ν represents the frequency difference, N the atomic number, A is a constant, n an integer, both of which vary with the system, m successive integers for the ν, $\nu_1 + \nu_2$, and ν_1 members of the system, and k a constant for all the systems considered.

Smith College,
Northampton, Mass.,
February, 1919.

ON THE CHARACTERISTICS OF ELECTRICALLY OPERATED TUNING FORKS.

By H. M. Dadourian.

Synopsis.—A series of experiments was performed to determine the conditions affecting the period of an electrically operated tuning fork with the following results: (1) The more massive the base of the fork and the table upon which it is placed the smaller is the period. This effect, however, is not greater than one part in 10,000. (2) A change in the constants of the electrical circuit containing the electromagnet affects the period. The effect is less than one part in 10,000 for moderate changes necessary for keeping the amplitude of vibration constant. (3) The period increases linearly with the increase in the length of the gaps between the contact springs and contact points. For the forks used this increase was about one part in 500 for a change of 0.1 mm. in the length of the gaps. (4) For a given fork there is an amplitude at which the period has a stationary value. This may be a maximum or a minimum depending upon the arrangement of the mounting of the contact springs. At small amplitudes the change in the period due to a variation in the amplitude may be considerable. (5) The temperature effect increases from 1.04×10^{-4} at $-25°$ C. to 1.43×10^{-4} at 56° C. The values corresponding to temperatures above 0° C. are from 20 to 40 per cent. greater than those obtained by other observers. There are no other published data giving the temperature effect below 0° C. (6) By keeping the temperature, the length of the gap and the amplitude constant, a well-made fork can be relied upon to give a constancy of rate accurate to one part in 50,000. (7) The theoretical expression for the period of vibration of a bar holds good for a tuning fork. (8) The velocity of sound in the steel of which the forks used were made is 5.49×10^5 cm. per sec. (9) The coefficient of modulus of elasticity of the steel is 19.10×10^{11}. (10) The temperature coefficient of the modulus of elasticity increases from -2.2×10^{-4} at $-25°$ C. to -2.96×10^{-4} at 56° C. These values were computed from the relation $\epsilon = -(2\theta + \alpha)$, where ϵ is the temperature coefficient of modulus of elasticity, θ the temperature effect upon the period of the fork, and α the coefficient of linear expansion of steel.

I. Introduction.

The determination of the frequency of tuning forks and the study of the causes affecting the frequency have been the subjects of investigation by a large number of physicists during the last hundred years or more. Consequently the author would not have thought of carrying out the researches described in the following pages had not circumstances led him to them. While working on problems of sound ranging with the Engineer Detachment of the United States Army in Princeton, it became part of the author's work to adjust and to determine the periods of vibration of a number of tuning forks. As these forks were slightly different in shape and in details of mounting from those used by previous investigators it was found desirable to know the extent to which the results obtained by them held good for these forks. A brief investigation was therefore planned with this end in view. But as the work progressed, interesting and important results were obtained which led the author to widen the scope of inquiry until the problem had been carefully studied in all its phases.

II. DESCRIPTION OF THE FORKS.

Most of the experiments described in the following pages were carried out with forks made by the Western Electric Company, of the type shown in Fig. 1. The fork proper is milled out of a solid piece of soft machine steel and is electro-galvanized to prevent rusting. The prongs of the fork are about 39.5 cm. long, 0.95 cm. thick, 1.90 cm. deep, and 2.90 cm. apart. The outer curves at the shoulders as well as the inner curve are circular. Consequently the prongs are thinner at the shoulders than the prongs of standard Koenig forks of the same general dimensions, and therefore vibrate about their fixed ends as axes more like rigid bars than do the prongs of the Koenig type of fork.

The fork is rigidly attached to the back of a brass casting which has the form of a rectangular trough. The electromagnet is provided with movable pole-pieces placed on the outside of the prongs. This feature had been introduced by the Western Electric Company to satisfy the special conditions for which the fork was originally designed and is not at all necessary for its general serviceability. In fact the common type of electromagnet with pole-pieces between the prongs is preferable for a fork for which constancy of frequency is the principal desideratum.

A steel contact spring, 0.28 mm. thick and 3.92 mm. wide, is attached to each of the prongs by means of a steel clamp screwed to the prong at a point about midway between its ends. The contact spring is reënforced with two shorter springs placed one on each side and clamped together. When the fork is adjusted the free end of each contact spring comes midway between two platinum contact points soldered to the ends of two brass screws. The latter are set in brass bars provided with set screws for the purpose of making the contact screws fast after adjustment. These bars are screwed to a piece of micanite which forms the top piece of the bridge over the prongs. In order to secure good electrical contact a circular piece of platinum foil is soldered to each side of the free end of each contact spring.

The scheme of electrical connections is shown in Fig. 2, where the arrowheads marked *A*, *B*, *C* and *D* indicate the four contact points at the ends of the horizontal brass screws shown in Fig. 1. The condensers

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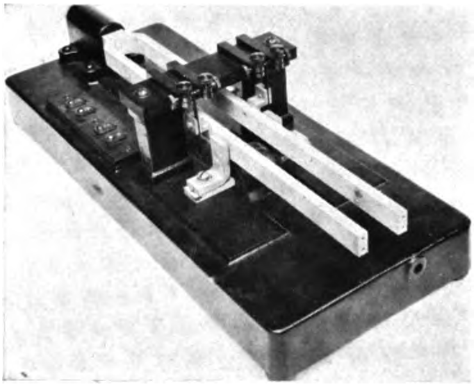

Fig. 1.

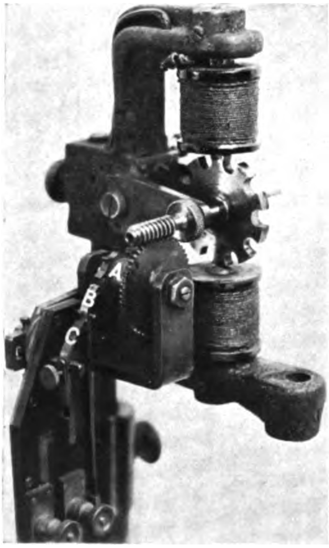

Fig. 4.

H. M. DADOURIAN.

marked *c* have each a capacity of 0.5 M.F., the resistance coils marked *r* have each a resistance of 150 ohms, and the field coil, *R*, of the electromagnet has a resistance of 225 ohms and about 6,300 turns of wire. The condensers and the resistance coils are attached to the under side of the brass base. The contact points *C* and *D* do not form parts of the electrical circuits, but they are necessary for the symmetry of the mechanical action of the contact points upon the contact springs. The contact point *D* is provided with the necessary connections, so that it can be made use of electrically in case it is desired to energize the electromagnet of the phonic wheel every half period instead of every full period, for instance. It may be stated here parenthetically that the wheel runs with greater smoothness between two than between four pole-pieces as the fork functions under normal running conditions remarkably free of sparking at the contact points, so that the pieces of platinum foil soldered at the ends of the contact springs last indefinitely.

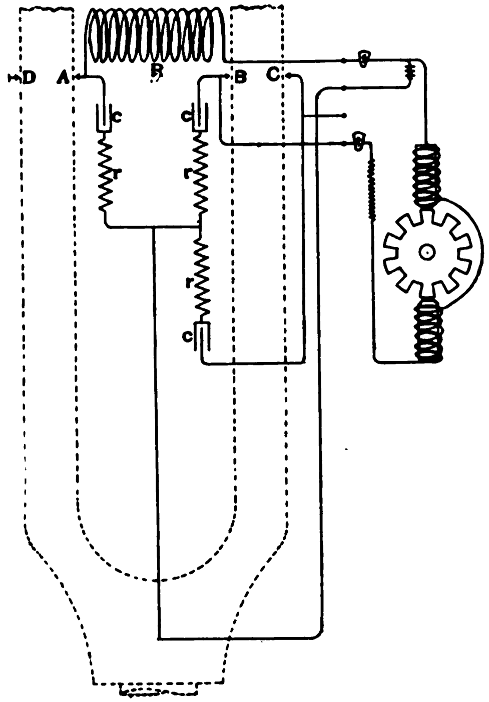

Fig. 2.

In addition to the W. E. (Western Electric) forks three others were used. One of these was a Leeds and Northrup fork kindly loaned by the Company. Another was a fork made by Pirard and Coeurdevache of

Paris. The third was an old 50 V. D. Koenig fork mounted upon the base of one of the W.E. forks. In order to adapt the Koenig fork to the W.E. base, contact springs were clamped to the sides of the prongs at points about one-fourth of the length from the free ends. The way in which this was done is indicated in Fig. 3, where *A* is a piece of steel channeled to a depth slightly less than the total thickness of the three springs between it and the prong. This piece was attached to the prong by means of two screws which pass through holes made in the springs. The arrangement is an improvement over that of the original W.E. forks, first because it is simpler and second because it does not unnecessarily modify the uniformity of the effective thickness of the prong in the neighborhood of the clamp, as the spring in contact with the prong has only one-third the width of the clamp in contact with the prong in the W.E. fork.

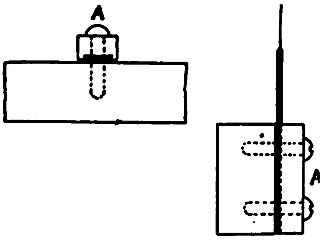

Fig. 3.

III. Method of Determination of the Period.

The period of vibration of the fork was determined by a method devised by Captain H. B. Williams. Chronographic records of every fiftieth complete vibration were compared with the records of the mean-time clock of the Princeton University Observatory. In order to record the vibration the phonic wheel and contact making device shown in Fig. 4 were used as intermediary between the fork and the chronograph. When the fork is set in vibration the circuit, of which the field coils of the phonic wheel form a part, is made and broken at the contact point *A*, Fig. 2, thus periodically exciting the electromagnet of the wheel. If the armature of the wheel is given a motion of rotation so that one of its teeth comes nearly in front of each of the pole-pieces of the magnet it falls in step with the magnetization of the electromagnet and rotates synchronously with the vibration of the fork. The rotation of the armature causes the toothed wheel *A*, Fig. 4, to rotate and to push against the rod *B* every time one of its teeth passes by the end of the latter. The rod *B* then comes into contact with the rod *C*, thus closing the circuit which operates the pen of the chronograph. The gearing intervening between the phonic wheel and the contact maker is such that this occurs once in every fifty complete vibrations of the fork. The short flat springs at the lower ends of the rods *B* and *C* hold them in position and connect them to the binding post at the lower left hand corner of Fig. 4 to which are attached the leads of the chronographic circuit. The time of contact may be made as short as desired by adjusting the position of the contact maker relative to the wheel *A*, and of the rod *C* relative to the rod *B*. The clock records were obtained by the intermediary of a quick contact making device, attached to the scope wheel of the observatory clock, of which the contact maker shown in Fig. 4 is a simplified copy.

The phonic wheel was invented by La Cour and Rayleigh, independently. They used it, however, to compare the frequencies of two forks by the stroposcopic method and not to obtain chronographic records of a fork. The method described in the preceding paragraph which may, properly, be called the *chronographic method*, is more convenient and is capable of yielding a greater accuracy than any other method hitherto used for determining the period of a tuning fork, as will be observed from the following statement.

First, in the chronographic method a record of two hours can be obtained without any more care on the part of the experimenter than that of winding up the clock-work of the chronograph. The record obtained can be read with great precision, and being permanent can be read at one's convenience and kept for future reference. The clock does not have to be adapted to the fork nor the fork to the clock; consequently the frequency of any fork can be obtained by comparing its record with that of the clock, the rate of which is practically constant and known very accurately. The rate of the mean time clock used in these experiments was known to one part in 200,000.

Second, in this method the recording arrangement does not affect the motion of the fork as in the case of the vibrographic method in which the variable pressure of the drum against the recording style must have an appreciable effect upon the period of the fork. In this connection it may be stated that an experiment was performed in which the period of W.E. fork No. 496 was observed while the mean current through the field coils of the phonic wheel was changed from 60 to 180 milliamperes, but no appreciable change in the period was observed.

Third, the method gives a direct comparison of the fork with a primary standard like a clock, and not with a secondary standard in the form of another fork.

The precision of the chronographic method depends upon (*a*) the accuracy with which the rate of the clock is known, (*b*) the accuracy of the positions of the fork signals relative to the positions of the clock signals on the chronographic record, (*c*) the length of the record, and (*d*) the precision with which it is read. The error due to the rate of the clock may be made negligible by using a clock the rate of which is accurately determined. The error due to the irregularities of the positions of the signals need not be more than .01 sec. in a well-constructed instrument. The chronographic record can be read with great accuracy by means of a differential scale often used by astronomers. So that even if an error of .05 sec. is made, due to the imperfections of the recording apparatus, the effect of this comparatively large error can be made negligible by taking a fairly long record. If the record is an hour long, for instance, an error of .05 sec. due to the end signals will introduce in the determination an error of one part in 72,000 only.

IV. EFFECT OF BASE.

It follows from theoretical considerations that the base upon which the fork is mounted should have no mass at all or it should have an infinite mass in order that it absorb no energy from the fork. It is to be expected therefore that the base of a fork will have some effect upon the period. In order to study this effect the following experiments were performed.

The W.E. fork No. 496 was placed upon a brick pier with a marble top and its period determined. A lead weight of 10 kg. was then placed upon the standard by means of which the fork proper is attached to its base and the period was again determined. The result of increasing the mass of the base by this means was to decrease the period by one part in 30,000.

A similar experiment was made with the Pirard and Coeurdevache fork and a decrease of one part in 1,000 observed. The reason for the comparatively large change in the period of this fork can be accounted for by the fact that the P. & C. fork weighs about 4 kg. while the W.E. forks weigh about 12 kg.

Next the effect of the table upon which the W.E. fork was placed was determined by observing its period while it was (*a*) upon the pier, (*b*) upon a wooden table and (*c*) upon a shaky stool. The result was a progressive increase in the period, the period on the stool being one part in 10,000 greater than on the pier.

The relatively large weight of the base and mounting of the W.E. forks is therefore a desirable feature, which makes the effect on the period of the mass of the table upon which it is placed relatively small.

V. EFFECT OF CHANGES IN THE CONSTANT OF THE ELECTRICAL CIRCUIT.

In Fig. 5 let oo' indicate the position of rest of the contact spring which is in series with the field coil of the electromagnet, bb' and cc' indicate the positions of extreme displacement, and aa' its position at the instant it comes into contact with A, Fig. 2, and closes the circuit of the electromagnet. On account of the self induction of the circuit the driving current does not attain its maximum value instantaneously but increases according to the well-known exponential law during the excursion of the spring from aa' to bb' and back to aa'. At aa' the spring leaves the contact point thus opening the circuit and permitting the current to die down. The values of the current at different parts of a complete vibration are roughly indicated by the curve $ab'a'ca$. The motion of the prongs is opposed by the magnetic field, while the spring moves from aa' to bb', is helped while moving from bb' to cc' and is opposed again while moving from cc' to aa'. Thus the motion of the prongs is opposed during the time the prongs come together and is helped the rest of the time. Therefore a periodic but non-harmonic force is impressed upon the prongs in addition to the forces of restitution and of damping. When a steady state of vibration is reached the energy supplied per cycle by this force equals the energy dissipated by the damping forces. This energy is a function of the area enclosed by the broken curve $ab'a'ca$. The area as well as the position of its center can be altered while the maximum value of the current is kept constant by changing the constants of the exponential curves and the position of the line aa'. In other words the energy applied per cycle and the effective phase of its application may be altered by changing the time constant of the circuit and the length of the gap between the spring and the contact point.

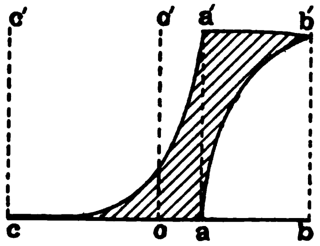

Fig. 5.

It will be seen in Part VII. that for constancy of frequency it is necessary to keep the amplitude constant. But to do this it may be necessary to regulate the driving current and hence it is of importance to know to what extent, if at all, a change in the time constant due to a change in the resistance of the circuit affects the period while the amplitude is kept constant. This point was investigated with the W.E. fork No. 488 in two experiments. In one of these the effect of an additional non-inductive resistance was observed, and in the other the effect of adding a self induction to the circuit was determined, the amplitude being kept constant in both cases. The results of the first experiment are shown by the curves of Fig. 6, where the abscissas denote the external non-inductive resistance added to the circuit of the electromagnet, and the ordinates denote the percentage increase in the period of the fork in ten-thousandths parts of the period corresponding to zero external resistance.

Curve I. was obtained with the length of the gaps between the contact springs and the contact points equal to .05 mm., while curve *II.* was obtained with the length equal to 0.3 mm. It will be observed that the effect is greater for the longer gap. But the effect is negligible for both, so long as the change in the resistance is small. If a change of 100 ohms is made in the resistance of the circuit to keep the amplitude constant, the change in the period is under one part in 100,000 for curve I. and under two parts for curve *II.*

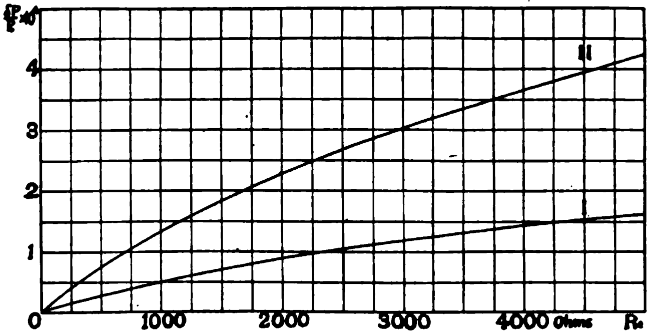

Fig. 6.

The effect of an increase in the self induction of the circuit was next observed by placing alternately an inductive resistance and a non-inductive resistance of equal value in series with the coil of the electromagnet and the change in the period was found to be less than one part in 10,000.

VI. Effect of the Length of the Gap.

It is evident from Fig. 5 that a change in the position of the line *aa'* would result in a change in the area of the cycle *ab'a'ca* and the position of the center of this area. Consequently changing the gap between the contact spring and the contact point introduces a change in the amount of energy applied to the fork per period and in the phase of application of this energy. Furthermore the circumstances of the mechanical action of the contact points upon the springs, and through them upon the prongs, is changed when the gaps are changed. Therefore an alteration in the length of the gaps is bound to affect the period of vibration of the fork.

This effect was studied by determining the period of vibration for gaps of different lengths. The results of an experiment with the W.E. fork No. 488 are shown by curve *I* of Fig. 7, where the abscissas denote the lengths of the gaps between the springs and the contact points in fractions of one millimeter, while the ordinates denote percentage increases in the period in ten-thousandths of the period corresponding to zero gap.

All four of the gaps were made of the same length, changing them from .04 mm. to .25 mm. in ten steps. It will be observed that the effect is very nearly linear and that it is comparatively large, being one fifth of one per cent. per one tenth of a millimeter change in the length of the gaps.

Curve *II* was obtained with the Koenig fork. In spite of the fact that the springs were less stiff than those of fork No. 488 the gap-effect of the Koenig fork is greater than that of No. 488. This can be accounted for by the fact that the springs were nearer the free ends of the prongs in the case of the Koenig fork and consequently the moment of the forces acting upon the springs was greater in proportion, and that the forks were relatively thinner and consequently their motion was more easily affected by this moment.

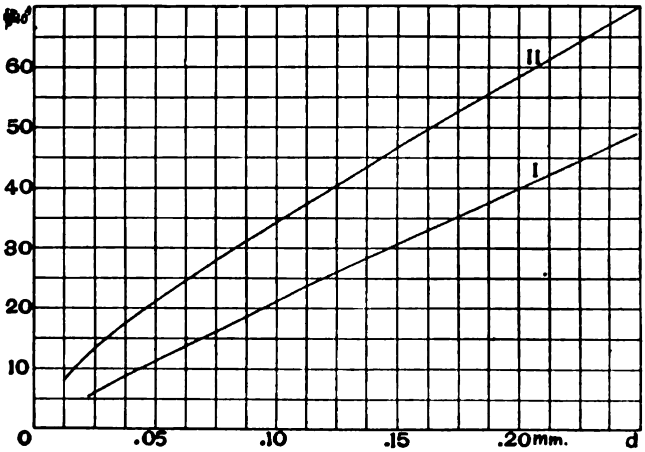

Fig. 7.

These results show that the gap-effect is considerable and that the gaps must be kept constant in length if constancy of frequency is desired. This fact does not seem to have been recognized by students and makers of tuning forks, as is evident from the form of contact springs of standard types of forks. Most of these forks have twisted platinum wires for contact springs which change their shape under the action of the contact points, making the length of the gaps variable. The springs of the W.E. forks can be relied upon to keep their shape and position so that the length of the gaps do not vary more than .01 mm.

It was found that the positions of the springs relative to the contact points were slightly changed when the position of the fork was changed. This was due to a redistribution of the weight of the fork among the four rubber feet of the base and the consequent strain in the frame of the base. In order to avoid possible errors from this source, the gaps were examined whenever the forks had to be moved and one of the feet raised by placing a few thicknesses of paper, thereby bringing the springs to positions midway between the contact points. This precaution can be made unnecessary by making the frame of the base more rigid and providing it with three instead of four feet.

The effect upon the period of fork No. 488 of eliminating the outer contact points was determined by observing the period alternately with the outer contact points at 0.1 mm. from the springs and at distances beyond the reach of the springs. The average results of four separate determinations was to change the time of 50 periods from 1.00096 sec. with the outer contact points at 0.1 mm. to 1.00369 sec. with the outer contact points beyond the reach of the springs, the inner contact points being kept at 0.1 mm. from the springs. In other words the mechanical action of the outer contact points upon the springs decreases the period by a fraction of about 27 parts in 10,000.

VII. EFFECTS OF CHANGES IN AMPLITUDE.

Mercadier[1] and Ettinghausen[2] have observed that the frequency of a fork increases with decreasing amplitude and approaches assymptotically the frequency corresponding to zero amplitude. Poske[3] and Heerwagen[4] on the other hand have found that the frequency increases linearly with decreasing amplitude. Heerwagen has expressed his results by the equation

$$n = n_0 - p\alpha$$

where n and n_0 denote the frequency, α the amplitude, and p a constant. Hartmann-Kempf[5] has observed, however, that the frequency increases more rapidly than Heerwagen's linear equation would imply and has expressed his results by the empirical relation

$$n = n_0 - (p + \Delta\alpha)\alpha.$$

The differences in the conclusions attained by these investigators can be accounted for in the light of the results obtained from the following experiments upon the effect of changes of amplitude upon the period. Throughout these experiments the temperature and the spark gaps were kept constant. The amplitude was measured by means of a traveling microscope provided with a vernier to read twentieths of one millimeter. The microscope was focused upon a bright spot at the end of one of the prongs and the double amplitude measured; the double amplitude of the other prong was then measured and one half of the average of the two observations taken as the amplitude of the fork.

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[1] Mercadier, C. R., 83, p. 800, 1876; Journ. de Phys., 5, p. 201, 1876.

[2] A. Ettinghausen, Pogg. Ann., 156, p. 337, 1875.

[3] H. Poske, Pogg. Ann., 152, p. 448, 1874.

[4] F. Heerwagen, Diss. Dorpat, 1890.

[5] R. Hartmann-Kempf, Ann. d. Phys., 13, p. 124, 1904.

Curves *I* and *II* in Fig. 8 show the results obtained with the W.E. fork No. 496. The abscissas denote single amplitudes in millimeters and the ordinates denote percentage increases in the period in ten-thousandths of the minimum period observed. The changes of amplitude of curve *I* were made by increasing the current in the field coils of the electromagnet while the pole-pieces were kept fixed. On the other hand, the changes in the amplitude of curve *II* were obtained by moving the pole-pieces relative to the prongs while the current was kept constant. The curves represent the averages of two sets of curves, one obtained by increasing the amplitude and the other by decreasing it. The two component curves did not quite coincide; each formed, however, a very smooth curve representing the experimental data.

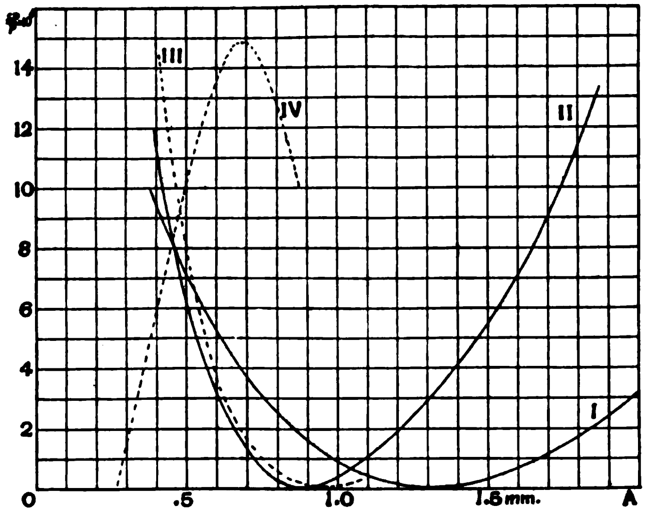

Fig. 8.

These curves bring out the interesting fact that there is an amplitude for which the period has a minimum value. In other words there is an amplitude in the neighborhood of which the period has a stationary value and consequently is not affected by small changes of amplitude. The value of this amplitude is different for the two curves because of the difference in the manner in which the changes were brought about. We shall see later that the amplitude corresponding to the stationary value of the period depends also upon the stiffness of the contact springs and upon the length of the gaps.

Curve *III* was obtained with the Leeds & Northrup fork. It will be observed that it has the general shape of curves *I* and *II*. This was to be expected because the Leeds & Northrup contact arrangements are somewhat similar to those of the W.E. forks. Curve *III* and curve *IV* could not be extended further to the right because the amplitude could not be increased beyond certain limits on account of the proximity of the prongs to the pole-pieces of the electromagnet.

Curve *IV* was obtained with a Pirard & Coeurdevache fork. In this case the period has a maximum instead of a minimum value because in the P. & C. fork the contact springs as well as the contact points are attached to the base and consequently the prongs are not in contact with the springs during a fraction of the period which forms a greater and greater portion of the period as the amplitude is increased. This explanation is sustained by the experiment described in the last paragraph of Part VI. where it was shown that relieving the prongs from the mechanical action of the outer contact points resulted in a considerable increase in the period.

Amplitude-period curves were also obtained for the Koenig fork and

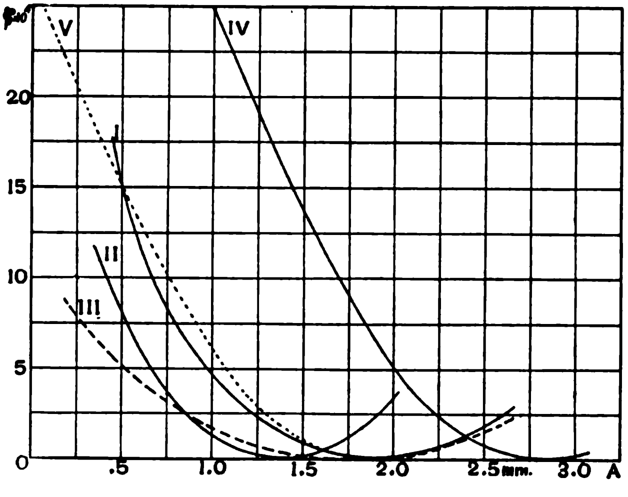

Fig. 9.

for the W.E. fork No. 488. Curve *III* of Fig. 9 was obtained with the Koenig fork, the changes in the amplitude being obtained by changing the driving current, while curves *I* and *II* were obtained with the W.E. fork No. 488. In the experiments corresponding to curve *I* the pole-pieces of the magnet were moved; in the experiment corresponding to curve *II* the driving current was changed. It will be observed that as in the case of fork No. 496 the amplitude corresponding to the minimum period is smaller for curve *I*.

Curve *IV* was obtained with the 488 fork under the same conditions as curve *I* except that the gaps were changed from .05 mm. in the case of curve *I*, to 0.30 mm. in the case of curve *IV*. In order to compare these two curves, curve *V* similar to curve *IV* was drawn so that its minimum point coincides with that of curve *I*. It will be observed that the rate of decrease of the period is greater for *I* than for *V*. This can be accounted for on the ground that for a given magnitude of the mechanical action of the contact points upon the springs, the energy of the prongs was greater in the experiments corresponding to curve *V*. This explanation is in accord with the fact that the curves are steeper to the left of the minimum point, where the amplitude is smaller and consequently the energy of the prongs is smaller.

From the foregoing considerations it will be expected that for a given length of gap and at a given amplitude less than the amplitude which corresponds to the stationary value of the period, the rate of decrease of the period will be smaller the smaller the stiffness of the springs. Certain experiments, the results of which are indicated by the curves of Fig. 10, sustain this conclusion. All three of these curves were obtained with fork No. 488 with gaps equal to .05 mm., the amplitude being altered by changing the driving current. In the experiment represented by curve *I* the fork had the same contact springs as in the experiment of curve *I* of Fig. 9, namely, springs with cross-sections of .41 mm. × 4.64 mm.

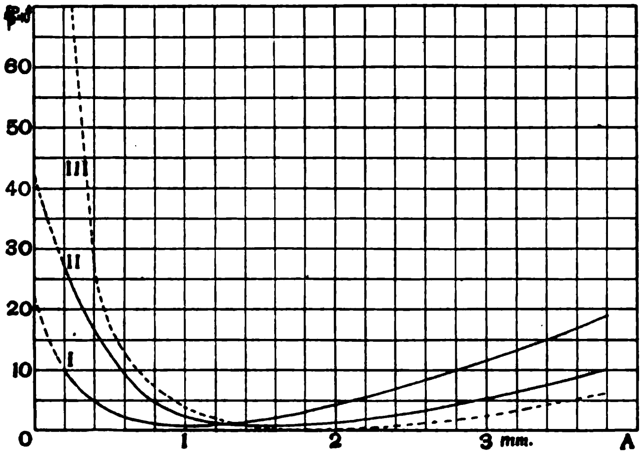

Fig. 10.

In the experiments represented by curves *II* and *III* the fork had springs with cross sections 0.28 mm. × 3.92 mm., and .48 mm. × 7.86 mm., respectively. A glance at these curves shows that the stiffer the springs the steeper are the curves on both sides of the minimum point, and the greater is the amplitude corresponding to the minimum period.

The results of other experimenters can be made to agree among themselves and with the results obtained by the author if it is supposed that the former have to do with that part of the period-amplitude curve which lies to the right of the minimum point and that the region investigated by Mercadier and Ettinghauser comes nearer the point of minimum period than the region studied by Poske, Heerwagen and Hartmann-Kempf.

The more important results of the experiments described in Part VIII. may be summarized in the following terms.

First, the period-amplitude curve passes through a stationary value and consequently there is an amplitude at which the fork can be operated with maximum constancy of period.

Second, the amplitude which corresponds to the stationary value of the period depends upon the stiffness of the contact springs and the length of the gaps.

From the results of the experiments described in Parts V. and VI. certain inferences may be drawn with regard to the best position for the contact springs. For a given contact pressure the action of the contact points upon the springs, and through them upon the prongs, is greater the nearer the springs are placed to the free ends of the prongs. Furthermore, for a given change in the amplitude the change in the contact pressure is greater the nearer the springs are placed to the free ends. On the other hand there is a limit to the extent to which the position of the springs can be approached to the shoulders of the fork. Hence there must be a position which is more favorable than any other. The determination of this position did not form a part of the investigations described here, but considerations based upon the mechanical action of the contact points upon the prongs, the position of center of percussion of the prongs, and the nodal points of the first harmonic of the vibration have led the author to the conclusion that the most favorable position for the contact springs must lie between the middle of the prong and one third of the distance from the free ends.

VIII. TEMPERATURE EFFECT.

A score or more physicists have determined the effect of temperature upon the frequency of tuning forks and have obtained concordant results.

These men, however, do not appear to have appreciated the importance of controlling certain factors which we have found to affect the frequency. It was deemed worth while, therefore, to redetermine the temperature effect under the favorable conditions which could be secured in the light of the experiments described in the preceding pages. Another reason which induced the author to carry his investigations into this phase of the tuning fork problem was the fact that he had at his disposal facilities to extend the investigation on the temperature effect to temperatures below zero degree centigrade, to which region the older investigators had not carried their work.

The work was carried on in the constant temperature rooms of the Palmer Physical Laboratory. The larger of the two rooms was provided with a thermostat which responded readily to changes of temperature as small as 0°.1. The smaller room had no temperature regulating device but its temperature could be lowered still further. Two separate determinations were made. The first, made in the larger room, covered the range of temperatures between – 3°.6 C. and 56°.8 C. The second, made in the smaller room covered the range of temperatures between – 25°.8 C. and 21°.2 C.

The W.E. fork No. 488 and the Koenig fork were placed side by side on a table in the middle of the larger room and a box with open ends was built over them. A microscope with a scale in the ocular was focused upon a bright spot at the end of one of the prongs of each fork, in order to observe and adjust the amplitude of vibration. Two thermometers made by R. Fuess with ranges of – 30° C. to 100° C. and scales divided into tenths of one degree were suspended so that the bulb of one was placed between the prongs and near the stem of one of the forks. A Callander recorder was set up to observe the temperature without going into the constant temperature room in use, but it was found to be not quite sensitive enough to give the temperature to the desired degree of precision; therefore it was used only to keep a rough record of the changes. The following general method of procedure was used during the first experiment: About one half hour after the temperature of the room had reached a desired value the room was entered and the gaps of the forks were examined to see if there had been any changes in the lengths of the gaps from the value .05 mm. to which they had been adjusted. Each fork was then set into vibration and the amplitude adjusted, if necessary; fork No. 488 was left vibrating and the open ends of the box were covered with pieces of woolen cloth. After about ten minutes the room was entered again, the thermometers read (this could be done without opening the box), and the chronograph started. Twenty minutes later the room was entered again, the thermometers were read, the Koenig fork was started and its record also taken for twenty minutes, after which the thermometers were read again. In this manner a large number of records were obtained which covered the range of temperatures between – 3°.6 C. and 56°.8 C. and extended over several days.

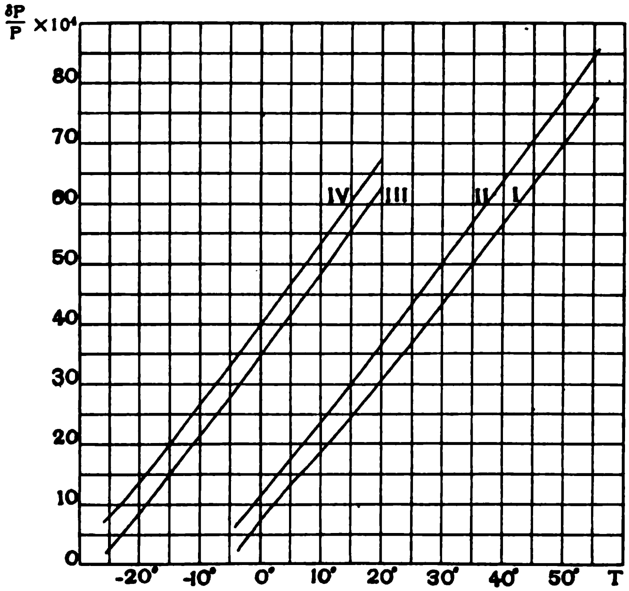

Fig. 11.

The results obtained with fork No. 488 were plotted on a large scale upon a millimeter cross-section paper with the temperatures as abscissas and the corresponding values of 50 complete periods as ordinates and a smooth curve was obtained with a continuously increasing slope. From this curve *II* of Fig. 11 was drawn, where the ordinates give the percentage increases in the period in ten-thousandths of the period at 0° C.

The period-temperature curve of the Koenig fork had an inflection point at about 11° and another at about 35°. There is no doubt that this behavior of the Koenig fork was due to slight variations in the spark gaps observed during inspections of the gaps made before taking records. The variation in the gaps must have been due to the expansion and contraction of the vulcanite piece which carried the contact points. In the regular W.E. forks this piece was of micanite which has a much smaller coefficient of expansion. Curve *I* which represents the results obtained by the Koenig fork is therefore of interest because it shows the importance of having spark gaps of constant length as well as because it supports the general conclusions derived from the other three curves of Fig. 11.

In the second experiment the Koenig fork was replaced by the W.E. fork No. 313 and the apparatus was set up in the smaller room. This room did not have a thermostat and consequently its temperature could not be kept constant; therefore the following method of procedure was adopted. The room was cooled several degrees below the temperature in the neighborhood of which it was desired to take a record; then the refrigerating machine was stopped and the temperature was allowed to rise for several hours and then records of the two forks were taken in the same manner as in the first experiment. This was repeated until ten pairs of records were taken while the temperature was lowered in the manner just described from 21°.2 C. to − 27°.C. The temperature of the room was then allowed to rise from − 27° to the normal room temperature and 15 more pairs of records were taken. The experiment took eight days and the rate of change of temperature was so slow during the intervals when records were taken that the temperature could be considered as constant. The results of this experiment are indicated by curves *III* and *IV* of Fig. 11, curve *III* corresponding to No. 488 and curve *IV* to No. 313.

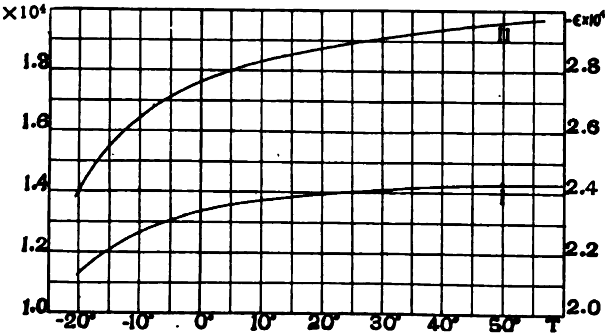

Fig. 12.

The results indicated by the curves *II*, *III* and *IV* were plotted accurately on a large scale and the different rates of increase in the period at different temperatures were determined for each curve. The average values for the three curves are given in Table I., where the numbers in the first column represent the temperatures and those in the second column the corresponding values of the temperature coefficient θ defined by

$$\theta = \frac{1}{P_0} \frac{\partial P}{\partial t},$$

or

$$P = P_0(1 + \theta t).$$

TABLE I.

t.	θ.	$-\epsilon$.	t.	θ.	$-\epsilon$.
− 20°	1.14×10^{-4}	2.39×10^{-4}	20°	1.38×10^{-4}	2.87×10^{-4}
− 10°	1.27	2.65	30°	1.40	2.91
0°	1.33	2.77	40	1.42	2.95
10°	1.36	2.83	50°	1.43	2.97
			− 20 to 50	1.34	2.79

These results are represented graphically by curve *I* of Fig. 12. The values of θ given in Table I. are from 10 to 40 per cent. higher than the average value of the results obtained by former observers as will be seen from Table II.

TABLE II.

Observer.	Range of Temp.	Temp. Coef.	Observer.	Range of Temp.	Temp. Coef.
Mercadier[1].......	3°–26°	0.96×10^{-4}	Michelson[5]......	12°–24°	1.00×10^{-4}
Kayser[2].........	0°–26°	0.98	Lang[6]...........	14°–19°	1.11
Koenig[3].........	3°–26°	1.11	Pierpaoli[7].......	0°–30°	1.05
Koenig[3].........	26°–56°	1.07	Woodruff[8].......	20°–200°	1.08
McLeod & Clarke[4]	15°–26°	1.10	Dadourian......	− 26°–57°	1.34

It will be shown in Part XII. that the effect of temperature upon the period of a fork is mainly due to its effect upon the elasticity of the fork and that only about 5 per cent. of the former effect is due to the increase in the dimensions of the fork resulting from a rise of temperature.

IX. CONSTANCY OF PERIOD.

The constancy of the performance of the W.E. fork No. 488 was tested under the following conditions: The pole-pieces of the electromagnet were set at 1.37 mm. from the prongs. The contact points were adjusted so that the length of the gaps was .05 mm. A potential of 24 volts was applied directly to the binding posts on the fork. Under these conditions an amplitude of 1.58 mm. was obtained which corresponds to the stationary period of the fork under the given conditions. The fork was operated continuously from 8.43 a.m. until 12.45 M., and chronographic records were taken. The period was then determined for different parts

[1] Mercadier, Journ de Phys., 5, p. 201, 1876; C. R., 83, p. 822, 1876.

[2] Kayser, Wied. Ann., 8, p. 444, 1879.

[3] Koenig, Wied. Ann., 9, p. 394, 1880.

[4] McLeod & Clarke, Phil. Trans. Roy. Soc., 171, part I., p. 1, 1880.

[5] Michelson, Am. Journ. Sci., 25, p. 61, 1883.

[6] Lang, Wied. Ann., 29, p. 132, 1886.

[7] Pierpaoli, Rend Linc., 4 (1), p. 714, 1888; 5 (2), p. 265, 1889.

[8] Woodruff, PHYS. REV., 16, p. 325, 1903.

of the run from careful measurements of the record, taking intervals of 20 minutes. The results of these determinations are given in Table III. where the numbers in the first column give the temperatures at the middle of the different 20-minute intervals, those in the second column are the corresponding observed values of 50 complete periods, the numbers of the third column are these values corrected to 20°.50 C. and those of the last column are the percentage deviations of the values of the periods thus determined from their mean value.

Table III.

Temperature.	50 P.	50 P_e.	$\delta P/P$.
20°.30	1.000885 sec.	1.000913 sec.	-8.4×10^{-6}
20°.34	1.000903	1.000925	3.6
20°.38	1.000910	1.000927	5.6
20°.46	1.000925	1.000931	9.4
20°.50	1.000336	1.000934	13.6
20°.56	1.000934	1.000926	4.6
20°.66	1.000923	1.000901	− 20.4
20°.69	1.000940	1.000913	− 8.4
20°.72	1.000954	1.000923	1.6
		1.0009214	± 8.4

Another set of determinations was made the next day with the spark gaps equal to 0.3 mm. the pole-pieces at 1.0 mm. from the prongs, and an amplitude of 1.75 mm. The fork was operated from 12.55 until 4.19 P.M. and a continuous record taken. The values of 50 periods were then determined as in the preceding experiment and the results given in Table IV. were obtained.

Table IV.

Temperature.	50 P.	50 P_e.	$\delta P/P$.
20°.77	1.004153 sec.	1.004134 sec.	-10×10^{-6}
20°.74	1.004168	1.004155	11
20°.72	1.004186	1.004175	31
20°.70	1.004181	1.004170	26
20°.67	1.004152	1.004148	4
20°.65	1.004153	1.004151	7
20°.63	1.004136	1.004137	− 7
20°.62	1.004126	1.004136	− 8
20°.61	1.004115	1.004122	− 22
20°.60	1.004114	1.004121	− 23
		1.0041443	± 14.9

A glance at the last columns of Tables III. and IV. shows the remarkable constancy of performance of the fork. In the first run the maximum deviation from the mean for a run of four hours is only one part in 50,000 while the average deviation is less than one part in 100,000. In the second run the deviation from the mean is larger. But in this case the deviation is systematic indicating that the conditions affecting the period were not kept as constant as during the first run. The great constancy of performance shown by the fork is all the more remarkable because it is a commercial article in the manufacture of which no great effort had been made to secure such important requirements as a high degree of uniformity of thickness of prongs and symmetry relative to the axis of the fork. This investigation shows conclusively that a proper control of the factors affecting the period is of prime importance.

X. EXPRESSION FOR PERIOD OF VIBRATION.

It has been shown by Mercadier[1] that the expression for the period of transverse vibration of a bar obtained from the theory of elasticity holds good for a tuning fork provided the projection of the median line of the prongs upon the geometric axis of the fork is taken for the length of the prongs and a small correction term is added to this. The expression for the period of vibration of a bar is

$$P = \frac{4\pi\sqrt{3}}{r^2 v}\frac{l^2}{a} \tag{1}$$

where l is the length and a the thickness of the bar, v the velocity of sound in the bar, and r the root of the equation

$$(e^r + e^{-r}) \sin 2r + 2 = 0 \tag{2}$$

and equals 1.87011.[2] For rods of the same material v is constant, therefore we can write

$$P = K\frac{l^2}{a} \tag{3}$$

Mercadier's modified expression for tuning forks is

$$P = K\frac{(l + \lambda)^2}{a} \tag{3'}$$

where λ is a correction term introduced in order to satisfy Mercadier's experimental results and equals $.012l$. Evidently the value of λ will depend upon the shape of the fork near the shoulders of the prongs. Since the W.E. forks had a slightly different shape from those used by Mercadier it was thought desirable to determine the value of λ for these forks. Using Mercadier's experimental value of K, namely, 1/81,827 sec./cm., the values given in the first column of Table V. were obtained for ten different forks.

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[1] E. Mercadier, C. R., 79, pp. 1001, 1069, 1874; Journ. de Phys., V., p. 201, 1876.

[2] Poisson, Traité Mecanique, II., p. 390.

Table V.

λ.	λ/l.	E.	$\delta E/E$.
+ 0.16 cm.	+ 0.0040	18.80×10^{11}	− 0.016
+ 0.07 "	+ 0.0018	18.92	− 0.009
+ 0.06 "	+ 0.0015	18.98	− 0.006
+ 0.06 "	+ 0.0015	19.03	− 0.004
+ 0.04 "	+ 0.0010	19.07	− 0.002
− 0.03 "	− 0.0007	19.16	− 0.003
− 0.05 "	− 0.0012	19.02	+ 0.004
− 0.09 "	− 0.0022	19.30	+ 0.010
− 0.10 "	− 0.0025	19.30	+ 0.011
− 0.17 "	− 0.0042	19.42	+ 0.017
− 0.005 "	− 0.0001	19.10	.0092

It will be observed (*a*) that the highest value of λ obtained is only one third of 0.48 cm. required for the W.E. forks by Mercadier's empirical relation $\lambda = 0.012\,l$, (*b*) that the values are fairly evenly distributed on the two sides of zero, and (*c*) that their algebraic sum differs from zero by an amount which falls within the experimental errors of the measurement of the lengths of the prongs. Therefore the unmodified equation (3) gives closer results for the W.E. forks than Mercadier's equation (3′).

XI. The Modulus of Elasticity and Velocity of Sound in Steel.

Replacing v in equation (1) by its expression in terms of the modulus of elasticity E and the density ρ we have

$$P = \frac{4\pi\sqrt{3}}{r^2}\frac{l^2}{a}\sqrt{\frac{\rho}{E}}. \qquad (4)$$

Putting in the last equation 1.87011 for r, 7.363 for ρ and the values of P, l, and a obtained from the ten W.E. forks the values of E given in the third column of Table V. were obtained. The value 7.363 for ρ was obtained from a careful determination of the density of four pieces which were cut off the ends of the prongs of different forks in adjusting their periods. Using this value of ρ and the average value of E given at the bottom of the third column, the value 5.093×10^5 cm. per sec. was obtained for the velocity of sound in steel.

The deviations of the values of λ and of E from their mean values are small and undoubtedly due to the facts that (*a*) the prongs were not as uniform as might have been reasonably expected, and that (*b*) some of the forks were cut in more at the shoulders than others.

XII. TEMPERATURE COEFFICIENT OF MODULUS OF ELASTICITY OF STEEL.

Let P_0, a_0, l_0, ρ_0 and E_0 denote the values at 0° C. of the quantities involved in equation (4), and P, a, l, ρ *and* E denote their values at any other temperature t, then we can write

$$\frac{P}{P_0} = \frac{a_0}{a} \cdot \frac{l}{l_0} \cdot \left(\frac{\rho}{\rho_0}\right)^{1/2} \left(\frac{E}{E_0}\right)^{1/2}. \tag{5}$$

Introducing in the last equation the temperature coefficients α, θ, and ϵ defined by

$$\begin{aligned} l &= l_0(1 + \alpha t), \\ P &= P_0(1 + \theta t), \\ E &= E_0(1 + \epsilon t), \end{aligned}$$

we obtain, to a first order of approximation, the relation

$$\theta = -\frac{\alpha + \epsilon}{2},$$

or

$$\epsilon = -(2\theta + \alpha). \tag{6}$$

Since α and θ are both positive magnitudes ϵ, the temperature coefficient of the modulus of elasticity must be negative. In other words the modulus of elasticity decreases with increasing temperature. Taking the coefficient of linear expansion of steel to be 1.1×10^{-5} and substituting in equation (6) the values of θ given in the second column of Table I. we obtain the figures of the third column for the values of ϵ corresponding to the temperatures given in the first column. The variation of ϵ with the temperature is shown graphically by curve *II* of Fig. 12. The unmistakeable increase of the absolute value of ϵ with the increase of temperature indicated by Table I. and curve *II* is in agreement with results obtained by Pizati[9] and Dodge.[10]

The relatively large differences among the values of ϵ obtained by different observers must be due to the variety of the specimens of steel used and also due to experimental errors. It may be stated that the indirect method used in this paper is more accurate than the more direct methods by which the other observers have determined the temperature coefficient of elasticity.

In conclusion the author wishes to acknowledge his indebtedness and express his thanks to Professor W. F. Magie for placing the facilities of the Palmer Physical Laboratory at his disposal; to Professor E. H. Loomis for helping him place the constant temperature plant in working order; to Professor H. N. Russell for the use of the observatory chrono-

TABLE VI.

Observer.	$-\epsilon$.	Observer.	$-\epsilon$.
Wertheim[1]	2.8×10^{-4}	Gray, etc.[7]	2.47×10^{-4}
Kohlrausch & Loomis[2]	4.6	Wassmuth[8]	2.64
Pizati[3]	1.95	Walker[9]	2.5
Mayer[4]	2.21	Dodge[10]	1.9
Miss Noyes[5]	3.4	Dadourian	2.79
Shakespeare[6]	3.87		

graph; and to Captain H. B. Williams, who had charge of the sound ranging work of which the present investigation formed a part, for his interest in the work.

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[1] Wertheim, Pogg. Ann., 2, p. 1, 1848.

[2] Kohlrausch & Loomis, Pogg. Ann., 141, p. 481, 1870.

[3] Pizati, Nuo. Cim., 1, p. 181; 2, p. 137, 1877; 3, p. 152, 1879; 5, pp. 34, 135, 145, 1879.

[4] Mayer, Brit. Assoc. Rep., p. 573, 1894.

[5] Miss Noyes, PHYS. REV., 3, p. 433, 1896.

[6] Shakespeare, Phil. Mag., 47, p. 539, 1899.

[7] Gray, Blyth and Dunlop, Proc. Roy. Soc., 67, p. 180, 1900.

[8] Wassmuth, Phys. Zeits., 6, p. 755, 1906.

[9] Walker, Proc. Roy. Soc. Edin., 28, p. 562, 1907.

[10] Dodge, PHYS. REV., 5, p. 373, 1915.

THE MATHEMATICAL STRUCTURE OF BAND SERIES II.

By Raymond T. Birge.

Synopsis.—In the first article of this series, the author proposed a hyperbolic formula, for the accurate representation of long band series. The present paper initiates a more general discussion of this formula, and of the experimental results which the author has since obtained.

Detailed analytic work has shown the new formula accurate, within limits of experimental error, for those band series showing the most radical deviations from the simple Deslandres' law. Preliminary work indicates that the hyperbolic formula is equally satisfactory in the case of series showing positive deviations from Deslandres' law, instead of the customary negative deviations.

A complete investigation of any band series is greatly facilitated by a preliminary study of its deviations from Deslandres' law. New methods have accordingly been developed for making this comparison in a rapid, yet accurate manner. In particular, there is developed a new Least Squares' formula, giving directly the value of the only desired constant in a Deslandres' formula, thus avoiding the more extended calculations necessary when all three constants are simultaneously evaluated.

A comparative study of band and line series formulæ suggests for band series a modified hyperbolic formula. This modified formula has been tested on several series and has been found very satisfactory. New line series formulæ are also suggested, but are not seriously recommended at this time.

Introduction.

In the first paper on this subject,[1] there was proposed a new formula for band series, together with the detailed quantitative results for the main (A_1) "singlet" series of the 3883 CN. band. It was shown also that preliminary work indicated that the formula would hold equally well for the other series of this band. In particular, the singlet series from the third head (C_1 series) exhibits the most radical deviations from Deslandres' law of any known band series. It therefore forms a crucial test for any new formula. This series has since been extended at both ends, the new lines having been identified from the author's own spectrograms. The complete quantitative results for this series were presented to the American Physical Society at the October, 1917, meeting.[2]

Certain remarkably systematic irregularities in the C_1 series, brought out by these computations, made it seem desirable to study also the B_1 series. This work has now been completed, much of the actual computing having been done by an assistant, for whose services the author is indebted to the Rumford Committee of the American Academy of Sciences. At the same time, a general survey was made of practically all band series which seemed to show large deviations from Deslandres' law. A very considerable amount of material, both quantitative and qualitative, has now been collected, and it has seemed desirable to present a general review of the progress of the work to date. Such detailed results as appear of sufficient importance will be published later.

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[1] Astrophysical Journal, 46, 85–103, 1917. In the future, it will be referred to as I.

[2] Phys. Rev., (2), 11, 136, 1918.

The present communication includes:

1. A discussion of methods of handling band series formulæ, in particular, Deslandres' parabolic formula.
2. A new viewpoint as to the original hyperbolic formula, together with a possible alternative formula in which the necessity of making a summation is avoided.

In the next paper there will be presented a review of typical band series, especially of those showing radical deviations from Deslandres' law, in a positive, as well as in a negative sense.

The Mathematical Analysis of Band Series.

It is well known[1] that all band series seem to conform to Deslandres' law[2] for at least an initial portion of their extent. The investigations of the author have convinced him that *no* band series strictly obeys Deslandres' law over *any* portion of its extent. But for short series (*i. e.*, series covering a small frequency interval) the deviations from this law may easily be far less than the experimental errors.[3] The first step however in the examination of any given series consists in determining how closely it conforms to the simple Deslandres' formula. Various methods for doing this have been used by previous investigators, but no general discussion of the matter has yet been published.

Deslandres' formula, in its original form,[4] states that the frequencies of successive lines of a band series are given by the parabola

$$\nu = \nu_0 + am + bm^2 \qquad (1)$$

or

$$\nu = A + B\,(m + c)^2, \qquad (1')$$

---

[1] For the most complete summarized description of all bands discussed in this series of articles, see Konen, "Das Leuchten der Gase und Dämpfe," pp. 199–278.

[2] Deslandres, Comptes Rendus, 103, 375, 1886.

[3] It may be remarked in passing that the amount of deviation from Deslandres' law seems to have no connection with the spectral extent of the series, nor with the number of terms in the series. Some series of 100 or more terms show a scarcely perceptible deviation, others of the same length a most radical deviation. Many series of few terms (30 or less) but of large spectral range also show radical deviations.

[4] For discussion see I., pp. 86–87.

where m takes successive integral values 0, 1, 2, 3, etc.[1] The successive first frequency differences are then given by the straight line

$$\Delta\nu = a + 2bm = 2B(m + c), \tag{2}$$

where m has the successive *fractional* values 0.5, 1.5, 2.5, etc., while the second frequency differences are given by the constant

$$\Delta\rho = 2b = 2B. \tag{3}$$

Any series for which the second frequency difference is constant therefore, per se, obeys Deslandres' law, and the first obvious method of testing this law is to compute these second differences.

But because of experimental errors, true perturbations, etc., these differences will never be constant, and in many cases it is difficult to tell from mere inspection whether they exhibit any definite trend from constancy. A far more delicate method of detecting any such trend, if present, is to plot the first frequency differences against m, on a fairly large scale. (50 × 50 cm. is recommended.) This curve, by Deslandres' law, should be a straight line, of slope $2b$ (*i. e.*, secondary frequency difference), and intercept ($\Delta\nu = 0$) $m = -a/2b = -c$. The author has called it the "slope" form, and has found it to be actually an hyperbola.

In practically all long series the slope curve exhibits a very definite trend from linearity. Yet in instance after instance investigators have analytically fitted the series to a Deslandres' formula, and have concluded that the agreement was satisfactory, when the slope curve (*if* it had been plotted) would instantly have shown the falsity of such a conclusion.

But let us assume that a casual inspection of the $\Delta\nu : m$ curve does not indicate any definite non-linearity. This condition is more likely to arise when the experimental errors are rather large (over 0.02 A). A case in point is the main series of the 3371 Nitrogen band, which Lewis[2] extended, by using self-induction in the circuit, until it entered the next band at 3159 A. The $\Delta\nu : m$ curve for this band is apparently a straight line and the equation given by Lewis is the best obtainable when the simplified Deslandres' formula

$$\nu = A + Bm^2 \tag{4}$$

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[1] Comparing (1) and (1') $b = B$; $c = a/2b$; $\nu_0 = A + Bc^2$. There seems to be some confusion as to just what is to be considered the "head" of a band. The ordinary idea, stated implicitly if not explicitly, is that this head is given by the line corresponding to $m = 0$, in any satisfactory formula. The author has tried in I. to follow consistently this usage. But (1') is often written $\nu = \nu_0 + B(m + c)^2$. This makes ν_0 (the head) correspond to $m = -c$. I doubt if anyone actually thinks of the head in that way, and so the form is objectionable. Of course when $c = 0$, the two view-points become identical.

[2] Astrophysical Journal, 40, 148, 1914.

is used. As has been remarked[1] (4) is *not* Deslandres' law, and its use is permissible only when the inaccuracy of the data makes the true determination of the intercept ($-a/2b = -c = -$ "the phase") impossible. In this case, the data *are* rather inaccurate, but a slight improvement can be made by using (1′) rather than (4).

The problem now becomes,—what is the best analytical method of handling the experimental data, in order to determine whether or not the Deslandres' formula holds, *i. e.*, whether a straight line is really the best simple smooth curve that can be passed through the experimental points of the $\Delta\nu : m$ plot, and if so, what is the slope of this curve? This problem is identical with that of determining the constancy of the acceleration of gravity, and its value, if constant, given the position of a series of crests on an Atwood's Machine curve, but with the time and point of initial descent not known. For this latter problem we have

$$s = s_0 + \nu_0 t + 1/2gt^2 \tag{5}$$

or

$$s = (s_0 - \nu_0^2/2g) + g/2(t + \nu_0/g)^2, \tag{6}$$

i. e.,

$$s = \alpha + \beta(t + \lambda)^2, \tag{6'}$$

where $\lambda =$ the "phase," *i. e.*, the fractional part of a second before $t = 0$ at which descent began.[2] In the case of Deslandres' law, (1) and (1′) correspond to (5) and (6′) respectively.

If a study of the actual plotted data, together with a knowledge of the experimental conditions, indicates that the deviations of the data from any simple smooth curve are due solely to chance experimental errors, then the method of Least Squares leads to the "most probable" values of the constants of the equation of the curve. In the case of spectral series, the conditions necessary for the legitimate application of Least Squares are actually present,—*i. e.*, in a well-measured series, using the new International standards, there is practically no opportunity for unsuspected systematic errors of any appreciable amount.

The full details of the method of Least Squares, as applied to the Deslandres' formula, are given by V. Carlheim-Gyllensköld.[3] The necessary calculations are, however, rather extended, especially in the case of long series. Moreover we do not, at the outset, wish to assume the truth of Deslandres' law. Instead, we wish to investigate whether or not the best smooth $\Delta\nu : m$ curve has a constant slope or a varying slope. But in the case of a long band series, such as the 3371 N series, the method of Least Squares can be applied, with great advantage, to this problem as well. For we may break up the $\Delta\nu : m$ curve into several sections, and determine, by Least Squares, the most probable value of the slope for each section. With 30 or more lines to a section, the probable error of this procedure is far less than that of any graphical method,—a fact which is frequently overlooked, but which the present series of investigations has repeatedly demonstrated.

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[1] I., p. 89.

[2] I. e., we assume $t = 0$ for the nearest even second to the time of starting, just as in band series we choose m so as to make c (the "phase") some fraction less than 0.5. This may possibly be the analogy Thiele had in mind when he called c the "phase." The question of the origin of the term has recently been raised. (Proc. Phys. Soc. of London, 30, 130, 1918.)

[3] Svenska Vet. Ak. Handl., 42 N : r 8, 1907.

The author has tried also various approximate methods for computing the second difference and of testing its constancy, but has come finally to the conclusion that the Least Squares solution is the only really feasible procedure in a case like that under consideration, where a plotted $\Delta\nu : m$ curve does not definitely indicate any curvature. Fortunately however it has been possible to very considerably reduce the amount of necessary calculations, so that the Least Squares method becomes but slightly longer than various approximate methods.

In equation (1) we have three undetermined coefficients, but in this particular case, we are interested only in the value of b. Moreover, the variable m takes on the regular series of values 1, 2, 3, etc. Now in the case of the simpler equation

$$y = A + Bx \tag{7}$$

it is possible to compute directly, by Least Squares, the value of B, when $x = 1, 2, 3, \ldots, n$. In fact[1]

$$B = \frac{(n-1)(y_n - y_1) + (n-3)(y_{n-1} - y_2) + \text{, etc.}}{1/6n(n^2-1)}. \tag{8}$$

The "physical meaning" of (8) is that we are to combine the first and last observations, the second and next to last, etc., and each difference thus obtained is to be multiplied by a factor equal to the length of the interval (in terms of the corresponding change in x). Finally, we divide the numerator by the total number of corresponding changes in x. Thus for 6 observations, (8) can be written

$$B = \frac{5(y_6 - y_1) + 3(y_5 - y_2) + (y_4 - y_3)}{5^2 + 3^2 + 1^2}. \tag{8$'$}$$

In the case of equation (1), it is to be expected that a similar formula for the value of b can be derived. But so far as the author has been able to ascertain, no such formula has been published. Accordingly this work has been carried out, and the desired expression obtained. As is to be expected, the necessary reductions and the final result are much more complicated than in the case of (7) and (8). This result is

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[1] See Kohlrausch, "Physical Measurements," p. 17 (English translation of 7th German edition).

$$b = \frac{30}{n(n^2-1)(n^2-4)} \Sigma \{ (y_r + y_{n+1-r})(A + [r-1]B + 6[r-1][r-2]) \}, \qquad (9)$$

where $A = (n-2)(n-1)$,

$B = 6(2-n)$,

n = number of observations (y) [$y = \nu$ of (1)],

$$r = \begin{cases} 1, 2, 3, \ldots\ n/2 \text{ for } n \text{ even.} \\ 1, 2, 3, \ldots\ (n+1)/2 \text{ for } n \text{ odd.}^1 \end{cases}$$

Thus for $n = 6$, we have

$$b = \frac{5(y_6 + y_1) - (y_5 + y_2) - 4(y_4 + y_3)}{56}. \qquad (9')$$

For $n = 5$

$$b = \frac{2(y_5 + y_1) - (y_2 + y_4) - 2(y_3)}{14}. \qquad (9'')$$

The "physical meaning" of (9) is this: The coefficients of the first and last observations, the second and next to last, etc., are the same. Hence the readings may be combined in pairs, as in the case of (8). (But note that the pairs of observations are added while in (8) they are subtracted.) The successive coefficients of these pairs of observations follow a parabolic law, in which the second difference is constant (= 12). Hence the calculation of the coefficients is very simple. In (9) A is the initial coefficient, and B the initial first difference. Thus formula (9) may be written

$$\begin{aligned} b = \frac{30}{n(n^2-1)(n^2-4)} \{ & A(y_1 + y_n) \\ & + (A+B)(y_2 + y_{n-1}) + (A + 2B + 12)(y_3 + y_{n-2}) \qquad (10) \\ & + (A + 3B + 36)(y_4 + y_{n-3}) + \text{, etc.} \}. \end{aligned}$$

In applying (9) to the N3371 series, the 84 observed frequencies (all weighted equally) were divided into two equal sets. For the first 42 (small m's) we obtain $b = 3.681$; for the second 42 (large m's) $b = 3.605$.[2] The mean is 3.643, very close to Lewis' value of 3.634, using the simplified equation (4).

Because of the large number of observations, the probable error in these results is very small. There is thus clear evidence of a variation

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[1] Note that for n *odd*, the last term in the summation contains but *one* y, *i. e.*, $y_{n+1/2}$ with the coefficient $(1-n)(1+n)/2$.

[2] The best approximate methods which the author has been able to devise give for the first 42, $b = 3.645$ or 3.660; for the last 42,—3.585 or 3.555,—showing the uncertainty of the methods.

of slope of about 2 per cent. But since 3.681 is the probable slope at the center of the first half of the series, while 3.605 applies to the center of the second half, we have a probable total variation, over the entire extent of the observations, of about 4 per cent. It is thus evident that the second difference *decreases*, and that the series shows deviations from Deslandres' law in the *negative* sense. Moreover, we have thus detected this 4 per cent. variation in a case where a really good analytic fitting of Deslandres' formula reveals practically no trace of systematic deviations. Even when such deviations *do* appear in the analytic work, they are difficult to interpret. This 3371 series has been used for illustration because it is the only really long series, among those studied, which from the plotted $\Delta\nu : m$ curve *seemed* to obey the parabolic law.

In the case of more complex formulæ, such as the hyperbolic law proposed by the author, it is practically necessary to use the ordinary methods of calculation. After plotting the data and drawing the best smooth curve through the results, a number of points on this curve, equal to the number of arbitrary constants in the assumed formula, are chosen and the theoretical curve is made to pass through these points, by solving the simultaneous equations of condition. The Obs-Calc value for each observation is then calculated and plotted (as ordinate). A smooth curve is drawn through the results (the so-called "Residual Plot"[1]). The object is then to "warp" the x-axis until it fits this smooth curve as well as possible. With practice one can predict roughly whether it is possible to so warp the axis as to give a really good agreement. The criteria depend entirely upon the form of equation used, and general rules cannot be given. Each warping means, of course, the determination of a new set of constants. In the case of the hyperbola, four trials are usually sufficient, if the data are all equally trustworthy. For the B_1 and C_1 series of the 3883 CN band, many more trials were necessary, since the systematic deviations, to be discussed later, made it difficult to determine, at first, just what would constitute the best agreement. Much time was thus lost trying to do (in the light of future knowledge) quite impossible things.

Also, the hyperbolic formula refers to the $\Delta\nu : m$ curve. The Obs-Calc values are however calculated and plotted for the $\nu : m$ curve. The warping of the x-axis of the Residual Plot then consists in giving it a definite new *slope* (*i. e.*, a definite new $\Delta\nu$) at selected points, and solving, instead of giving it a definite new *ordinate*. The judging of the proper slope is far more difficult than the judging of the proper ordinate, and this is the one real objection to the use of the "slope" form of the equation. But the greater simplicity of this form more than compensates. The fact that, for the proposed hyperbolic formula, the $\nu : m$ equation is a summation, instead of an integral, is an objection to this particular formula but not to the use in general of the $\Delta\nu : m$ curve, rather than the $\nu : m$ curve.

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[1] Goodwin, Precision of Measurements, p. 60.

THE HYPERBOLIC FORMULA, AND A POSSIBLE ALTERNATIVE FORM.

As shown in Fig. 2, page 92 of I., the $\Delta\nu : m$ curves[1] for the A_1, B_1, and C_1 series of the 3883 CN band look very much like hyperbolæ, instead of the straight line demanded by Deslandres' formula. These three series have now been found to fit accurately hyperbolic curves, save for certain systematic perturbations (previously known) and for small systematic irregularities, newly discovered. These latter amount to a change of frequency of only one part in 300,000 as a *maximum*, and so are inconsequential as far as the general form of the curve is concerned. Aside from the real periodic irregularities, the observed and computed curves agree to at least 0.005 A throughout a series 200 A long, *i. e.*, on a $\nu : m$ curve extending ten feet in each direction, the agreement is to less than 0.1 mm. throughout, indicating the great accuracy of the hyperbolic formula.

The deviations from Deslandres' law in all of these series are in a *negative* sense, in that the successive frequency differences increase *less* rapidly than demanded by the law, *i. e.*, the second differences *decrease*. Series of this type will be referred to as *negative series*, and bands containing them as "bands of the negative type." The phrase "negative bands" is already in use, with a distinctly different meaning. In I. the statement was made that in only one band are series known in which the deviations are in a contrary sense, *i. e.*, the 2370 band of air. This statement is quite incorrect. At the time of discovery, these 2370 series *were* the only known examples of what we shall call *positive series*. Since then a number of similar series have been found. It is these series which the author has been particularly studying. In the 2370 band there are four positive series, and four negative series. We shall call such a band a "*neutral band.*" At present the 2370 band is I believe the only known example.

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[1] In all detailed quantitative work, the author has used the frequency in vacuo, but for the qualitative work to be described in the following paper the reduction to vacuo is not necessary and has not been made. Wave-lengths should never be used, except incidentally. In this connection it might be added, since it seems not known everywhere, that there is published a table of reciprocals (Cotsworth's Reciprocals,—McCorquodale and Co., London.) giving to seven figures the reciprocals of all numbers to 100,000. By linear interpolation to hundredths one can obtain the reciprocal of any 7 figure quantity to seven figures. The maximum error is one unit in the last figure, the probable error considerably less than half a unit. The rule for interpolating the seventh figure, as given in the tables, is obviously incorrect. The author has found it more convenient to use a slide rule for all interpolations. When a succession of adjacent frequencies is desired, as in a band series, each can be obtained in a very few seconds,—quite as rapidly, in fact, as by the use of the best calculating machines.

In the only detailed study of this band,[1] a very good drawing but no data on the observed wave-lengths are recorded. (Schniederjost[2] resolves only a few lines of this band.) The only data given are the Obs-Calc values for every fifth line, for four of the series, together with the equations used. With this data it is possible to work back to the observed frequencies, for every fifth line, and thus to determine roughly the form of the $\Delta\nu : m$ curves. These curves show but slight deviations from a straight line, the maximum variation in slope being about 9 per cent. (for the II_2 and IV_2 series). That is, the final slope (highest m) is about 109 per cent. or 92 per cent. respectively, of the initial slope (at $m = 0$). The $\Delta\nu : m$ curve for the A_1 series of the 3883 band has been given in Fig. 2 of I. (page 92) and for convenience in comparing other series the slope at various points, in terms of the initial slope (*i. e.*, the relative second frequency differences) is given herewith.

$m =$ 0	100 per cent.	$m =$ 101	76.5 per cent.
21	98 per cent.	121	61.5 per cent.
41	95 per cent.	141	29.0 per cent.
61	91 per cent.	151	14.0 per cent.
81	85.5 per cent.	156.5	0 per cent. (max. $\Delta\nu$)

In respect to the magnitude of the departure from Deslandres' law, the 2370 series are thus not in a class with the 3883 series, or with many others studied, and the fitting of them to a six constant equation, such as that proposed by the author, would be a simple matter. The striking point about these 2370 series is however the fact that four of them show positive deviations, and four negative, and the deviations in corresponding series are *equal*. This shows that if the *negative* series are hyperbolic (and the $\Delta\nu : m$ curves look quite similar to the initial portion of the A_1 3883 curve) the *positive* series are *also* hyperbolic. It also suggests a new point of view on the entire subject.

We have assumed in I. that the hyperbolic law is due to the fundamental field of force of the molecule (all spectroscopists now agreeing that band series are due primarily to molecular structure, rather than to atomic). Let us assume however that the *tangent to the* $\Delta\nu : m$ *curve at* $m = 0$ represents this fundamental field of force, *i. e.*, a Deslandres' law, and that the deviations from this straight line indicate disturbing effects. Let us denote any $\Delta\nu$ on the straight line by $\Delta\nu_s$ and any $\Delta\nu$

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[1] Deslandres and Kannapell, Comptes Rendus, 139, 584, 1904.

[2] Zeitschrift für wiss. Photographie, 2, 265, 1904.

on the hyperbola by $\Delta\nu_n$. Let $\Delta\nu_s - \Delta\nu_n = D$. Then if the $\Delta\nu_n : m$ curve is truly an hyperbola, the $D : m$ curve will also be an hyperbola (and conversely) since we have made only a linear transformation of variables in which the derivatives $ab - 4c^2$ remains invariant. (The transformation is mathematically the same as that from fixed to moving axes.) Moreover, the curve in $-D$ and m is also an hyperbola, by symmetry, and thus the curve in $\Delta\nu_s - (-D) = \Delta\nu_p$ is an hyperbola. (The converse transformation of variables.) Thus the four *positive* series ($\Delta\nu_p : m$ curves) are hyperbolæ, if the four *negative* series ($\Delta\nu_n : m$ curves) are. Also, we can state that the systematic deviations (D) from the fundamental field of force of the molecule (assumed to be such as to give a Deslandres' law) are of such nature as to yield hyperbolic $D : m$ curves, instead of saying that the field of force is such as to yield hyperbolic $\Delta\nu : m$ curves. The two statements are actually only different ways of viewing the phenomena, for the physical facts are the same, in either case.

The chief use that has been made of the above ideas is in the handling of positive series. It is difficult to recognize the hyperbolic form, in such series, and I have therefore drawn the tangent at $m = 0$, reversed the deviations (D) from this line, and drawn the resulting curve, thus obtaining the corresponding negative $\Delta\nu : m$ curve. This curve, in all cases, has appeared to be hyperbolic, as nearly as could be judged by mere inspection. This procedure also gives a simple method of judging, in the case of positive series, how radical are the departures from Deslandres' law. In only one series (one of those in the 3064 water band) does the corresponding negative $\Delta\nu : m$ curve extend past the point of maximum $\Delta\nu$. In this one case it has about the extent of the A_1 3883 curve. Thus no positive series shows anything like the deviation of the C_1 3883 series.

The so-called positive series are thus found (qualitatively, at least) to be no exception to the hyperbolic law, but to represent simply *positive* hyperbolic deviations from Deslandres' law, instead of *negative* deviations. Needless to say, the structure of *positive* band series is somewhat different from that of *negative* series. The former has no point of maximum $\Delta\nu$ and no "tail." The remarks on page 101 of I. should therefore be considered as applying only to *negative* series.

The hyperbolic law proposed by the author has the form

$$\Delta\nu^2 + B\Delta\nu . m + Cm^2 + D\Delta\nu + Em + F = 0, \qquad (11)$$

where $F = 0$ for the A_1 3883 series.

Solving for $\Delta\nu$ we have

$$\Delta\nu = -Bm/2 - D/2 \pm\sqrt{\left(\frac{B^2-4c}{4}\right)m^2+\left(\frac{BD}{2}-E\right)m+\frac{D^2}{4}-F}, \quad (12)$$

where $m = 0.5$, 1.5, 2.5, etc., and the negative sign before the radical is to be used. Then

$$\nu_n = \nu_0 + [\Sigma\Delta\nu]_{m=0.5}^{m=n-0.5}. \quad (13)$$

Since $\Delta\nu$ is given by an hyperbola we might write (13) as

$$\nu = \nu_0 + \Sigma \text{ hyperbola}. \quad (14)$$

As explained in I., pages 100–101, the best convergent series that approximates this function is probably

$$\nu = a + bm^2 - cm^4 + dm^6, \text{ etc.}, \quad (15)$$

which Kilchling[1] uses as an empirical formula, but which Ritz[2] derived on theoretical grounds.

The formal analogy between line and band series has frequently been pointed out. The approximate formula for line series (Rydberg) is

$$\nu = \nu_\infty - \frac{N_0}{(m+\mu)^2}, \quad (16)$$

or

$$\frac{N_0}{\nu_\infty - \nu} = (m+\mu)^2. \quad (17)$$

The approximate formula for band series is

$$\nu = A + B(m+\mu)^2 \quad (1')^3$$

or

$$\frac{\nu - A}{B} = (m+\mu)^2. \quad (18)$$

(18) and (17) have the same functional form, and hence the formal analogy, previously stated by various authors. The following suggestions however are believed to be mainly original.

There seem to be two possible methods of modifying the $f(m)$ occurring in (17) and (18), in order to obtain more accurate formulæ. (1) Compute successive differences, in (17) and (18). This gives

$$\Delta\left(\frac{N_0}{\nu_\infty - \nu}\right) = 2(m+\mu), \quad (19)$$

$$\Delta\left(\frac{\nu - A}{B}\right) = \frac{\Delta\nu}{B} = 2(m+\mu). \quad (20)$$

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[1] Zeitschrift für wiss. Photographie, 15, 293, 1916.

[2] Ritz (Weiss) Comptes rendus, 152, 585, 1911, or Astrophysical Journal, 35, 75, 1912.

[3] Here, as explained, A is *not* exactly equal to ν_0 (the true head) but differs from it by Bc^2. In giving the formal analogy between line and band series, equation (1′) is usually written with ν_0 in place of A. But to avoid inconsistency, this has not been done here.

But the author has found that $\Delta\nu$ = hyperbola, instead of $\Delta\nu$ = straight line. By analogy we might expect for line series

$$\Delta\left(\frac{N_0}{\nu_\infty - \nu}\right) = \text{hyperbola}. \tag{21}$$

The other method is (2) Compute the square roots of (17) and (18). This gives

$$\sqrt{\frac{N_0}{\nu_\infty - \nu}} = m + \mu, \tag{22}$$

$$\sqrt{\frac{\nu - A}{B}} = m + \mu. \tag{23}$$

In the case of line series, this second procedure has always been followed. The $\sqrt{N_0/\nu_\infty - \nu}$ instead of being a straight line, is actually a much more complex $f(m)$. Nicholson[1] concludes that it must be some $f(m + \mu)$. Ritz uses $f(m) = m + a + b/m^2$. Hicks uses $m + a + b/m$. Nicholson,[1] in order to get accurate agreement for even the simple Helium series, has had to use

$$m + \mu + \frac{a}{m + \mu} + \frac{b}{(m + \mu)^2} + \frac{c}{(m + \mu)^3}.$$

But *possibly* this $f(m)$ is an hyperbola. By analogy we would have, for band series,

$$\sqrt{\frac{\nu - A}{B}} = \text{hyperbola}, \quad \text{or} \quad \nu = A + (\text{hyperbola})^2 \tag{24}$$

where B has been incorporated into the hyperbola. By comparing (24) with (14) it is seen that we have thus replaced the summation by a square, and have thus greatly simplified the necessary calculations.

Formula (24) has been carefully tested on the A_1 and C_1 series of the 3883 band. It *seems* to fit these series almost as well as does (14). More trials might possibly result in an equally good agreement. If either the A_1 or C_1 series were slightly longer, it would be possible to decide definitely between the two formulæ. The C_1 series extends much further (in terms of curvature) than the A_1, but because of the greater size of the perturbations and other irregularities of the C_1 series, there is a greater uncertainty as to the best position of the theoretical curve. The two formulæ can be made to give practically identical results for small values of m, but not for larger values.

The methods of handling the two formulæ are quite different, but only the following brief statement will be made at this time. With formula (24) it is possible to handle the data directly in terms of ν and m.

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[1] Proc. Roy. Soc., (A), 91, 255, 1915.

But since the computations necessary in solving six simultaneous equations are several times as extended as those necessary for five, the author has preferred to work with the hyperbola, *i. e.*, with the curve in $\sqrt{\nu - A}$ and m. Now the experimental values of $\sqrt{\nu - A}$ are indeterminate, since A is as yet unknown. In the case of line series [compare (22) with (23)] it is possible to determine ν_∞ with great accuracy, regardless of the type of function used. The same thing is roughly true for A in band series. Any simple formula, such as Deslandres', applied to the known smaller values of m, will give an approximate value of A. It is moreover known that the curve in $\sqrt{\nu - A}$ and m must approach linearity, with decreasing m, since all band series approach Deslandres' law in this region, and this curve is very sensitive to small changes in A, when m is small. It is thus possible to determine accurately the value of A, necessary to get even approximate linearity. All computations can be made with a 20-inch slide rule. It is not necessary to have a good agreement between the observed and computed $\sqrt{\nu - A} : m$ curves, for m small, since in this case a relatively large change in $\sqrt{\nu - A}$ produces a relatively small change in $\nu - A$. The converse is true for large values of m, so that in this region the observed and computed $\sqrt{\nu - A} : m$ curves must agree very exactly.

By using the above method, the time required to determine a given set of constants is the same for the two formulæ. But for the new formula the time required for testing the constants is far less, and this constitutes, with the old formula, the major portion of the work. Since either of the formulæ give practically satisfactory results for the 3883 series, it is immaterial which form is used, in testing other simpler series. But since the original form allows the data for the hyperbolic curve ($\Delta\nu : m$) to be computed directly from the observations, this form has been used in all the general work to date. Each form has six coefficients, or five, if the hyperbola cuts the origin.

In lines series, by analogy, we would have instead of (22)

$$\nu = \nu_\infty - \frac{N_0}{(\text{hyperbola})^2}. \tag{25}$$

The author has plotted the experimental curves, for certain well-known line series, in terms of $\Delta(N_0/\nu_\infty - \nu)$ and m, and also in terms of $\sqrt{N_0/\nu_\infty - \nu}$ and m. The hyperbolæ (if either of the curves are such) are reversed, compared to band series, approaching linearity for large values of m, while the vertex occurs near the origin.[1] Unfortunately, line series do not yield themselves to graphical methods, for if an error of 0.01 A in term $m = 2$ is represented by, let us say, 0.01 mm., an equal error in $m = 30$ may be represented by about 5 mm. or more.

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[1] The limiting slope ($m = \infty$) equals *two*, for the first form, or *unity*, for the second, regardless of the equation used, and thus should be the same for all line series, if N_0 is truly a universal constant. (Rydberg Constant). In band series one finds a great diversity of nitial slopes and there is no such corresponding universal constant.

The author is not prepared, at the present time, even to suggest the use of either (21) or (25) for line series. The present trend of line series work is to derive formulæ directly from theoretical considerations. Because of the recent success of such efforts, by the use of various modifications of the Bohr atom,[1] it does not seem desirable to introduce any new purely empirical formula. In the case of band series, we should expect, from theoretical considerations, a formula of the type of (14) rather than of (24), since any application of the quantum idea must involve a summation, implicitly if not explicitly.

ADDENDUM.

The author has just received a reprint of a long paper by T. Heurlinger entitled "Untersuchungen Uber Die Struktur der Bandenspektra" (Lund, 1918). A small portion of Heurlinger's work duplicates work (unpublished) of the author. In general however the ground covered is different. Heurlinger is not primarily interested in a new type of formula. He uses simply the power series (15), or a modification, and compares the numerical constants, for the various series. His main object is to relate together all the series in any one band, or group of bands, and to thus formulate general rules regarding band structure. The paper contains several ideas on band structure, radically new, but very suggestive, and apparently supported by a large amount of experimental evidence. The author intends to discuss these ideas more fully in a future paper.

UNIVERSITY OF CALIFORNIA,
December, 1918.

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[1] See particularly H. S. Allen, Proc. London Phys. Soc., 30, 127, 1918. Vegard, Phil. Mag., 35, 293, 1918. Ishiwara, Proc. Math. Phys. Soc. of Tokio, (2), 8, 106, 1915; (2), 8, 173, 1915; (2), 8, 540, 1916; (2), 9, 20, 1917; (2), 9, 160, 1917.

THE PASSAGE OF PHOTOELECTRONS THROUGH METALS.

BY K. T. COMPTON AND L. W. ROSS.

SYNOPSIS.—Theoretical expressions are derived giving the rate of escape of photo-electrons from the surface of a thin metallic film as a function of its thickness. These expressions result from various assumptions regarding the probability of escape of an electron photoelectrically excited at a given depth within the metal.

The photoelectric currents from films of platinum and of gold of various thicknesses are measured for different wave-lengths of light, and the results are found to be consistent with the view that the average distance in any direction which an excited electron may move without losing its ability to escape is about $2.67\,(10)^{-7}$ cm. in platinum and $5.0\,(10)^{-7}$ cm. in gold, and that the probability of going a given distance without losing its ability to escape falls off exponentially with the distance. This average distance is shown to be independent of the initial kinetic energy of the electron. There is evidence that platinum is first deposited in a relatively unstable and electropositive state, and that it spontaneously changes into the ordinary stable form.

A critical discussion and comparison of methods of measuring film thicknesses shows that they may be relied upon if proper precautions are taken. Various interesting properties of these thin films are illustrated and discussed.

INTRODUCTION.—Ladenburg[1] was the first to prove that electrons, excited photoelectrically within a metal by ultra-violet light, may travel some distance through the metal before escaping from its surface. He deposited nickel electrolytically on glass, and showed that the photoelectric currents from the films increased with increasing film thickness up to about $2(10)^{-4}$ cm. This result was qualitatively confirmed by Rubens and Ladenburg,[2] who showed that the ratio of the photoelectric currents on the incident and emergent sides of a piece of gold foil was about 100 : 1, while the ratio of the incident and emergent light intensities was about 1,000 : 1.

In 1913, Partzsch and Hallwachs[3] published a criticism of the quantitative results of Ladenburg's measurements, and also made a theoretical and experimental study of the optical and photoelectric properties of thin metal films deposited on quartz. The theoretical treatment in the present paper is based on, and is an extension of, the work of Partzsch and Hallwachs.

The experimental measurements here recorded are in many respects similar to measurements published by Robinson,[1] and in the main confirm his results. Certain refinements had to be added to his experimental procedure, however, in order to attain sufficient accuracy to permit a quantitative application of the theory.

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[1] Ann. d. Phys., 12, p. 558, 1903.

[2] Ber. d. D. Phys. Ges., p. 749, 1907.

[3] Ann. d. Phys., 41, p. 247, 1913.

Method.—The general plan of the experiment was to allow ultraviolet light of definite wave-length to enter a metal film, deposited on a quartz plate, from the quartz side and to measure the total photoelectric emission from the film. This was done with films of various thicknesses, maintaining other conditions constant, so that the photoelectric emission was obtained as a function of the film thickness. The nature of the variation of emission with thickness depends on the distance rate of absorption of light in the film and on the probability that an electron, photoelectrically excited by the light at any point in the film, may reach and escape from the surface of the film. The rate of absorption of light in the film is known, or may easily be measured. Thus the experimental results depend ultimately on the probability of the escape of an electron from any point in the film. Various assumptions regarding the form of the expression of this probability, associated with various assumptions regarding the passage of electrons through the metal, may therefore be tested by the experimental results. In the following section several such assumptions are put into forms suitable for quantitative tests.

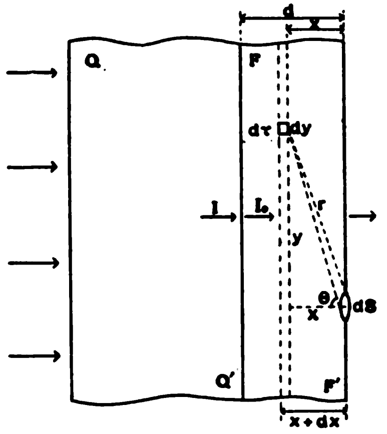

Fig. 1.

Theory.—Let QQ' and FF' in Fig. 1 represent sections of the quartz plate and metal film, respectively, the thickness of the latter being d. Light of intensity I strikes the film from the quartz side, but part of it is reflected, so that I_0 represents the intensity of the light which enters the film. Since the reflecting power of the film varies with its thickness, I_0 is a function of d.

Let dx be the thickness of a layer of the film distant x from its free surface. The intensity of the light in this layer is $I_0\epsilon^{-\alpha(d-x)}$, where α is the coefficient of absorption of the light. If we let ν represent the number of electrons which are photoelectrically excited per unit time, per unit volume, per unit light intensity, then the rate at which electrons are excited in the layer dx is

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[1] Phil. Mag., 25, p. 115, 1913; *ibid.*, 32, p. 421, 1916.

$$\nu A I_0 \epsilon^{-\alpha(d-x)} dx,$$

where A is the area of the film. From symmetry it is evident that half of these start moving toward the free surface.[1]

Let $F(x)$ be the probability that an electron will move a distance whose component normal to the surface is x without losing its ability to escape. Then the rate of escape from the film of electrons which start from the layer dx is

$$dN = \frac{\nu A I_0}{2} \epsilon^{-\alpha(d-x)} F(x) dx.$$

The total rate of emission of electrons is therefore

$$N = \frac{\nu A I_0}{2} \epsilon^{-\alpha d} \int_0^d \epsilon^{\alpha x} F(x) dx. \tag{1}$$

SPECIAL CASES.—(*a*) *Assume that the number of electrons which retain ability to escape falls off exponentially with distance moved normally to the surface, or*

$$F(x) = \epsilon^{-\beta x}, \tag{2}$$

where $1/\beta$ is the average distance which an electron can move normally to the surface without losing its ability to escape. Then the integration in equation (1) is easily performed, giving

$$N = \frac{\nu A I_0}{2} \frac{\epsilon^{-\alpha d} - \epsilon^{-\beta d}}{\beta - \alpha}, \tag{3}$$

which is identical with the equation derived by Partzsch and Hallwachs.[2]

(*b*) *Assume that the number of electrons which retain ability to escape falls off exponentially with the distance moved* (*in any direction*).—Consider those electrons which start toward the surface from an element of volume $d\tau$ in the layer dx and escape from an element of the free surface dS. Of these, the fraction $dS \cos \theta / 2\pi r^2$ start toward dS; but of these, only the fraction $\epsilon^{-\gamma r}$ succeed in escaping, according to our present assumptions. Thus

$$\frac{dS \cos \theta}{2\pi r^2} \epsilon^{-\gamma r}$$

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[1] This assumes that the electrons begin to move with equal probabilities of motion in all directions and therefore in directions which are independent of the orientation of the electric vector in the light wave. There is possibly some experimental evidence that this assumption is not strictly true. If the electron is ejected from the atom by a direct pull from the electric field of the light wave the present assumption is probably seriously in error. If, however, the photoelectric emission is a secondary effect caused by a storing up of energy in the atom and its consequent instability, the assumption is valid.

[2] *Loc. cit.*

is the probability that an electron, starting toward the surface from $d\tau$, will escape from dS. It is to be noted that this probability is the same for electrons starting from any point in dx at a distance between y and $y + dy$ from O. Thus the probability that an electron, starting toward the surface from any point in the layer dx, will escape from dS is

$$\frac{1}{A}\int_{y=0}^{y=\infty}\frac{dS\cos\theta}{2\pi r^2}\epsilon^{-\gamma r}\cdot 2\pi y\,dy,$$

since the layer may be considered to extend to infinity in comparison with the depth x. Finally, the probability that an electron, starting toward the surface from dx, will escape, is found by substituting A for dS, and is

$$F(x) = \int_0^\infty \frac{\cos\theta}{r^2}y\epsilon^{-\gamma r}dy.$$

This expression may be written

$$F(x) = x\int_x^\infty \frac{1}{r^2}\epsilon^{-\gamma r}dr,$$

of which the solution is

$$F(x) = \epsilon^{-\gamma x} + \gamma x Ei(-\gamma x), \tag{4}$$

where $Ei(-\gamma x)$ represents the exponential integral

$$Ei(-\gamma x) = -\int_x^\infty \frac{1}{r}\epsilon^{-\gamma r}dr,$$

whose value for various values of (γx) is given in Laska's "Sammlung von Formeln."

On this assumption, therefore, the total rate of electron emission from the film should be, by equation (1),

$$N = \frac{\nu A I_0}{2}\epsilon^{-\alpha d}\int_0^d \epsilon^{\alpha x}[\epsilon^{-\gamma x} + \gamma x Ei(-\gamma x)]dx. \tag{5}$$

The integral of this expression may be easily evaluated by graphical methods.[1]

---

[1] This equation may be integrated by parts after substituting for $Ei(-rx)$ its alternative form given above. In performing this integration it is necessary to express the limits of integration of one of the terms by

$$\int_0^d f(x)dx = \int_0^\infty f(x)dx - \int^\infty f(x)dx$$

and to remember that

$$\int_0^\infty \frac{\epsilon^{-rx} - \epsilon^{-(r-\alpha)x}}{x}dx = \log\frac{r-\alpha}{r}.$$

We thus find for the solution

$$N = \frac{\nu I_0 A}{2}\left\{\frac{1}{\alpha}(\epsilon^{-rd} - \epsilon^{-\alpha d}) + r\left(\frac{d}{\alpha} - \frac{1}{\alpha^2}\right)Ei(-rd)\right.$$
$$\left. + \frac{r}{\alpha^2}\epsilon^{-\alpha d}Ei(-(r-\alpha)d) + \frac{r}{\alpha^2}\epsilon^{-\alpha d}\log\frac{r}{r-\alpha}\right\}.$$

In this expression, $1/\gamma$ represents the average distance in any direction which an electron may move without losing its ability to escape.[1]

(*c*) *Assume that an electron loses energy in proportion to the distance moved through the metal.*—Let σ be the energy lost per centimeter path. The electron starts with an amount of energy $h\nu$, where h is Planck's constant and ν is the frequency of the exciting light; of this energy, an amount $h\nu_0$ is necessary to enable it to escape from the surface of the metal. Therefore

$$\sigma r_0 = h(\nu - \nu_0) \tag{6}$$

determines the greatest distance r_0 which an electron can move in the metal without losing its ability to escape. As in case (*b*), the fraction $dS \cos \theta / 2\pi r^2$ of the electrons starting toward the free surface from $d\tau$ move toward dS. All these will escape if $r < r_0$, and none if $r > r_0$. Thus the probability that an electron starting toward the surface from the layer dx will escape from dS is

$$\frac{1}{A}\int_0^{\sqrt{r_0^2-x^2}} \frac{dS \cos \theta}{2\pi r^2} \cdot 2\pi y dy = \frac{dS}{A}\left(1 - \frac{x}{r_0}\right)$$

if $x < r_0$ and is zero if $x > r_0$. Therefore

$$\left.\begin{aligned} F(x) &= \left(1 - \frac{x}{r_0}\right) \quad \text{if} \quad x < r_0; \\ &= 0 \qquad\qquad\quad \text{if} \quad x > r_0. \end{aligned}\right\} \tag{7}$$

By equation (1), if $d > r_0$,

$$\begin{aligned} N &= \frac{\nu A I_0}{2}\epsilon^{-\alpha d}\int_0^{r_0}\left(1 - \frac{x}{r_0}\right)\epsilon^{\alpha x} dx \\ &= \frac{\nu A I_0}{2}\epsilon^{-\alpha d}\left[\frac{1}{\alpha^2 r_0}(\epsilon^{\alpha r_0} - 1) - \frac{1}{\alpha}\right], \end{aligned} \tag{8}$$

Although this equation permits numerical computations to be made, such computations are neither as rapid nor as accurate as those made by a graphical solution of equation (5). The inaccuracy is due to the fact that the value for N is expressed as the *difference* between relatively large terms, and the tables of $Ei(-rx)$ are not sufficiently accurate to give the difference with high percentage accuracy.

[1] We have implicitly assumed that the "ability to escape" depends on the velocity but not on the direction of motion of an electron at the surface. This is objectionable, since the chance of escaping against the attraction of the induced positive charge left on the film is certainly dependent on direction as well as speed of motion of an electron. The mechanism of "escape" and the nature of the work done in escaping is so little understood, however, that we have neglected this factor. If most of the work done in escaping is done within distances from the surface which are of atomic order of magnitude, the present assumption will probably not be seriously in error. It can furthermore be shown that the relations between N, α and r are such as to reduce any error from this cause to a minimum.

and if $d < r_0$

$$
\begin{aligned}
N &= \frac{\nu A I_0}{2}\epsilon^{-\alpha d}\int_0^d\left(1 - \frac{x}{r_0}\right)\epsilon^{\alpha x}dx \\
&= \frac{\nu A I_0}{2}\left[(1 - \epsilon^{-\alpha d})\left(\frac{1}{\alpha} + \frac{1}{\alpha^2 r_0}\right) - \frac{d}{\alpha r_0}\right].
\end{aligned}
\tag{9}
$$

Thickness of Film giving Maximum Photoelectric Emission.—It is much easier, in practice, to determine or test values of β, γ and σ by finding the thickness of film giving maximum emission than by attempting to fit the entire graphs of the theoretical equations to the experimental curves. We shall therefore discuss the conditions under which maximum emission is obtained in the three cases considered above.

(*a*) By equating to zero the derivative of equation (3) with respect to film thickness d we find the thickness d_m for maximum emission to be given by

$$
d_m = \frac{1}{\beta - \alpha}\log\frac{\beta - \left(\dfrac{1}{I_0}\dfrac{\partial I_0}{\partial d}\right)_m}{\alpha - \left(\dfrac{1}{I_0}\dfrac{\partial I_0}{\partial d}\right)_m}, \tag{10}
$$

in which the term $1/I_0\ \partial I_0/\partial d$ arises from the variation of the reflecting power of the quartz-film surface with thickness and may be determined experimentally, as described later. Taking experimental values of this term and of the coefficient of light absorption α we find the values of d_m for various values of α and β shown graphically in Fig. 2. If d_m and α are determined experimentally, the appropriate value of β may be found from these curves.

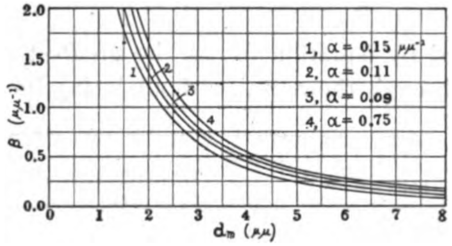

Fig. 2.

(*b*) To find the maximum value of N from equation (5) the easiest method is to actually plot the equation, as is done in Fig. 3 for various values of γ.[1] From these and similar curves the values of d_m appropriate to various values of γ and α have been determined and plotted in Fig. 4, whence γ may be found if α and d_m are experimentally determined.

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[1] In Fig. 3 the curves for various values of r are drawn so as to merge into the curve giving the percentage of transmitted light, thus calling attention to the fact that for thick films the photoelectric emission varies with thickness in practically the same manner as does the transmitted light. To do this, however, the four curves had to be plotted to four different scales. If drawn to the same scale the maximum values of N for the four curves 1, 2, 3 and 4 would be 21.3, 39.9, 70.9 and 121.7 respectively.

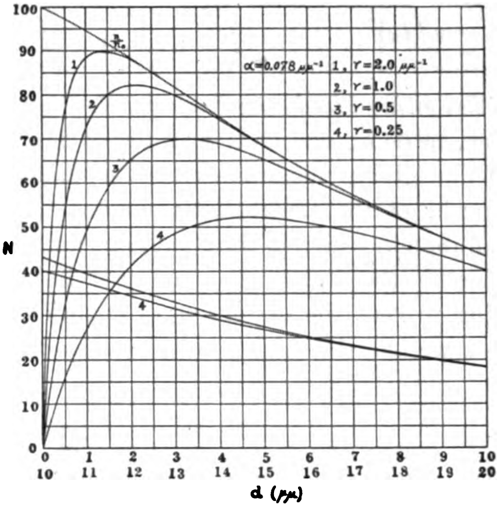

Fig. 3.

(c) In case energy is lost in proportion to the distance moved through the metal, the thickness for maximum emission is given by differentiation of equation (9), whence

$$d_m = \frac{1}{\alpha} \log (1 + \alpha r_0), \tag{11}$$

where r_0 is defined by equation (6) as the greatest distance which an electron can go through the metal without losing its ability to escape.

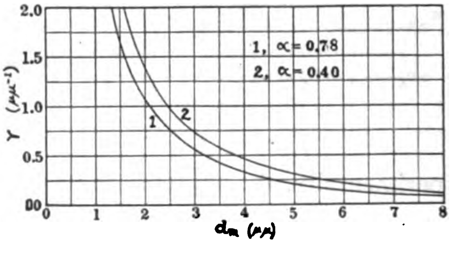

Fig. 4.

Since αd_m is an extremely small quantity, equation (11) may be written without appreciable error, $\alpha d_m = \alpha r_0$, whence, by equation (6)

$$d_m = \frac{h(\nu - \nu_0)}{\sigma}. \tag{12}$$

If this were the actual type of energy loss, the thickness of film giving maximum current should be proportional to $(\nu - \nu_0)$, which is the difference between the frequency of the exciting light and the least frequency of light which will produce photoelectric emission.

Apparatus.—In order to test the foregoing assumptions and to measure some of the quantities involved, metallic films were deposited on quartz plates by the method of "sputtering," in a vessel whose essential features are shown by Fig. 5. Transparent quartz plates one centimeter square and one millimeter thick were carefully cleaned in boiling caustic potash, nitric acid and distilled water and mounted by a touch of sealing wax over holes in a brass disk, as at *P*. Twelve such plates were mounted on the disk, which was then placed in the vacuum chamber shown in Fig. 5, so that the plates could be placed in turn under the cathode *C* by turning the ground glass joint *J*. A stationary brass shield *SS'* was held just above the disk *NN'* so as to shield from the discharge all the plates except the one *P* below the cathode. The vessel was rendered air tight by melting wax in the groove *GG'*. Great care was taken to keep the inside of the vessel free from wax or grease, and where wax was necessary it was entirely shielded from the discharge. The exhaust tube was connected through a liquid air trap to a McLeod gauge and Gaede pump. The source of the discharge current was a Snook high potential transformer designed to give a very steady discharge.

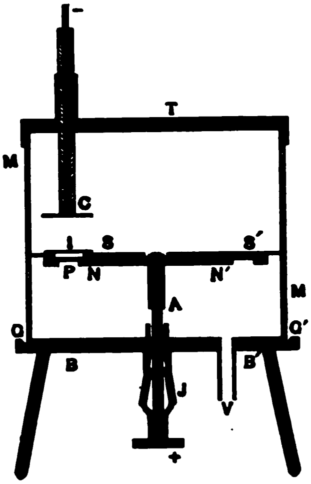

Fig. 5.

Measurement of Thickness of Films.—Three methods were employed to measure the thickness of the films. Of these, the most direct method was to weigh the thickest film, thence calculate its thickness and take the thickness of the other films proportional to their times of exposure to "sputtering." B. Pogány[1] has shown that this method is accurate if the sputtering conditions are maintained constant. In order to have constant conditions we adopted the following procedure. An iron disk *I* was placed over the hole in the shield *SS'* so that none of the quartz plates was exposed to the discharge. The discharge in the vessel was then continued for about two hours, until no more gas could be detected coming from the walls of the vessel under the influence of the discharge.

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[1] Phys. Zeit., 15, p. 685, 1914.

Then the iron disk I was removed by a magnet and the plates were exposed successively to the discharge. They were exposed in the following order: 1, 2, 3, ···, 11, 12, 12, 11, ···, 2, 1, 1, 2, ···, 12, 12, ···, 1; so that the effect of any change in sputtering conditions would be evenly distributed among the plates. During the entire process the gas pressure was maintained constant by a regulating valve. Ordinarily, the times of deposit ranged from 1 minute to 100 or more minutes.

The second method employed involved a measurement of the absorption of light in the films. The disk NN' was mounted on an optical bench so that light from a mercury arc could be focused on any one of the films. The reflected or transmitted light was focused on the slit of a Hilger monochromatic illuminator provided with a linear thermopile, which served to measure the intensity of the light. By thus determining the intensity of the incident light, the reflection and absorption by the quartz plate and the intensities of the lights transmitted and reflected by the film covered plates, we found the fraction of the light entering the film which was absorbed by it. The coefficient of absorption of light of the wave-lengths used, 576.9 $\mu\mu$ and 579.1 $\mu\mu$, is accurately known, whence the thickness of each film could be calculated.

The third method involved the use of data concerning the variation of reflection of ultra-violet light with thickness of platinum films on quartz, published by Partzsch and Hallwachs.[1] When light strikes a platinum film from the quartz side, they found that the intensity I_0 of light entering the film is less than the intensity I of light incident on the quartz by an amount shown by curve 1, Fig. 6, for films of various thicknesses d. Ninety-two per cent. of the incident light is transmitted by a blank plate. From this we find that the ratio of the intensity of light transmitted by a blank plate to that transmitted by a plate with film of thickness d is given by

$$\frac{n}{n_0} = \frac{I_0}{.92}\epsilon^{-\alpha d}, \tag{13}$$

in which α is known with considerable accuracy for the wave-length 253.6 $\mu\mu$ as a result of a slight extra-polation of measurements made by Meier.[2] Taking $\alpha = .078\ \mu\mu^{-1}$, we obtain curve 2, Fig. 6. The intensity of the light transmitted by any film was taken to be proportional to the photoelectric current produced when this light fell upon a plate P (Fig. 7) which could be placed in the path of the light. The ratio of this current to that produced by light which has passed through a blank plate was therefore equal to n/n_0. Thus the thickness of the film d could be found directly from curve 2, Fig. 6, as soon as the ratio n/n_0 was determined.

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[1] *Loc. cit.*

[2] Ann. d. Phys., 31, p. 1017, 1909.

The reliability of these methods of estimating film thickness is proven by a comparison of their results when applied to the same set of films.

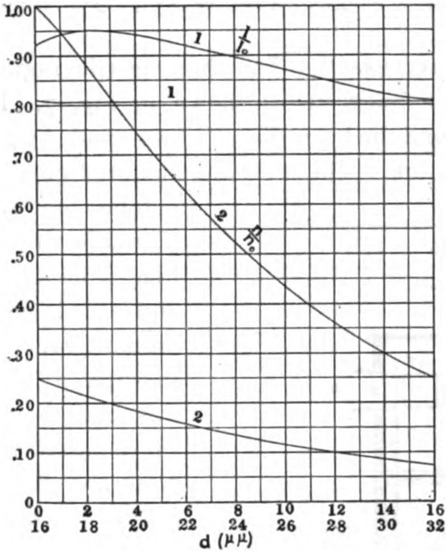

Fig. 6.

In Table I., t represents the time of sputtering and d_1, d_2, d_3 are the thicknesses calculated by the three methods, respectively. The area of plate no. 12 was 0.95 sq. cm. and the mass of film deposited on it was 0.000925 gm. This film was so opaque that no estimates of its thickness could be made by the second and third methods. The experimental

TABLE I.

No.	t (sec.).	$d_1(\mu\mu)$.	$d_2(\mu\mu)$.	$d_3(\mu\mu)$.
1	0	0	0	0
2	20	1.5	1.8	1.7
3	30	2.3	2.5	2.5
4	40	3.1	3.0	3.0
5	50	3.8	3.8	3.9
6	60	4.5	4.4	4.6
7	70	5.3	5.5	5.5
8	80	6.0	6.5	6.2
9	90	6.8	6.8	6.6
10	120	9.0	8.6	8.6
11	150	11.3	11.8	11.6
12	6000	452.0	?	?

data which were used, with Curve 2, Fig. 6, to find the values of d_2 are found in Table II.

Of these methods, the third was most convenient, and was used for most of the work with platinum films. The data for its use in dealing with gold films are lacking, so that we employed the second method in this case.

Photoelectric Measurements.—The photoelectric properties of these films were tested in the brass vessel shown in Fig. 7. Electrical contact between the films and the brass disk NN' was made by gold leaf, cemented with a little India ink. The disk was mounted on a shaft A in such a manner that any film F could be placed in the path of ultraviolet light from a quartz mercury arc and a Hilger monochromatic illuminator. This light entered through a quartz window W, was brought to a focus at the slit S, passed through the quartz plate and film and entered the hollow brass box B. This box was designed to reduce the amount of light returning to the film after reflection from the box, and a calculation proved that less than one per cent. of the photoelectric current from the film could be due to light reflected from the box.

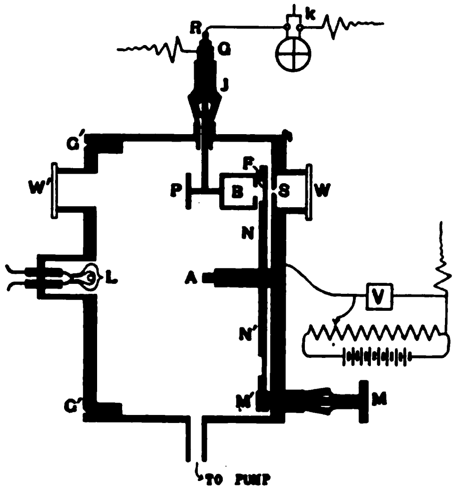

Fig. 7.

The photoelectric current N from the film F passed to the box B which served as the receiving electrode, and was connected with an electrometer which was used at a sensitiveness between 2,000 and 50,000 mm. per volt, depending on the type of measurement being made. The vessel and films were charged to a negative potential of about 10 volts, indicated by the voltmeter V, and the insulation of the electrometer system was protected by the guard ring G.

In order to measure the intensity of the light transmitted by the film, the platinum plate P was turned to the position of the box B and the potential of the vessel changed to + 10 volts, so that the electrometer measured the photoelectric current n from the plate.

To facilitate quick substitution of one film for another, a second toothed wheel M', which could be rotated by the outside handle M, was made to engage the toothed disk NN'. A small lamp L illuminated the inside of the vessel, and the adjustment of any film to the position F was visible when looking through the window W'.

Experimental Results. Platinum.—Ten sets of platinum films were tested, each set being examined to determine (*a*) the thickness of film giving maximum photoelectric emission, (*b*) the effect of using different wave-lengths of exciting light and (*c*) the nature of changes which occur in the films after they are deposited on the quartz plates. These points will be taken up separately.

(*a*) *The Thickness of Film Giving Maximum Photoelectric Emission.*—Table II. contains typical experimental measurements. The numbers in the columns marked N refer to the electrometer deflections per half minute due to the photoelectric currents from the films. Those marked n refer to currents from the plate P and measure the intensity of the light transmitted by the films; these data are used with Fig. 6 to determine the film thickness d. In order to avoid error due to "time" changes, such as photoelectric fatigue, all measurements were taken in order 1, 2, 3, ···, 11, 12 and back again 12, 11, ···, 2, 1, as shown in the columns marked with subscripts 1 and 2. The means of these measurements are given in columns N and n.

Table II.

No.	N_1.	N_2.	N.	n_1.	n_2.	n.	$d(\mu\mu)$.
1	0	0	0	191	186	188	0.0
2	47	45	46	170	168	169	1.7
3	74	68	71	162	158	160	2.5
4	83	82	82	153	152	152	3.0
5	81	81	81	145	139	142	3.9
6	77	72	75	134	131	133	4.6
7	67	68	67	122	124	123	5.5
8	65	62	63	117	113	115	6.2
9	67	67	67	112	112	112	6.6
10	58	60	59	92	93	93	8.6
11	31	29	30	71	71	71	11.6
12	0	1	1	0	0	0	?

These results are shown graphically in Fig. 8 (*c*). It is seen that there are two maxima, instead of the one predicted by the theory. It might be thought that this is due to some erratic photoelectric condition of films 9 and 10, if it were not for the fact that they were found in every one of the ten sets of platinum films tested. Other examples are shown in curves (*a*), (*b*) and (*d*), in which the ordinates have been changed by constant factors in order to avoid confusion in plotting. It appears that there is one maximum at $d = 3.75\ \mu\mu \pm .25\ \mu\mu$ and another in the neighborhood of $d = 7.5\ \mu\mu \pm 1.0\ \mu\mu$.

Numerous tests were made in an attempt to discover the cause of the second maximum. For one set of films mercury vapor was allowed free access to the sputtering and testing vessels, while in other tests all parts were carefully cleaned and mercury vapor excluded by liquid air traps, but no difference was observed. Apparently it made no difference whether the films were allowed to stand in air or in *vacuo*. We have concluded that both maxima are really due to properties of the platinum films. The interpretation of these maxima is suggested by experiments discussed in the following sections.

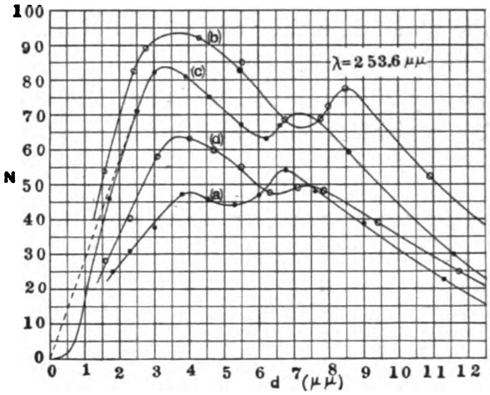

Fig. 8.

One other feature of these curves is of interest. All the curves near the origin show an inflection, as if the photoelectric currents increased with thickness of film, first slowly and then more rapidly. If the quantity ν in our theoretical equations were constant, this portion of the curve would be straight, indicating proportionality between emission and thickness. Robinson,[1] who has noticed this phenomenon, has supposed that it indicates less intrinsic photoelectric sensitiveness for thin than for thicker films. We have been able to prove, however, that ν really is constant, and that the inflection in the curves is due to a secondary cause, viz., lack of continuity of very thin films. When a very thin film is first exposed to the ultra-violet light, the rate of electrometer deflection falls off rapidly as the exposure is continued. If the light is shut off the film regains its original apparent sensitiveness only after a lapse of some minutes. If the *initial* rate of deflection instead of the total deflection in a relatively long time interval is used to determine N, it is found that the values of N tend to lie on the theoretical straight line, such as the dotted line in curve (c). This is much more marked in the case of gold films than in the case of platinum films—which is in line with the observation that a greater thickness of gold than of platinum is required to give an electrically conducting layer.[1] The upward concavity of the curves near the origin is apparently to be explained, therefore, by the effect of isolated regions of the film which quickly charge up to such positive potentials as to reduce the rate of emission from these and contiguous parts of the film. This phenomenon is easily detected by the failure to get electrometer deflections which are proportional to time, but it was never observed in the case of a platinum film thicker than 2 $\mu\mu$.

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[1] *Loc. cit.*

(*b*) *The Effect of Using Different Wave-lengths of Exciting Light.*—The influence of the wave-length of exciting light upon the character of the emission is shown by the curves of Fig. 9. All show the double maximum, but there are two points of difference. The second maximum is relatively smaller and the first maximum is displaced toward the left when dealing with the shorter wave-lengths. Every set of films tested showed these characteristics, although they were more pronounced in some sets than in others.

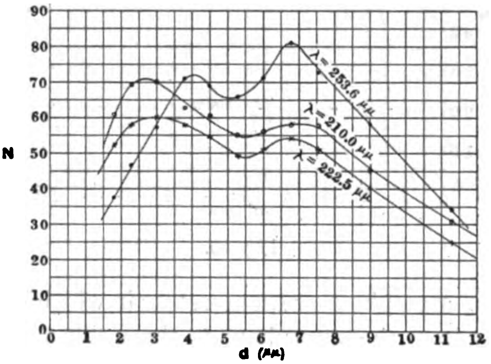

Fig. 9.

(*c*) *Time Changes.*—An examination of Fig. 8 shows that even for a given wave-length, the two maxima are not always of the same relative magnitude. This is due to a slow time change in the properties of the films, an actual example of which is shown by Fig. 10. The second maximum becomes relatively less important as the film stands, and it makes no apparent difference whether the films stand in *vacuo*, or exposed to air. Heating the films in an oven at about 150^c C., however, greatly accelerates the change, and after this treatment the second hump is scarcely noticable.

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[1] King, Phys. Rev., 10, p. 291, 1917.

Discussion of Results. *Interpretation of the Two Maxima.*—Apparently the two maxima of the curves must be ascribed to platinum in two conditions. Of these conditions, the one giving rise to the right hand maximum is evidently more unstable and more electropositive than the other, since it is relatively more important soon after the formation of the films and relatively more sensitive to light of long wave-lengths. We therefore take the first maximum to be characteristic of platinum in its ordinary stable state, while the second maximum is due to platinum in the amorphous or spongy state in which it is first deposited. Since the change from the amorphous to the stable state is spontaneous, the latter state must be one of less internal potential energy, which is in accord with the observation that the platinum in this state is more electronegative. It is also seen, by reference to Figs. 2 and 4, that the average distance, $1/\beta$ or $1/\gamma$, moved by an electron without losing its ability to escape is greater in the less compact state. These deductions are all consistent with each other and are also in line with well known facts regarding the abnormal resistivity of freshly sputtered films and the change to normal resistivity produced by "aging." We shall therefore confine the remaining discussion to the information regarding ordinary platinum which is given by a study of the first maximum.

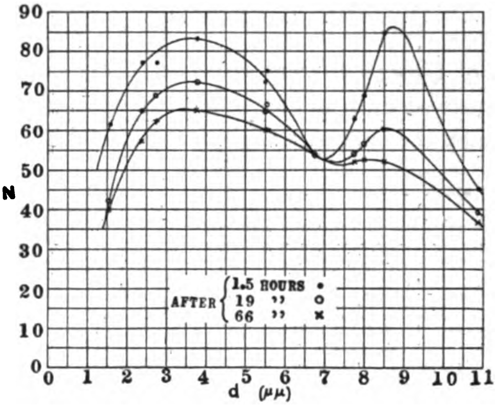

Fig. 10.

Comparison of the Three Assumptions Regarding the Process by which an Electron Loses its Ability to Escape.—The third assumption, viz., that an escaping electron loses energy in direct proportion to the distance traveled through the metal, may at once be discarded. According to it, the thickness giving maximum emission should be directly proportional to $(\nu - \nu_0)$ by equation 12, or to $(1/\lambda - 1/\lambda_0)$, where λ is the wave-length of the exciting light and λ_0 is the longest wave-length of light which will cause photoelectric emission. λ_0 is known to be about 280 $\mu\mu$ for platinum.[1] The values of λ employed ranged from 210 $\mu\mu$ to 265 $\mu\mu$. Thus the assumption requires that d_m should be four or five times larger for the shorter than for the longer wave-lengths. Not only is this not the case, but what little variation of d_m with λ is found is in the opposite direction to that required by the assumption.

As far as we can see, either the first or the second assumptions are in conformity with the facts. We are unable to decide between them experimentally, since either equation (2) or equation (5) satisfies the experimental results within the limits of uncertainty introduced by the presence of the second maximum. Of the two assumptions, however, the more reasonable one is the second—viz., that the probability that an electron will retain its ability to escape decreases exponentially with the distance traveled from its starting point.

On this assumption, we may determine $1/\gamma$, the average distance which an electron may move without losing its ability to escape. For $\lambda = 253.6\ \mu\mu$, we may take $d_m = 3.75\ \mu\mu$ as the mean of all our determinations. Taking $\alpha = .078\ \mu\mu^{-1}$, we find γ, from Fig. 4, to be $0.375\ \mu\mu^{-1}$.

For shorter wave-lengths, d_m has slightly smaller values. This may be due to a larger value of α for shorter wave-lengths or to a larger value of γ. This latter possibility is very unlikely, since it would mean that the rapidly moving electrons have less average penetrating power than the slow ones. We attempted measurements of α for the shorter wave-lengths and did succeed in proving that α increases as λ is decreased below about 245 $\mu\mu$, but our measurements were not accurate enough to prove that the shift of d_m with wave-length is due entirely to this cause. We are, however, safe in saying that there is no evidence that $1/\gamma$ *increases* as λ decreases, *i. e., there is no evidence that electrons with large initial velocities retain their ability to escape over longer paths than the slower electrons.*

This result is rather surprising and is of considerable importance, for it proves that an electron loses its ability to escape at a single catastrophe, and not by gradual process or a succession of small energy losses. If we define one of these catastrophes as a "collision," then we may identify $1/\gamma$ with the "mean free path" of the electron in the metal. This "mean free path" is apparently independent of the velocity of the electron, within the limits of velocity of photoelectrons. It is not easy to see how the theory of free electrons moving like molecules of a perfect gas in a metal can be reconciled with these results which indicate definite collisions of an inelastic type. These experiments are therefore consistent with the facts of specific heats of metals in indicating that electrons cannot exist in metals freely and independently sharing the thermal energy with the atoms.

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[1] Compton and Richardson, Phil. Mag., 26, p. 549, 1913.

The magnitude of the mean free path $1/\gamma$ is $2.67\ (10)^{-7}$ cm., which is greater than, but of the same order of magnitude as the distance between atomic centers in platinum. It is therefore probable that the electrons lose at least the greater part of their kinetic energy at encounters with atoms, and they may entirely lose their freedom at these encounters.

Patterson[1] has calculated the mean free path of conducting electrons by data regarding the Hall effect in platinum and also by applying J. J. Thomson's theory of the resistance of thin films. By each method he finds the mean free path to be about $6(10)^{-7}$ cm., not far different from the value we have calculated. Strict comparison is inadmissible, however, because the "mean free path" does not mean quite the same thing in the two cases, and because the assumptions underlying the formulæ used by Patterson cannot be accepted in their present form.

GOLD FILMS. In dealing with gold films the experimental difficulties were so great as to preclude the possibility of any great accuracy, and no extensive tests were made. The difficulties arise from the high resistence of films of the thickness necessary for the present tests, and are apparently due to lack of continuity of the films. Isolated portions of the films quickly charge up to retarding potentials, causing the rate of electrometer deflection to decrease rapidly. This phenomenon was noticed with only the very thinnest films of platinum.

In order to be more independent of errors from this cause, the films were kept shielded from light until the electrometer key was opened and we were ready to make an observation. A shutter was opened, so as to expose the film for two seconds. The deflection due to the electrons liberated in this interval was taken to represent the relative photoelectric sensitiveness of the film. The results of these tests are shown in Fig. 11. There is no evidence of a double maximum, though the determinations may not be accurate enough to make this conclusion certain. The mean free path $1/\gamma$ may be calculated by Fig. 4 to be about $5.0\ (10)^{-7}$ cm. The value of α used in this calculation is given by Meier[2] as $\alpha = .056\ \mu\mu^{-1}$ for wave-length $253.6\ \mu\mu$. Patterson has suggested a value $16(10)^{-7}$ cm. for the mean free path of the conducting electrons, while Partzsch and Hallwachs suggest

$$\frac{1}{\beta} = 11.6(10)^{-7} \text{ cm.}$$

(See the second assumption of the present paper.)

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[1] Phil. Mag., 3, p. 655, 1902.

[2] *Loc. cit.*

Summary.—The experiments described in this paper indicate that:

1. Photoelectrons, excited within a metal, lose their initial kinetic energy as the result of single and definite catastrophes, or "collisions," rather than by a gradual process or a succession of small energy losses.

2. The average distance which an electron can move before it loses its energy by one of these "collisions" is independent of the initial velocity and is of the same order as, but slightly larger than, the distance between atoms. The measurements indicate a mean free path of about 2.67 $(10)^{-7}$ cm. in platinum and 5.0 $(10)^{-7}$ cm. in gold.

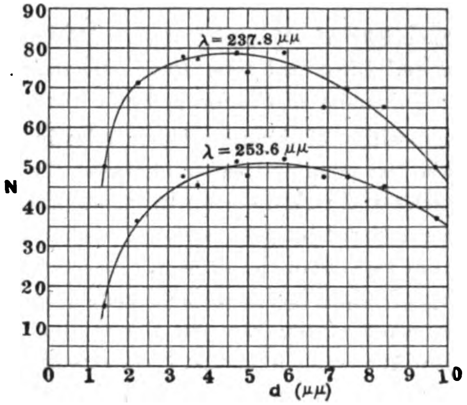

Fig. 11.

3. Platinum films thinner than 2 μμ and gold films thinner than about 8 μμ are apt to be imperfectly conducting due to isolated regions which are not in conducting contact with the main body of the film. Platinum films therefore appear to be more homogeneous than gold films.

4. The small penetrating power of photo-electrons renders it extremely improbable that the thermionic emission from metals may be accounted for by considering it to be a photoelectric effect from the entire body of the metal integrated over the complete radiation spectrum.[1]

[1] This possibility is discussed in detail in Richardson's "The Emission of Electricity from Hot Bodies," pp. 95–102.

[2] Since the above paper was written, a paper dealing with the same general subject has been published in the February number of this Journal by Dr. Otto Stuhlman, Jr. His experimental results are not in good agreement with those in this paper, and his interpretation of them is quite different. We are at present unable to suggest a satisfactory explanation of the apparent lack of agreement.

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NEW BOOKS.

The Fundamental Equations of Dynamics and its Main Coördinate Systems Vectorially Treated and Illustrated from Rigid Dynamics. By FREDERICK SLATE. Berkeley: University of California Press, 1918. Pp. vii + 233.

This is a critical exposition of dynamical principles in the form of a commentary rather than a text-book or treatise, covering the concepts associated with systems of particles and rigid bodies with some passing reference to deformable media. One declared purpose is to utilize the values that appear in some of the more recent aspects of the subject, especially the point of view of energetics, the theory of relativity, and the formulation in terms of vector algebra. Mention is made of the Einstein theory, but the only relativity actually considered is of the Galileo-Newton type, extended to cover the study of motion relative to rotating frames of reference and the forces of Coriolis. Extended treatment is given the Euler coördinates for a rigid body, which also supplies the main illustration given of the Lagrange equations. At the end of the volume are a series of notes, partly historical, partly on comparison references. The style is somewhat verbose and seems stilted at times, perhaps because of a conscious effort at sufficient novelty of wording to avoid the risk of conventionalism which is so real in such a familiar subject. For this reason and in view of its apparent purpose the value of the book will appear best to those who already have acquaintance with the matter and are interested to think through its foundations in a fresh way.

A. C. L.

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THE

PHYSICAL REVIEW.

THE FUNCTION OF PHASE DIFFERENCE IN THE BINAURAL LOCATION OF PURE TONES.

By R. V. L. Hartley.

Synopsis.—This paper offers an explanation of the sound images observed when pure tones of the same frequency and intensity but of different phase are applied to the two ears. Use is made of theoretical curves calculated by Stewart and Fry giving the relation between the position of an actual source and the resulting phase difference P of the sound at the two ears. On the assumption that the listener subconsciously perceives the phase difference and places the image in the position that experience has taught him to associate with that phase difference, most of the observed phenomena follow directly from the curves.

For frequencies below about 600 cycles they account for (1) the observed rotation of the image with continuously varying P; (2) its motion toward the ear followed by a range of uncertainty as P approaches 180°; (3) the increase with frequency of the value of P for maximum lateral displacement of the image. For higher frequencies they explain the progressive decrease in definiteness of the image with increase in frequency as due to the presence of a number of simultaneous images. These multiple images as given by the curves are in very good agreement with those observed in the rather unique experiments of Bowlker.

The completeness with which the assumed theory of direct perception of phase difference explains the observed phenomena is strong evidence in its favor against the rival theories based on cross conduction through the head.

Introduction.

The problem of the location of a source emitting a pure tone presents features which differentiate it quite sharply from the general problem of sound location. In the first place, pure tones are much less easily localized than are complex sounds. And in the second place, persons who are deaf in one ear have almost no sense of location for pure tones (provided the head is fixed), even though they may locate other sounds almost as well as a person with normal hearing.

Hence, while both monaural and binaural effects may be utilized in locating complex sounds, the location of a pure tone is essentially binaural, and so must depend upon differences in the characteristics of

Mistake in original

the sound reaching the two ears. A sustained pure tone is completely determined by its frequency, amplitude and phase, and of these characteristics only the last two are subject to variation with the position of the source. It follows therefore that the sense of location can be dependent only on differences of intensity or phase or both. The existence of a connection between relative intensity and location was probably recognized before that of phase difference, but both have been known for a comparatively long time. In what follows it will be assumed that the effect of relative intensity is reduced to a minimum by keeping the intensities equal at the two ears.

Stewart,[1] in his article on Binaural Beats, includes a very good review of the literature on phase difference as affecting location. The experimental results may be very briefly summarized here. Two general methods have been used. In one, a particular phase difference is maintained by means of an adjustable difference between the paths from a single source to the two ears. In the other the phase difference is made to vary continuously by exciting the two ears from sources of slightly different frequency. The rate of variation is determined by the difference in frequency.

It is commonly observed that with zero phase difference, the apparent source of the sound, or for brevity the image, is in the median plane. Its location varies with different experimenters but more often appears to be in front. For frequencies below about 600 cycles it is agreed that the image lies on the side for which the phase at that ear leads. With continuously varying phase difference the image is described as starting from a position straight in front for zero phase difference, moving more or less in a circle to one side, till opposite the ear, then moving in toward the ear, entering the head, jumping quickly to the other ear for 180°, and continuing on around to the front along a similar path on that side. For higher frequencies most observers report a falling off in the definiteness of the location and finally a complete loss of the sense of direction.

In attempting to explain the mechanism by which the phase difference determines the position of the image, two general theories have been advanced. One of these, which we may term the direct perception theory, assumes that the listener is able to take cognizance of a difference in phase, as such, and by virtue of his past experience, or that of his ancestors, is able to associate any particular phase difference with the direction from which he has been accustomed to receive sounds having that phase difference.

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[1] G. W. Stewart, PHYS. REV., June, 1917, p. 502.

Lord Rayleigh[1] has attempted to explain in a general way the observed facts on the basis of this theory. He pointed out that for a fork of 128 cycles, moving around the head, the phase difference would never exceed about one eighth of a period, and commented on the fact that in the laboratory experiment the location effect persisted for phase differences right up to half a period. He noted also that at a frequency around 512, the phase difference with the source opposite one ear should be 180° and concluded there could be no discrimination at that frequency for sounds directly right or left. For somewhat higher frequencies he pointed out that the same phase difference could be produced by a sound on either the right or the left, and considered it fortunate that our sense of direction fails us at the point where it would begin to give misleading results. It should be noted that in arriving at the phase difference for an actual source at one side he made use of the very rough assumption that the difference in the paths to the two ears is one foot. He also drew other conclusions from the unjustifiable assumption that the sense of lateralness is a maximum when the phase difference is 90°. On the whole, his results were not particularly conclusive.

Against this theory certain psychologists advanced the argument that it involved a violation of the general principle that the sensation resulting from the stimulation of a sensory nerve is independent of the nature of the stimulus. In other words, they held that the characteristic phase of the sound stimulus at each ear could not be preserved in the nervous impulses reaching the brain as would be necessary for a perception of difference in phase. Apparently, however, this general principle, has been deduced from rather limited experimental evidence, and it is not considered by psychologists generally to be very definitely established. This objection, therefore, is not sufficient to warrant a rejection of the theory, provided it can be shown to be satisfactory in other respects.

Nevertheless, it was probably this argument which led to attempts on the part of physicists to show how differences in phase could give rise to differences in the intensity of the stimuli, which latter would cause a location on the side of the stronger stimulus. This resulted in various theories being advanced which we may designate as cross-conduction theories, since they are based on the assumption that an appreciable fraction of the sound entering each ear is conducted through the head to the other ear and there combines with that entering directly.

Myers and Wilson[2] present such a theory. In order to make the directions fit the experiments they find it necessary to introduce an arbitrary phase reversal in passing through the head. Even then their theory explains only the most general features of the experiments, since it makes the displacement of the image proportional to the sine of the phase difference. More recently Stewart[1] has embodied the same idea in a more elaborate theory and uses it to account for the location when the phase difference is between 90° and 270°. For the rest of the cycle he assumes direct perception of phase.

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[1] Rayleigh, Phil. Mag., 13, 1907, pp. 214 and 316.

[2] C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 80, 1908, p. 260.

Against this type of theory numerous objections have been raised. Probably the most important of these is the experimental evidence showing that the amount of sound actually reaching the ear by cross-conduction is far too small to produce the effects ascribed to it. A good summary of this evidence is given by Peterson.[2] Further, if the location effect due to phase difference were determined purely by the resulting intensity relation it should be very strongly affected by changes in the relative intensity of the sounds coming to the two ears. Under certain conditions, however, it is found to be practically unaffected by very considerable changes of this sort. In order to give even a qualitative explanation of the phenomena it is necessary to introduce a purely arbitrary assumption of phase reversal within the head. To make the agreement at all quantitative even over a limited range of frequency, it has been necessary to set up a very complicated theory involving a rather large number of assumptions.

To put the situation briefly then, the cross-conduction theory, while it does give something of an explanation, is open to very serious objections. The direct perception theory, on the other hand, while it has nothing very serious against it, has so far had no very positive evidence advanced in its favor, such as would be furnished by a quantitative explanation of the experimental facts. The most obvious line of attack then is to find out whether or not any such explanation is possible on the basis of direct perception.

PURPOSE.

It is the present purpose to attack this question by attempting to trace a correlation between the phase differences produced at the ears by actual sources in various positions and the positions of the images which result when sounds of equal intensity and predetermined phase differences are applied experimentally to the two ears.

DATA AND DISCUSSION.

For this the first essential is a knowledge of the phase at the ear as a function of the position of the source for various frequencies. Stokes[3] has developed a method whereby it is possible to calculate, for any point on a great circle of a rigid sphere, the phase of a sound whose source is in the plane of the great circle. The wave-length of the sound and the distance of the source are expressed in terms of the radius of the sphere. Stewart[1] has applied this method to the calculation of certain special cases. At the time this work was undertaken, T. C. Fry, a colleague in the Western Electric Co., had, independently, applied the method to certain other cases. A study of these two sets of results led to the conclusions described below. Later Fry extended his calculations securing the results from which have been plotted the curves given in Figs. 1 to 5.

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[1] G. W. Stewart, PHYS. REV., June, 1917, p. 514.

[2] Joseph Peterson, Psychol. Rev., Vol. 23, No. 5, p. 333, 1916.

Lord Rayleigh, Theory of Sound, Ch. 17.

Stewart assumes the head to be such a rigid sphere of 60 cm. circumference, the ears being at opposite ends of a diameter. For a single case, corresponding to a source, of frequency about 280 cycles, distant about 480 cm., he plots the difference in phase at the two ears as a function of the direction of the source. The only deduction which he draws from this curve is that, "The phase difference changes the most rapidly when the source of sound is in front or behind the hearer."

In plotting the curves of Figs. 1 to 5, the circumference of the head was assumed to be 55 cm. (radius, $r = 8.75$ cm.). The angular separation of the ears in the horizontal plane was taken as 165° and the velocity of sound as 340 m. per second. The figures represent progressively increasing frequencies. The individual curves of each figure are for sources at various distances R from the center of the head, ranging from $2r$ to infinity. (Unfortunately, the method of calculation was not adaptable to sources at the surface of the head where $R = r$.) Abscissæ represent the angular displacement ϕ of the source from a point straight in front, being taken positive to the right and negative to the left, 180° being at the back. Ordinates represent the phase lead P at the right ear over the left. Negative values then mean that the phase is leading at the left ear. A phase lead $180° + x$ ($x < 180°$) at one ear is equivalent to a lead of $180 - x$ at the other ear. In other words, those parts of the curve where the phase lead exceeds 180° in either direction must have 360° either added or subtracted so as to keep the ordinates between $-180°$ and $+180°$. This has been done in Figs. 3, 4 and 5.

From this theoretical information it is possible to predict, on the basis of the direct perception theory, where a listener should place the image when the phase difference at the ears is given any arbitrary value. For it is only necessary to read from the curve corresponding to the frequency of the sound, the value of ϕ corresponding to the given value of P.

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[1] G. W. Stewart, Phys. Rev., 1914, 4, p. 252.

As the curves give only those values of ϕ lying between $-90°$ and $+90°$, they give only the images located in front of the observer. If extended to $\phi = 180°$ in each direction the curves would be nearly symmetrical with respect to $\phi = \pm 90°$. (The departure from symmetry is due to taking the ears as 165° apart rather than 180°.) This means that any particular phase difference could be produced by a source either in front of or behind the observer and hence we should expect the image to be in either one or both of these positions. As already noted some experimenters find it in one place and some in the other. Inasmuch as the majority seems to find the image in front, the discussion will be limited to those quadrants with the understanding that the same general conclusions hold for images at the back.

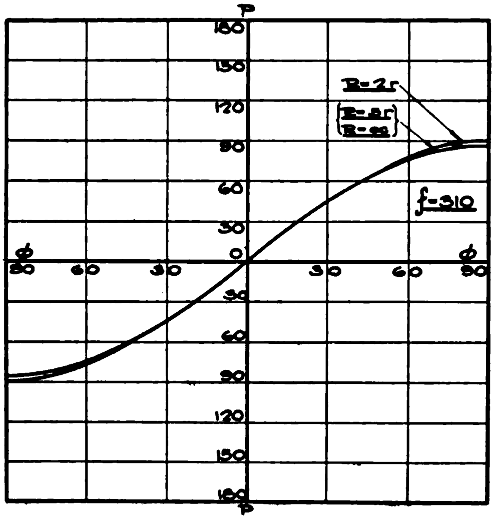

Fig. 1.

Consider first the case of the low frequency note of 310 cycles, shown in Fig. 1. For values of phase difference less than about 86°, we may for each ordinate choose an abscissa lying on any one of an infinite number of curves distributed between those of $R = r$ and $R = \infty$. This means that so far as phase alone is concerned there is in this region nothing to fix the distance of the image, while its angular position is determined, within limits, by this distance. The observed fact that the image is generally at a considerable distance is probably to be explained by the equality of intensity. Generally speaking, for any given direction the intensities at the ears become more nearly equal the greater the distance of the source. This then would explain why the image in moving around from the front would stay at a considerable distance till it was opposite the ear for a phase difference of 86°. For a further increase in phase there is no longer a corresponding abscissa on the curve for $R = \infty$ so it is necessary to go to the curve for the greatest distance which will still give a point. This means that once the image has reached a position near 90° its direction remains unchanged, but in spite of equal intensity, it moves in toward the ear, reaching it for a phase difference somewhat greater than 90°, being the maximum value of the hypothetical curve for $R = r$. This is in agreement with the observations quoted above for the case of continuously changing phase difference. Beyond this point there is no corresponding position for an actual source and hence the curves tell us nothing as to where an image is to be expected. However, it does not follow that such phase differences are foreign to the experience of the listener. They may very easily be produced by the reflections which occur in an enclosed space. Or they may occur when the source is within the head, or when the latter is in direct contact with the sounding body. Most observers describe the image as being located within the head in this case. Bowlker,[1] however, speaks of the image as being spread over a considerable angle on either side of 90°. In any case the definiteness of the sense of location should be much less than for small phase differences. It is, therefore, not surprising that, as Stewart has observed, the position of the image is affected much more by unequal intensities when the phase difference is near 180° than when it is near zero.

Consider next the effect of increasing the frequency. Up to something over 600 cycles (Figs. 1 and 2) the curves are of the same general form and we should expect the same general behavior of the image, as is found to be the case. The main variation is that the phase difference at which the image reaches its maximum lateral displacement increases with the frequency. Thus for 620 cycles (Fig. 2) the sense of lateralness should be a maximum for about 170°, instead of 90° as Rayleigh assumed. In support of this variation in the value of P for maximum image displacement, may be cited the results obtained by Myers and Wilson[2] using a variable path arrangement. They give curves in which the sensation of lateralness, measured in arbitrary units, is plotted against the path difference, and attempt to show a connection between this apparent displacement and the sine of the phase difference. An inspection of the curves, however, shows that for a frequency of 128, the maximum displacement occurs for a phase difference considerably less than 90°, while above 90° the displacement shows a decided falling off. At 384 cycles, the maximum is more nearly at 90°, while for 512, it is on the side toward 180°. What is perhaps a more striking confirmation is a statement made by Bowlker,[1] in connection with some experiments in which various path differences were obtained by placing tubes of different lengths over the two ears and listening to a distant sound. He says "A wave-length of about 36 inches (frequency about 370) . . . gave an image displaced 90°. Sound with longer wave-length gave a displacement of 90° before a phase-difference of half a wave-length was reached, and the sound image then seemed to spread over a continually increasing length of arc on each side of 90°."

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[1] T. J. Bowlker, Phil. Mag. (6), 15, p. 318, 1908.

[2] C. S. Myers and H. A. Wilson, *loc. cit.*

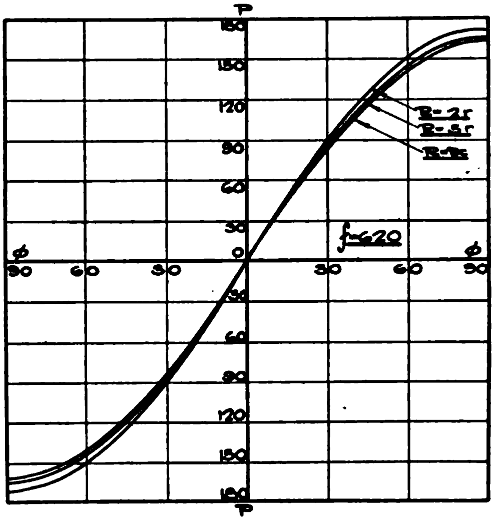

Fig. 2.

As the frequency is still further increased a marked change occurs in the nature of the curves owing to the fact that an actual source may then cause a phase difference greater than 180°. For a frequency of 930 cycles, Fig. 3 shows that for ordinates above 126° there are, on the curve for $R = \infty$, two abscissæ, one positive and one negative. This means two images, one on the right and one on the left. In the range between 126° and 106° there can be no image at infinity on the left, but there can be one at some distance greater than $2r$. (As before, the equality in intensity should tend to keep the image as far away from the head as is consistent with the phase relations.) For P less than the critical value corresponding to $R = r$, there is only one image as at lower frequencies.

Starting then with $P = 0$, the image is in front. As P increases the image moves rather slowly to the right keeping at a considerable distance.

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[1] Bowlker, *loc. cit.*

When P is somewhat less than 106° it has moved out about 20°, when a second image appears at the left ear. This new image moves quite rapidly outward from the ear, reaching a great distance by the time the first has moved to 33° ($P = 126°$). It then swings around in a circle toward the front, at first rather rapidly and then more slowly, arriving at 51° on the left at the same time that the other image reaches the same position on the right ($P = 180°$). To complete the cycle P must be varied from − 180 to 0°. The image on the right now moves on around the circle with increasing speed till opposite the ear, when it moves in to the ear and disappears. Meanwhile, the left image moves slowly toward the front reaching there for $P = 0$. That is to say, the two images follow the same path relative to their respective sides but in opposite directions. The one on the side where the phase-lead is increasing begins in front and disappears at the ear, the one on the other doing the reverse. It will be noticed that the paths followed are exactly the same as for the low frequencies discussed above. The difference lies in the magnitude of the variation of P required to cover the path.

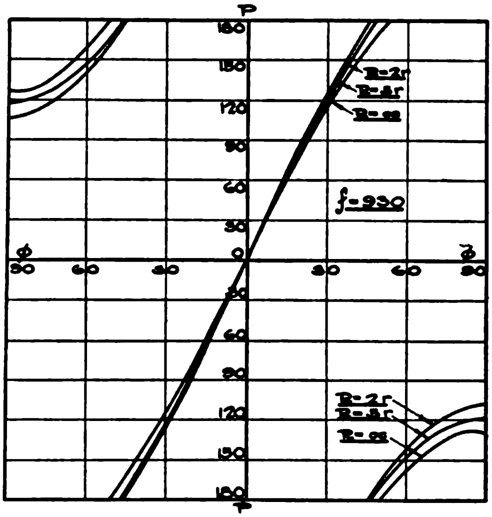

Fig. 3.

For still higher frequencies the range on either side of 180° in which two images are present increases progressively (see Fig. 4) until the whole cycle is covered. For higher frequencies still, where the possible phase difference exceeds 360°, there should be a third image for values of P close to zero, being on the right for small positive values of P and on the left for negative. Fig. 5 shows a case where this third image is present for most of the cycle. For still further increasing frequencies it follows from the above that the greatest number of images in the field during any part of a cycle increases by one each time the maximum possible phase difference increases by 180°.

Before the foregoing picture of the sound images for high frequencies can be used to predict the experimental results to be obtained at those frequencies, there must be something known or assumed as to the ability of the listener to perceive and localize a plurality of simultaneous images, giving to each the location assigned to it by the phase relations. It is quite conceivable that the listener would be unable to do this, in which case he might either have the sensation of a single image in some sort of average position or he might have no sense of direction at all. In either case his judgment would be much less certain than for a single image. Under these circumstances he would probably be influenced considerably by the equality of intensity, which would tend to cause him to locate the image in the median plane, particularly since at high frequencies the variation of intensity ratio with the position of the source is more marked than at low. This assumption appears to be in agreement with the results of most observers, who find that from about 600 cycles upward the sense of location by phase difference becomes progressively less trustworthy. That the change should appear to be gradual follows from the fact that the region of two images is at first confined to a small part of the cycle and gradually extends over the remainder. So far then, the theoretical deductions are not inconsistent with the experiments, on the assumption that more than one image can not be located separately.

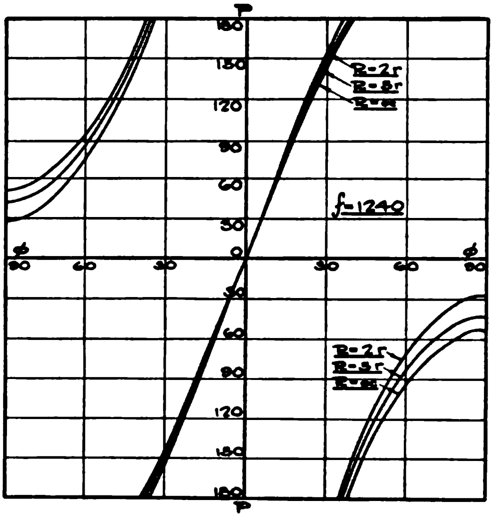

Fig. 4.

That this is not universally true, however, is shown by a series of experiments by Bowlker,[1] in which he was able to recognize two and even three separate images, in the field at once. Whether this was due to some peculiarity of his apparatus or of his individual sense of hearing is not known, but in either case the experiments are of special interest as being probably the only ones available for testing the above theory regarding multiple images.

The method used was an improvement of that referred to above in which tubes of unequal length were used. In order to eliminate possible inequalities of intensity due to the unequal lengths of the tubes, or to sound shadows cast by the head, he used exactly similar tubes on the two ears, bending them upward so as to make the openings horizontal in a plane well above the top of the head. The difference in path was then secured by facing in various directions relative to the source. The latter consisted of an organ pipe about 30 feet away. The distance L between the openings was 31 inches. The path difference was given by L into the sine of the angle turned through from the position facing the source. With this arrangement he was able to locate the direction of the sound image with considerable exactness, especially near the front. Due to some asymmetry either of the tubes or the ears, the image, when facing the source, was not always exactly in front. It was necessary for some frequencies to turn through as much as 4° to bring it in front. This position was then used as the corrected zero. At moderately low

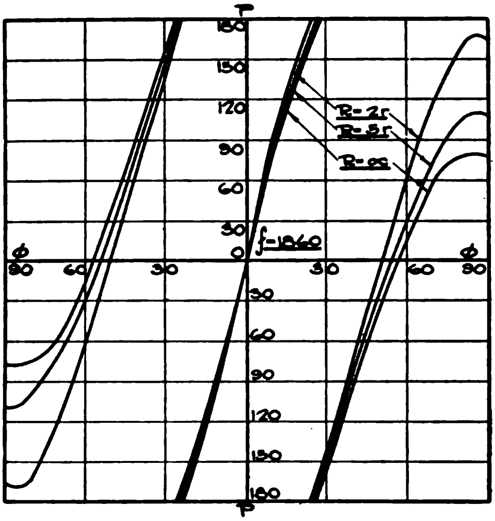

Fig. 5.

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[1] Bowlker, *loc. cit.*

frequencies the image moved around toward the side of leading phase, until at a point corresponding to a path difference of half a wave-length ($P = 180°$), it jumped to a similar position on the left. Continuing the rotation till this new image was straight in front the angle was generally found to agree quite closely with a path difference of a whole wave-length. The position of the image just before and after it crosses to the other side he calls the crossover angle. At frequencies of 263 and 335 he gives this as "90° (wide image)," which agrees with his results for unequal tubes. At higher frequencies it decreases progressively, having the following values; for 485 cycles, 40° to 50°; 690 cycles, 36°; 970 cycles, 18°, and 1290 cycles, 15°. He speaks of this cross-over angle as being "near the maximum displacement that phase will produce with the particular wave-length under observation."

At this point he reduced the separation of the openings to 15 inches so as to avoid the necessity of measuring very small angles. Repeating the experiment with 1,290 cycles he found for the cross-over-angle, 20°. He also observed that for all positions corresponding to phase differences between 180° and about 100° both images were present in the field at once. At 1,675 cycles he says: "Two images were evident during nearly the whole range—there were practically always two and sometimes three images evident, though I had some doubts whether the central image was always real or a result of attention to the two side images." In place of the cross-over angle he gives the maximum displacement as 12°.

He then reduced the apertures of the tubes from 2 inches to 0.6 inches. For a note of 2,090 cycles he says, "now three images could always apparently be heard together when facing one of them." The maximum displacement is here given as about 9°. At 2,310 cycles, "the image further to the left seemed stronger and tended to draw off one's attention." At 3,050 cycles, "it was very difficult to determine even approximately the position of an image, the one to the left of the two or three in the field of view seeming the loudest as a rule. I only felt sure that phase was still playing a part in fixing the maxima and minima which gave rise to the centers of the sound images."

The general agreement of these experiments with the theoretical results is obvious. The order of the changes in the images with increasing frequency is in striking agreement. First at low frequencies there is the progressive increase in the value of P to give maximum lateral displacement, accompanied by a narrowing of the region in which the image appears diffuse. This represents the range of frequency in which the maximum of the $P - \phi$ curve is less than 180°. Next comes a range in which the displacement of the image for $P = 180°$ is less than 90° and decreases progressively with increasing frequency. This corresponds to the diminishing value of ϕ for $P = 180°$, over the frequency range in which the values of maximum phase difference exceed 180°. Along with this comes the appearance of a second image on the opposite side, the part of the cycle for which it is present increasing progressively. When it has covered the whole cycle a third image appears, corresponding to the condition where the maximum value of P exceeds 360°.

The quantitative agreement is, however, not exact. The greatest angular displacement in the case of multiple images is always given as quite small and never reaches 90° as the theory would indicate. Also the frequencies at which the additional images appear are rather higher than the theoretical values. The second of these discrepancies is probably a result of the first. Suppose, for example, that in Fig. 4 the observer could not detect the second image when its displacement exceeded say 45°. The second image ($R = \infty$) would then appear when P reached 170° instead of 50° as it otherwise would. In order to have it appear at 50° under the assumed conditions it would be necessary to go to a higher frequency. That the second image should not have been detected under certain conditions is not surprising, when it is considered that practically all the other experimenters have failed to observe it at all. Furthermore, the assumptions underlying the theoretical calculations are such that we should not expect a very close agreement with experiment. The head is not a sphere nor is its circumference likely to be exactly 55 cm. The ears do not lie on a great circle and in most cases are not separated by exactly 165°. In fact, the differences, both physical and psychological, among the different observers are so great that there should be wide variations in the experimental results themselves. It should be strongly emphasized, therefore, that although fairly definite quantitative values have been given, it has been done chiefly for the sake of clearness in exposition, and there is no justification for using these values to predict with equal definiteness the results of any given experiment.

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AN EXTENSION OF THE ELECTRON THEORY OF METALS.
I. THERMOELECTRICITY AND METALLIC CONDUCTION.

BY A. E. CASWELL.

SYNOPSIS.—Using as a starting-point the simple equations deduced from the electron theory by J. J. Thomson and others for thermo E.M.F., thermoelectric power, Peltier E.M.F. and the Thomson effect, and assuming that the number of free electrons in unit volume of a metal is an exponential function of the temperature, the author shows that (1) the thermoelectric power is a linear function of the temperature, (2) the equations relating thermo E.M.F. and temperature, and Peltier E.M.F. and temperature, represent parabolas having their axes perpendicular to the axis of temperature, and (3) the Thomson effect is different for different metals and may be either positive or negative, but will usually have a positive temperature coefficient. Likewise, using the equations for electrical and thermal conductivity and making the second assumption that the number of positive centers with which the electrons collide changes with the temperature (these centers being atoms, molecules or clusters of molecules), it is found that (4) the electrical conductivity of pure metals decreases with increase of temperature but is not exactly inversely proportional to the absolute temperature, (5) the peculiar behavior of the electrical resistance of alloys can be accounted for, and (6) the thermal conductivity may either increase or decrease with the temperature, the temperature coefficient depending both upon the temperature and the material. A third assumption, viz., that the positive centers take part in the conduction of heat but not of electricity, leads to the conclusion that (7) the usual value deduced for the Wiedemann-Franz-Lorenz ratio is too small, and the variations in the value of this ratio at ordinary temperatures is accounted for. The above theoretical results, especially (1), (2) and (3), are, at least within the ordinary range of temperatures, in substantial agreement with experiment, since the constants involved in the theory can be determined from experimental data, and so the author concludes that (8) the concentration of electrons in a metal is an exponential function of the temperature, and (9) the number of positive centers changes with the temperature, the exact relation being somewhat uncertain. The number, however, must generally increase with the temperature.

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1. Introduction.

THE theory outlined in this paper is an attempt to amplify, or expand, the electron theory of metals as applied to thermoelectricity and metallic conduction so as to bring it more nearly into harmony with the experimental facts without at the same time doing violence to such portions of the theory as are not treated here. Three simple assumptions, relating the electrons, molecules and temperature, are made which will be studied in detail later. Certain mathematical forms are employed to express the assumed relations. It is not claimed that these expressions are more than first approximations to the true functions, neither is it claimed that they are even first approximations in the neighborhood of the absolute zero and temperatures near which marked changes in the structure of the material occur. It is claimed, however, that the theory presented here offers, at least within the range of ordinary temperatures, as good an explanation of the experimental facts here treated as previous theories do, and it explains some facts which have been unaccounted for hitherto.

At the outset the writer wishes to state that much of such merit as the present paper possesses is due to the fact that he has had the advantage of constant consultation with his colleague, Professor W. P. Boynton, whose familiarity with the kinetic theory of matter has made his numerous suggestions and criticisms of primary importance.

Throughout this discussion the symbols employed have the following meanings:

m = mass of an electron,
u = "mean square velocity" of the electrons,
T = absolute temperature,
t = Centigrade temperature,
α = the gas constant for a single electron, being defined by the equation $\frac{1}{2}mu^2 = \alpha T$,
ϵ = charge on an electron,
λ = mean free path of the electrons,
N = number of free electrons in one c.c. of a metal,
n = number of positive centers in one c.c. of a metal,
P = Peltier E.M.F.,
Q = thermoelectric power,
E = thermo E.M.F.,
σ = Thomson E.M.F., or so-called "specific heat of electricity,"
κ = electrical conductivity,
ρ = electrical resistivity, and
k = thermal conductivity.

The other symbols used are constants whose significance is obvious from the context. The subscripts 1 and 2 are used to refer to two different metals except where some other convention is specifically stated.

A theory to be completely satisfactory should account for the following facts.

1. The thermo E.M.F. between any two metals can usually be expressed by an equation of the form $E = At + \frac{1}{2}Bt^2$.

2. Similarly, the thermoelectric power of any two metals can usually be represented by a equation of the form $Q = A + Bt$.

3. The Peltier E.M.F. must satisfy the equation $P = QT$.

4. The Thomson effect may be either positive or negative and is different for different metals.

5. The Thomson effect has a temperature coefficient.

6. The electrical conductivity of pure metals at ordinary temperatures is almost inversely proportional to the absolute temperature, or the resistivity is nearly proportional to the absolute temperature. Usually the resistivity increases somewhat more rapidly than the absolute temperature.

7. At ordinary temperatures the electrical conductivity of alloys is frequently nearly independent of the temperature and occasionally increases with increase of temperature.

8. The thermal conductivity of a number of metals is nearly constant at ordinary temperatures and may either increase or decrease as the temperature is raised.

9. The ratio of the thermal to the electrical conductivity at all ordinary temperatures is nearly the same for all metals, the ratio being nearly proportional to the absolute temperature. In the case of pure metals the ratio usually increases somewhat faster than the absolute temperature, but in the case of alloys it usually increases more slowly.

Besides the facts just enumerated which are explained in the present paper there are number of galvanomagnetic, thermomagnetic and electronic emission phenomena which must ultimately be explained. No attempt is made to account for these effects, their treatment being reserved for a subsequent paper. So far as known none of the hypotheses advanced here in any way interferes with the explanation of any of these effects.[1]

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[1] For general discussions of the above phenomena, including the subject matter of the present paper, the reader is referred to J. J. Thomson's Corpuscular Theory of Matter, p. 49 et seq.; O. W. Richardson's The Electron Theory of Matter, p. 407 et seq.; E. Bloch's La Théorie Electronique des Métaux (in a series of essays published under the title Les Idées Modernes sur la Constitution de la Matière, Villars, Paris); K. Baedeker's Die Elektrischen Erscheinungen in Metallischen Leitern (Braunschweig); also N. R. Campbell's Modern Electrical Theory, pp. 54–86.

2. Previous Theories and Their Limitations.

The equations deduced from the electron theory by Sir J. J. Thomson, and which are taken as a starting-point in the present discussion, are:

$$P_{12} = \frac{2\alpha T}{3\epsilon} \log \frac{N_1}{N_2}, \tag{1}$$

$$Q_{12} = \frac{2\alpha}{3\epsilon} \log \frac{N_1}{N_2}, \tag{2}$$

$$\sigma = \frac{2\alpha}{3\epsilon} \left\{ T \frac{d}{dT} (\log N) - \frac{1}{2} \right\}, \tag{3}$$

$$\kappa = \frac{\epsilon^2 N \lambda u}{4\alpha T}, \tag{4}$$

and

$$k = \tfrac{1}{3} \alpha N \lambda u. \tag{5}$$

From equations (4) and (5) we obtain the Wiedemann-Franz law that the ratio k/κ is a constant for all metals at any given temperature and the Lorenz law that this ratio is proportional to T, which laws together give

$$\frac{k}{\kappa T} = \frac{4}{3} \left(\frac{\alpha}{\epsilon} \right)^2. \tag{6}$$

The thermoelectric equations satisfy the following equations which were deduced from thermodynamical considerations by Lord Kelvin, namely,

$$P = QT, \tag{7}$$

and

$$\sigma_1 - \sigma_2 = T \frac{dQ_{12}}{dT}. \tag{8}$$

The constant multiplying factors on the right-hand side of equations (1) to (5), inclusive, have been given different values by different theorists, but in any case they only differ by simple numerical factors. Thus Drude[1] replaces $2\alpha/3\epsilon$ by $4\alpha/3\epsilon$ in equations (1), (2) and (3), and Lorentz,[2] assuming the Maxwellian distribution of velocities among the electrons, finds instead of equation (4) the equation

$$\kappa = \sqrt{\frac{2}{3\pi}} \left(\frac{\epsilon^2 \lambda N u}{\alpha T} \right), \tag{4a}$$

and instead of equation (5) the equation

$$k = \frac{8}{9} \sqrt{\frac{2}{3\pi}} (\alpha N \lambda u). \tag{5a}$$

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[1] Ann. Phys., 1, 566, 1900 and 3, 369, 1900.

[2] Lorentz, The Theory of Electrons, pp. 63–67.

From (4*a*) and (5*a*) it follows that

$$\frac{k}{\kappa T} = \frac{8}{9}\left(\frac{\alpha}{\epsilon}\right)^2. \tag{6a}$$

The value of the ratio $k/\kappa T$ thus found is but two thirds of that found from the simple theory by Thomson and Drude. Since these differences in multiplying constants are of only secondary importance in connection with the present discussion they will be disregarded for the present. It may be remarked, however, that experimental data may, with the aid of the theory presented in this paper, enable one to discriminate between the different methods of approach to the problem and to select that one which agrees best with experiment.

It has usually been assumed either that N remained constant or else was either directly or inversely proportional to the square root of the absolute temperature. The first assumption was made in order to explain the optical properties of metals, the second was made by J. J. Thomson to account for the Thomson effect, and the last was made to account for the fact that at ordinary temperatures the temperature coefficient of resistance of a number of metals is approximately equal to 1/273 per Centigrade degree. From equation (4) it follows that if the electrical conductivity is inversely proportional to the absolute temperature, either N or λ must be inversely proportional to the square root of the absolute temperature, since $u = \sqrt{2\alpha T/m}$. We see no reason why λ should vary to any appreciable extent if the number of molecules does not change and the alteration in volume is quite small. Then N must change with temperature. But from the optical properties of metals it seems that N does not change.[1] So one has his choice: either N changes or it does not. But whether N is constant or is proportional to some power of the absolute temperature, the proportionality factor depending upon the material, certain results follow which are not in harmony with experiment. Some of these discrepancies are:

1. The thermo E.M.F. is a linear function of the temperature.
2. The thermoelectric power is constant for any given pair of metals.
3. The Thomson effect is the same for all metals at all temperatures.
4. The thermal conductivity of any metal is independent of the temperature.
5. The ratio $k/\kappa T$ is the same for all metals at all temperatures and within the range of ordinary temperatures is usually too small.

On the whole, the most that can be said for the theories so far published is that they show that these properties are to be expected, but they are wholly inadequate when it comes to quantitative measurements.

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[1] Drude, Ann. Phys., 14, 951, 1904.

3. Statement of the New Theory.

The fundamental assumptions upon which this theory is based are

(1) That the number of free electrons in a metal *changes* with the temperature,

(2) That the number of positive centers with which the electrons collide, whether these centers are atoms, sub-atoms, molecules or groups of molecules, *changes* with the temperature, and

(3) That these positive centers take part in the transfer of heat but not of electricity.

The mathematical forms which have been selected to express the relations between the numbers of electrons and positive centers are

$$N = ae^{xT}, \tag{9}$$

and

$$n = be^{yT} \tag{10}$$

where a and b are positive constants, and nothing is postulated regarding the signs of x and y.

We are led to the first assumption by the fact that unless equations (1), (2) and (3) are entirely wrong, N must depend upon T. The exponential form of equation (9) is suggested by the logarithmic form of these three equations. The excellent agreement of the resulting equations with experiment indicates the probability that equation (9) is an accurate representation of the facts.

The assumption that the number of positive centers with which the electrons collide changes with the temperature is somewhat startling.[1] For our purposes it is unimportant whether these centers are atoms, sub-atoms, molecules or clusters of molecules. A great deal of uncertainty exists as to the states of aggregation in solids and liquids, and there is no *a priori* reason why the state of aggregation in a substance should not vary with the temperature of the substance. It is not necessary that all the centers shall be alike. A continual process of disintegration and recombination of the centers may be going on. If a theory based on this assumption should harmonize better with experiment than others, this circumstance would lend color to the assumption.

Remembering that, as in the kinetic theory of gases, $\lambda = f/nr^2$, where r is the radius of the positive centers and f is a constant, and substituting be^{yT} for n in equation (10), we obtain

$$\lambda = ce^{-yT}, \tag{11}$$

where $c = f/br^2$.

When $T = 0$, $N = a$, $n = b$ and $\lambda = c$. But when $T = \infty$, then

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[1] Cf. Richardson's The Electron Theory of Matter, p. 467.

N and n are both infinite, and $\lambda = 0$. In practice, of course, T never approaches infinity, and since both x and y prove to be small, the values of N and n at, say, 2000° C. will probably seldom exceed 50 to 100 times the corresponding values at the absolute zero.

The third assumption seems reasonable from the fact that insulators conduct heat even though they do not conduct electricity, and have very few, if any, free electrons.

The only assumption involved in the theory of thermoelectricity here presented is the first, namely, that the number of free electrons is a function of the temperature. The second assumption is introduced to account for electrical conductivity, namely, that the number of positive centers is a function of the temperature. The third assumption, that the positive centers take part in the conduction of heat, is involved only in those portions of the theory which deal with thermal conductivity.

Equations (1) to (5), inclusive, are taken as the starting-points in the present discussion. This is not to be interpreted as indicating a preference on the part of the writer for one physicist's theory rather than that of another. In the light of our present knowledge, however, it seems that these equations are likely to be as valid as any of the others. Changing the constant factors has no effect upon the qualitative results of the theory presented here, and only to a minor extent upon the quantitative results.

4. PELTIER E.M.F.

Sir J. J. Thomson obtained equation (1) given above for the Peltier E.M.F. by assuming that electrons under the influence of an electric force flow across the boundary between two metals until the electric force is in equilibrium with the opposing pressure gradient at the boundary. The expression for the Peltier E.M.F. on the basis of the present extension of the simple theory is readily obtained by substituting in equation (1) the values of N_1 and N_2 given in equation (9), namely, $N_1 = a_1 e^{x_1 T}$ and $N_2 = a_2 e^{x_2 T}$. This gives

$$P_{12} = \frac{2\alpha T}{3\epsilon}\left\{\log\frac{a_1}{a_2} + (x_1 - x_2)T\right\}. \tag{12}$$

This equation may be rewritten in the form

$$P = AT + BT^2, \tag{12a}$$

or

$$P = P_0 + (273B + Q_0)t + Bt^2, \tag{12b}$$

where

$$A = \frac{2\alpha}{3\epsilon}\log(a_1/a_2),\ B = \frac{2\alpha}{3\epsilon}(x_1 - x_2),\ Q_0 = A + 273B,\ P_0 = 273Q_0,$$

and $t = T - 273$. P_0 is the Peltier E.M.F. at 0° C. and, as we shall see in the next section, Q_0 is the thermoelectric power of the pair of metals at 0° C.

The equations (12), (12*a*) and (12*b*) represent a parabola with its axis perpendicular to the axis of temperature. We should expect the Peltier E.M.F. to have a maximum value at the absolute temperature.

$$T' = -A/2B, \tag{13}$$

or in Centigrade degrees,

$$t' = -A/2B - 273 = -(273B + Q_0)/2B. \tag{13a}$$

The value of the Peltier E.M.F. at the temperature T' should be

$$P' = -A^2/4B. \tag{14}$$

If we knew the experimental form of the parabola it would be possible for us to calculate the values of A and B, and from these values we could obtain the value of the ratio a_1/a_2 and also of the difference $(x_1 - x_2)$. In practice, however, it will be very much simpler to obtain these values from the thermo E.M.F.

5. THERMOELECTRIC POWER.

The equations for the thermoelectric power are readily obtained from the corresponding ones for the Peltier E.M.F. by dividing by T (see equation (7)). From equations (12), (12*a*) and (12*b*) we thus obtain

$$Q_{12} = \frac{2\alpha}{3\epsilon}\left\{\log\frac{a_1}{a_2} + (x_1 - x_2)T\right\}, \tag{15}$$

$$Q = A + BT, \tag{15a}$$

and

$$Q = Q_0 + Bt, \tag{15b}$$

where A, B and Q_0 have the same values as in the preceding section. These equations represent quite accurately, by a suitable choice of constants, the thermoelectric power of a thermocouple consisting of almost any pair of metals.

6. THERMO E.M.F.

The expression for the thermo E.M.F. is obtained from that for the thermoelectric power by integrating the latter with respect to T. If E is the thermo E.M.F. between two metals 1 and 2, when their junctions are at the temperature T_1 and T_2, then

$$E = \int_{T_1}^{T_2} Q\,dT,$$

whence

$$E = \frac{2\alpha}{3\epsilon}\left\{\left(\log\frac{a_1}{a_2}\right)(T_2 - T_1) + \frac{1}{2}(x_1 - x_2)(T_2^2 - T_1^2)\right\}. \tag{16}$$

If we put $T_1 = 0$ and $T_2 = T$, then equation (16) becomes

$$E = \frac{2\alpha}{3\epsilon}\left\{T\log\frac{a_1}{a_2} + \frac{1}{2}(x_1 - x_2)T^2\right\}, \tag{17}$$

where E is the thermo E.M.F. between two junctions at 0° and $T°$ absolute, respectively. Equation (17) may be rewritten in the form

$$E = AT + \tfrac{1}{2}BT^2. \tag{17a}$$

Substituting $t + 273°$ for T_2 and 273 for T_1 in equation (16) we obtain

$$E = Q_0t + \tfrac{1}{2}Bt^2, \tag{16a}$$

where E is the thermo E.M.F. between two junctions at 0° C. and $t°$ C., respectively.

Equations (17), (17*a*) and (16*a*) represent a parabola with its axis perpendicular to the axis of temperature. The maximum value of E occurs at the neutral temperature, which is therefore given by the equation

$$T' = -A/B, \tag{18}$$

or

$$t' = -Q_0/B. \tag{18a}$$

The thermo E.M.F. of the couple when one junction is at 0° C. and the other is at the neutral temperature is given by the equation

$$E' = -Q_0^2/2B. \tag{19}$$

Experimentally it is found that the thermo E.M.F. between any two metals is quite accurately represented by a parabola with its axis perpendicular to the axis of temperature, that is, by equations of the type deduced above, where A and B may either positive or negative. Thus we see that the equations here deduced for thermoelectric power and thermo E.M.F. are in agreement with the experimental facts. And since equation (7) has been shown to be true both experimentally and theoretically, the equations for the Peltier E.M.F. must also be in agreement with the experimental facts. From our theory we see that A is positive or negative according as a_1 is greater or less than a_2. Similarly, B is positive or negative according as x_1 is greater or less than x_2.

From the known values of A and B obtained from experiment we may calculate the values of a_1/a_2 and $(x_1 - x_2)$ by means of the following equations:

$$a_1/a_2 = e^{3\epsilon A/2\alpha}, \tag{20}$$

and

$$x_1 - x_2 = 3\epsilon B/2\alpha. \tag{21}$$

It may be remarked in passing that if N were a constant different for different metals, or if N were proportional to any power of the absolute temperature, the proportionality factor being different for different metals, then the thermoelectric power would be constant and the thermo E.M.F. would be proportional to the difference between the absolute temperatures of the two junctions. These conclusions, however, are contrary to experiment.

7. Thomson Effect.

When we substitute ae^{xT} for N in equation (3) we obtain

$$\sigma = \frac{2\alpha}{3\epsilon}(Tx - \tfrac{1}{2}). \tag{22}$$

Since for practically all metals the absolute value of σ is considerably less than $\alpha/3\epsilon$, it follows that x must be positive. Hence, we may conclude that *in order to account for the Thomson effect it is necessary to assume that the number of free electrons in a metal increases with the absolute temperature.* So long as x is positive, σ will have the same sign as $(Tx - \tfrac{1}{2})$, and the absolute value of σ, when it is negative, cannot be greater than $\alpha/3\epsilon$. On the other hand, σ may have any positive value when x is positive. In the case of certain alloys[1] it appears that σ is negative and greater than $\alpha/3\epsilon$. In such cases x must be negative.

Differentiating equation (22) with respect to T, we obtain

$$\frac{d\sigma}{dT} = \frac{2\alpha x}{3\epsilon}. \tag{23}$$

From equation (23) it appears that *the Thomson effect has a temperature coefficient, and that this temperature coefficient is normally positive.* Qualitatively at least, this result is in harmony with some of the best experimental results such, for example, as those obtained by Berg[2] on copper. In the case of the alloys mentioned above, which have abnormally high negative values of the Thomson effect, Laws found that the temperature coefficients were negative. This is in harmony with our theory.

From the value of the Thomson effect at any temperature it is possible to calculate the value of x. Solving equation (22) for x we obtain

$$x = \frac{3\epsilon\sigma + \alpha}{2\alpha T}. \tag{24}$$

Generally speaking, Tx cannot be very far different from $\tfrac{1}{2}$ at ordinary temperatures, since σ is usually quite small in comparison with $\alpha/3\epsilon$. This being so, let us assume that $Tx = \tfrac{1}{2}$ and that $T = 300°$ absolute (27° C.). Then $x = 0.001667$. Substituting this value of x in equation

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[1] Laws, Phil. Mag., 7, pp. 560–578, 1904, and Caswell, Phys. Rev., N.S., XII., pp. 231–237, 1918.

[2] Ann. Phys., 32, pp. 477–519, 1910.

(9), we find the ratio of the value of N at any temperature $(T + 1)$ to its value at the temperature T to be 1.0017. Or, to use a somewhat larger temperature interval, the ratio of N at $(t + 100)°$ C. to its value at $t°$ C. is 1.181. For a temperature interval of 1000° C. the ratio is 5.294. Assuming 0.002 as a probably maximum value of x for a pure metal, we find the corresponding values of the ratio for a temperature interval of one degree Centigrade to be 1.0020, for a 100° C. interval 1.2214, and for a 1000° C. interval 7.389.

If our theory is correct, the Thomson effect is a linear function of the temperature which approaches the common value for all metals at the absolute zero of $\sigma = -\alpha/3\epsilon$. In case we should have used $4\alpha/3\epsilon$ instead of $2\alpha/3\epsilon$ in equations (1), (2) and (3), then σ should have a different limiting value. The same is true if any other factor is used instead of $2\alpha/3\epsilon$. If this limiting value of σ can be determined, then we have a means of selecting the best from among the different equations proposed.

It follows at once from equation (22) that

$$\sigma_1 - \sigma_2 = \frac{2\alpha T}{3\epsilon}(x_1 - x_2), \tag{25}$$

or

$$\sigma_1 - \sigma_2 = BT, \tag{25a}$$

these equations being in the form of equation (8). It is obvious that the metal having the more positive value of x will have the more positive value of σ, and *vice versa*. Consequently, if we were to arrange a series of metals in the order of the slope of the corresponding lines in a thermoelectric power diagram, the metals should also be arranged in the order of their Thomson effects at any given temperature.

8. ELECTRICAL CONDUCTIVITY.

In the preceding sections the only assumption which has been employed is that the number of free electrons is a function of the temperature, and the satisfactory agreement between the theory and experiment seems to prove the wisdom of selecting the expression ae^{xT} to represent this number at any temperature T. No assumption was made as to the sign of x, but we have found that to account for the Thomson effect in most substances x must be positive. To this assumption we now add a second, namely, that the number of positive centers is a function of the temperature, and, in consequence, the mean free path of the electrons is a function of the temperature. Any discrepancies between theory and experiment which appear in this section must, therefore, arise not from the first assumption, but from the second.

Substituting the value of N from equation (9), the value of λ from equation (11), and $u = (2\alpha T/m)^{1/2}$ in equation (4), we obtain as the value of the electrical conductivity

$$\kappa = \frac{\epsilon^2 ac}{2\sqrt{2\alpha m}} e^{(x-y)T} \cdot T^{-1/2}, \tag{26}$$

which we shall rewrite in the form

$$\kappa = Ce^{-zT} \cdot T^{-1/2}, \tag{26a}$$

where $C = \epsilon^2 ac/2(2\alpha m)^{1/2}$, and $z = y - x$. Both C and z are constants for any given metal.

Since the electrical conductivity of pure metals within the range of ordinary temperatures is roughly inversely proportional to the absolute temperature, it follows from equation (26a) that the exponential factor on the right-hand side of the equation must decrease with increase of temperature. In order that this shall be so, z must be positive or $(x - y)$ must be negative. But since x is positive in the case of pure metals, y must also be positive and greater than x. Hence, we conclude that *in the case of pure metals both the number of free electrons in the metal and the number of positive centers with which they collide increases with the temperature, the number of positive centers, in general, increasing faster than the number of electrons.*

In the case of alloys the electrical conductivity cannot be said to bear any general relation to the temperature. In fact in the case of some alloys, such as manganin, the temperature coefficient is exceedingly small. In such cases the exponential factor may increase with the temperature. That is, if x is positive, then y is either positive and less than x or else y is negative, but if x should happen to be negative, as we have seen is probable in the case of some alloys, then y is also negative but numerically greater than x. The case when both x and y are positive seems the most likely to occur, since this is the rule for pure metals. In any case we may conclude that *in some substances, particularly alloys, the number of free electrons increases with the temperature faster than the number of positive centers with which they collide.*[1]

The electrical conductivity of carbon, which though not a metal conducts metallically, increases rather rapidly with the temperature. This means that z for carbon has a comparatively large negative value. This probably indicates a large positive value of x with a smaller positive, or possibly negative, value of y. An abnormally large value of x corresponds to an abnormally large positive value of the Thomson effect.

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[1] A good account of the electrical properties of alloys is given in Baedeker's Elektrischen Erscheinungen in Metallischen Leitern. Lord Rayleigh (Scientific Papers, Vol. IV., p. 232) has suggested that the resistance of alloys is unduly high owing to a "false resistance" arising from the Peltier effect between non-homogeneous parts of the metal.

Since ρ is the reciprocal of κ it follows that

$$\rho = \frac{1}{C} e^{zT} \cdot T^{1/2}, \tag{27}$$

whence

$$z = \frac{1}{T} (\log C + \log \rho - \tfrac{1}{2} \log T). \tag{28}$$

If ρ is proportional to T, as is assumed but is not strictly true to fact, then

$$z = \frac{1}{T} (\log (CD) + \tfrac{1}{2} \log T), \tag{28a}$$

where ρ is put equal to DT.

From equation (28a) it appears that either ρ is not proportional to T or else z is not constant as assumed, but is a function of T. Whenever the value of z is greater than the right-hand side of equation (28a) the resistivity increases faster than the absolute temperature, but when the reverse is true the resistivity increases more slowly than the absolute temperature.

From equation (27) it is obvious that the resistivity of a metal approaches zero at the absolute zero, and increases continuously with the temperature providing z is positive. In case z is negative the resistivity should have a maximum value when $T = -(1/2z)$.

Since the factors a and c, which are contained in the value of C, are as yet undetermined, we cannot use equation (28a) to calculate the value of z. But since we know how ρ varies with T we may write

$$\frac{\rho_1}{\rho_0} = e^{z(T_1 - T_0)} \left(\frac{T_1}{T_0} \right)^{1/2}, \tag{29}$$

where ρ_1 and ρ_0 are the resistivities at T_1° and T_0° absolute, respectively. Putting $T_1 = T_0 + 1$, and solving for z, we obtain

$$z = \log \frac{\rho_1}{\rho_0} - \tfrac{1}{2} \log \left(1 + \frac{1}{T_0} \right). \tag{29a}$$

If we denote the temperature coefficient of resistance at T_0 by β, we may rewrite equation (29a) thus:

$$z = \log (1 + \beta) - \tfrac{1}{2} \log \left(1 + \frac{1}{T_0} \right), \tag{29b}$$

or as a first approximation which gives a result about one half per cent. too high

$$z = \beta - \frac{1}{2T_0}. \tag{29c}$$

We find the temperature coefficient of resistivity of copper given as 0.00428 at 18° C., *i. e.*, $T_0 = 291$, and so z is found to be 0.002545 if we use equation (29*b*), but is found to be 0.00256 if we use equation (29*c*).

From the ratio of the resistivities of two metals at any temperature T we may determine the ratio c_1/c_2, thus:

$$\frac{c_1}{c_2} = \frac{\rho_2 a_2}{\rho_1 a_1} e^{(z_1 - z_2)T}. \tag{30}$$

Since all the quantities on the right can be determined the ratio c_1/c_2 can be calculated. It should be noted, however, that the numerical values which we obtain for a_1/a_2, and consequently for c_1/c_2, depend upon whether we use $2\alpha/3\epsilon$ or some other multiplying factor in equations (1), (2) and (3).

9. Thermal Conductivity.

We now proceed to introduce the third assumption, namely, that the molecules or, more generally, the positive centers, whether these are atoms, sub-atoms, molecules or groups of molecules, take part in the transmission of heat, although they have no part in the transmission of electricity in solid bodies. In the simple equation (5), namely, $k = \frac{1}{3}\alpha N\lambda u$, N, λ and u refer specifically to the electrons. A fundamental assumption of the electron theory is the equi-partition of energy among the electrons and molecules, but there are certain reasons why we are at present unable to assign values to the molecular quantities corresponding to N, λ and u. Indeed for our present purpose it is unimportant whether such values can be assigned. Obviously, there is the probability that heat is transmitted by the molecules, or positive centers, by radiation as well as by conduction. What fraction of the total heat transmitted is transmitted by radiation we are unable to determine. For the present it remains an open question whether the equi-partition of energy law applies to the molecules or to the positive centers.

It is rather obvious that the mean free paths and the velocities of the positive centers, or of the molecules, will vary in the same directions as λ and u, respectively. We shall assume, for simplicity, that λ_1 is directly proportional to λ, and that u_1 is directly proportional to u, where λ_1 is the mean free path of the positive centers, or molecules, as the case may be, and u_1 is the corresponding mean square speed. We may then rewrite equation (5), substituting $N + n$ for N and introducing a constant ω which includes the proportionality factors connecting λ and λ_1, and u and u_1, and also the ratio of the total amount of heat transmitted by the molecules to that conducted by them. We thus obtain

$$k = \frac{1}{3}\alpha u\lambda(N + \omega n). \tag{31}$$

Equation (31) simply means that in view of our ignorance of the behavior of these positive centers, we feel justified in assuming that the fraction of the total heat transmitted by them is proportional to their number.

Replacing N by ae^{xT}, n by be^{yT}, λ by ce^{-yT} (see equations (9), (10) and (11)), and u by $(2\alpha T/m)^{1/2}$, equation (31) reduces to

$$k = \frac{\sqrt{2\alpha}}{3\sqrt{m}} ac \left(e^{(x-y)T} + \frac{\omega b}{a} \right) T^{1/2}. \tag{32}$$

It is at once apparent that in the case of pure metals the exponential factor in the first term on the right-hand side of equation (32) tends to make k decrease as T increases, since $(x - y)$ is negative, while the factor $T^{1/2}$ tends to make both terms increase as T increases. So k may either increase or decrease with increase in temperature.

10. THE WIEDEMANN-FRANZ AND LORENZ LAWS.

We have already seen that the Wiedemann-Franz law coupled with that of Lorenz leads to the relation that $k/\kappa T$ = *a constant*. Substituting the values of k and κ from equations (32) and (26), respectively, we obtain

$$\frac{k}{\kappa T} = \frac{4\alpha^2}{3\epsilon^2}\left(1 + \frac{\omega b}{a} e^{yT} \right). \tag{33}$$

When $\omega = 0$, this equation reduces to equation (6) as a special case. Equation (33) leads us to expect that the experimentally determined value of the ratio will always be greater than that calculated from the simple theory. Assuming the validity of equation (6), the experiments of Jaeger and Diesselhorst[1] on thirteen metals and three alloys gave values in every case, except that of aluminum, greater than we should expect from the simple theory; but if we assume the validity of equation (6*a*) the discrepancy is much greater, even aluminum being about thirty per cent. too high. It has been objected that the amount of heat transmitted by the positive centers, whatever they may be, must be much too small to account for the discrepancies between Jaeger and Diesselhorst's experiments and the simple theory. But this objection is based on the assumption that the heat so transmitted must be approximately the same as that transmitted by an insulator. This does not necessarily follow. If there are practically no free electrons in an insulator and there are large numbers of them in conductors, is it not reasonable to suppose that there may be more freedom of movement among the molecules in a conductor than among those in an insulator?

The minimum value which the right-hand side of equation (33) can have is $4\alpha^2(1 + \omega b/a)/3\epsilon^2$. This seems out of harmony with the results obtained by Meissner[1] at very low temperatures, since he found that for copper at 20° absolute the value of the ratio $k/\kappa T$ was but one seventh of its value at 0° C., the value at 0° C. being only slightly greater than $4\alpha^2/3\epsilon^2$.

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[1] Sitzungsber. Berlin, 1899, p. 719.

If the subscripts 1 and 2 refer, respectively, to the temperature T_1 and T_2, then

$$\frac{k_2/\kappa_2}{k_1/\kappa_1} = \frac{\left(1 + \frac{\omega b}{a} e^{zT_2}\right) T_2}{\left(1 + \frac{\omega b}{a} e^{zT_1}\right) T_1}. \tag{34}$$

According to the simple theory this should be

$$\frac{k_2/\kappa_2}{k_1/\kappa_1} = T_2/T_1. \tag{34a}$$

Jaeger and Diesselhorst, working between the temperatures of 18° C. and 100° C., *i. e.*, between 291° and 373° absolute, found instead of the theoretical value of the ratio on the left-hand side of equation (34*a*), viz., $T_2/T_1 = 1.28$, that eight of the thirteen metals tested gave values greater than 1.28, two of them, platinum and palladium, giving values of 1.35, two gave values of 1.28, one of 1.27, one of 1.26 and one, bismuth, of 1.12. The three alloys tested gave results less than 1.28. Whenever y is greater than x, as we have seen is the case in pure metals, equation (34) leads us to expect the ratio to be greater than 1.28. In the majority of cases this seems to be true. But in the case of an alloy, such as manganin, which has a very small temperature coefficient of resistance, x is greater than y, and equation (34) leads us to expect the ratio to be less than 1.28. This is actually the case. In the case of manganin the ratio is 1.21.

11. Discussion of the Theory.

In this section the author wishes to make some general observations concerning the theory here presented. In the first place, we see that, since the equations are of the proper form and the values of the constants involved can be determined from experimental data, the agreement of the theory with the phenomena of the Peltier and Seebeck effects is not only qualitative but quantitative as well. Not only do the values of the constants so determined indicate that, in general, the number of free electrons increases with the temperature, but this is what we might expect, since the kinetic energy and, consequently, the agitation of the molecules increases with the temperature.

In a previous paper the author has shown that either (1) the electrons which take part in thermoelectric phenomena are different from those which take part in electric conduction, or (2) the mean free path of the electrons in an alloy is noticeably different from that in a pure metal, one of the constituents of the alloy. The former statement seems unreasonable, while the latter is plausible if the formation of an alloy involves the regrouping of the atoms or molecules. Thus the writer was led to the conclusion that in some way a change is brought about in the mean free path of the electrons. The introduction of the exponential value of N from equation (9) into equation (4) makes the resulting change in the electrical conductivity in the opposite direction to that in which it actually occurs. So we have adopted the hypothesis that the mean free path of the electrons as well as their number is a function of the temperature. The change in the mean free path may be thought of as being brought about in either of two ways, always assuming that the number of collisions between electrons is negligible in comparison with the number between electrons and the positive centers. Either the size of the positive centers may change or their number. Assuming that the size changes but not the number we are led to conclusions which do not agree as well with experiment as do the conclusions which we arrive at by assuming that the number of positive centers is a function of the temperature. The author is inclined to believe that the exponential form of the function which has been employed in this paper is a fair representation of the facts, if temperatures near which marked changes in the structure of the metal occur, such as eutectic-points and melting-points, are excluded. In this connection it should be noted that the Peltier E.M.F. and the thermo E.M.F., both of which we assume depend only upon the concentration of the electrons, are not discontinuous functions of the temperature at the melting-point, but it appears that the electrical conductivity of a metal changes markedly as it passes from the solid to the liquid state. In the case of a number of metals, *e. g.*, cadmium, lead, potassium, sodium, tin and zinc, the electrical conductivity in the solid state at the melting-point is about one half the value in the liquid state at the same temperature. Exceptions are bismuth and antimony. In both of these the conductivity is approximately doubled as the substances passes from the solid to the liquid state. The usual case, that of decrease in conductivity in passing from the solid to the liquid state, is apparently due to a sudden increase in the number of positive centers. This is what one might expect since there is, apparently, a reduction in the forces of cohesion.

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[1] Deutsch. Phys. Gesell., Verh. 16, 5, pp. 262–272, 1914.

It seems probable that at low temperatures the molecules of a metal are gathered in clusters which act as single positive centers and that as the temperature is raised the increased activity of the molecules causes these clusters to break up slowly. The complete theory should, therefore, take account of the change in the cross-sections of the clusters as they break up as well as the change in their number. What the change in the cross-section is must depend upon the number and arrangement of the molecules in each cluster, and any general expression must represent a mean rate of change in the cross-sections. Since the number of positive centers must increase faster than their average cross-section decreases, the change in the former is more important than that in the latter.

In the theory as developed in this paper no allowance has been made for the expansion of the metal as the temperature is raised. The complete theory ought to take account of this expansion. The correction, however, is likely to be quite small.

It is obvious that an exhaustive study of the quantitative relations of this theory and experiment ought to be made. We have shown that qualitatively it is capable of explaining all of the phenomena considered if we exclude very low temperatures. It is also capable of explaining each of these phenomena separately quantitatively, but it remains to be seen to what extent the values of the constants involved are identical when different phenomena are compared. The writer is now carrying out the necessary computations to put the theory on a quantitative basis and hopes to publish the results of this work in the near future.

Physical Laboratory,
University of Oregon,
February, 1919.

THE AUDION AS A CIRCUIT ELEMENT.

By H. W. Nichols.

Synopsis.—The functional equations for the three-element audion are used to deduce the actions taking place in any circuit containing the audion. The equations for small variations in the general network containing audions are developed and simple equivalent circuits to replace the tube are shown. Next the effects of the capacities between elements are worked out and also some examples of amplifiers and oscillation generators.

THE three-element thermionic tube has come into such general use in radio telegraphy and telephony as well as in wire telephony and other fields that it seems worth while at the present time to publish some of those relations, holding between its characteristics and those of the connected electrical circuits, which are used in the design of amplifying, oscillating and detecting systems.

In this paper will be considered only those actions which may be sufficiently well described by supposing the variations in plate and grid currents and voltages to be small. In this discussion the methods adopted in a previous paper[1] will be shown in their application to vacuum tube circuits, in which application, as a matter of fact, their greatest practical use has been found.

The first figure shows a conventional circuit drawing of an audion in which the grid and plate potentials, with filament potential as zero, are denoted by v and V respectively, while the corresponding currents are denoted by i and I. A discussion of the operation of this device is unnecessary, since several papers have appeared describing the construction and operation in considerable detail. It has been found that, as long as there is an excess of electrons supplied by the filament so that temperature saturation is maintained, the plate current is represented with considerable accuracy by a function of a *single* variable, thus:

$$I = F(V + \mu v + c),$$

in which c is a constant which depends partly upon the contact differences of potential of the elements in the tube and μ is another constant which is characteristic of the structure. In many forms which the device may take the plate current is very approximately proportional to the square of the variable above, but for the purpose of this discussion it is not necessary to so limit it.

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[1] Phys. Rev., Aug., 1917.

The grid current depends also upon the two potentials, but its functional form is not known. For this discussion it is sufficient to recognize that it is such a function and write it:

$$i = f(V, v).$$

The problem to be considered in this paper is now the following. An audion is connected into an electrical network in any manner, that is, its three terminals are joined by any system of impedances; certain grid and plate voltages are impressed upon it with the result that certain currents, determined by the functional equations above, flow to plate and grid; and then it is required to describe the small variations about this steady state which will take place when the system is acted upon by arbitrary electromotive forces or otherwise disturbed. This is the problem of amplifiers and oscillation generators with the limitation that the amplitude of oscillation shall be small. While this limitation is serious in some cases, for example if it is desired to find the maximum power output of the device, still a great deal of information concerning the operation of the actual device may be obtained by thus studying the ideal case, which, in fact, is the only one which lends itself to simple and general treatment from the point of view of electrical networks.

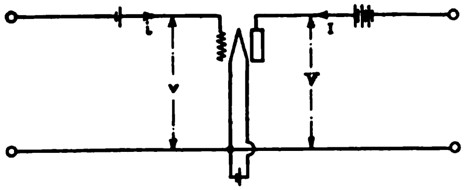

Fig. 1.

In Fig. 2 is shown the audion connected into the network. In this figure the impedances Z_1, Z_2, Z_3, may be of any form whatever and are not restricted to single conducting paths as indicated. They are supposed only to be such circuits that it is possible to write an equation of the form $ZI = e$, connecting the current entering the terminals and the E.M.F. producing this current when the circuit is isolated. Certain electromotive forces, E, are supposed acting in each branch and the three circuits are coupled to one another by mutual impedances, M_{jk}, between impedance j and impedance k. These mutual impedances are defined, as usual, by the statement that a unit current in branch j induces an E.M.F. M_{jk} in branch k. Both self and mutual impedances are in the general case operators, functions of $p = d/dt$, as used in the paper above cited.

If in this network the three currents are chosen as shown, we may wirte down the network equations in terms of them as below:

$$\begin{aligned} Z_1(I_1 + I_3) + v &= E_1 + M_{12}(I_2 - I_3) - M_{13}I_3 \\ Z_2(I_2 - I_3) + V &= E_2 + M_{12}(I_1 + I_3) + M_{23}I_3 \\ Z_3I_3 + V - v &= - M_{13}(I_1 + I_3) + M_{23}(I_2 - I_3). \end{aligned}$$

These equations are of course exact and hold for all conditions; however, because they are not linear, their solution is somewhat complicated and it is preferable to obtain the first and approximate idea of the behavior of the network by considering only small changes in the currents from their steady or mean values. This amounts to considering current variations over a part of the characteristic curve so small that it may be taken to coincide with the tangent at the given operating point and consequently the solutions obtained will not describe any effects which depend upon the curvature of the characteristic, such effects, for example, as the detection of signals.

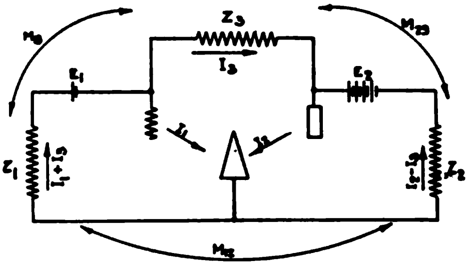

Fig. 2.

Suppose then that in the departure from the steady values I_1 changes to $I_1 + x_1$, etc., these changes being small and produced by superposed electromotive forces e_1, etc. We shall then have

$$\delta v = \frac{\partial v}{\partial I_1} x_1 + \frac{\partial v}{\partial I_2} x_2, \qquad \text{etc.}$$

Substituting these values in the network equations, there will result a set of linear differential equations, with independent variable t, of the type:

$$Z_1(x_1 + x_3) + \frac{\partial v}{\partial I_1} x_1 + \frac{\partial v}{\partial I_2} x_2 = e_1 + M_{12}(x_2 - x_3) - M_{13}x_3$$

. . . .

which set may be conveniently represented in the following tabular form:

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
Z_1+\frac{\partial v}{\partial I_1} & -M_{12}+\frac{\partial v}{I_2\partial} & Z_1+M_{12}+M_{13} & e_1 \\
-M_{12}+\frac{\partial V}{\partial I_1}\ . & Z_2+\frac{\partial V}{\partial I_2} & -Z_2-M_{12}-M_{23} & e_2 \\
-M_{13}+\frac{\partial V}{\partial I_1}-\frac{\partial v}{\partial I_1} & -M_{23}+\frac{\partial V}{\partial I_2}-\frac{\partial v}{\partial I_2} & Z_3+M_{13}+M_{23} & e_3
\end{array}$$

These are the network equations for small disturbances but it is convenient to throw them into another form which takes account of the physical meaning of some of the terms. To do this, note that $\partial v/\partial I_1$ is evidently the resistance, r, of the grid circuit under the given operating conditions, and $\partial V/\partial I_2$ is the resistance, R, of the plate circuit under the same conditions. From the characteristic equations we find by the usual methods, the further relations:

$$\frac{\partial v}{\partial I_1}=\frac{F'}{J}=r, \qquad \frac{\partial v}{\partial I_2}=\frac{-1}{J}\frac{\partial f}{\partial V},$$

$$\frac{\partial V}{\partial I_1}=-\frac{\mu F'}{J}=-\mu r, \qquad \frac{\partial V}{\partial I_2}=\frac{1}{J}\frac{\partial f}{\partial v}=R,$$

where

$$J=F'\left(\frac{\partial f}{\partial v}-\mu\frac{\partial f}{\partial V}\right).$$

Now since $\partial V/\partial I_1$ is the E.M.F. induced in the plate circuit through the tube by unit current in the grid circuit and rx_1 is the corresponding increment, δv, in grid potential, it follows from the above relations that one action of the tube is to introduce into the plate circuit an effective E.M.F. equal to μ times the grid voltage variation.

Also, since $\partial v/\partial I_2$ is the E.M.F. introduced into the grid circuit through the tube by unit current in the plate circuit, we find that another action of the tube is to introduce the voltage

$$\frac{\partial v}{\partial I_2}x_2=-\frac{1}{J}\frac{\partial f}{\partial V}x_2=-Rx_2\frac{\partial f/\partial v}{\partial f/\partial V}=-\delta V\cdot\frac{\partial f/\partial V}{\partial f/\partial v}$$

into the grid circuit. Thus the two mutual impedances of the tube are in general different in the two directions, which accounts for the amplifying properties of the device as explained in the paper already mentioned. This reverse action, from plate circuit to grid circuit, is often ignored in discussions of the audion and it is assumed that, since the plate current is a function of $V+\mu v$ only, the action of the device is summed up in the statement that the grid is μ times as effective as the plate in producing current changes. This action is, however, the only one if, for example, the grid current is independent of plate potential: there is then no reaction upon the grid circuit (neglecting capacity from grid to plate, which will be considered later) and the tube is a purely unilateral device. Since in practice this condition often holds very approximately, it is often legitimate to neglect $\partial v/\partial I_2$ and consequently to consider the tube as a device which, for small current changes, simply introduces into the plate circuit (just where will develop later) an E.M.F. equal to μ times the change in grid potential. When this condition holds the last equations are still further simplified into:

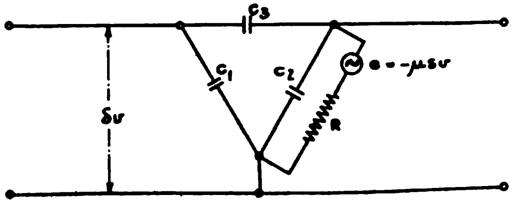

Fig. 3.

x_1	x_2	x_3	
$Z_1 + r$	$-M_{12}$	$Z_1 + M_{12} + M_{13}$	e_1
$-M_{12} - \mu r$	$Z_2 + R$	$-Z_2 - M_{12} - M_{23}$	e_2
$+M_{13} - (\mu + 1)r$	$-M_{23} + R$	$Z_3 + M_{13} + M_{23}$	e_3

Equivalent Circuit of the Tube.—When the grid current does not vary apprecaibly with plate voltage it is possible to use a very simple equivalent circuit for the audion itself. This circuit is shown in Fig. 3 in which the resistance R of the plate circuit is shown between plate and filament and the three condensers, c_1, c_2, c_3 represent respectively the internal capacities between grid-filament, plate-filament and grid-plate. These capacities must obviously be taken into account at sufficiently high frequencies. Under the given conditions r is effectively infinite. Located in the impedance of the plate circuit is an alternator whose E.M.F. is equal to μ times the voltage, δv, produced across the grid-filament terminals. If this circuit is worked out it will be found that it is the exact equivalent of the actual tube under the given conditions. The feature of the circuit is the location of the fictitious E.M.F. between filament and plate, and the network will serve for the solution of tube problems as long as the electromotive forces impressed upon the network are small enough not to violate the conditions. The use of this equivalent avoids the necessity of setting up the complete equations.

It is obvious that, as a closer approximation, an equivalent circuit may be drawn in which another alternator is used to represent the reaction upon the grid circuit, but for practical use it has been found that this complication is in most cases unnecessary. Except at the very highest frequencies this network may be still further simplified by omitting the condenser c_2 since its capacity is but a few micro microfarads and it is shunted by the resistance R. The other condensers are, however, very important, even at moderate frequencies, and this will be shown by two examples.

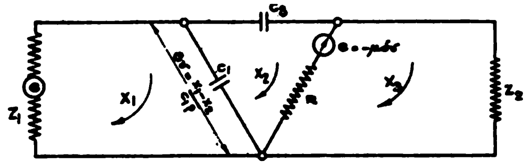

Fig. 4.

1. *The Input Impedance of an Audion Amplifier.*—Fig. 4 represents the equivalent circuit of an audion for frequencies below a few million and with the grid maintained negative with respect to the filament so that the input *resistance* is infinite. The output circuit is closed through an impedance Z_2. If there were no connection between the plate and grid circuits the impedance of the amplifier, as measured from the input terminals, would be simply that of the condenser c_1. The grid-plate capacity, however, modifies this result greatly. This impedance will be worked out and to do this the network equations, in terms of the mesh currents x_1, x_2, x_3 are given below with an E.M.F. e in the input circuit. p represents d/dt as usual and the fictitious E.M.F. in the tube resistance is μ times the voltage drop, $(x_1 - x_2)/c_1p$, over the condenser c_1. The sign is such that a positive grid tends to increase the plate current.

$$
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
Z_1 + \dfrac{1}{c_1p} & -\dfrac{1}{c_1p} & 0 & 0 \\[2ex]
-\dfrac{1+\mu}{c_1p} & \dfrac{1+\mu}{c_1p} + \dfrac{1}{c_3p} + R & -R & 0 \\[2ex]
\dfrac{\mu}{c_1p} & -R - \dfrac{\mu}{c_1p} & R + Z_2 & e
\end{array}
$$

If D is the operational determinant of this system and D_{11} the minor of the element (1, 1) the input impedance is

$$Z = D/D_{11} \quad \text{with} \quad Z_1 = 0,$$

and this works out to be

$$Z = \frac{1}{c_1 p} \frac{1 + W c_3 p}{1 + W c_3 p + \frac{c_3}{c_1}\left(1 + \mu - \mu \frac{W}{Z_2}\right)},$$

in which W is equal to $RZ_2/(R + Z_2)$, the impedance of R and Z_2 in parallel. Note that if the grid-plate capacity is zero, this reduces to $1/c_1 p$ which is the impedance of the grid-filament condenser. This impedance is in the operational form; to find the impedance to steady alternating currents of frequency $n/2\pi$, p is to be replaced by in where i is the imaginary unit.

To save space only a few consequences will be noted, since it is easy to draw conclusions from the formula. In the first place, even at low frequencies (when the terms containing p as a factor are negligible) the impedance is charged into

$$\frac{1}{c_1 p} \frac{1}{1 + \frac{c^3}{c_1}\left(1 + \mu - \mu \frac{W}{Z_2}\right)}; \quad p \text{ small.}$$

In some cases this impedance only remotely resembles a capacity reactance; for instance if the output impedance, Z_2, is inductive, the impedance may have a negative resistance component. This means that the amplifier is likely to sing under these conditions, and in fact the tendency of a tube to oscillate "through its internal capacities" has often been observed.

At extremely high frequencies the formula shows that the impedance again approaches a capacity reactance, as it obviously should do since then the condenser c_1 practically short-circuits the current entering the input terminals. It is instructive to work out the impedance under various simplifying assumptions, and in the case of amplifiers which sing the solution of the difficulty will often be found in this manner.

2. *Amplification at High Frequencies.*—As a second example of the use of the equivalent circuit of the tube, suppose the circuit of Fig. 4 is to be used to produce in the impedance Z_2 an amplified current under the influence of the generator e in the input circuit. It is clear from inspection that for very high frequencies the condenser c_3 will have such a low impedance that the amplifying action of the tube will be seriously impaired. To get a quantitative measure of the effect of varying the frequency, consider what is meant by amplification. If the tube were a perfect unilateral amplifier a current x_1 in mesh 1 would produce a certain E.M.F., e_3, in mesh 3, while a current x_3 in mesh 3 would produce no E.M.F. in mesh 1. On the other hand, if the network were a simple

transforming one (no amplification), the same E.M.F.'s would be produced by the same currents in either direction through the network. Hence, defining the two mutual impedances in the usual way as

$$M_{13} = \frac{e_3}{x_1}, \qquad M_{31} = \frac{e_1}{x_3},$$

it is clear that for a perfect unilateral amplifier $M_{31}/M_{13} = 0$, while for a perfect transforming network this ratio is unity. The above ratio of the two mutual impedances is therefore a measure of the failure of the tube as an amplifier.

From the network determinant, already used, we find this ratio to be

$$\frac{M_{31}}{M_{13}} = \frac{D_{31}}{D_{13}} = \frac{1}{1 - \dfrac{\mu}{c_3 Rp}},$$

or for simple harmonic currents of frequency $n/2\pi$:

$$\frac{M_{31}}{M_{13}} = \frac{1}{1 + \dfrac{i\mu}{Rc_3 n}}.$$

It will be seen that for $c_3 = 0$ this ratio is zero, as it should be, and for c_3 infinite it is unity. Also, since the frequency appears in the ratio, the amplification will be different for different frequencies, so that the tube will distort and tend to eliminate higher frequencies.

With certain tubes and for certain frequencies for which the ratio μ/Rc_3n is comparable with unity, the amplification is greatly impaired. This difficulty may, however, be avoided at any given frequency by the device of shunting the grid-plate capacity with an inductance of suitable value to make the impedance between grid and plate infinite at the given frequency. In doing this it is of course necessary to put in series with the inductance a large condenser to avoid conductive connection of grid and plate. It will be found that this method permits greater amplification at one frequency but it is open to the objection that variation of frequency is not convenient. Of course care must be taken not to couple this antiresonant mesh with others.

The Network as an Oscillation Generator.—In my Physical Review paper the general theory of oscillation generators was given and it was shown that the possibility of sustained free oscillation of a network depends upon the existence of certain types of unsymmetrical terms in in the characteristic determinant. The vanishing of the determinant then determines both the frequencies of all the possible modes of free oscillation and also the relations which must hold among the circuit constants in order that the oscillations shall be sustained (or have given damping). Since the solution for any oscillation generator requires only the ability to write down the network equations and find the roots of the determinant, it is hardly necessary to go into the detailed solution for any special case. As an example, however, the network of Fig. 5, purposely simplified to avoid long equations, will be handled by the method of the simplest equivalent circuit. The plate-grid capacity is included in the capacity C and the mutual inductance of the two coils is m.

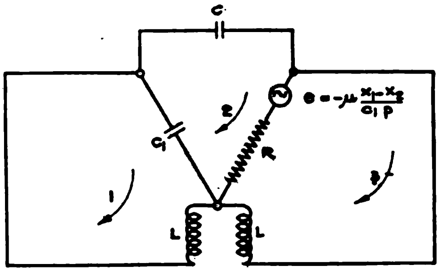

Fig. 5.

Choosing the mesh currents as shown, the network equations are

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
\frac{1}{c_1 p} + Lp & -\frac{1}{c_1 p} & -mp & 0 \\
-\frac{\mu + 1}{c_1 p} & \frac{\mu + 1}{c_1 p} + \frac{1}{Cp} + R & -R & 0 \\
mp + \frac{\mu}{c_1 p} & -R - \frac{\mu}{c_1 p} & Lp + R & 0
\end{array}$$

Note that these are the same as the last set with the addition of the transformer coupling. Now if we require that the oscillations be sustained, p must be taken equal to in, in which n is 2π times the frequency of oscillation. Then the vanishing of the determinant, which is complex, will require that the real and imaginary parts separately shall vanish, that is, that the even and odd powers of p shall separately be zero. As explained in the paper above cited, this determines both the frequencies of all the modes of oscillation and the circuit adjustments which must be made to permit the oscillations.

It will be noticed that in this determinant all the even powers of p appear in the coefficient of R; consequently by combining rows and columns until R appears in but one element, the even powers of p will be contained in the minor of that element, which minor, equated to zero, will determine the frequencies of sustained oscillation. This process leads to the frequency equation:

$$(L^2 - m^2)p^4 + \left(\frac{L}{C} + 2\,\frac{L - m}{c_1}\right)p^2 + \frac{1}{Cc_1} = 0,$$

and there are therefore two possible frequencies of sustained oscillation, corresponding to the two values of $p^2 = -n^2$. Whether or not these oscillations will actually take place depends upon whether the circuit constants are adjusted to make the odd powers of p vanish. This second equation, obtained by equating to zero the imaginary part of $D = 0$, is:

$$(L^2 - m^2)\left(\frac{c_1}{C} + \mu + 1\right)p^2 + \frac{L + \mu m}{C} = 0.$$

If, when one of the values of p^2 obtained from the first equation is substituted in this, it is satisfied, the oscillation corresponding to that value of p will take place. This fact explains a peculiarity of this kind of circuit, namely the common experience that in changing the frequency of an oscillator circuit the frequency may suddenly jump to some entirely different value for a small change in the constants. The reason is that in the process of changing the frequency the circuit constants are so changed that the relation holding among them becomes more nearly correct for another oscillation which would not before satisfy that relation. For example, in circuits of the type above, the most stable frequency is, for most ordinary adjustments, approximately that determined by the transformer and condenser C alone; but for some relations of L, m, μ, etc., the circuit will not maintain this frequency but will pass to the other. It is possible to design a network in which, as a constant is varied, the frequency will jump discontinuously from one value to others. The converse problem is to design an oscillation circuit in which the condition for sustaining a given oscillation holds for all variations of the element which is used to vary the frequency. Such a circuit will be stable over the whole frequency range.

Returning to the particular circuit above, it will be seen that if the capacity c_1 is very small the frequency of sustained oscillation will be that for which

$$n^2 = \frac{1}{2(L - m)C},$$

namely that corresponding to the single resonant circuit of the transformer and condenser C. The second equation then shows that this oscillation will be sustained if $\mu = 1$. If the inductances of the two windings had not been taken equal, μ would have turned out to be a function of L_1, L_2, m. It is worth noting that this circuit will oscillate even if there is no mutual inductance provided the circuit constants are properly adjusted. It then becomes identical with the amplifier, discussed above, with an inductive load.

It is probably unnecessary to mention the fact that the solution for all three-mesh oscillation generators may be derived directly from the determinant appropriate to this one by simply inserting the proper impedances. The solution for any case is theoretically just as simple, but practically the algebra is usually troublesome when there are more than three degrees of freedom. An exception is the case of so-called "iterative networks" which have periodic structures and certain other symmetrical networks.

As has been said, these methods of treating the audion in a circuit are approximate only, because of the limiting assumptions made, but considerable use of them in connection with experimental circuit work has shown that they are useful. The problem of finite amplitudes cannot be handled so generally, but when it is necessary to consider such refinements special methods are available which, it is hoped, may be given in another paper.

RESEARCH LABORATORIES OF THE
WESTERN ELECTRIC CO., INCORPORATED,
February, 1919.

THERMIONIC AND PHOTO-ELECTRIC PHENOMENA AT THE LOWEST ATTAINABLE PRESSURE.

By C. F. Hagenow.

Synopsis.—The object of these investigations was fourfold:

1. To study the photo-electric effect of tungsten at the lowest possible pressures, especially after the illuminated plate had been denuded of occluded gases by continued electronic bombardment of such energy as to maintain it at a white hot temperature.

The pressures attained were as low as 3.5×10^{-7} mm. of mercury. Upon denuding a tungsten plate of its occluded gases by raising it to a bright yellow heat by electronic bombardment, the photo-current rose to many times the value obtained before such denuding and further heating produced no further change.

2. To find, by direct observation, the short wave-length limit for tungsten and to observe how this changes, if at all, with the removal of occluded gases.

The upper wave-length limit of tungsten was located in the region between 2100 and 2300 Å. According to Einstein's equation, the corresponding values of the work necessary to free an electron from the metal would then be between 5.7 and 6.3 volts. These are larger than the corresponding values in the case of thermionic emission.

3. To observe, with the aid of the electric vacuum gauge, recently devised by O. E. Buckley, the progress of the clean-up effect, particularly at extremely low pressures.

Under the conditions employed it was not found possible to reduce the pressure by means of the clean-up alone lower than 10^{-5} mm., which was not as low as could be attained with the pump itself.

4. To study pressure and other changes accompanying the appearance of the "blue haze."

A gradually increasing potential difference was applied between the tungsten plate and a glowing filament until the blue haze appeared. *No pressure change whatever was observable at the moment of its appearance.* Certain current and voltage discontinuities, which occur under the experimental condition just described, were found to have characteristics similar to those observed by O. W. Richardson and Charles B. Bazzoni in the case of mercury vapor. The clean-up of the blue haze is very rapid and shows a striking increase in its rate of absorption as the pressure approaches the final minimum. This behavior is in marked contrast to the clean-up with a lighted filament, but without a field, and is quite similar to that observed by S. Brodetsky and B. Hodgson in their experiments with a vacuum tube discharge under the condition of abnormal cathode fall. At the pressures employed in these trials, about .01 mm. of mercury, this final pressure remained unchanged in the presence of the field, and did not change further even in those cases in which a trace of the blue glow remained in the tube.

I. EFFECTS OF ELECTRONIC BOMBARDMENT UPON PHOTO-CURRENTS FROM TUNGSTEN.

THE question of the part played by occluded gases in photo-electric phenomena has received the attention of many observers. Among those who have contended that the photo-electric effect is essentially dependent upon the presence of occluded gases may be mentioned Paech,[1] Fredenhagen,[2] Küstner[3] and Wiedemann and Hallwachs.[4] In particular, Küstner finds a total absence of the effect in the case of zinc when gases were removed. The zinc was scraped in vacuo and all "reacting" gases were removed by a spark discharge over potassium.

These conclusions are contradicted by experiments made by Pohl and Pringsheim,[5] Hennings,[6] Piersol,[7] Dushman,[8] Millikan and Souder,[9] and Welo.[10] For example, Piersol finds an increased sensitiveness in a number of metals when the occluded gases are removed by heating the electrode with an electric current. He also observed a superposed maximum effect "dependent on the activity of the gaseous surface," which disappeared after the highest temperature was reached.

This author states that a vacuum of .0002 cm. of mercury was used and does not mention the use of liquid air, so that the degree of exhaustion which he attained was not of the order aimed at in this experiment.

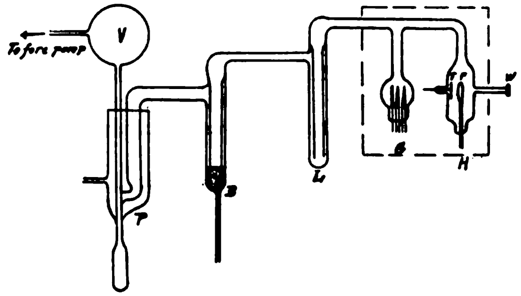

Fig. 1.

As the technique for the removal of the occluded gases and production of higher vacua has been improved, the evidence has become more and more conclusive for the contention that the photo-electric effect is an intrinsic property of the pure metal. This paper furnishes further convincing support of this point of view.

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[1] G. Paech, Ann. der Phys., 43, 135, 1913.
[2] K. Fredenhagen, Phys. Zeit., 15, 65, 1914.
[3] H. Küstner, Phys. Zeit., 15, 68, 1914.
[4] G. Wiedemann and W. Hallwachs, Ber. d. D. Phys. Ges., 16, 107, 1914.
[5] R. Pohl and P. Pringsheim, Ver. d. D. Phys. Ges., 16, 336, 1914.
[6] A. E. Hennings, PHYS. REV., 4, 228, 1914.
[7] R. J. Piersol, PHYS. REV., 8, 238, 1916.
[8] See I. Langmuir, Phys. Zeit., 15, 524, 1914.
[9] R. A. Millikan and W. H. Souder, Proc. Nat. Acad. Sci., 2, 19, 1916.
[10] L. A. Welo, PHYS. REV., 4, 251, 1918.

The apparatus (Fig. 1) was exhausted by the mercury diffusion pump *P* (kindly furnished by the Western Electric Company), working against a fore-vacuum of .01 mm. of mercury, or less, produced by a Töpler pump. Once established, no further pumping was necessary, as the volume, *V*, was quite large compared with that of the rest of the apparatus. A barometer tube, *B*, served as a valve to shut off the part to the right of it. *L* is the liquid air trap, which was kept continually in liquid air while the apparatus was set up. At no time was there any evidence of mercury vapor in the tube *H*. For measuring the pressures the extremely sensitive ionization gauge, *G*, due to Buckley,[1] was used. The photo-electric tube, *H*, contained the tungsten plate,[2] *T*, opposite which, at a distance of about a centimeter, the tungsten filament, *F*, and, in line with both, a tube sealed with a quartz window, *W*; the last being fastened with De Khotinsky wax. The connecting tubing had a bore of about 2.5 cm. and the whole apparatus was made as compact as possible. The source of illumination was, for the most part, a spark between zinc terminals. To avoid all electromagnetic disturbances, the spark gap, together with the induction coil, storage cell and Leyden jar, were all enclosed in an iron box.[3] A Dolazelek electrometer served for measuring the photo-currents, a sensibility of about 300 mm. per volt being usually employed. An electric oven could be slipped over a region indicated in the figure by the dotted line.

The ionization gauge, calibrated by Buckley against both a McLeod and a Knudsen manometer, gives the pressure in millimeters of mercury according to the formula (for air),

$$p = .05\frac{I_C}{I_A},$$

where I_A is the current, in amperes, set up by a coated filament in a field of 200 volts and I_C the resulting flow of positive ions. The former can be read on a milli-ammeter, at least for the larger values, and the latter by means of a galvanometer. In order to read the smaller currents for I_A at the higher pressures more accurately, the writer employed the following null method of measuring these small currents. Fig. 2 shows the connections. A known resistance, R_1 is placed in series with the circuit whose current it is desired to measure. Connected to one end, A, of this resistance, is a circuit containing a variable resistance, $R_2 + R_3$ and a cell, whose E.M.F. need be known only to the degree of accuracy of the gauge itself. A Leeds and Northrup portable galvanometer was put at G. The adjustment consists simply in finding a point B' so that on closing the key, K, no deflection resulted. B and B' being then at the same potential, we have, calling the currents in R_1 and R_2 respectively C_1 and C_2,

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[1] O. E. Buckley, Nat. Acad. Sci. Proc., 2, 683, 1916.

[2] This was a thorium free sample kindly furnished by Dr. Langmuir and of the same sort used by him in the determination of Richardson's "b."

[3] W. H. Kadesch, Phys. Rev., 3, 367, 1914.

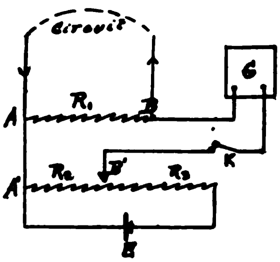

Fig. 2.

$$C_1R_1 = C_2R_2.$$

Also

$$C_2 = \frac{E}{R_2 + R_3},$$

Therefore

$$C_1 = \frac{R_2E}{R_1(R_2 + R_3)}.$$

As actually used, the resistances were set in a fixed relation and the heating current of the glower of the gauge regulated for zero deflection of the galvanometer. Since I_A need not be changed very often (*i. e.*, only for a large variation in the range of pressure) a few previously calculated values of R_1, R_2 and R_3 enabled one quickly to set on any desired current.

The limit of this instrument to indicate low pressures is set only by the possibility of being able to measure the current I_C. On the other hand, it was found that a maximum reading was obtained at .019 mm. This was located by plotting the values of I_C against time as the apparatus was gradually exhausted by a Gaede mercury pump.

The photo-currents measured were the saturation currents for a timed duration of the spark. The plate T (Fig. 1) was connected to one of the quadrants of the electrometer, the other being earthed. A positive potential was applied to the filament, F, the other pole being earthed, and the liberated electrons thus removed from the tube. A field of 10 to 20 volts was sufficient for saturation.

With the apparatus as first set up and no further exhaustion than simply pumping with the diffusion pump, the following readings were taken.

The photo-currents are expressed in millimeters deflection on the basis of 300 mm. per volt sensibility and 2-second illumination when the electrometer alone is used. (Sometimes an auxiliary condenser had to be connected.) These are given under C in the last column.

TABLE I.

Date.	No.	p.	C.	Date.	No.	p.	C.
May 10	1	4.8×10^{-3}	9.6	May 13	8	7.7×10^{-3}	9.8
11	2	"	7.2	14	9	5.2×10^{-3}	8.7
12	3	"	7.1	"	10	3.8×10^{-4}	7.7
"	4	"	6–6.8	"	11	1.4×10^{-4}	8.1
"	5	"	8.2	"	12	8.0×10^{-5}	8.9
13	6	5.0×10^{-3}	7.9	15	13	5.1×10^{-6}	7.8
"	7	3.2×10^{-4}	8.7	"	14	8.5×10^{-6}	7.3

As is to be expected from all preceding photo-electric work there is here no evidence for a relation between pressure and photo-currents. The table includes every run that was made until the apparatus was opened. On one or two occasions the writer has noticed fluctuations in excess of those here recorded, but the foregoing are representative of the sort of constancy that readings, taken days or weeks apart under apparently the same conditions, would show.

It may be mentioned, parenthetically, that readings were sometimes taken at zero fields, the filament F being earthed. Both T and F being of the same metal there should be no contact potential present. The results were very variable in this case as might be expected under the conditions. The number of electrons caught by the filament would depend on their particular distribution at any particular time, and this would be influenced by the charging of the inner surface of the glass tube. It was observed that illuminating the plate repeatedly, even when it was earthed at the time, would cut down the ensuing current greatly, doubtless because of such accumulation of charge on the walls. The effect would disappear after a short while. This phenomenon practically ceased at the saturation voltage, as it should do if the foregoing explanation is correct.

The effect upon the photo-currents, C, of denuding the tungsten plate of gases is shown in Table II. The headings have the same significance as in Table I. The denuding was done by glowing the filament while the negative potential indicated in the last column was applied to it.

So far no parts of the apparatus had been baked out or the metal parts heated, except the illuminated plate as described.[1]

No further readings were taken for four months. A new tungsten plate was mounted, though from the same original piece. Then the data contained in Table III. were taken.

The increase in no. 3 is not as large was as expected in view of the

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[1] The ionization gauge and the tungsten filament in the photo-tube had been renewed since the readings in Table I. were taken.

TABLE II.

Date.	No.	*p*.	*C*.	Remarks.
June 9	1	1.5×10^{-3}	1.4	Just after set up.
"	2	"	187.6	After first bombardment, for 10 minutes up to 960 volts D.C. No observable heating of the plate.
11	3	1.9×10^{-3}	133.0	No further treatment since last observation.
12	4	2.0×10^{-3}	433.0	After bombardment with 1,500 volts, about 30 minutes.
13	5	1.9×10^{-3}	449.0	No further treatment since last observation.
		(Apparatus opened.)		
July 6	6	2.6×10^{-3}	94.0	Note decrease after exposure to the air.
"	7	2.0×10^{-3}	1885.0	After bombarding at 1,500 volts. Not visibly heated but surface slightly altered in appearance.
"	8	2.0×10^{-5}	1900.0	After pumping down to 1/100 value of preceding pressure. Practically no effect on the photo-current.
7	9	1.6×10^{-3}	1715.0	Day after a bombardment up to 1,400 volts, about 30 minutes. First time plate at red heat in spots.
		(Apparatus opened.)		
18	10	2.9×10^{-3}	4.0	Decrease after exposure to air.
"	11	4.8×10^{-4}	968.0	After bombardment at 3,500 volts A.C. about 30 minutes. Plate bright red in central portion, duller at edges.

effect of other cases of bombardment. Also the spontaneous increase in no. 6 is hard to account for, unless the fact mentioned in the last column may have had some bearing on it. However in view of the results of all these observations, it appears that there is an enormous increase of photo-sensitiveness after the occluded gases are removed. No. 7 shows the final reading at the lowest pressure attained for photoelectric observations in the course of these experiments. The tube and guage had been baked out, the metal parts all glowed (the collector plate of the gauge had shown no visible heat during the bombardment, but a subsequent bombardment that caused it to glow did not influence the pressure attained). It can then fairly represent the conditions of a pure metal in a very low pressure thoroughly out-gassed. The close agreement of the final values of *C* in Tables II. and III. is probably largely accidental. On account of a rearrangement of the set up, necessitated by the addition of a monochromatic illuminator, the intensity of illumination of the plate in Table III. was slightly changed.

According to Table II. the maximum photo-current occurs after only

TABLE III.

Date.	No.	p.	C.	Remarks.
Nov. 30	1	2.2×10^{-3}	50	
Dec. 1	2	"	93	After a slight bombardment to try a new transformer.
Jan. 1	3	6.2×10^{-6}	112	After bombarding 15 minutes at 3,500 volts A.C. Plate at red heat all over.
2	4	9.0×10^{-4}	106	No further treatment since last observation.
3	5	1.1×10^{-6}	111	After baking tube and gauge for one hour at 300° C.
7	6	1.2×10^{-4}	407	This was after the tungsten filament had been brightly glowed for one hour for a clean up experiment.
8	7	3.5×10^{-6}	964	After bombarding 15 minutes at 2,200 volts A.C. Plate at a bright orange yellow heat. Now appears for the first time a bright metallic lustre.

a moderate heating of the plate. This maximum does not appear in Table III. and is probably due to some accidental surface condition in the first case. Piersol,[1] however, has observed a similar phenomenon in the case of a number of other metals. When new trials along this line were undertaken some weeks later, troubles unfortunately developed with the mountings of the tungsten plate that made further tests of this kind impossible.

Pressure readings were made with this apparatus down to 3.5×10^{-7} mm., with the photo-tube taken off. This is below the lowest pressure which it was possible to maintain for photo-electric observations. The De Khotinsky wax, used to hold the quartz window, may account for this difference. Also, as has been mentioned above, it was impossible to bake out every part of the apparatus. An attempt to heat the parts not inclosed by the oven with a Bunsen burner, while the diffusion pump was in operation, proved a failure. With the gauge and tube in the electric oven, and the diffusion pump going, mercury vapor diffused over into the tube.

It was also found that, though taking off the liquid air caused the pressure to rise rapidly, the vacuum could be as quickly restored by replacing it, with the pump in operation. Thus in one case the pressure rose almost immediately from 5×10^{-6} to 1.2×10^{-2} mm. and returned to 5×10^{-6} mm. It may be remarked that the low pressures mentioned just above were read after mercury vapor had diffused through the tubing as described.

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[1] *Loc. cit.*

II. DIRECT OBSERVATION OF LONG WAVE-LENGTH LIMIT OF TUNGSTEN.

To determine the long wave-length limit of sensitivity of the tungsten a Hilger monochromatic illuminator was set up before the quartz window; a Heræus quartz mercury lamp being the source of light. As used with this apparatus, this lamp with direct illumination of the tungsten plate, gave a scale deflection about three times as large as that obtained previously with the zinc spark, *i. e.*, about 1,000 mm. in two seconds of illumination when no condenser was used. When the light was passed through the monochromator it was found impossible to obtain any deflection at all with any wave-length which would pass through the instrument. However some idea was gained of the limits of sensitiveness by means of two absorption media.

A solution of methyl alcohol placed in the path of the direct beam from the source[1] cut out the effect entirely. The first trials were made after the runs no. 2 in Table III. Other trials after no. 3 gave the same results. Finally after the last run, no. 7, a small deflection was observed. This was less than one thirteen hundredth of the deflection caused by the unfiltered light and about twice that which could still be observed with certainty. Since this sensitiveness was not attainable before the plate was thoroughly out-gassed, any conclusions as to a real change of the wave-length limit based on the absence of any observable effect under those conditions would be correspondingly doubtful.

The empty absorption cell itself, by virtue of the absorption of its fused quartz windows, cut down the deflection to only one tenth of its value, both before and after the plate was out-gassed.

Mt. M. J. Kelly, at this laboratory, found, photographically, that methyl alcohol had a sharply defined absorption region beginning with decreasing wave-length, at 2370 Å, and that the fused quartz windows had a similar absorption region beginning at 2160 Å. Now the above experiment showed that about nine tenths of the total light effect must have been due to wave-lengths shorter than 2160 Å, and all but one part in thirteen hundred to wave-lengths less than 2370 Å. The question arises whether this residual effect is due to active wave-lengths longer than 2370 Å or to a very small amount of light of shorter wave-length that was not entirely absorbed by the methyl alcohol. The latter is the more natural assumption, so that the indication of this experiment is that the long wave-length limit of tungsten is between 2100 and 2300 Å.

We can compare these results with the minimum frequency for which tungsten is light-sensitive, as calculated from Einstein's equation,

$$h\nu_0 = w_0$$

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[1] The zinc spark was also used at times.

where w_0 is the work done by an escaping electron against the forces which tend to retain it in the metal. If we assume that w_0 has the same value as in thermionic emission, we can use its value as obtained from thermionic experiments. Lester[1] has made some experimental verifications of the identity of the "work function," φ, of an electron escaping from a metal and "b" in Richardson's equation,

$$i = a\theta^{1/2}e^{-b/\theta}.$$

The relation is

$$\varphi e = bk,$$

where e is the charge on an electron, k the gas constant for one molecule. φ was calculated from experiments made on the "cooling effect." Now if we assume that w_0 in Einstein's equation has the same value as in thermionic emission, using Lester's value for tungsten, $\varphi = 4.478$ volts, we have

$$\nu_0 = \frac{\varphi e}{h} = \frac{4.478 \times 4.774 \times 10^{-10}}{300 \times 6.547 \times 10^{-27}} = 1.088 \times 10^{15},$$

or

$$\lambda_0 = 2757 \text{Å}.$$

On the assumption that the experiment with the methyl alcohol light filter does really indicate a total absence of light sensitiveness to wave-lengths longer than 2370 Å, the difference is seen to be at least 400 Å. This is much larger than the variations in either the values of φ as observed by Lester, or the values of b obtained by Langmuir when proper precautions were taken to have all occluded gases removed.

It may be mentioned that Lester's value of b ($= \varphi e/k$) is 52,130. This agrees well with the weighted mean value, $b = 53{,}130$, which that writer has calculated from some measurements by Smith[2] and Langmuir.[3]

Lester[4] also found that the effect of residual gases was to increase the value of φ. This he tested by sealing off a tube containing a tungsten filament and allowing the glowing filament[5] to clean up the active gases present. Inert gases do not affect φ. If this fact has any direct bearing on the present experiment it would mean that the occluded gases responsible for the higher value of φ were such that they were not driven out by the treatment the plate received as described above.

It is, of course, not established that the work function is the same in photo-electric as in thermionic emission, or that these phenomena deal with the same type of electron. Concerning the case of platinum

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[1] Horace Lester, Phil. Mag., 31, 197, 1916.

[2] K. K. Smith, Phil. Mag., 29, 802, 1915.

[3] I. Langmuir, PHYS. REV., 2, 450, 1913.

[4] *Loc. cit.*

[5] *I. e.*, the same filament which was the subject of investigation.

Hughes[1] makes the following comparison: Richardson[2] gives 5.34 volts as representing the work done (per unit charge) when an electron escapes from hot platinum. But from photo-electric observations, using the most probable value of λ_0, viz., 2910 Å, V_0 comes out 3.86 when calculated from $V = k\nu - V_0$. When instead of k the corresponding value from the quantum theory, h/e is used, V_0 becomes equal to 4.32 volts.

The results of the present experiments indicate a value of the work function. calculated from Einstein's equation, as lying between 5.7 and 6.3 volts. The work required to remove an electron from tungsten photo-electrically thus seems to be larger than that required in the case of thermionic emission.

III. CLEAN-UP EFFECTS.

The fact that an incandescent filament tends to improve the vacuum has been noted by many observers. It has been subjected to detailed study by Langmuir[3] for a number of gases.

The present attempt offers nothing new, except as it might serve to make a comparison with another sort of clean-up to be discussed later in connection with the blue haze experiments.

The mercury valve, B (Fig. 1), was closed and the filament glowed a little more brightly than in the ordinary Mazda lamp. The following readings were then taken.

TABLE IV.

Time (Minutes).	p.	**Absorption Cu. mm. × 10^3v.**	**Per Minute.**
0	1.2×10^{-4}		
5	6.8×10^{-5}	6.84	1.37
10	3.5×10^{-5}	4.55	.91
20	1.9×10^{-5}	2.20	.22
30	1.6×10^{-5}	.40	.04
40	1.4×10^{-5}	.26	.026
50	1.3×10^{-5}	.13	.013
60	1.25×10^{-5}	.07	.007

The absorbed air is given in cu. mm. at atmospheric pressure per cu. mm. of the volume, v, of the apparatus used. The last column contains the average rate per minute in the same units.

The next day but one following this run the pressure was found to have risen to 2.8×10^{-5} mm., when the filament was again glowed at a higher temperature, bringing the pressure down to a little below 10^{-5} mm. The graph is shown in Fig. 3.

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[1] A. L. Hughes, Photo-electricity, Cambridge University Press.

[2] O. W. Richardson and K. T. Compton, Phil. Mag., 24, 576, 1912.

[3] I. Langmuir, J. Am. Chem. Soc., 34, 1310, 1912; 35, 107, 1913; 35, 931, 1913.

Table IV. shows that the rate of clean-up, as shown in the last column, diminishes to 1/200th of its initial value while the pressure drops to 1/10th of its original amount. According to Langmuir[1] the rate of clean up for oxygen is proportional to the pressure. For nitrogen[2] it is different according to the range of pressure experimented with, but is independent of the pressure in the interval from .0065 mm. to .0001 mm. of mercury.

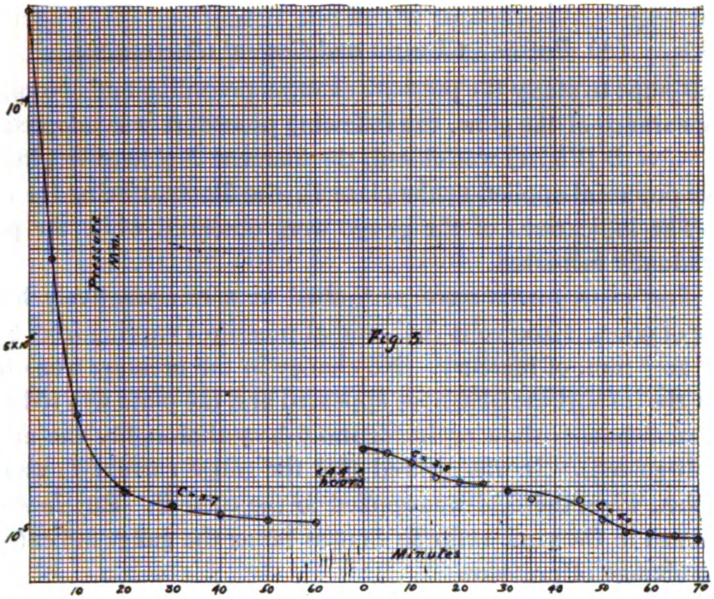

Fig. 3.

The present experiment indicates that the rate of clean up of air decreases at a greater rate than the pressure and that the latter reaches a fairly well defined limiting value, which is manifestly not due alone to the inert gases. Langmuir found a "fatigue effect" in the case of hydrogen, so that a continual treatment of the glowing wire in that gas brought the clean-up practically to a standstill. In view of these experiments and the fact that the rate of clean up depends on the temperature of the filament in a way widely different for the different gases, no general conclusions can be drawn from experiments on a mixture such as air.

IV. Phenomena Accompanying the Appearance of the "Blue Haze."

The bluish glow, known as the "blue haze" has often been remarked.[3]

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[1] I. Langmuir, J. Am. Chem. Soc., 35, 107, 1913.

[2] *Loc. cit.*, 35, 931, 1913.

[3] W. R. Whitney, Proc. Am. Inst. El. Eng., 31, No. 1, 921, 1912. Saul Dushman, Phys.

It can be seen, at a certain state of exhaustion, in the ordinary incandescent lamp. Whitney[1] observed that it "cleaned up" gases and was accompanied by a disintegration of the lighted filament.

The present investigation may be grouped under three heads: (A) Pressure changes and general observations; (B) Current-voltage relations; (C) Clean-up effects.

(A) *Pressure Changes and General Observations.*

To attempt to study the pressure changes preceding and accompanying the appearance of the glow, the following procedure was adopted.

The valve *B* (Fig. 1) was closed, the filament *F* glowed brightly and a positive potential applied to the plate *T*; the filament being the negative pole. The indications of the gauge were observed while the potential was gradually increased.

The first experiments were performed at a comparatively high field voltage of 900 to 2,100 volts A.C. and at pressures varying from 6×10^{-4} to 5×10^{-5} mm. of mercury. No change in the pressure was observed, even at the instant the blue haze appeared and the plate was red hot with the bombardment (showing incidentally how thoroughly the metal had been out-gassed). It was noticed that the gauge current I_A was always diminished when the field was applied to the tube *H*. But I_C was found to diminish in the same ratio, so that as far as the gauge indicated, there was no change of pressure.

In order to bring the necessary potential within lower and more accurately measurable limits, it was decided to use a pressure as high as .01 mm. A potentiometer supplied the field up to the available 220 volts, D.C.

It was then observed that the blue haze appeared very sharply, accompanied by a sudden fluctuation in the voltage downward and a break upward in the electronic current. This phenomenon has been described by Richardson and Bazzoni,[2] in their experiments on mercury vapor.

At the first one of such fluctuations a blue glow appears directly behind the anode; it can be made to appear and disappear within a variation of potential too small to be observed on a Weston 30-volt voltmeter, with which a change of one fifth volt could be easily detected. On increasing the field the glow spreads into the tube and, a little later, at a voltage which seemed to be constant for all pressures tried, a second "kick" occurred. Here the glow suddenly spread further into the tubing above

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REV., 4, 123, 1914. A. Wehnelt and E. Liebreich, Phys. Zeit., 15, 557, 1915. O. W. Richardson and Charles B. Bazzoni, Phil. Mag., 32, 426, 1916.

[1] *Loc. cit.*

[2] *Loc. cit.*

the bulb *H* and also increased greatly in intensity. At this point the glow was very sharply defined and a column of it could be pushed along the tubing at will by controlling the field.

The gauge showed no change in the pressure at any time, except that after some minutes it would begin to register the falling pressure due to the above-mentioned clean-up effect. The blue glow then began gradually to recede into the tube *H* and finally to disappear altogether. The rate of disappearance depends on the pressure of the absorbable gases present, and for the lower pressure, the rapid disappearance of the glow, once it begins to withdraw, is very striking. As shown at the end of the paper, this disappearance was accomplished at times in an interval of a second. In these experiments also, as in the first trials with A.C. fields, it was observed that the gauge current I_A was always diminished when the field was applied in the tube *H*. But I_C diminished in the same ratio, so that the gauge recorded no change in pressure.

(B) *Current-Voltage Relations.*

In order to maintain the pressure as nearly constant as possible during a set of observations the mercury seal *B* was left open. A constant P.D. of seven volts was maintained across the terminals of the filament. To the apparatus mentioned above was added a milliammeter. A typical set of readings is shown in Table V. V_1 is the first "kick" voltage, when the blue color first appears behind the anode; C_1 the corresponding current change. In this set the initial current was not readable on the scale used. V_2 is the voltage drop at the second kick, already referred to above. The voltages are measured at the positive end of the glowing filament, the average would thus be 3.5 volts greater. Under C_2 are given the current readings corresponding to the voltages, V_2.

TABLE V.

No.	p.	V_1.	C_1.	V_2.	C_2.
1	.0101	18 → 14	0 → .005	26 → 22	.130 → .133
				30 → 25	.135 → .140
2	.0086	22 → 16	→ .006	22 → 20	.113 → .115
3	.0068	24 → 16	→ .007	22 → 20	.118 → .122
4	.0047	33 → 18	→ .013	25 →	
5	.0023	54 → 24	→ .026		
6	.0010	92 → 44	→ .045		

Owing to the fact that even with the considerable volume in use the pressure would slowly diminish, it was usually impossible to duplicate any reading during a run. The kicks are rather hard to catch accurately and rapid fluctuations are apt to occur at these points of instability, especially at the lower pressures. Thus in case of no. 6 the voltage and current made the jump back and forth twice, finally remaining at the "before the kick" value. This meant, of course, that the clean-up had progressed so far that the higher potential of V_1 was not sufficient to start the blue glow again.

Fig. 4 is the plot from the above data. No. 1 seems to have three points of instability, though that may well be the result of the manipulation of the potentiometer. It did not occur in the other sets. The currents before the first break are not shown in the plot; they increase gradually from zero until the break. Thus the whole phenomenon resembles that described by Richardson and Bazzoni[1] in the case of mercury, even to the order of magnitudes involved. These observers mention also the second point of instability, but are of the opinion that it is due to the "discharge wandering in the tube" and did not regard it as of as much interest as the first. Coming as it does at approximately the same voltage for all the pressures would suggest that it is a function of the apparatus rather than of the other conditions. This would remain to be investigated.

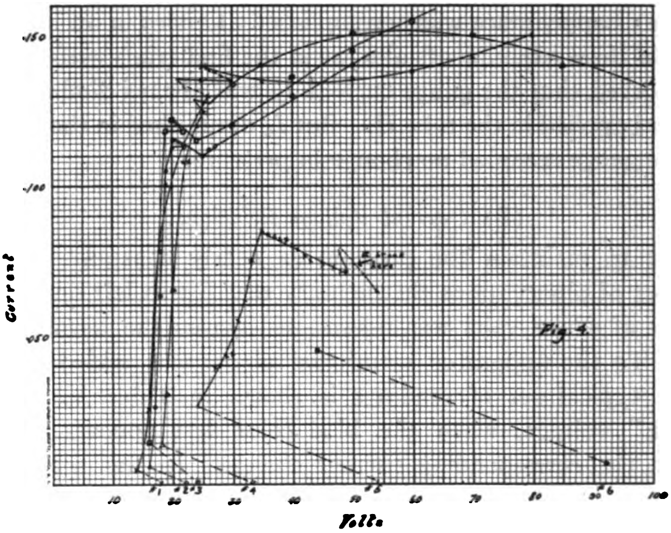

Fig. 4.

Referring again to the curves, it seems as if in no. 5 the first break had overtaken the second, being just at that voltage; it shows also another break later. But the conditions at that stage were too variable to make any general statement. This is also evident from an inspection of the progress of the currents, after the first very rapid increase immediately after the first break. No. 6 consisted of only the first jump.

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[1] *Loc. cit.*

A second table is added to show the duplicability of the phenomenon. The seventh column gives the ratio of the currents, before and after the first kick. This was used by Richardson and Bazzoni as a measure of the magnitude of the kick. It shows a maximum as found by them.

Table VI.

No.	p.	V_1.	$C_1 \times 10^4$.	V_2.	C_2.	Ratio of C_1's.
1	.0120	17 → 14.5	.75 → 5.9	26 → 24	.104 → .107	7.9
2	.0113	19 → 16	1.5 → 12.0	28 → 22	.136 → .148	8.0
3	.0070	26 → 16	1.8 → 40.0	22 → 21	.105 → .108	22.2
4	.0026	60 → 26	2.7 → 50.0	28 → 26		18.5
5	.0013	114 → 42	45 → 600			13.3
6	.0009	206 → 105	110 → 1270			11.5

The behavior of the voltage V_1, also, is similar to that found by those observers for mercury. Thus, an attempt to lower the voltage after the first "kick" results in a sudden drop to the initial part of the curve; *i. e.*, to a smaller current and larger voltage.

As is evident from Tables V. and VI., the minimum potential necessary to maintain the glow (which is practically the lower reading in the column under V_1) approaches a limiting value as the pressure increases. In these experiments the minimum value for V_1 was never below 14 volts, with a pressure up to .02 mm., the highest readable on the gauge. Taking account of the potential difference across the terminal of the glowing filament, the average potential would be 17.5 volts. Compared to the ionizing potential for nitrogen (using 7.7 volts), this is roughly in the same ratio as the corresponding quantities Richardson and Bazzoni found for mercury, viz., 11.5 volts for the glow as compared to minimum ionizing potential of 4.9 volts. However, in the light of some recent experiments by Hebb,[1] who has shown that pure mercury vapor will arc at 4.9 volts, the above comparison may not have any significance. The present trials were necessarily complicated by the fact that they were made on a mixture of gases subject to a sorting-out effect due to the clean-up. A detailed study of this phase of the ionization problem on pure gases would seem to be a profitable undertaking.

The writer does not lose sight of the fact that where there is very probably a selective absorption of the gases present in air, the gauge constant would be subject to change. Nevertheless, such a variation would not obliterate the general trend of the phenomenon in question. This also applies to the experiments on clean-up effects, which follow.

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[1] T. C. Hebb, Phys. Rev., 11, 170, 1918.

(C) *Clean-Up Effects.*

The clean-up effect of the blue haze was very marked, as was the wearing away of the lighted filament. The walls of the tube became covered with a shiny brown coating, showing interference colors. Langmuir[1] refers to a "clear brown" coating as being WN_2. This clean-up is characteristically different from that produced by the lighted filament without a field between the filament and a neighboring electrode. As will be shown more in detail later, the rate of clean up is quite different and increases with decreasing pressure just before the blue glow disappears. Also it operates at a considerably higher pressure than the ordinary clean-up. Thus it was always possible to evacuate with the blue haze at .02 mm. pressure, though a glowing filament, without the field, had absolutely no effect at this pressure.

In order to accelerate the progress of the phenomenon a smaller volume was used, *B* being closed. The first observations were taken immediately after the experiments described in part "B" above were completed; that is, after the air in the apparatus had presumably been deprived of its more easily absorbed constituents.

The filament being glowed as before and the maximum field applied (not over 220 volts) pressure readings were taken at intervals of a minute, during the first two runs. As soon as the minimum pressure was attained, *B* was opened a moment, thus equalizing the pressure throughout the apparatus, and *B* again closed for the next run.

The tube and about half of the connecting tube to the gauge was filled with intense purple glow which turned to a pure blue as the exhaustion proceeded. After a time the glow would begin to recede into the tube and, usually, disappear altogether. In the instances where a glow remained there was no difference in the pressure observed. The rapid decrease in the time required for glow to disappear as run after run was made was very striking. Table VII. gives a summary of these trials. The initial and final pressures are denoted by p_1 and p_2 respectively; t is the time taken for the glow to disappear, which was also the interval between the two pressure readings, as nearly as they could be taken. No great accuracy is claimed for the time in the shorter intervals, but they serve sufficiently to indicate the progress of the phenomenon. The quantities in the last two columns are the same as the corresponding ones in Table IV.

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[1] I. Langmuir, J. Am. Chem. Soc., 35, 931, 1913.

TABLE VII.

No.	p_1.	p_2.	t, Seconds.	Absorption Cu. mm. × $10^4 v$.	Absorption Per Min.
1	.0110	.0015	360	12.5	2.10
2	.0076	.0011	240	8.55	2.14
3	.0072	.0010	30	8.16	16.3
4	.0065	.0009	20	7.37	22.1
5	.0062	.0015	4	6.2	93
6	.0058	.00061	2	6.8	205
7	.0053	.00068	1	6.0	360
8	.0049	.00099	—	5.4	—
9	.0046	.0011	—	4.6	—

Thus as the proportion of absorbable gases decreases with each run, the rate of clean-up increases. The total pressure at the beginning of the final run is only a little less than half that at the commencement of the experiment.

The matter was next tested by admitting a fresh lot of air into the apparatus. The valve *B* was left open and the entire apparatus evacuated in the above manner. Readings were taken every minute or two until a steady state was reached at a pressure of .0008 mm. at the end of 45 minutes.

The results are best represented in a graph, shown in Fig. 5. The curve is practically a straight line up to a pressure of .0022 mm., when there is a rapidly increasing acceleration until the minimum pressure is attained. It is at that point the glow begins to recede and disappear.

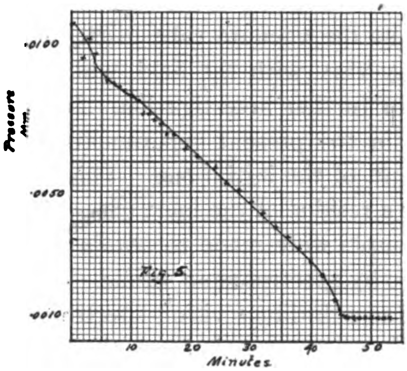

Fig. 5.

The field and current strength were recorded. Both remained at a fairly steady state during the greater part of the time, *i. e.*, until the bend in the curve was reached, when the voltage rose and the current fell rapidly.

In the present case we are dealing with a mixture of gases, hence, since the rate of absorption increases with the diminution of the total pressure, it must vary still more rapidly with the partial pressure of the more absorbable constituents.

According to Langmuir[1] the oxygen atoms collide with electrons in the tungsten filament by virtue of the energy of agitation of the former, which then take on a negative charge and combine with positive tungsten atoms. A field might conceivably aid or retard this process; negative oxygen ions being repelled before chemical combination could take place; or, positive ions of the gas impelled to the negative filament.

In an investigation on the absorption of gases in vacuum tubes, Brodetsky and Hodgson[2] have given evidence for a mechanical theory of the phenomenon. Glass vacuum tubes were used, the current being obtained from a high potential battery. A curve for air shown by them has the general characteristics of Fig. 5.

According to their work the absorption, together with the disintegration of the cathode, is conditioned on the presence of an abnormal cathode fall: neither of the two effects appearing when the cathode fall is normal. The fact is cited (supporting the mechanical theory) that the inactive gases helium and argon are also readily absorbed. Hodgson[3] shows curves, similar to the above, for helium. Whatever may be the mechanism of the process, the absorption in the case of the blue haze is certainly of the same nature as the vacuum tube discharge. Just how selective it is and how it may vary with the field, which affects the ionization, remains a subject for further investigation. In all cases of absorption with the blue haze in the present experiments the minimum pressure was about 10 per cent. of the initial one. This is of course too large to be accounted for by a residue of the inert gases alone.

A number of attempts were made to obtain some information of the nature of the process by spectroscopic means.

A Schmidt and Haensch direct vision spectrometer (after Hoffmann) was employed. As has been stated above, the color of the glow changes from a sort of purple to a pure blue toward the end of the exhaustion. The spectrum of the blue haze at .01 mm. pressure showed the brighter nitrogen and oxygen lines and some of the nitrogen bands of the first and second Deslandres series as described by Kayser. No hydrogen lines were seen. After a clean-up, whenever there remained a visible glow, it was found that the nitrogen bands of the first group had disappeared and some of the second. H_α and H_β were observed.

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[1] I. Langmuir, J. Am. Chem. Soc., 35, 107, 1913.

[2] S. Brodetsky and B. Hodgson, Phil. Mag., 31, 478, 1916.

[3] B. Hodgson, Phys. Zeit., 13, 595, 1912.

It was thought that the removal of a great deal of the nitrogen and oxygen might allow other lines to become more prominent, such as those of argon. In order to make possible a more careful and detailed study of the spectrum, a table was constructed showing the observed lines and the most prominent lines of the common and the noble gases in parallel columns. A number of new lines did appear that corresponded to those of argon within the limits of accuracy of the spectroscope, but they were not among the prominent ones, and could usually be duplicated by either nitrogen or oxygen, though these latter had not been seen before the clean-up. Thus it was practically impossible to draw any safe conclusions from the character of the spectrum. Also the feeble luminosity, in the bright glare of the lighted filament, made observation difficult.

In conclusion the writer wishes to take this opportunity to thank Dr. Millikan, at whose suggestion these experiments were undertaken, for his very kind assistance and numerous helpful suggestions.

RESIDUAL GASES AND VAPORS IN HIGHLY EXHAUSTED GLASS BULBS.

BY J. E. SHRADER.

SYNOPSIS.—The investigation of high vacua has been extended by making observations on the effect of the sealing off of highly exhausted glass bulbs, and noting the subsequent change in pressure with time over an extended period. On sealing off the bulbs and gauges by melting the glass at a constriction there is a comparatively large evolution of gases and vapors which can be partly removed by pumping during the time of sealing off, but the pressure in the bulbs is always four or five times as large as the pressure before sealing off.

BY the use of the mercury diffusion pump and the Knudsen absolute gauge pressures of the order of 1×10^{-8} mm. Hg have been produced and measured.[1] These pressures have been measured during the operation of the pump and it has not come to the attention of the author that any data have been obtained showing what pressures can be maintained in bulbs which have been pumped to this degree of exhaustion and sealed off. To secure information on this subject four glass vessels were exhausted to a high vacuum and sealed off and their change in pressure with time was observed. The system for this investigation consisted of a mercury diffusion pump with liquid air trap, a sensitive Knudsen absolute gauge of about 500-c.c. capacity and the bulb to be tested of about 1,500-c.c. capacity. A narrow constriction was made in the tube connecting the bulb and gauge to the pump so that they could be sealed off readily by heating a limited amount of glass. The gauge and bulb were so disposed that they could be heated by an electric oven to a short distance beyond the constriction. At the same time all the glass connecting tubing was heated with a bunsen flame to release occluded gases and vapors.

It has been recognized that the vacuum obtainable with the diffusion pump is limited practically by the heat treatment of the glass, though it is true that the greater the speed of the pump the lower the pressure that can be reached. Equilibrium will be established when the speed of the pump is equal to the rate at which gases and vapors are given off from the glass. If the escape of the gases and vapors is accelerated by heat treatment, this equilibrium is reached sooner and the ultimate pressure is lower. The effect of heat treatment on the vacuum obtainable after pumping until equilibrium was reached at that temperature is shown by the following results:

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[1] Shrader and Sherwood, Production and Measurement of High Vacua. PHYS. REV., Vol. XII., No. 1, July, 1918.

Temperature........	20	100	200	300	300	500
Pressure in mm. Hg..	1×10^{-5}	1.9×10^{-6}	1.7×10^{-7}	1.2×10^{-7}	2×10^{-7}	2.4×10^{-8}

While these results are only relative the importance of heat treatment is clearly shown.

The cleaning of the glass is very important in high vacuum production It was found that the removal of grease by potassium bichromate solution followed by a prolonged washing with boiling water to remove soluble salts proved most effective.

The electric oven in which the gauges and bulbs were placed was kept at a temperature of 500° C. while the system was pumped for two hours.

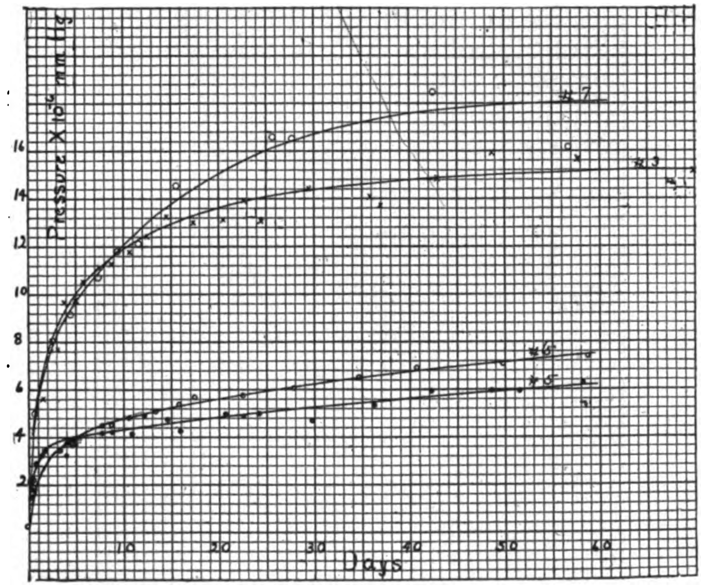

Fig. 1.

The glass would easily stand this temperature though it would collapse at 550° C. This margin of temperature was kept for safety. After the two hours heating and pumping the electric oven was removed and the bulbs and gauges were allowed to cool. After the pressure had been observed the sealing-off process was accomplished by heating the constriction with the pointed flame of a blowpipe. When the glass was near the melting point the gauge indicated a considerable increase in pressure.

The glass was kept heated near the melting point for about ten minutes while the pump continued to operate. The pressures before and immediately after sealing are given in the table below for four bulbs and gauges which had been given identical treatment.

	No. 3 (702-P).	No. 5 (702-E.J.).	No. 6 (702-P.).	No. 7 (702-E.J.).
Pressure before sealing off...	2.7×10^{-7}	$3\ \times10^{-7}$	8.5×10^{-8}	9.7×10^{-8}
Pressure after sealing off....	1.1×10^{-6}	1.5×10^{-7}	3.4×10^{-7}	8.6×10^{-7}

These bulbs and gauges were securely mounted and the changes in pressure with time were observed. These results are shown in Fig. 1. It is noted that in all cases there is a rapid initial rise of pressure with time followed by a slower and more uniform change. No conclusions can be drawn from these data on the different grades of glass as to its suitability for vacuum work. Bulbs 3 and 6 were of Corning glass 702-P. while 5 and 7 were of 702–E.J. or Pyrex. It is observed that 5 and 6 and 3 and 7 almost duplicate each other. It is also observed that bulbs 6 and 7 were pumped to the highest degree of exhaustion yet the pressure in no. 7 is the highest of all after a few days. It is noticed, however, that 5 and 6, which had the lowest pressure after sealing off, still have the lowest pressure after sixty days.

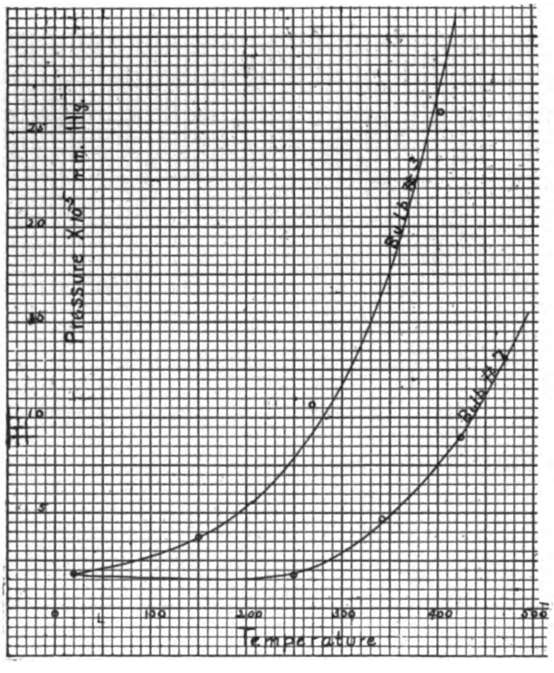

Fig. 2.

To determine the effect of subsequent heating, two of the bulbs and gauges were replaced in the oven and heated for one hour at increasing successive temperatures and the increased pressure resulting from each heating was determined. These results for bulbs 3 and 7 are shown in Fig. 2. It is seen that heating 7 to 250° did not change the pressure while for 3 the pressure increased with all increases of temperature. The changes of pressure at the lower temperatures have always been observed to be less rapid than at the higher temperatures.

This investigation shows that the vacuum in sealed vessels deteriorates with time, rapidly at first and then more slowly, and that subsequent heating even at temperatures lower than the heat-treating temperature results in increase of pressure due to further liberation of gases and vapors from the glass. No connection between different samples of the same glass or different glass can be established. It is quite probable that there are variations in the properties of different samples of the same glass quite as great as the variations between different glasses of about the same grade, for example, Corning 702-P and Pyrex.

THE JOULE-THOMPSON EFFECT FOR AIR AT MODERATE TEMPERATURES AND PRESSURES.

BY L. G. HOXTON.

SYNOPSIS.—In the experiments as described air was pumped first through a set of tubes designed to remove carbon dioxide and moisture, then through the throttling device or "porous plug" and finally back to the pump. The plug and tubing immediately leading to it were immersed in an oil bath thermostat. The bath temperatures, for the most part, lay between 0° and 100° C., the mean pressures, from 4.5 to 6.4 meters of mercury, while the pressure-drop varied between 0.25 and 0.80 meters of mercury. The temperature-drop was measured by platinum resistance thermometers differentially connected. The plug was of the radial flow type originated by Regnault, roughly cylindrical in form, through the walls of which the air flowed toward the axis, escaping finally through one end. The accidental errors were apparently of an order under one per cent.

The Joule-Thomson coefficient (μ) was found to have a decreasing linear variation with increasing mean pressures. Its dependence upon both temperature and pressure are embodied in either of two empirical formulas, namely:

$$\mu = +0.3935 - 0.00691p - 0.001835t + 0.00000134t^2 + 0.0000325pt$$

$$\mu = -0.2599 + 182.0\frac{1}{\tau} - 552.4\frac{p}{\tau^2},$$

all pressures being measures in meters of mercury, t in degrees C., while $\tau = 273 + t$.

These formulas are applied to calculate the temperature of the ice point on the thermodynamic scale, to reduce to the same scale the readings of the constant pressure air thermometer and to discuss the variations of c_p with pressure. Good agreement with the results of other work is found in the second and third cases, while in the first, the result, in common with that found from other Joule-Thomson data, is higher than the result found from the characteristic gas equations.

The paper includes a brief historical summary of previous experimental work, a discussion of several theoretical points and of sources of error, especially heat leakage.

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Historical Outline.
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Preliminary Experiments with Radial-flow Plugs.
Final Results and Conclusions.
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Bibliography.

INTRODUCTION.

A NEED for new determinations of the Joule-Thomson effect has long been recognized by students of thermodynamics, and it has found expression occasionally in the literature touching this subject.

Such data, as is well known, still preserve their original importance for the study of molecular attractions in gases and for the establishment of the thermodynamic scale of temperatures. More recently they have acquired interest for the engineer, namely for the computation of steam tables, ammonia tables and the like. The latter aspect has been discussed by Davis.[1]

Aside from the desirability of experimenting over a wide range, increased accuracy is obviously one of the first improvements in the data that is needed. The published results have been considered too uncertain on the whole, and in many cases too difficult of interpretation to be of great value. The present paper is devoted largely to the latter phase of the problem and is therefore somewhat limited in scope.

At first it was planned to work with several gases and cover a considerable extent of temperatures and pressures. However, a year's experimentation at the Johns Hopkins University with carbon dioxide disclosed many difficulties in the way of obtaining results at all consistent with one another. It was decided, therefore, to confine the work for the time being to an attempt to develop a reliable method and to proceed to this end by ways that seemed the most direct. Side problems were to be avoided wherever possible. Hence the range of temperatures and pressures were restricted within the limitations of the resources already at hand. Air was chosen as the gas to be experimented upon, chiefly because of its availability and the small amount of attention to contamination and slow leaks in the apparatus that its use would entail. Results from pure gases are, of course, to be preferred in general, yet those obtained from air are not without interest as regards thermometry and for comparison with work on air in this and related fields.

It is hoped that in the experiments to be herein described, most of the larger sources of error have been overcome. The accidental errors are small for work of this kind, being of an order considerably under one per cent. Perhaps there is a systematic error outstanding, indicated possibly by the relatively large value computed for the thermodynamic temperature of melting ice. The sources of such error, if existent, have so far not been found. Further experiments with other gases should throw light upon this matter, especially those gases whose expansion coefficients have been determined with great care by Chappuis, namely, nitrogen, hydrogen and carbon dioxide. The author would welcome, in the meantime, any suggestions occurring to the minds of those interested in this subject.

Historical Outline.—Although a historical summary of experimental work down to 1905 has been given by Kester, in a paper to be taken up later, a complete outline would seem desirable here, first to supply a few omissions and secondly to include a considerable amount of new material.

After the original classical experiments of Joule and Thomson,[2] carried out in the years 1852 to 1862 on air, nitrogen, oxygen, carbon dioxide and hydrogen, whose work is too well known to need further comment, one of the early experimenters to enter this field was Regnault.[3] His work has often been overlooked. The most interesting feature of his apparatus is the ingenious design of its porous wall or throttling partition, which was not in the form of a "plug." It was a hollow enclosure through the wall of which the gas flowed from the outside to the interior. A hole in the wall was provided for the continuous escape of this gas. The obvious purpose of this design was to provide thermal protection. In this respect it seems greatly superior to the plug form used by Joule and Thomson. Yet Regnault abandoned the work as futile with an expression of regret at the waste of time spent on a research so "sterile." He found that the temperature readings within the enclosure were too uncertain to have any meaning. In the light of subsequent advances in methods of measurement, several possible causes for these troubles might be assigned, but, especially since his descriptions are not full in detail, their discussion here might not be profitable.

The forms of throttling device or "plug" introduced in these two investigations are typical. Most of the subsequent experimenters have employed the form used by Joule and Thomson, possibly because the work of Regnault, as far as it went, has had little attention directed to it. In 1910 Burnett and Roebuck,[4] in a paper entitled "On a Radial Flow Porous Plug and Calorimeter," proposed a plug on the Regnault principle adding improvements in detail. Their work, however, was entirely independent. Later Roebuck[5] used apparatus of this type on liquid water in order to determine the mechanical equivalent of heat. Only two other experimenters so far have published work with similar apparatus with gases, namely, Trueblood[6] and the present author. Following the usage of Trueblood the term "axial flow" will be applied to the plug form used by Joule and Thomson to distinguish it from the other or "radial flow" type.

One should not fail to note in this connection that certain protective features of the radial flow principle have come into use more or less generally in continuous flow calorimetry, as for example, in the work on C_p by Swann[7] and by Scheel and Heuse.[8]

Hirn[9] carried out throttling experiments upon steam, using an orifice. Work of others on steam, namely, Griessmann,[10] Grindley,[11] Peake[12] and Dodge,[13] has been discussed by Davis.[14] As Davis states, the first named used a porous plug very much like that of Joule and Thomson, while the others used a throttling calorimeter, their work being undertaken for the "purpose of determining the variation of the specific heat of steam with pressure and temperature, an investigation which has since been more satisfactorily accomplished in other ways."

Cazin[15] in 1870 published an account of experiments which were essentially modifications of the Gay-Lussac free-expansion experiment. His apparatus was complicated, and, as he admits, his results were qualitative.

Natanson[16] in 1887 described porous plug experiments on carbon dioxide at 20° C. only. His pressure-drop was about one atmosphere and his highest absolute pressure 25 atm.; and between the lowest and highest pressures he found a decided dependence of the cooling effect upon pressure, whereas Joule and Thomson had not noted (up to 6 atm.) any such effect. This dependence does not agree with that found later by Kester and is in the opposite sense from that found for air in the present paper.

Mention is made here of work reported by Witkowski,[17] who carried out experiments on air at high pressures, passed through a succession of porous plugs. Only a brief abstract of his paper has been available, which contains no quantitative results. There is no obvious theoretical advantage to be gained by employing a succession of plugs. All the data could be had from a single plug by properly varying the pressures and bath temperatures.

Kester[18] in 1905 extended the work of Natanson on carbon dioxide to temperatures between 0° and 100° C. finding also a dependence upon pressure. The results of the two observers are not in agreement, Kester's results at the same temperature (20° C.) being independent of pressure up to the highest pressure employed at that temperature, namely, 41 atm., while the work of Natanson yielded a positive pressure coefficient. At one pressure, 26.4 atm., Kester found an indication of a positive coefficient.

Olszewski[19] performed experiments (using not a porous plug but a throttle valve) on hydrogen in order to determine its "inversion temperature" as bearing upon the problem of its liquefaction. The high pressure of the experiment varied from 117 to 170 atm. and the low pressure was uniformly one atmosphere. The one datum of this experiment was $-$ 80.°5 for the inversion temperature under these conditions. The effect observed was, of course, an integrated one and leaves much to be desired for application of thermodynamic equations.

Five years later[20] (1907) he did the same thing for nitrogen and air, finding a series of inversion points for magnitudes of the pressure-drop varying from 160 atm. to 30 atm.

Rudge[21] carried out (1909) a few experiments on carbon dioxide supplied on the market in the form of "sparklets" for making carbonated water. The duration of each experiment was but a few minutes and their author does not attach great importance to the results, but offers them in view of the scanty known data. It would seem that the ideal conditions for continuous-flow methods could hardly be realized in so short a time.

Bradley and Hale[22] in the same year carried out experiments upon "the nozzle expansion of air at high pressure," *i. e.*, from initial pressures of 70 to 200 atm. to final pressures of 1 atm. Their temperatures extended from 0° C. to − 120° C. approximately.

Their apparatus was a liquefier of the Hampson type and the air expanded through a specially constructed "nozzle." Their particular problem was the process of liquefaction; and their results were in qualitative agreement with those of Joule and Thomson. These experiments would seem subject to error from heat leakage.

Again in the same year (1909) Dalton[23] carried out at the Leyden laboratory similar nozzle experiments in air at 0° C. only and for pressure drops from about 5 to 40 atm. His results agree with the Joule and Thomson figures at 0° C., and he notices no pressure effect, although his curve would indicate an extremely slight negative coefficient.

Vogel,[24] whose original papers have been unavailable, carried out porous plug experiments in the Laboratory of Technical Physics of the Techniche Hochschule in Munich. He was looking for a pressure effect and found a negative coefficient. The temperature of his experiments was 10° C.

Noell,[25] whose original paper likewise has been unavailable but an abstract[26] of which is at hand, continued the work of Vogel in the same laboratory. His pressure-drops were 6 and 8 atmospheres while the arithmetic mean of the absolute high-side and low-side pressures varied from 25 to 150 atm. His high-side temperatures varied from − 55° C. to + 250° C. He worked with air only. He found a pressure as well as a temperature effect which he embodied in the interpolation formula:

$$\frac{\Delta T}{\Delta p} = \frac{50.1 + 0.0297\,p}{T} + \frac{14830 - 1.674\,p}{T^2} + \frac{366000 - 19093\,p}{T^3} - (0.122 + 0.0000157\,p).$$

This is evidently an empirical expansion in descending powers of T, whose coefficients are linear functions of p.

The pressure units seem to be kg. per cm.^{2} in the left-hand member and atmospheres on the right. The difference in any case would not be more than about 3 per cent., but the author does not make it clear. The deviations of his observed values from those calculated by this formula vary from 2 per cent. to 6 per cent. at the lower temperatures, to 50 per cent., and over at the highest temperatures. Nevertheless his work seems to be more systematic and satisfactory than that of his predecessors, certainly for air.

Cuts of the apparatus of Vogel and Noell may be found in Jellineks' Lehrb. d. Ph. Chem., Vol. I., 1914. They show two forms of plug, the earlier being of the axial flow type, the later, a combination of the axial and radial flow types in which an inner axial flow plug was surrounded by a jacket containing the inflowing high side air at bath temperature.

Buckingham[27] has proposed a modified form of the porous plug experiment in which the process is to be rendered isothermal by supplying heat, say electrically, to the plug in just the sufficient amount to prevent the temperature-drop. If there should be a heating effect, as with hydrogen, there would have to be devised some way of abstracting heat from the plug. No extended experiments by this method have been published.

Trueblood[6] carried out experiments at about the same time as those of the present author, which are described in a paper entitled "The Joule-Thomson Effect in Superheated Steam; I. Experimental Study of Heat Leakage." He limited the work to a single temperature and pressure and carried out a thorough and effective study of the heat leakage. Extensive tests, under varying conditions, were made upon a number of different plugs of the axial and radial flow types. He concluded the latter to be the superior and that it is capable of yielding reliable results. A remarkable form of indirect heat leak was found for the radial flow type, which he named the "regeneration effect." This matter will come up later under a separate head.

The experimental work in this field, thus briefly summarized, lends itself only in small measure to the application of thermodynamic equations owing to the difficulty or impossibility of attaching a definite meaning to a large part of it. Thus the only work found available for the calculation of the thermodynamic scale of temperatures hitherto published[28, 29, 30, 31] has been that of Joule and Thomson principally, with later computations including the results of Natanson, Kester and, in one instance, of Olszewski.

It has seemed to the author that in some cases the programs of experimentation have been, perhaps, too ambitious in the direction of extreme temperatures and pressures with the result that an oversight of vital details affecting accuracy was quite possible. The intention to limit the present work in the way mentioned at the beginning was confirmed by considerations of this sort.

General Conditions of Present Experiment.—The range of temperatures was confined approximately to the fundamental interval—actually from 15° C. to 90° C., although experiments had been started at 110° C. when the apparatus called for repair. The term "pressure" in the title of this paper refers to the arithmetic mean of the absolute pressures on the two sides of the plug. The range lay from about 6 to 8½ atmospheres, or more precisely, from 4.5 to 6.4 meters of mercury. The pressure-drop ranged from 0.25 to 0.80 meter of mercury. Pressures throughout this paper will be expressed in meters of mercury. For the immediate purpose of these experiments outlined above, the obvious gas to be chosen was, of course, air.

REMARKS ON THEORY.

The general theory of the plug experiment has been given so often in treatises and in journal articles that a full treatment of it here would be out of place. As much, however, should be given as is sufficient to define the necessary symbols and to develop such special formulæ as are needed in the computation of results.

The function $u + pv$, the "total heat" or so-called "*enthalpy*" of Kamerlingh Onnes, is supposed to be the same in the regions of equilibrium above and below the plug. Let us call it H. Then the Joule-Thomson coefficient (μ) is defined as the limiting value of the ratio of the temperature-drop to the pressure-drop, when these each approach zero, under the condition $H = const.$, that is,

$$\mu = \left(\frac{\partial T}{\partial p}\right)_H {}^{1} \qquad (1)$$

Graphically represented this is the slope of the tangent to the curve $H = const.$ on the (p, T) diagram. From (1), through the application of the second law of thermodynamics, follows the general relation always associated with the porous plug experiment, namely,

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[1] It may be worth while to note, in passing, that inasmuch as one often finds confusion of statement between the free expansion or Gay-Lussac effect and the Joule-Thomson effect, the distinction between the two may be brought out by expressing the two as

$$\left(\frac{\partial T}{\partial p}\right)_u \quad \text{and} \quad \left(\frac{\partial T}{\partial p}\right)_H$$

respectively.

$$T\left(\frac{\partial v}{\partial T}\right)_p = v + \mu c_p.$$[1] (2)

It will be shown later that μ was found experimentally to be a function of temperature and pressure. In the search for an empirical formula to express this relation and one which would have a little more rational basis than a mere power series, resort was had to the equation of van der Waals. While this equation does not strictly fit the measurements on isotherms, yet the author is in accord with Berthelot[32] in thinking of it as a valuable index to broad features of the properties of gases and as being something more than an empirical interpolation formula.

Applying to van der Waals' equation, namely

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

the relation (2) above, we get, keeping only terms of the *second order* in the small quantities a and b, the relation

$$\mu c_p = -b + \frac{2a}{RT} - \left(\frac{6ab}{R^2T^2} - \frac{4a^2}{R^3T^3}\right)p. \qquad (3)^2$$

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[1] Thus calling s the entropy

$$Tds = du + pdv = dH - vdp,$$

or, putting T = constant,

$$T\left(\frac{\partial s}{\partial p}\right)_T = \left(\frac{\partial H}{\partial p}\right)_T - v.$$

But

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \text{ ("thermodynamic relation")}$$

and

$$\left(\frac{\partial H}{\partial p}\right)_T = -\left(\frac{\partial H}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_H = -c_p\mu.$$

$$\therefore \quad T\left(\frac{\partial v}{\partial T}\right)_p = v + \mu c_p. \qquad (2)$$

A short proof quite similar to this has been given by Callendar.[29]

[2] Expanding van der Waals's equation,

$$pv + \frac{a}{v} - bp - \frac{ab}{v^2} = RT,$$

and applying the operator $(\partial/\partial T)_p$ to both sides,

$$\left[p - \frac{a}{v^2} + \frac{2ab}{v^3}\right]\left(\frac{\partial v}{\partial T}\right)_p = R.$$

$$\begin{aligned}\therefore \mu c_p = T\left(\frac{\partial v}{\partial T}\right)_p - v &= RT\left[p - \frac{a}{v^2} + \frac{2ab}{v^3}\right]^{-1} - v \\ &= \left[RT - pv + \frac{a}{v} - \frac{2ab}{v^2}\right]\left[p - \frac{a}{v^2} + \frac{2ab}{v^3}\right]^{-1} \\ &= \left[-b + \frac{2a}{pv} - \frac{3ab}{pv^2}\right]\left[1 - \frac{a}{pv^2} + \frac{2ab}{pv^3}\right]^{-1},\end{aligned}$$

on substituting the value of $RT - pv$ from the expanded equation of van der Walas and

Now the variation in c_p within the limits of the present investigation is small or at most may be considered a linear function of p. If then we neglect the term in T^3, μ should be a function of the form

$$\mu = A + \frac{B}{\tau} + p\frac{C}{\tau^2}, \tag{4}$$

where $\tau = 273 + t$.

The first two terms are those of the complete formula of Rose-Innes.[30] It will be noted that the pressure effect is linear and even then only comes in if we retain second order terms. Later this formula as well as a five-constant power series linear in p and of the second degree in t° C. will be fitted to the observations. Obviously the second power of τ in (4) could be replaced by the first power and the adjustment be empirically just about as good within the small range of observed temperatures, but the second power is retained to keep closer to the original basis.

DESCRIPTION OF APPARATUS.

The Air Circuit.—The air-circuit and the disposition of the various parts are shown in Fig. 1. Air is drawn in by the compressor at the intake A which is kept open all the time in order to replenish the supply of air lost by slow leakage. The air then passes upward through the driers D and D', the former containing a column of about a meter of calcium chloride, the latter about half a meter of soda-lime to remove carbon dioxide, followed by a half meter of calcium chloride to remove the water put in by the reaction of the carbon dioxide with the soda-lime. Water trapped in the lower end of D from the air under compression is

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factoring out p. Let us now expand the right hand bracket and in all expressions retain no order in a and b higher than the second. Then

$$\mu c_p = \left[-b + \frac{2a}{pv} - \frac{3ab}{pv^2}\right]\left[1 + \frac{a}{pv^2} - \frac{2ab}{v^3} + \cdots\right]$$

$$= -b + \frac{2a}{pv} - \frac{4ab}{pv^2} + \frac{2a^2}{p^2v^3} + \cdots.$$

Now, in the second order terms, pv may be replaced by RT alone. In the first order term we must put $pv = RT - a/v + bp$.

$$\therefore \frac{2a}{pv} = \frac{2a}{RT}\left[1 - \frac{a}{vRT} + \frac{b}{RT}p\right]^{-1}$$

$$= \frac{2a}{RT} + \frac{2a^2}{vR^2T^2} - \frac{2ab}{R^2T^2}p + \cdots$$

$$= \frac{2a}{RT} + \frac{2a^2}{R^3T^3}p - \frac{2ab}{R^2T^2}p + \cdots$$

$$\therefore \mu c_p = -b + \frac{2a}{RT} - \frac{6ab}{R^2T^2}p + \frac{4a^2}{R^3T^3}p \cdots$$

Q. E. D.

drawn off by the valve *B*. This was done usually once during a run which lasted some four or five hours.

From the driers the air is carried through a quarter-inch gas pipe to the plug apparatus proper in a neighboring room. Here it passes first through a series of throttling needle-valves *t*, then through a third drying tube *D''* containing phosphorus pentoxide. From this it passes through copper "automobile tubing" of about 5 mm. outside diameter, into the bath, under the surface of which the automobile tubing continues for about two meters, connecting thence with a coil of larger pipe of brass and about one cm. internal diameter and 6 meters in length. This coil terminates in the plug casing *P*. After passing through the porous plug, the air leaves the casing by way of another length of automobile tubing much shorter than the first and soldered parallel with it for a purpose stated below.

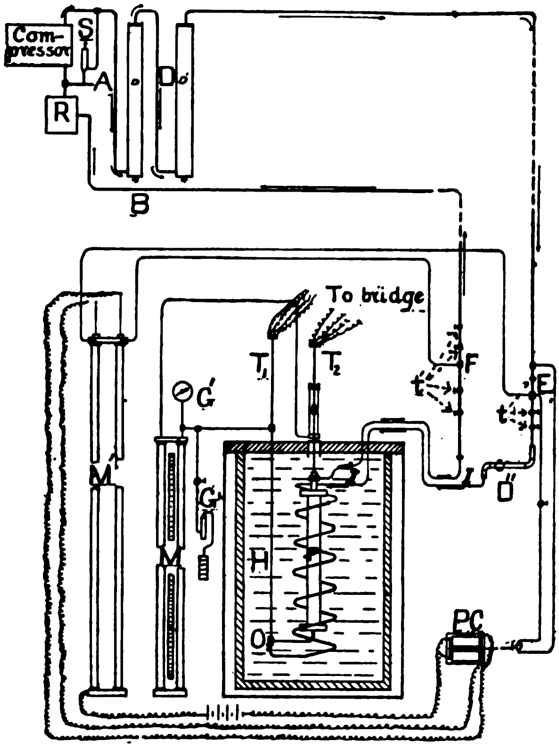

Fig. 1.

Interchanger.—For a length *I*, *I* of about a meter the tubes are soldered together side by side, forming thus a heat interchanger, so that when the bath and the room were at considerably differing temperatures the air leaving the bath would be brought to approximately room temperature and the air entering the bath would be brought to approximately the bath temperature in advance. This device was found to be necessary for two reasons: first, it conserved the heat of the bath and improved its temperature regulation, and secondly, it prevented the throttles t' in the return line from progressively warming, a condition which was found seriously to interfere with pressure regulation. This warming of the valves would gradually change their opening and hence change the rate of flow of air through them.

After leaving the low pressure throttles t' the air returns to the compressor by way of a receiver R of about 10 liters capacity. Curiously enough, this receiver, when placed on the low-pressure side, was quite effective in reducing the pulsations in the air-circuit produced by the intermittent action of the compressor while no receiver was needed at all on the high pressure side.

Pressure Regulation.—It is obviously necessary to provide means to keep constant the pressures at all points, and especially the pressure-drop in passing through the plug. The heat of compression liberated here might quite mask the Joule-Thomson effect. It was observed, for instance, on one occasion, that a fluctuation in the pressure-drop of 0.1 per cent. resulted in a fluctuation of about 5 per cent. in the temperature-drop and consequently by 5 per cent. in the observed Joule-Thomson effect. Special precautions therefore were necessary to provide good pressure regulation. Further, this had to be automatic on account of the necessity for the observer to work alone. Fluctuations of 20 per cent. in the power service had to be cared for.

Rough pressure regulation was attained as a preliminary by placing an adjustable spring safety-valve S (Fig. 1) in a by-pass from the high pressure to the low pressure lines near the compressor. This was usually set so that the difference on the two sides of the valve would be from 80 to 160 lbs., gauge reading, depending upon the pressure drop at the plug.

Superposed upon this was a finer method of regulation which was designed to control the pressure drop between the points E and F. This was carried out by means of an electrically controlled valve mechanism actuated through contacts in a mercury manometer M', which was connected in at the points E and F. The valve mechanism is indicated at PC which is connected as a by-pass to the first or upper throttle t so that when opened the flow of gas is increased.

The two electrical contact-points of the manometer M' are set at any desired difference of level such that the tops of the mercury columns play just under them when the device is functioning properly. If for some reason the pressure difference decreases, the lower contact will be made, and this will actuate one of the electromagnets at *PC*, which, through setting in motion a train of gears will open the by-pass needle-valve at *PC* very slowly, thus supplying an increased flow of air and bringing the pressure difference back to its proper value. A similar action in the reverse direction will take place if the pressure drop exceeds the limits set by the positions of the contact points. When the pressure is at its proper value, the valve at *PC* is at rest.

There were, of course, small fluctuations in pressure remaining, even though this device gave much finer regulation than did the safety-valve *S*. These residual fluctuations were practically damped out by the throttles *t* and *t'* lying between the points *EF* and the plug. The greatest variations in the pressure drop at the plug were of the order of 0.1 mm. of mercury and usually too small to be observed.

The throttles *t* and *t'* together with the safety-valve *S* served, in addition, as a means of setting the absolute pressures in the system to any desired values.

Mounting of Thermometers.—T_1 and T_2 (Fig. 1) are brass tubes housing the stems of platinum resistance thermometers, one of which is situated in the elbow *O*, on the high-pressure side of the plug, and the second in the interior of the plug casing *P*. Emerging from the tops of these tubes and passing through sealed joints in appropriate hard rubber insulation are the leads which connect through heavy lamp-cords to the bridge. Into the latter they are connected differentially as will be described later. Thermometer tube T_2 has a telescoping joint indicated in the diagram whose purpose is to permit one to shift vertically the thermometer and so to explore the region on the low-pressure side of the plug in order to study the distribution of temperature therein.

The Differential Manometer.—From the sides of the stems of the thermometers T_1 and T_2 lead two small tubes connected to the manometer *M* as shown in Fig. 1. The latter is thus put in communication with the spaces occupied by the thermometer coils themselves and is intended to measure the pressure-drop between the high and low pressure sides of the plug. The manometer was somewhat over a meter in height and was furnished with a brass meter-bar of the "H" form made by the Société Genèvoise. This was placed between the tubes and read by a micrometer telescope from a distance of about one meter, the tube and scale being in the same field of view at one time. The crosshair was leveled by setting on the image of the level surface of mercury placed in a glass cell with plane sides at the same distance from the telescope as the manometer. The heights of meniscus were read off in order to supply data for the capillary correction which never exceeded 1 mm., and if omitted, would not have introduced an error exceeding 1/500 of the observed pressure drop, and even that would be within the accuracy demanded, for errors somewhat less than one per cent. in the Joule-Thomson effect came in from other sources. The corrections, however, were applied making use of the tables given in Kohlrausch, "Physical Measurements," 3d edition (English), p. 440. The internal diameters of the tubes varied from 5.0 mm. to 5.7 mm. along the portions where readings were made.

The readings were likewise reduced to 0° C., making use of tables given in Landolt and Börnstein's "Tabellen," 3d Aufl., pp. 34 and 35. Pressures throughout this paper are expressed in meters of mercury at 0° C. at the University of Virginia. The value for g (980 cm./sec^2) is needed only in final computations.

Pressure Gauges.—To the high-pressure side of the differential manometer were connected a piston gauge and a dial gauge, G and G' (Fig. 1) respectively. The latter was used for setting to the desired absolute pressures; while the former yielded their correct measure. The piston was of 3/8″ drill rod mounted in a snugly fitting brass cylinder provided with a leather packing at the top, so curved as to become tighter as the pressure increased. This also insured that the effective area of the piston was its own cross section and not a compromise between this and that of the bore of the cylinder. Provision was made for rotating the piston, in a manner similar to that carried out by Amagat, in order to eliminate the static friction.

The diameter of the piston measured by two micrometer calipers came to 0.9536 cm. giving an area of 0.7142 sq. cm. Thus 1 kg. on the piston corresponded to a pressure of 1.030 meters of mercury at 0° C. and at the same locality.

The weights were adjusted by a set of platinum plated weights furnished by Rüprecht of Vienna, and brought to within 0.02 per cent. of accuracy or about one fiftieth of the errors of reading of the gauge when in use. Great accuracy was not needed, for it will be seen that the effect of pressure on the Joule-Thomson coefficient is small.

Bath and Thermostat.—The bath was about 90 cm. deep and 30 cm. in diameter with triple walls of sheet brass. The space between the middle and inner walls was filled with asbestos, and that between the middle and outer was designed for the circulation of steam, but not so used. The outer wall was lagged with asbestos. The cover was lagged with about 10 cm. thickness of cotton waste which has been all that was necessary for temperatures up to 130° C. "Renown engine oil" made by the Standard Oil Co. was used to fill the bath and, on account of its great thermal expansion, a siphon overflow had to be provided. Rapid circulation was provided by two motor-driven propellers on one vertical shaft rotating in a well. The stirrer and heating coils are not shown in the diagram.

Heating was applied electrically from a 220-volt D.C. circuit, through four coils totaling a capacity of one kilowatt. These could be cut in independently in series or in parallel, various combinations being used, including auxiliary resistances, according to the temperature of the bath.

Thermostatic control was obtained by keeping a steady current in some of the resistances and varying the current in the remainder. This was done automatically by the wheatstone bridge type of thermostat similar in principle to that described by Darwin[33] and by Randall[34] and later made use of by the Leeds and Northrup Co. Its performance was quite good, keeping the bath, under the best working conditions, constant to 0°.001 C.; and in but few instances was the fluctuation more than 0°.004 C. It is a remarkably convenient form of apparatus for setting at any desired temperature.

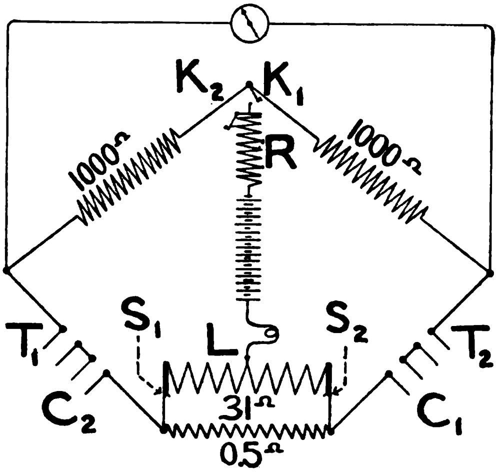

Fig. 2.

The use of a 220-volt D.C. supply is perhaps open to criticism; but with care in insulation, trouble from leakage through the galvanometers from this source was avoided. It was, moreover, the only kind of power-supply at hand.

The Wheatstone Bridge.—Resistance measurements were made in terms of a decade resistance box manufactured by the Leeds and Northrup Co. of Philadelphia. The coils needed (from 30 ohms down) were calibrated twice, two years apart, and found to remain constant to well within the required accuracy.

The thermometers on the high and low-pressure sides of the plug were connected into the bridge differentially as shown in Fig. 2, where T_1 and T_2 represent the wires leading to the thermometer coils and C_1 and C_2 the compensation leads which will be described below.

The quantity desired is the difference in the effective resistances of the two thermometers. The Carey-Foster[25] method was employed because it measures directly the difference between resistances and eliminates all fixed contact- and lead-resistances. The mercury commutator for effecting the transposal of the resistances to be compared, which in this case are $T_1 + C_2$ and $T_2 + C_1$ is omitted from the diagram.

It will be noted that the arrangement of battery and galvanometer is not that usually published in the text-books. By putting, as indicated, the battery instead of the galvanometer in series with the sliding contact and by having no key in series with the galvanometer the error of thermal E.M.F.'s was eliminated. This arrangement follows the practice of the Bureau of Standards. It has worked admirably and obviated the necessity of reversing the battery current.

The slide wire S_1LS_2 was a "Students' Potentiometer" manufactured by W. G. Pye & Co., of Cambridge, England, and consists of a wire wound in a helix on a slate drum and with sliding contacts S_1S_2 at the ends. It has 10,000 divisions from end to end and a resistance of 31 ohms approximately. This being too high for direct use, it was shunted by a resistance of about half an ohm made of manganin wire. With this arrangement and by repeated tests at different dates the value of one division was determined to be 0.00005666 ohms or, in terms of the particular thermometers used, equivalent to about 0.0005° C. The precise conversion of bridge readings into degrees will be given later.

The errors due to the variability of the sliding contact resistances S_1 and S_2, being connected in series with a resistance as high as 31 ohms, proved to be negligible. For, by test, the uncertainty was not over 0.12 division (0.00006° C.) while, in the final readings, only the nearest whole division was taken into account. All other junctions in the four arms were either amalgamated or soldered. The wire S_1LS_2 was calibrated several times and was found uniform to a small fraction of a division.

For resistance thermometry, and with the battery connected between the thermometers as shown, the more sensitive arrangement is to have high-resistance ratio-arms; hence a high value for these, namely, 1,000 ohms, was employed.

The key K_1 opened and closed the battery circuit while K_2 in conjunction with the resistance R, permitted the observer to vary the battery current for the purpose of obtaining corrections due to the heating effect of the measuring current in each thermometer.

The Platinum Resistance Thermometers.—These were of the four-lead compensated type introduced by Callendar.[36] The ends of the compensating leads were united by a short length, about 1 cm., of the same sort of wire as that used for the thermometer coils. The thermometers were connected differentially into the bridge, thermometer No. 1 and compensating-lead No. 2, being connected in series on one side, while thermometer No. 2 and compensating-lead No. 1 were in series on the other side. These connections (Fig. 2) are lettered T_1, C_2 on the left, T_2 and C_1 on the right. In differential connections, the need for the compensation leads is very much reduced, but, in the present instance, they were kept in to be on the safe side, especially in view of the fact that the depth of immersion of the two thermometers was not quite the same.

The wire for these thermometers was obtained from the Cambridge Scientific Instrument Co., Ltd., very pure and recommended by them for thermometer work, in which they make a specialty. The diameter of the wire was about 0.03 mm., and its length such as to give a change in resistance of about 11 ohms for a change in temperature from 0° C. to 100° C. It was wound upon the usual crossed mica frames in the form of a compact cube approximately 5 mm. each way. The fineness of the wire and the compactness of its mounting was for the purpose of allowing exploration of the space on the low-pressure side of the plug.

The four leads were sealed into the ends of a glass tube as appears at T_1 or T_2, Fig. 3, and passed thence up the tube and out through a wax-sealed joint into the room whence they were connected to the bridge by heavy No. 14 lamp-cord about 3 meters long each.

The thermometer coils were exposed to the flowing air in some of the preliminary experiments, but later, copper foil caps indicated by the dotted lines at T_1 and T_2 were slipped on. This made the galvanometer considerably steadier and did not introduce an objectionable lag because the copper caps should take the temperature of the gas before the walls of the enclosure.

The glass tubes containing the leads, after emerging from the air channels, were enclosed in larger brass tubes of ⅛″ pipe size (the brass tube for T_1 shown in the diagram is larger than natural size), which came in contact with the bath and allowed air-connection to the differential manometer. The immersion of T_1 was about 44 cm. and that of T_2

about 34 cm. under the bath. The latter immersion was extended by a lagged air-space above the bath's surface by about 12 cm. This was thought sufficient to eliminate the effect of heat conduction along the leads inside the glass tubes in spite of the stagnant air-space by which the leads were surrounded.

The electrical insulation proved sufficient at all times. Repeated tests were made; in fact, the bridge wiring was arranged so that the thermometer insulations between pairs of leads could be coupled in parallel and the combination thrown in series with the galvanometer and a battery of 30 volts. This test, as a matter of precaution, was made a part of the routine of observation. In dry weather the galvanometer was not sensibly deflected, while in one or two instances in damp weather a film seemed to form on the outside insulation whose resistance was of the order 20 megohms. This obviously would not affect the measurements.

Several tests were made for a possible effect of pressure on the resistance of the thermometers but none was found.

Application of Callendar's Formulæ.—These were adapted to the work in hand, as embodied in equation (8) below, in the following manner, beginning with the formulæ themselves, namely:

$$t_p = 100\frac{R - R_0}{R_{100} - R_0} \text{ (defining the ``platinum scale'')} \tag{5}$$

and

$$t - t_p = \delta\left(\frac{t}{100} - 1\right)\frac{t}{100} \text{ (correcting to the hydrogen scale),} \tag{6}$$

where δ is a quantity to be determined by experiment for the sample of platinum wire used.

Now, if Δt is a small temperature change, we get from (6)

$$\begin{aligned}\Delta t &= \Delta t_p\left\{1 - \frac{2\delta}{10000}(t - 50)\right\}^{-1}\\ &= \Delta t_p\{1 + 0.0003(t - 50)\}\end{aligned} \tag{7}$$

to the first order of approximation, if we put $\delta = 1.50$. This value, given by the Cambridge Scientific Company, who furnished the wire, was accepted as sufficiently accurate. For it is evident that its omission altogether could not alter the right hand member of (7) by more than 1.5 per cent. in the range from 0° to 100° C. covered in these experiments.

If μ_{t_p} therefore, is the value of the Joule-Thomson coefficient observed in terms of the platinum scale and μ in terms of the hydrogen scale, then we have

$$\mu = \mu_{t_p}\{1 + 0.0003(t - 50)\}, \tag{8}$$

where μ_{t_p} is the observed Joule-Thomson coefficient in terms of platinum degrees, and ${}^{g}\mu$ this coefficient calculated in terms of the gas-scale. (The distinction between the hydrogen and nitrogen scales need not be drawn for these measurements.)

The values of the fundamental intervals, i. e., $R_{100} - R_0$:

	Thermometer T_1	Thermometer T_2
May 12, 1915............	10.952 ohms.	10.930 ohms.
" 14, "	10.963 "	10.932 "

Accurate knowledge of the fundamental interval of thermometer T_2 is all that is necessary, a matter that will come up later. The weighted mean for T_2 is 10.932 ohms.

The above tests were made with currents of air flowing past the coil at two speeds and likewise with the air stagnant. No variation of consequence occurred. The thermometers, moreover, showed themselves more than sufficiently stable during tests covering a year previous. The actual data then obtained cannot be used, however, on account of subsequent slight alteration in the thermometers.

Correction for Heating Due to Current.—The bridge current, of course, heats to some extent the thermometer coils themselves and this must be allowed for. Since the liberation of heat is proportional to the square of the current, the resulting small rise in temperature should follow the same law. Callendar's method is to double the bridge current and subtract one third the difference of the two corresponding settings from the first setting. The author found it a little more convenient to increase the bridge current in the ratio $1 : \sqrt{2}$. Thus the correction is immediately the difference in the two bridge settings.

Moreover, in order to make certain that this rule held under the present conditions a direct test was made with bridge currents in the ratio $1 : 2 : 2.5 : 4$. The linear relation between the bridge readings and the square of the current was satisfactory. This held also in varying currents of air. Thus, where the rates of flow were 0, 2.9, 19, 36, 80, 132 and 250 cc./sec. the corrected bridge readings were 0.°016, 0.°018, 0.°015, 0.°015, 0.°015, 0.°015, 0.°014 respectively (reckoning from an arbitrary zero).

The order of magnitude of the heating correction in the final runs was 0.°01 C. The lower value of the battery current was 6 milliamperes.

Incidentally, the above data show that the length of the coil of pipe in the bath was sufficient to give the gas the temperature of the bath before it reached the plug.

Conversion of Bridge Readings to Degrees.—After the heating correction has been applied, the differential reading may be converted into degrees by a multiplying factor which was obtained as follows:

The unit for reading off settings was taken as one revolution of the slide wire drum and is called the "bridge unit" hereafter. Repeated standardizations of the wire at four different dates in terms of the calibrated ohm gave the value 0.005666 ohms per bridge unit.

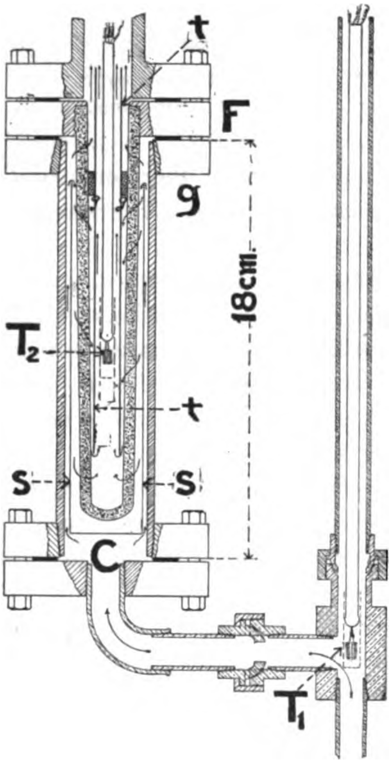

Fig. 3.

Combining this with the figure just given of 10.932 ohms per 100° C. we get the factor 0.05178 degrees per bridge unit.

The values for μ obtained from the observations in terms of bridge units per meter of mercury are convertible into their proper values in terms of degrees per meter of mercury by the application of first this factor and then formula (8).

The Porous Plug.—The form of plug finally adopted was suggested by the research of Regnault already alluded to. Its longitudinal section is shown in Fig. 3 (dotted shading) the material being earthenware—a Pasteur filter. As may be seen, the flow of gas through the plug is radial toward an axis, and after passing through the plug wall its outflow takes place parallel to the axis. The purpose of this construction, as already pointed out, is to furnish thermal protection.

The whole apparatus shown in Fig. 3 is immersed in the bath, whose surface is 44 cm. above the point T_1. Air after traversing about 8 meters of pipe, in the bath, passes T_1, enters the plug casing at C and travels upward between the wall of the casing and a bright tinned radiation shield *ss*, as is indicated by the arrows. After passing over the top of the radiation shield it fills the space immediately outside the wall of the plug, through which it flows in two streams, divided by the "guard ring" *gg*. Within the plug is a glass tube *tt* which constrains the lower of these two streams to pass, all of it, past the thermometer T_2 while the upper stream is kept from passing the thermometer. Both streams pass out finally at the top.

The purpose of the guard ring is to avoid errors arising from conduction of heat along the material of the plug from its mounting F and ultimately reaching the thermometer inside.

The temperature of the plug in general is different from that of the casing. Radiation between the two would introduce an error. The radiation shield is to prevent this. As will be seen later, the radiation shield proved to be unnecessary but it was retained as a precaution, and is free from objection.

The construction of the plug casing (Fig. 3) permits the use of the principle of interchangeable parts. It may be seen that different plugs may be bolted in and that different sizes of casing as well may be employed. This proved a valuable feature in the preliminary experiments.

PRELIMINARY EXPERIMENTS.

Early Experiments with the Axial Flow Plug.—A number of experiments were carried out at the Johns Hopkins University in 1905–6, on carbon dioxide with a straight-flow plug of the type used in the Joule-Thomson experiments. Platinum resistance thermometers were used and so arranged as to be movable. The pressure-drops were of the order of one atmosphere. Plugs of various materials were tried, eider down, cotton, silk and paper. A vacuum-jacket was attempted and, what seemed better, the guard ring principle contained in a suggestion by Buckingham[27] was carried out. The porous plug was coaxial with a second annular plug designed to eliminate lateral temperature gradients.

The last arrangement improved things somewhat, giving one result in agreement with those of Joule and Thomson, but the general characteristic of the experiments was their lack of reproducibility.

On displacing the low-side thermometer, for instance, a steep temperature gradient along the axis was observed. The observed cooling effect continually fell off as the thermometer was further withdrawn. A shift, for instance, from 1 to 4 and 6 cm. distances cut the effect to about one half and one third respectively.

With varying rates of flow also the effect varied, but with a tendency toward a limit for the higher speeds. Natanson observed a variation with flow also. Perhaps we can thus account for the partiality of observers for high flows and plenty of gas. This feature and the ample dimensions of the apparatus of Joule and Thomson undoubtedly contributed to the accuracy of their results.

In addition, an enormous amount of time had to be consumed before a steady state could be attained, amounting to an hour and a half before a reading could be taken. Apart from other considerations, this would lead to a large chance for error unless the pressure regulation were watched very closely. A damped oscillation in the cooling-effect similar to that in the Joule-Thomson experiments was also observed.

It was decided, then, as already mentioned, to narrow the field and to concentrate on developing a method for the attainment of reliable results. What was to be desired was thermal protection, fine pressure regulation, and good temperature control. These problems have been taken up in the manner already described. The later experiments have been carried out in the Rouss Physical Laboratory of the University of Virginia.

Preliminary Experiments with Radial Flow Plug—Exploration.—Referring to Fig. 3, it will be remembered that the thermometer T_2 was capable of being shifted throughout the entire length of the plug. In the initial explorations the inner tube *tt* had not been introduced. The temperature distribution was fairly uniform except above the middle, and the correction for the heating of the thermometers due to the bridge current was much larger at the bottom of the plug, where the air-flow was slow, than at the top where the air-flow was faster. This led to the introduction of the tube and guard ring with the result that it not only reduced this heating correction but rendered the temperature distribution along the tube uniform to the order of 0.°001 C. Closer tests than this were not pressing nor possible, for time-fluctuations due to bath and pressure fluctuations were of this order of magnitude.

The final arrangement of the plug with tube and guard ring is almost identical with the arrangement published by Burnett and Roebuck (*loc. cit.*).

Rate of Flow.—This was separately measured by collecting the air over a pneumatic trough. Roughly the linear speed was found to be directly proportional to the pressure drop and independent of the absolute pressures, and hence of density. The result can therefore be simply expressed by a single number, namely 2.3 cm. per sec. per cm. drop of mercury. In the final runs this would mean a maximum speed within the tube *tt* of 180 cm. per sec., a little more than this at the bottom of *tt* (Fig. 3) and about double this around the thermometer caps. In the coil leading into the plug-housing, the speed was about the same.

Velocity Cooling.—The upper limit of the possible errors arising from the change in kinetic energy involved in the above variation of speeds was computed to be of the order of magnitude of 0.°004 C. Actually none was observable. For, in a run with the plug removed and with speeds of flow in the ratios 50 : 100 : 200 : 330 the bridge readings (with reference to an arbitrary zero) were 0.°037, 0.°038, 0.°038, 0.°038 respectively. The fastest flow exceeded the greatest speed of the air in the final runs. This run, moreover, was taken with the thermometer caps on, when the error from velocity cooling should be a maximum, as is plain from a glance at Fig. 3. For, when the caps are removed, the speed of air past the thermometers would be less.

The formula used in this computation, namely,

$$c_p(T_1 - T_2) = \tfrac{1}{2}(w_2^2 - w_1^2) + g(h_2 - h_1)$$

was not strictly applicable, for, as in all deductions of this sort the temperatures could be given only by thermometers *moving with the gas.* Stationary thermometers would be expected to indicate smaller temperature differences. The preliminary experiments of Joule and Thomson, as well as those just outlined, are in agreement with this idea.

Aspirator Effect.—This is an effect cognate with velocity cooling. For to measure the pressure of a stream of fluid the manometer likewise should be moving with the fluid, strictly speaking. A test for error arising from this cause was made by taking readings on the differential manometer with the plug removed while the air was run through at velocities higher than those employed in the runs. The manometer showed zero reading throughout.

Effect of Varying Rates of Flow upon the Observed Temperature Drop.—These observations are depicted graphically in Fig. 4. Curve 2 was taken with the plug whose characteristic rate of flow has just been given,

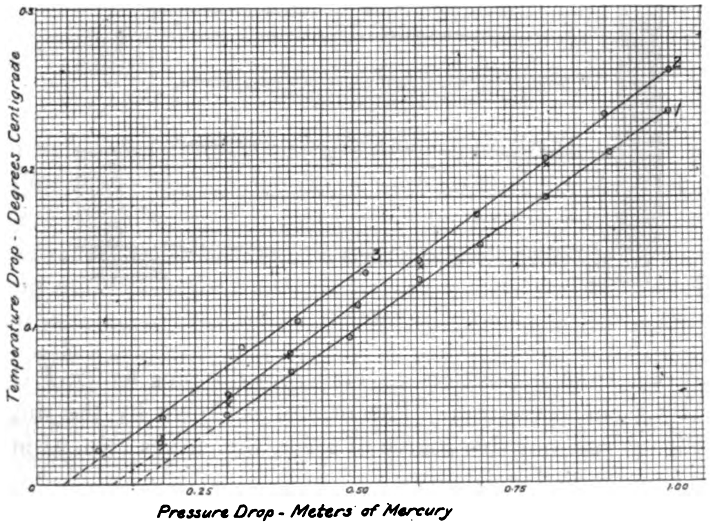

Fig. 4.

Preliminary Experiments, Varying Rate of Flow and Position of Guard Ring.

Curve 1: Relative Flow 0.7—Slope 0.28
" 2: " " 1.0— " 0.30
" 3: " " 3.0— " 0.29
Crosses: Guard Ring shifted 4 cm.—flow as in 2.

namely, 2.3 cm. per sec. per cm. of Hg pressure drop. Curve 1 applies to the same plug wrapped with rubber bands till its flow was 0.7 as great, while curve 3 is for a plaster of Paris plug whose flow was three times as great.

The first essential feature of these curves is that they are straight lines within the error of experiment, except near the origin. (This is true not only in the preliminary but in the final runs.) Near the origin the points were so irregularly distributed that they have been omitted from the diagram.

The second essential feature is that the more rapid the flow the nearer the curve approaches to a limiting line passing through the origin. A third feature to be noted is that these lines are sensibly parallel. These features will be taken up later, in discussing the effects of a residual heat leakage.

Heat Leakage by Conduction.—The nature of this residual heat leakage was sought in conduction, radiation and convection. Conduction through the gas does not seem probable. The only remaining path would be along the material of the plug. This point was tested by shifting the guard-ring from a position 1.5 cm. from the plug flange *F* (Fig. 3) to a point 5.5 cm. therefrom. The results as shown by the rings and crosses (Fig. 4) rule out this source of error, the curves (No. 2) in these two cases being indistinguishable.

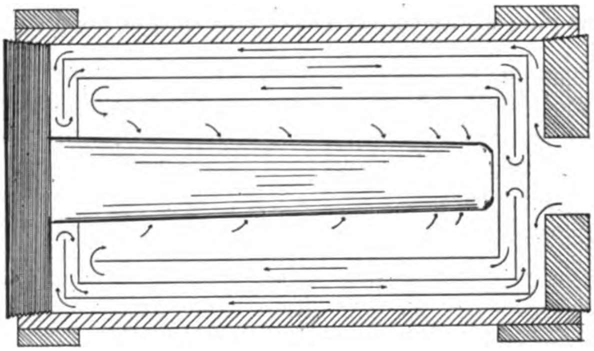

Fig. 5.

Heat Leakage by Radiation.—Since the surface of the plug must be at a different temperature (lower in case of air) from that of the walls of the plug casing, heat may pass by radiation across the annular space between them and thus reach the interior without first being absorbed by the gas itself, and so an error may be introduced. This was first tested by comparing runs with the dark iron casing walls lined with bright tin and then without the tin. There was no systematic difference. Again a trial was made with three radiation shields on the one hand, disposed as in Fig. 5, in a large casing and on the other, with no radiation shields at all (the casing being that shown in Fig. 3). Fig. 6 shows that these results also are indistinguishable. Radiation, then, is not to be looked for to explain the depression of these lines nor as a serious source of error.

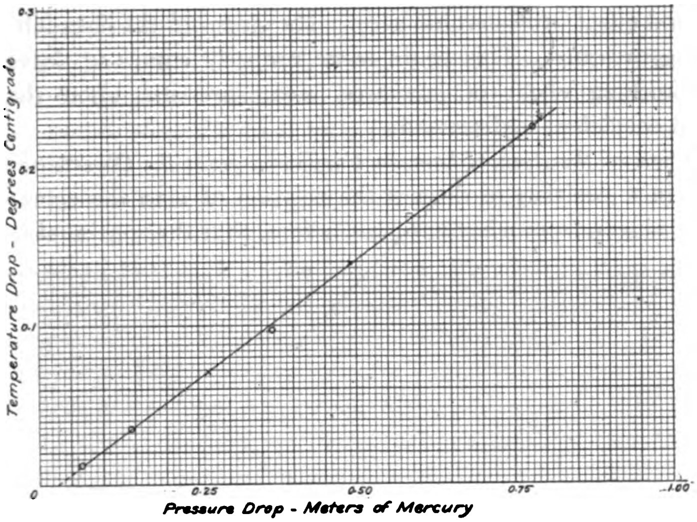

Fig. 6.

Preliminary Experiments, Effect of Radiation Shields.

Circles: No radiation shields
Crosses: 3 " "

Heat Leakage by Convection.—A turbulent motion of the gas surrounding the plug could possibly convey heat across from the walls of the casing without permanent absorption. If so, an increase in the inside diameter of the casing should reduce this error and shift the lines nearer to the origin. Now in the experiments of Fig. 6 (no shield), the annular space between the walls and the plug was about 6 mm. thick or about twice as thick as for the experiments of Fig. 4. The curve of Fig. 6 passes much nearer to the origin. Consequently convection must play a part.

It was not thought worth while to push these experiments further, the practical limit being apparently reached in the experiments of Fig. 6.

Influence of Density.—Where the mean density of the air is increased by operating at increased mean pressures, one should expect a minimizing effect upon the errors in the temperature readings, owing to the increased volumetric thermal capacity of the gas and the fact that the heat leakage should not increase in the same ratio, if at all. This was verified by experimental runs, whose results are shown in Fig. 7, where it will be seen that curves corresponding to the higher pressures pass nearer the origin. Further confirmation in the final results were found and will be noted when these come up.

A second effect is observable here. The slopes of the lines fall off with increasing pressure. This again more properly comes up later, where the diminishing effect of increased mean pressure upon the Joule-Thomson coefficient is discussed.

Derivation of the Joule-Thomson Coefficient from the Observations.—We are now in a position to discuss the relation of μ to these preliminary data. It is seen from what has gone before, that the continual reduction of error by first reducing the heat-leak in obvious ways and secondly by increasing the mass of gas affected, has led in the main to the shifting of the curves of the type shown in Figs. 4, 6 and 7 so that their rectilinear portions (produced) are made to pass nearer to the origin without, however, changing their directions sensibly (exclusive, of course, of the pressure effect, which is entirely independent). The limiting ideal case, therefore, would be represented by a straight line passing through the origin, whose slope would be the true value of μ. The obvious step then

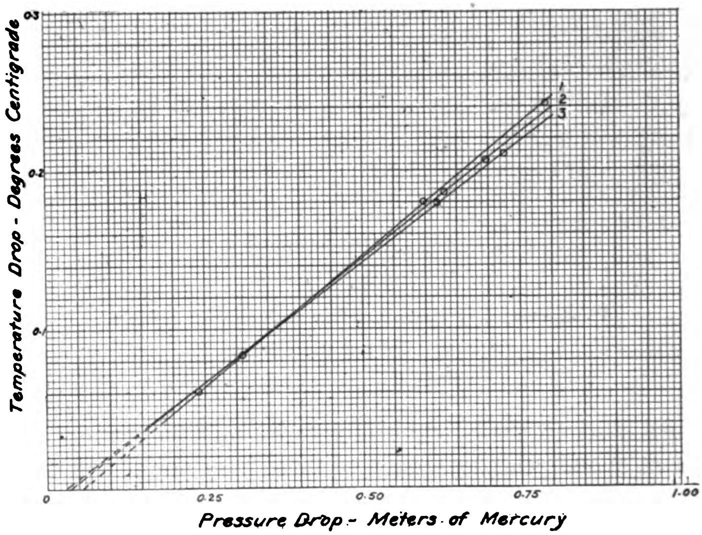

Fig. 7.

Preliminary Experiments, Varying Densities.

Curve 1: Mean pressure 2.5 meters mercury
" 2: " " 4.5 " "
" 3: " " 6.4 " "

to take is to make the actual runs under conditions as nearly ideal as feasible; and for the values of μ adopt the values of the slopes of these straight lines without reference to their position. These values are taken to be as near the truth as the data are capable of yielding it.

The question may now be asked: Can anything be said as to the constancy of $u + pv$ or H along these plots? It will be remembered that after the gas has passed through the plug the increment in the function H is equal to the gain of heat per gram of gas during its passage. Now it is conceivable that the heat-leak (measured in calories per second) might be of such magnitude as to make the slope of the plots too steep for a locus of constant H, or again, such as to make it too flat.

Start, for example, with one of the smaller pressure drops and suppose there is a small heat leak. The resulting temperature drop will be too small. Let this observation be represented by a point on the (p, t) diagram, say, near the origin on a plot like that of Fig. 6. The value of H for this point will be greater than its value, H_0, for the origin, which represents the state of the gas on the high pressure side of the plug. A straight line, then, drawn from the origin to a point A, say, when there is a heat leak, will pass from regions of lower to regions of higher H and will be too flat. Now suppose that the experiment be carried to a higher pressure drop and a second point B is determined. As a different and an extreme case of heat-leak, let us suppose, for the moment, that from the point A onward it remains constant. Then, since the rate of flow has been increased, due to the increased pressure-drop after we leave A, the gain per gram falls off. The line AB, therefore, passes from higher to lower values of H and is too steep. Now let us try another hypothetical law for the heat-leak, namely, that from the point A onward, it is proportional to the rate of flow of gas. Then the gain in heat per gram remains fixed and the two points A and B are characterized by the same value of H. A line plotted under this condition would be, therefore, a line of constant H, although this value would be greater than H_0. In general, then, starting from the point A: if the heat-leak varies more rapidly than the first power of the rate of flow of gas, the plot is too flat; if, less rapidly, the plot is too steep. In call cases the plot will lie under the line $H_0 = \text{const.}$

The hypothesis that the heat-leak is proportional to the rate of flow of the gas seems a reasonable one to make for actual Joule-Thomson experiments, at least as a first approximation. For, as has been seen, the rate of flow is approximately proportional to the pressure-drop, which in turn is very closely proportional to the temperature-drop, and the latter again in turn would be expected to be proportional to the heat-leak, certainly as far as conduction and radiation are concerned, and possibly convection too. In the case of convection, however, there seems to be a possibility that the leak would be speeded up in the case of turbulent motion and cut down in the case of stream-line motion, thus producing deviations from the proportionality relation on one side or the other. At present there is no way of deciding this point but the deviation in any case should be small.

If, then, the total heat-leak be small and if its deviation from being proportional to the rate of flow be likewise small, the deviations of our plots from true curves $H =$ const. should be small to quantities of the second order of errors. It is hoped that this may be reasonably claimed for the final runs described in this paper.

In the light of what has just been developed, the actual plots are now susceptible of a further interpretation. The curve of Fig. 6, if completed by connecting up with the origin by the experimental determination of the requisite further points would of course curve upward in this region. This was actually done in some instances although some of the points were uncertain. This curvilinear portion of the plot, then, would represent a heat-leak varying more rapidly than a direct proportion of the flow-rate, while the remainder would represent a heat-leak proportional to this rate. Both the excess heat-leak and the uncertainty of the points near the origin are to be expected on account of the very slow, almost stagnant, flow of the air in experiments with such small pressure-drops.

As a matter of fact it is physically impossible for anything but an excess heat-leak to occur for this region. Otherwis we should have two curves, H and H_0 say, for which $H - H_0$ is finite, approaching intersection as a limit at an arbitrary point, namely, the origin of the diagram. We have, then, in other words, the surprising inference that it is impossible, in a throttling process, for a heat-leak to be continuously proportional to the rate of flow of gas below a finite limit of the flow and, likewise, of the pressure drop, although this would seem quite possible otherwise.

Referring now to Fig. 4, where the rate of flow of the air has been varied, other things being unchanged, by altering the porosity of the plug, the plots are explicable on the ground that the heat-leak is proportional to the rate of flow of the air in each case but that the constant of proportionality is smaller the greater the porosity of the plug.

Zeros of the Differential Thermometers.—In the determination of the slope of the lines just discussed, it is obviously unnecessary to know the precise value of the bridge setting when the pressure-drop is zero. This knowledge was useful in the preliminary experiments in affording an index of the magnitude of the heat-leak. In the final run, therefore, the determinations of the zeros were discontinued. For they required time and at best were uncertain. The conditions for accuracy in a body of stagnant air in the pocket-like interior of the plug in the position shown in Fig. 3 are obviously unfavorable. The only way to put this accuracy on a par with the other readings would be to provide a way of opening the plug and sending a current of air through.

A matter well worth noting here is that, when this method of observation is employed, one is relieved of the necessity of a close adjustment to equality of the differential thermometers. The high side thermometer serves in reality merely as a sort of compensator for error due to fluctuations in the bath temperature, and an accurate knowledge of it even is unnecessary. For example the fundamental intervals of the two thermometers T_1 and T_2 differed by 0.3 per cent. Now suppose the temperature of the bath during a run changed by as much as 0.01 degree. Then the differential reading, after the steady state had been reached, would be changed by 0.00003 degree.

Values given for the Temperatures and Absolute Pressures.—In what follows μ will be discussed as a function of temperatures and pressures. The pressures will be the means of the pressures above and below the plug. On the diagram they will be represented by the mid-points of the limited straight lines there plotted. As to temperatures, the variations are too small to make it worth while to correct the bath temperatures.

Error from Moisture.—Tests made at a fixed temperature and pressure, in which the air from the room was passed through long-used calcium chloride first; then the drying material was omitted; finally fresh calcium chloride was put in. The values obtained for μ were 0.35, 0.34, and 0.33 degrees per meter respectively, showing a small increase due to moisture (and the poor quality of old calcium chloride). One should expect, then, that a final drying with phosphorus pentoxide would suffice. In the final runs this was used, with the exception of those at 30° C., and these, in spite of that, seem to fit well into the empirical formula derived from all the observations, as will appear later. The phosphorus pentoxide could be continually observed from the outside during the runs, and it was only at the ends of the longer ones that it had a gummy appearance.

Conduction of Heat along the Thermometer Leads.—After the final runs had been made, it was suspected that there might be an error arising from this cause. A new set of runs with the thermometer caps removed, exposing the thermometer wire to the stream of gas would test this point.

A number of observations were, therefore, made covering the same range of temperature and pressure. The galvanometer, of course, was quite unsteady under these conditions, but the agreement between the new and the old data was satisfactory. Thus the mean of ten runs with the caps off gave for μ a value 0.265 while the mean of ten runs at the same temperatures and pressures with the caps on gave 0.267 degrees C. per meter of mercury pressure-drop. This was well within the accidental error of the runs with caps off.

Regeneration Effect.—After the experiments described in this paper had been completed, the paper by Trueblood, referred to in the beginning, appeared, in which is described a form of heat-leak arising from the peculiar geometry of the radial flow apparatus. The nature of this regeneration effect can best be described by quoting from Trueblood's paper: "At its open end, the plug is necessarily brought, for purposes of mechanical support, into good thermal contact with the metal walls of the case. As a result, fluid which has passed the low side thermometer is provided with a thermally short path of communication with fluid which has not yet passed through the plug. This results in a depression of the temperature of the latter, which, joining the low side fluid at at a temperature therefore lower than then exists there, contributes to a still further depression of the temperature of the fluid which has not yet passed. This cumulative action, similar in principle to that taking place in certain types of liquid air machines, is limited by the ability of the bath to supply heat to the low side fluid. . . . The effect of the action just described on the observed temperature drop is indirect, since the fluid directly affected does not pass the low side thermometer. But the temperature depression produced in the neighborhood of the open end produces indirectly, by conduction through the walls of the plug, the walls of the case, the inside lagging and, doubtless, to some extent through the fluid itself, a depression of the temperature of fluid which does pass the low side thermometer; that is, there is an outward leak of heat from fluid between thermometers to fluid which has passed the low side thermometer with the result that the observed temperature drop is too high."

In the author's apparatus the regeneration effect, he believes, was negligible, although, as a matter of fact, its possibility was not foreseen. For, under the heading *heat leakage by conduction*, it was shown that a shift in position of the guard ring did not lead to a change in reading. Now since the temperature depression of the low side gas is due to the "regeneration" process should be manifest in greater and greater degree at points nearer and nearer to the mechanical support, it should have been shown up by the test in question, which, as has just been seen, was not the case. This result may have been due to the unusually long and narrow proportions of the plug used. A natural remedy for the regeneration effect or any effect arising from conduction would seem to be the employment of a long plug.

FINAL RESULTS.

Between August, 1915, and January, 1916, fourteen determinations of the Joule-Thomson coefficient (μ) were made at five temperatures between 0° C. and 100° C. and at mean pressures of 2.5, 4.5 and 6.4 meters of mercury.

In Table I. are collected the data summarized from each run. The first four columns need no comment. The "bridge readings" of column 5 correspond to the temperature-drop, without being reduced to degrees.

TABLE I.

1. Date.	2. Bath Temp.	3. Pressure (Meters).	4. Pressure Drop (Meters).	5. Bridge Readings.	6. Slope (Bridge Units per Meter).	Residuals.		9. Wt.	10. Intercepts (Bridge Units).
						7. Bridge Units.	8. Degrees.		
1916									
Jan. 10	15°.9	2.45	.763	14.26	0.698	—	—	0.2	
			.260	10.75					
" "		6.50	.694	13.53	0.622	—	—	0.3	
			.263	10.85					
1915									
Aug. 20	30°.0	2.49	.792	4.67	0.635	−0.01	−0.0005	1.0	−0.34
			.599	3.48		+0.02	+0.001		
			.311	1.63		0.00	.0000		
			.241	1.18		−0.01	−0.0005		
" 23	"	4.48	.697	3.98	0.607	−0.01	−0.0005	1.0	−0.24
			.631	3.59		0.00	0.0000		
			.298	1.60		+0.03	+0.0015		
			.199	0.94		−0.03	−0.0015		
" 26	"	6.40	.726	4.07	0.588	−0.01	−0.0005	1.0	−0.19
			.620	3.46		−0.02	−0.001		
			.310	1.63		0.00	0.0000		
			.255	1.32		0.01	+0.0005		
Nov. 24	50°.0	2.50	.747	4.12	0.560	+0.02	+0.001	1.0	−0.08
			.653	3.54		−0.03	−0.0015		
			.343	1.84		+0.01	+0.0005		
			.264	1.39		0.00	0.0000		
Nov. 29	"	4.50	.820	4.37	0.535	−0.05	−0.0025	0.5	+0.03
			.689	3.77		+0.05	+0.0025		
			.309	1.71		+0.03	+0.0015		
			.193	1.03		−0.03	−0.0015		
Dec. 1	"	6.33	.790	4.28	0.529	−0.02	−0.001	1.0	+0.12
			.711	3.90		+0.02	+0.001		
			.307	1.74		0.00	0.0000		
			.193	1.14		+0.01	+0.0005		

TABLE I. (*Continued*).

1.	2.	3.	4.	5.	6.	Residuals.		9.	10.
Date.	Bath Temp.	Pressure (Meters).	Pressure Drop (Meters).	Bridge Readings.	Slope (Bridge Units per Meter).	7. Bridge Units.	8. Degrees.	Wt.	Intercepts (Bridge Units).
1915									
Dec. 6	70°.0	2.53	.769	3.93	0.496	+0.02	+0.001	1.0	+0.1
			.500	2.54		−0.04	−0.002		
			.259	1.41		+0.02	+0.001		
Dec. 8	"	4.36	.747	3.84	0.480	−0.01	−0.0005	1.0	+0.26
			.497	2.66		+0.01	+0.0005		
			.247	1.44		−0.01	−0.0005		
Dec. 13	"	6.40	.718	3.72	0.468	0.00	0.0000	1.0	+0.36
			.498	2.70		+0.01	+0.0005		
			.255	1.55		0.00	0.0000		
Dec. 16	90°.0	2.44	.746	4.87	0.441	0.00	0.0000	1.0	+1.57
			.509	3.83		0.00	0.0000		
			.257	2.71		0.00	0.0000		
" "	"	4.52	.747	4.91	0.421	−0.01	−0.0005	1.0	+1.78
			.495	3.88		+0.02	+0.001		
			.248	2.81		−0.01	−0.0005		
Dec. 15	"	6.35	.743	3.51	0.406	−0.02	−0.001	1.0	+0.51
			.506	2.60		+0.04	+0.002		
			.253	1.52		−0.02	−0.001		

This reduction is more simply done later. Column 6 gives the slope of the straight line that would be formed by plotting after the manner of Figs. 6 and 7, but actually they have been calculated by the method of least squares. The residuals in temperature, shown in column 8, give no evidence of a systematic deviation from the straight line and, moreover, the order of their magnitude is seldom over 0°.001 C. The weights (column 9) assigned to the several runs are the same for all except three, where experimental difficulties of a temporary sort turned up during the respective runs. In the two runs of January 10 it was impractical to get more than two points for each straight line, so there was no least square reduction. That carried out on November 29 suffered from a derangement of the pressure controlling apparatus. The later runs included one less point each but the gain in observing experience about balanced the omission of one point, and so the weights for these were kept at unity. The "intercepts" (column 10) on the temperature axis, are known to within additive constants only, except for the runs at 30° C., where zero readings of the differential thermometers were taken. The only purpose these data serve here is to furnish further evidence for the conclusions stated under the head "*influence of density*." It will be seen that with ascending pressures, hence increasing densities, the intercepts progressively rise along the temperature axis, with one exception.

The Joule-Thomson Coefficient Expressed as a Function of Temperature and Pressure.—The values of the slopes (column 6) reduced to the proper units are taken as the values of μ for reasons already given. This reduction is now made by the use of the multiplying factor 0.5178 together with formula (8). μ is then expressed in degrees C., hydrogen scale, temperature-drop per meter of mercury pressure-drop.

The fourteen values of μ thus obtained are classified in Table II.

TABLE II.

Approximate Pressure. (Meters.)	15° C.	30° C.	50° C.	70° C.	90° C.
	.358	.327	.290	.258	.232
2.50	(+.008)	(+.002)	(−.002)	(−.002)	(+.002)
	(+.002)	(+.001)	(.000)	(−.001)	(+.001)
		.312	.277	.250	.220
4.50		(−.001)	(.000)	(−.001)	(−.001)
		(−.002)	(−.002)	(.000)	(−.002)
	.319	.302	.274	.244	.213
6.40	(−.006)	(.000)	(+.002)	(+.002)	(−.001)
	(−.010)	(.000)	(+.004)	(+.003)	(−.002)

Upper line of residuals—five-constant formula.
Lower line of residuals—three-constant formula.

according to temperature and pressure. To these values were fitted, by least square computation, two empirical formulas, one a five-constant formula in t, the other a three-constant formula in $t + 273$ or τ. In the same table are included in parentheses, under each observed value of μ, the residuals (obs.-calc.) obtained with each of these formulas, the upper one applying to the former, the lower, to the latter formula.

The five-constant formula was tried following upon an initial inspection of the observations, which showed that the variations of μ were linear in p but not linear in t. Consequently a second degree term in t was included. The formula is

$$\mu = +0.3935 - 0.00691p - 0.001835t + 0.00000134t^2 + 0.0000325pt, \quad (9)$$

the units being as above specified.

Five constants make an unwieldy formula and seemed quite too many for the limited range of temperatures and pressures here employed. Resort was then had to the three-constant formula (4) suggested by the use of the equation of van der Waals. The least square reduction for this gave

$$\mu = -0.2599 + 181.96\frac{1}{\tau} - 552.4\frac{p}{\tau^2}, \quad (10)$$

where $\tau = 273 + t$. This formula is plotted in Fig. 8, which shows likewise the observed points.

The five-constant formula agrees a trifle more closely with the observations than does the other, as should be expected, having two more constants, but there is little to choose between them in this respect. All things considered, the three-constant formula is to be preferred; it is more rational and is simpler.

The computations of these formulæ were subjected to the standard numerical checks throughout. The smallness of the residuals, being on the whole less than one per cent. would indicate, at least for the present observations, that a better formula is not worth looking for.

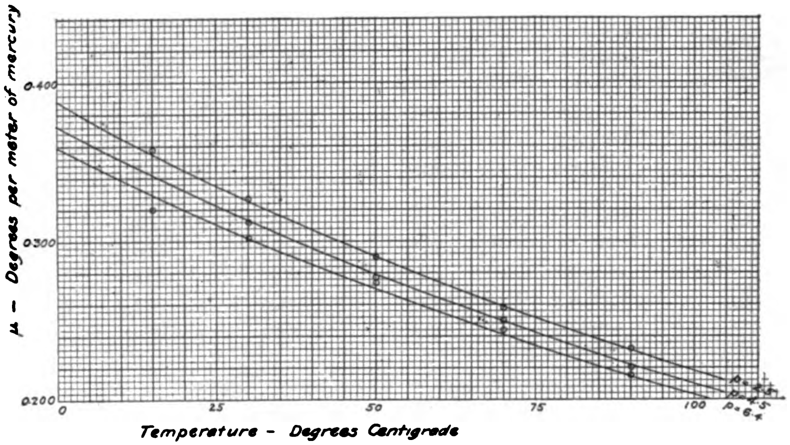

Fig. 8.

Joule-Thomson Coefficient, as a Function of Temperature and Pressure.

Curves are graphs of the formula:

$$\mu = -0.2599 + 181.96\frac{1}{\tau} - 552.42\frac{p}{\tau^2}$$

Circles are observed values.

Comparison with Other Results.—Joule and Thomson gave as their formula for air

$$\mu = 0.92\left(\frac{273.7}{T}\right)^2 \text{ degrees C. per 100 inches of mercury,}$$

which, by reduction, is

$$\mu = 0.362\left(\frac{273.7}{T}\right)^2 \text{ degree C. per meter of mercury.}$$

Their data have been embodied by Rose-Innes (*loc. cit.*) in the formula

$$\mu = -0.275 + 178\frac{1}{T} \text{ reduced to degs. C. per meter of mercury,}$$

which resembles (10) omitting the pressure term. The observations of Joule and Thomson, it will be remembered, did not indicate an effect of pressure.

Buckingham, by making use of the results of Kester on carbon dioxide and of Joule and Thomson on oxygen, air, nitrogen, and hydrogen, applied the law of corresponding states and found two formulas of similar type, one of which is

$$\mu = \frac{a_1}{\tau - b_1} - d_1, \qquad \tau = 273 + t.$$

A few numerical values calculated by the aid of this formula and the values given for the constants, will be found further on in Table III.

The only experiments published showing a pressure effect for air are those of Vogel and of Noell. The latter, being performed subsequently in the same laboratory may be considered as superseding the former. However, it is well to state Vogel's formula, namely,

$$\mu = 0.268 - 0.00086p \quad \text{(pressures in kg. per cm}^2\text{)},$$

which holds for the fixed temperature only.

The lengthy formula of Noell is quoted near the beginning of this paper. The pressure coefficient at 10° C. is practically the same in both cases. At 0° C. Noell's pressure coefficient is -0.0015 while that found by the author is -0.0074 the units in both figures being degrees C. and meters of mercury. The agreement here leaves something to be desired but it is well to note that the greater portion of the data to which Noell's formula was adjusted lay in a range of much higher temperatures and pressures than the temperatures and pressures of the present paper. It is interesting to note that Noell's formula is also linear in p and that both coefficients are negative.

Further corroborative evidence as to the negative sign may be found in the calculations of Dalton (*loc. cit.*) for hydrogen based on the empirical equations of state of Kamerlingh-Onnes. He plots numerous curves of the integrated Joule-Thomson effect, or in other words curves $H =$ const. over a wide range in temperature and pressure, *i. e.*, from 1 to 100 atm. and $-215°$ C. to $+17°$ C. These curves in the (p, t) plane were all concave downward, showing a negative pressure coefficient of μ for hydrogen, over the whole field, which covered some inversion points and both positive and negative values of μ. One would expect that an application of the law of corresponding states might lead to the same thing for all gases, although a general statement of this sort could not be made. In fact, accepting van der Waals' equation for the moment, a glance at equation (3) shows that the sign of the expression

$$\left(6b - \frac{4a}{RT}\right)$$

determines the sign of the pressure coefficient. In regard to carbon dioxide, Natanson found a positive coefficient, and Kester found no dependence upon pressure except at 26.4 atm. and here an increase in μ over its value at lower pressures was indicated. The matter on the whole for carbon dioxide seems undecided.

In Table III. are set forth for comparison a few numerical values for μ calculated from the formulæ of other observers above quoted (omitting those of Vogel and Rose-Innes). In general the values of the present paper exceed the other values. The former, at 4.5 meters pressure are in closest agreement with those of Buckingham. It is of interest to note, however, that, choosing a temperature of 0° C. for example, the integrated cooling effect over a drop of about 7 atmospheres calculated by the author's formula agrees with that set down for "corresponding states." This was roughly the pressure-drop in the experiments of Joule and Thomson.

TABLE III.

t° C.	Joule and Thomson.	Corresponding States. (Buckingham.)	Noell $p = 1m.$	Present Paper.		
				$p = 1m.$	$p = 4.5m.$	$p = 6.4m.$
0	.362	.374	.365	.399	.373	.359
50	.259	.270	.244	.298	.278	.270
100	.194	.212	.165	.224	.210	.203

Calculation of the Temperature of the Ice Point on the Thermodynamic Scale.—The general relations are

$$\frac{v_{100}}{T_{100}} - \frac{v}{T_0} = \int_{T_0}^{T_{100}} \frac{\mu C_p}{T^2} dT \text{ (pressure = const.)}, \qquad (11)$$

$$v_{100} = v_0(1 + 100\,\alpha),$$

$$T_{100} - T_0 = 100.$$

The first of these is one of the two ways of integrating equation (2); the second equation defines, in terms of experiment, the mean temperature coefficient at constant pressure of the gas used in the experiment; while the last is a mathematical definition. The limits of integration are the two temperatures of the ice point and the steam point.

If we factor out a mean value of c_p (which varies but slightly within these limits) calling it $\overline{C}_p$ and call the value of the remaining definite integral $(I_{100} - I_0)$, noting that $1/v_0 = \rho_0$ the density of the gas at the given constant pressure and at the ice point, then the combination of these three relations results in

$$T_0 = \frac{1}{\alpha} + \frac{T_0(T_0 + 100)}{100\alpha}\, \rho_0 \bar{C}_p (I_{100} - I_0)$$

without approximation.

The first term on the right is the temperature of the ice point on the scale of the gas in question. The second term is the quantity that must be added thereto in order to derive the same temperature on the thermodynamic scale. It is small and may be called a "correction." Its relative magnitude will be seen to come out only about 0.0037 times the first term.

Now the accuracy of this term is limited by that of the determinations of μ, namely an order somewhat under one per cent. Expressed in degrees this would come to about 0°.01 C. The following two approximations therefore are permissible: First, T_0 may be replaced by 273; secondly, the evaluation of the integral

$$\int_{T_0}^{T_{100}} \frac{\mu}{T^2} dT,$$

in which formula (9) or formula (10) must be used, t may be treated as if it were $T - 273$ in formula (9) and in formula (10) τ may be treated as if it were T. Further, or successive approximations are unnecessary. The above formula, with these approximations, then becomes

$$T_0 = \frac{1}{\alpha} + \frac{273 \times 373}{100\alpha}\, \rho_0 \bar{C}_p (I_{100} - I_0). \tag{12}$$

The value of $(I_{100} - I_0)$ has been computed and found to be

+ 0.000306 by (10), the three-constant formula for μ,

and

+ 0.000308 by (9), the five-constant formula for μ,

the unit of pressure still being one meter of mercury at 0° C. in this locality.

The expression of this result in terms of absolute units and the calculation of the remainder of the formula were carried out by means of the following data:

density of mercury = 13.595 gm. per cm^3,
g = 980 cm. per sec^2,
$\bar{C}_p$ = 241 cal per gm. per deg.,
1 cal. = 4.187×10^7 ergs.
$\rho_0 = 0.001293 \frac{100}{76}$ gm. per cm^3,
$\alpha = 0.00367282$.

The value for g is rounded off from 979.937, the figure given as determined in this locality by the U. S. C. and G.S.; that for $\bar{C}_p$ is a mean taken from the papers of Swann and Scheel and Heuse already mentioned. The value for α was determined with great care and precision by Chappuis[37] at the standard thermometric pressure of one meter of mercury. The reciprocal of this is 272.270.

The correction term in (12) obtained by use of these data are

1.093 by the three-constant formula

and

1.100 by the five-constant formula,

whence, for the thermodynamic temperature of melting ice, we get

$T_0 = 273°.36$ by the three-constant formula

and

$T_0 = 273°.37$ by the five-constant formula.

The other most recent calculations are by Buckingham (1908) using the Joule-Thomson coefficient for various gases adjusted into a single reduced formula by the "law of corresponding states," and by Berthelot (1907) using the characteristic equation. Previously in 1903, he had computed values on the basis of the Joule-Thomson effect. These results are collected in Table IV.

TABLE IV.

Author.	T_0.	Gas.	Method.	Reference Used Here.
Buckingham	273 273	Air	J-T effect	Buckingham[31] (1908)
(1908)	273.286	N_2	"	"
	273.049	H_2	"	"
	273.267	CO_2	"	"
Berthelot	273.13	H_2	"	Buckingham[31] (1907)
(1903)		N_2		
		CO_2		
		Air		
Bertholet	273.04	H_2 in Pt-Ir (1)	Characteristic	Berthelot[32] (1907)
(1907)	273.07	" " " (2)	equation	
	273.10	" in *verre dur*		
	273.09	N_2		

The value arrived at in this paper is higher than any of these; and, further, it will be noted that the method by the Joule-Thomson effect gives higher values than does the method using the characteristic equation of the gas. This fact has been noted by Buckingham and others but has not been considered to be satisfactorily explained. Rose-Innes and Berthelot (the latter while discussing his values obtained from hydrogen) have suggested the possibility of a surface action at the walls of the containing bulb, such as the formation of a layer of gas whose density is different from that to be found in the interior. This, of course, affects the main term $1/\alpha$.

Pressure Coefficient of α.—A partial comparison may be made here by taking into consideration the pressure coefficient of α found by Chappuis[37] for nitrogen and that deducible for air from the results of this paper. Reference to equation (12) will show that the correction term is proportional to the density ρ_0 (neglecting the pressure effect in μ, an influence of the second order) and hence proportional to the pressure, which, in the present units, has the value 1. T_0, being a constant, this term is, therefore, the pressure coefficient of the reciprocal of α. The pressure coefficient for α, then, is computed to be 0.0000147 for air, while Chappuis finds the value 0.0000119 for nitrogen, or about four fifths as much, an agreement which, at least, is qualitative.

Correction to the Constant-pressure Air Thermometer between the Ice and Steam Points.—Here, since by definition, the centigrade gas and centigrade thermodynamic scales must agree at both end-points, a closer agreement among the results from various sources is to be expected.

We have, as before, the general relations

$$\frac{v}{T} - \frac{v_0}{T_0} = \int_{T_0}^{T} \frac{\mu c_p}{T^2}\,dT \equiv \bar{c}_p(I - I_0)$$

and

$$v = v_0(1 + \alpha t),$$

defining t on the gas scale in question.

Now let

$$T - T_0 = t_a$$

defining t_a on the thermodynamic scale and

$$T_0 = \frac{1}{\alpha} + \eta$$

where η stands for the correction term in (12).

Therefore, since $v_0 = 1/\rho_0$,

$$T_0(1 + \alpha t) - T = TT_0\rho_0\bar{c}_p(I - I_0).$$

The left-hand member becomes

$$- t_a + T_0\alpha t = - t_a + t + \alpha\eta t.$$

Therefore the formula at once reduces to

$$t_a - t = \alpha\eta t - TT_0\rho_0\bar{c}_p(I - I_0),$$

in which $t_a - t$ is the correction sought.

Replacing η now by its expression in (12) and noting that, for the same reasons as before it is permissible, in the right-hand member, to identify t with t_a, c_p with C_p, and to replace T_0 and $1/\alpha$ by 273, the equation finally becomes

$$t_a - t = 273\rho_0\bar{c}_p\{3.73(I_{100} - I_0)t - (I - I_0)(t + 273)\}, \qquad (13)$$

which, inspection shows, vanishes at both the ice and steam points, as, of course, it should. The values of $\bar{C}_p$ and ρ_0 are the same as those given for the computation of (12). $\bar{C}_p$ and $\bar{c}_p$, strictly speaking, are not identical, owing to the differing limits of their respective integrals, but it is obvious that their difference is not sufficient to affect the computations.

Corrections $(t_a - t)$ calculated by this formula for every 10° C. are to be found in the last column of Table V. The other columns are copied from Buckingham's article in the Phil. Mag., 1908.[31] The corrections are negative, that is, the numbers in the table are to be subtracted from the constant-pressure readings. The new values and those of Berthelot in this instance are seen to be practically identical. Chappius (*loc. cit.*), moreover, states that the air and nitrogen scales are sensibly the same in this interval; so that we have here a real basis for comparison. This is agreement within the accuracy of gas thermometry.

TABLE V.

Thermodynamic Corrections to the Constant Pressure Nitrogen Scale Obtained by Various Writers, Compared with the Corrections to the Constant Pressure Air Scale Deduced from the Joule-Thomson Effect. 1,000 mm. = pressure.

t°C.	Rose-Innes, 1901.	Callendar, 1903.	Berthelot, 1903.	Buckingham, 1907.	Mean.	This Paper (Air), 1916.
10	0.0120	0.0109	0.010	0.0078	0.010	0.0100
20	0.0205	0.0188	0.017	0.0137	0.017	0.0170
30	0.0261	0.0236	0.022	0.0179	0.022	0.0212
40	0.0288	0.0260	0.024	0.0203	0.025	0.0235
50	0.0289	0.0260	0.024	0.0209	0.025	0.0238
60	0.0269	0.0240	0.022	0.0198	0.023	0.0218
70	0.0228	0.0204	0.019	0.0172	0.020	0.0189
80	0.0168	0.0151	0.014	0.0129	0.015	0.0141
90	0.0092	0.0081	0.007	0.0071	0.008	0.0068

Variation of c_p with Pressure.—Recent work of Holborn and Jacob[38] on c_p for air at 60° C. and at pressures from 1 to 200 atmospheres affords a further comparison of the present work with that of other observers in that the Joule-Thomson effect can be utilized to evaluate the pressure coefficient of c_p.

Writing equation (2) in the form

$$\mu c_p = T\left(\frac{\partial v}{\partial T}\right)_p - v$$

and differentiating with respect to T at constant p, we get

$$\mu\left(\frac{\partial c_p}{\partial T}\right)_p + c_p\left(\frac{\partial \mu}{\partial T}\right)_p = T\left(\frac{\partial^2 v}{\partial T^2}\right)_p = -\left(\frac{\partial c_p}{\partial p}\right)_T, \tag{14}$$

the last member following directly from a well-known equation in thermodynamics.

Now the left-hand member can be evaluated by using either formula (9) or (10) for μ and (say) Swann's values (*loc. cit.*) of c_p and $(\partial c_p/\partial T)_p$. It should be noted that, in regard to units, c_p may be expressed in calories per gm. per deg., since the equation is homogeneous in that physical quantity. Also since μ is here expressed in degrees per meter of mercury and since the data of Holborn and Jacob for $(\partial c_p/\partial p)_T$ are expressed in atmospheres for the pressure unit, the left hand member must be multiplied by 0.76 to make the two sets of results comparable. Finally T must be put equal to $273 + 60 = 333$, the temperature of Holborn and Jacobs' experiments.

Now Swann gives at one atmosphere,

$$c_p = 0.241 \quad \text{and} \quad \left(\frac{\partial c_p}{\partial T}\right)_p = 0.000016,$$

then by the 3-constant formula (10) also at 1 atmosphere

$$\mu\left(\frac{\partial c_p}{\partial T}\right)_p = 0.0000047,$$

$$\text{and} \quad c_p\left(\frac{\partial \mu}{\partial T}\right)_p = -\,0.0003901.$$

The sum of these two (of which the first is almost negligible) is $-\,0.0003854$. Multiplying by 0.76 we get

$$\left(\frac{\partial c_p}{\partial p}\right)_T = 0.000293 \text{ cal. per gm. per deg. per atm.}$$

The value of this quantity from Holborn and Jacobs' empirical interpolation formula, namely,

$$10^4 c_p = 2413 + 2.86p + 0.0005p^2 - 0.00001p^3,$$

is

0.000286 cal. per gm. per deg. per atm.

and from their observations at their lowest range of pressures alone (namely 1 to 25 atmospheres) it is

0.000312 cal. per gm. per deg. per atm.

The agreement here is about as good as would be expected. Moreover it is corroborative as regards a comparison between two widely different forms of continuous flow experiments.

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ERRATA.

Vol. XIII., April, 1919, in the article by A. F. Kovarik, entitled "The Automatic Registration of α-particles, β-particles and γ-ray and X-ray Pulses," page 277, second line from bottom, the first P_2 should read P_1; page 278, line 28, 4.77×10^{-11} should read 4.77×10^{-10}; page 280, line 32, first word "or" should read "for."

Vol. XIII., April, 1919, in the abstract by Irwin G. Priest, entitled "A New Formula for the Spectral Distribution of Energy from a Complete Radiator," in equations (2) and (3), pages 314 and 316, $A - \frac{1}{3}$ should read $A^{-1/3}$; line 6, page 317, "are" should read "an"; line 10, page 317, D^2 should read D_2.

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PHYSICAL REVIEW.

THE MEASUREMENT OF HEAT AND THE SCOPE OF CARNOT'S PRINCIPLE.

By Arthur C. Lunn.

Synopsis.—This paper outlines a symmetric form of exposition of the principles of thermodynamics made possible through recognizing on a parity from the outset the two kinds of conservation naturally called after Black and Carnot, each of which is physically valid under proper conditions, and which lead to the energy and entropy scales of measurement of heat; thus exhibiting both of these, together with the thermodynamic scale of temperature, as calorimetric concepts. In the two parts of the paper are given respectively an experimental and a deductive treatment.

In part I. the experimental meaning of conservation is considered in terms of certain conditions of consistency; first in connection with single-temperature transfers of heat, where those conditions underlie the uniqueness of meaning of an arbitrary scale of heat for various temperatures; second in connection with passages of heat between bodies at different temperatures, which reveal the dual nature of a quantity of heat, here illustrated by some analogies.

In part II. are investigated the deductive consequences of Carnot's hypothesis of the universal ratio of efficiency. With only the single-temperature conservation of heat assumed to be known experimentally (probably the mildest basis on which this principle is intelligible), it is shown that there follows practically the entire theory of reversible thermodynamics, in particular the existence of thermal energy and entropy and the equivalence of thermal energy and work.

SINCE the time when the basic principles of thermodynamics were given a standard form, chiefly by Clausius, Kelvin and Rankine, it has become customary not only to speak of "first" and "second" laws but to adopt a corresponding order of thought in the development of their more immediate consequences. It is recognized however that the two principles are at least to a certain extent independent, so that an inverse order of development is conceivable; as illustrated for instance by the fact that Kelvin was able, without using the principle of equivalence, to point out the implication in Carnot's theory of a universal or absolute scale of temperature.[1] Moreover, while the principle of equivalence enters chiefly on an experimental basis, the second law is made to rest to a large extent on an a-priori hypothesis, in the form of Carnot's principle or its equivalent, to be judged by experimental test of its deductive consequences. An unsatisfactory result is that while the notion of thermal energy, already familiar from experimental calorimetry, becomes still more concrete through equivalence to mechanical work, the notion of entropy, introduced by theorem and definition, appears comparatively abstract and difficult of physical interpretation.

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[1] Kelvin, Camb. Phil. Soc. Proc., June 5, 1848.

Still, this order of ideas is undoubtedly natural in presence of the general understanding that Carnot's original theory, though contributing the invaluable concepts of the reversible cycle and the universal ratio of efficiency, is in contradiction with experiment through assuming the conservation of caloric in the cases contemplated, the substitution in these cases of the principle of equivalence of heat and work having opened the way for the development of thermodynamics in its now classical form. To make the first law intelligible it is of course assumed that the measurement of heat by itself is already on an experimental basis. This is taken to have been accomplished through the processes of calorimetry, as initiated by the pioneer work of Black, which may be said to rest on the hypothesis or fact, according to the point of view, of the conservation of caloric in certain cases of conduction and the like where it is now known to be an essential feature that no mechanical work is done.

It has not escaped notice however that in some ways Carnot's "caloric" shows distinct resemblance to entropy rather than to thermal energy.[1] Detailed examination shows that most of the propositions of Carnot's essay, and of the formulas in Clapeyron's mathematical commentary and Kelvin's "account," [2] though so often called erroneous and even meaningless by commentators, have a meaning and are correct if "caloric" is taken to mean entropy. It is certain therefore that the amount of inconsistency with experimental results supposed to be present in Carnot's theory has usually been exaggerated and its exact nature not clearly set forth. From the point of view to be explained in this paper the only error of importance appears to be that, not realizing the type of conservation required by his theories to be physically distinct from that of Black, he borrowed numerical data obtained on the energy scale and applied them in cases where they should have been given on the entropy scale. For example, at constant volume the capacity of an ideal gas for change of thermal energy is constant but its capacity for change of entropy varies inversely as the absolute temperature.

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[1] Callendar, presidential address, Phys. Soc. Proc., 23, 153–189, 1911.

[2] Clapeyron, J. de L'Éc. Pol., 14, cah. 23, 152, 1834. Kelvin, Trans. Ed. R. Soc., XVI., 1849.

One may say then that the hypothesis of conservation of caloric was used effectively by Black and Fourier in cases where one would now say that thermal energy is conserved and entropy increases, while Carnot assumed it for the case of his reversible engine where thermal energy changes but entropy is constant. The parallel though incomplete is unmistakable. It suggests that the whole basis of thermodynamic theory might be recast in a more symmetric form, securing at the same time a more completely experimental setting, by considering from the outset on a parity the two measurable properties of a quantity of heat and the corresponding scales of measurement based on two types of conservation, each valid in the respective class of cases whereby the experimental establishment of the scale is achieved. Such a formulation is attempted in part I. of this paper where the term "quantity of heat" is for convenience used, in a sense more generic than usual, as denoting a relatively primitive concept not really simple, but which rather may be said to bifurcate into the two distinct and more specific concepts, each of which corresponds in the appropriate cases to the respective types of conservation identified by the names of Black and Carnot.

Such a bifurcation during the process of refinement into scientific precision of a primitive and relatively vague concept is by no means a rare event in the history of science, though as in the theory of heat it is often obscured by the continuance in use, to denote one only of the emerging concepts, of a term previously more comprehensive and indefinite. During the formative period of dynamics search was made for an acceptable measure of the amount of motion in a moving body and the controversies led to the appearance of two important and distinct concepts, momentum and kinetic energy; some traces of vagueness of language remain in the Principia of Newton, though technically he chose to restrict the term "quantity of motion" to the first of these. The "degree of electrification" of a body might be taken to include reference to both charge and potential; the coldness of metal on a winter day is an uncertain blend of low temperature and good conductivity; the distinct concepts quantity of heat and temperature arose from the scrutiny of judgments of "hot" and "cold." In a similar way it seems just to use the term "quantity of heat" in a generic sense, to comprise reference in combination to both of the two independent measurable properties, thermal energy and entropy, although since the foundation of calorimetry the expression has come to be restricted to one of them as a matter of custom. Further examples could easily be cited, but for the present purpose the point of view suggested is especially well illustrated by the following example, which will be seen to furnish a rather vivid and far-reaching analogy to the thermodynamic relations.

The term "amount of gas" may be treated as somewhat indefinite, referring to volume or mass or both jointly according to the interpretation intended, as in the specification of percentages in a gaseous mixture. Suppose then an extensible balloon containing air, with attached tube and shut-off permitting control of contents, be allowed to rise through water in which it is immersed. If the shut-off be closed the mass of air is constant while the volume increases; if the contents be allowed to escape at a suitable rate the volume may be kept constant while the mass diminishes. Here mass, volume, depth, pressure in water, atmospheric pressure at surface, correspond respectively to thermal energy, entropy, temperature on some empirical scale, temperature on thermodynamic scale, and temperature of conventional zero above absolute zero.

In the typical case of Black's calorimetry the heat leaving a body *A* at a certain temperature and that entering another body *B* at a lower temperature are such that the thermal energies are the same, but the entropy entering *B* is larger than that leaving *A*. In the typical case of Carnot's theory, where the transfer of heat occurs by mediation of a reversible engine working in a simple Carnot cycle between the two temperatures, the entropies involved are equal but *B* receives less thermal energy than *A* gives up. Thus the justice of the term "conservation of caloric," in two different senses, seems to be exactly the same in the two cases.

The incompleteness of the parallel with respect to other features may also be instructive. In both cases there is known to be conservation of energy, because the variations of mechanical energy in its various forms compensate those of the thermal energy; while there is not yet recognized any physical magnitude whose variations compensate those of the known thermal entropy, so as to allow the formulation of a law of conservation of entropy in some wider sense. Still it is conceivable, as Callendar suggests, that there may be other and equivalent forms of entropy, and especially does there seem to be a chance for speculation on this point in connection with the thermodynamics of radiation. Perhaps also there may be other and independent aspects of heat, beside energy and entropy, whose conservation could be established in some experimentally intelligible sense; for instance in connection with the aggregate result of the molecular Döppler effects, since energy and temperature alone do not determine spectrum of radiation if the black body condition be not imposed. Such novel aspects or magnitudes could probably not be sifted out by the study of bodies, like fluids in gross, whose state depends on only two independent thermodynamic coördinates.

As underlying the interpretation of thermal measurements the notion of conservation or equivalence appears in various forms. These may be analyzed in terms of three fundamental types, corresponding to the following three elementary cases of experimental comparisons: (1) comparison of quantities of heat entering or leaving different bodies at the same temperature, (2) comparison of quantities of heat at different temperatures, (3) comparison of heat with other physical entities, especially completely available energy in some form.

The treatment given in Part I. shows that on the basis of the first two only, including the suggested dual aspect of the second, it is possible to exhibit entropy as well as thermal energy as a calorimetric magnitude, thus reaching a more symmetric experimental setting than the familiar forms of the theory. The equivalence of thermal energy and work is then introduced as an additional experimental result, while the existence of an analogous non-thermal equivalent for thermal entropy appears merely as a question.

As an alternative development a symmetrically deductive treatment is given in Part II. Here the only experimental comparisons assumed are of the type (1) above, while Carnot's hypothetical principle of the universal efficiency function is stated as a postulate. The existence of thermal energy and entropy, identifiable with the experimental concepts, is deduced, so that for reversible processes there follow both first and second laws as commonly understood. Thus, on what appears to be a minimum basis of experiment sufficient to make the measurement of heat an intelligible notion at all, Carnot's principle alone proves to be an adequate hypothetical basis for the entire general theory of the thermodynamics of reversible processes, having in particular the principle of equivalence of heat and work as a corollary. Distinct indications of the possibility of such a conclusion will be found in the apparently little known memoir by Reech,[1] in which the development is such that it can be specialized further by assuming either conservation of energy or conservation of caloric, considered as mutually incompatible. Results partially similar to those now obtained will also be found in the recent papers by Larmor and Raveau,[2] dealing with Carnot's principle apart from the principle of equivalence. Some reference will be made later to corresponding features in these papers, but the detailed comparison will be left to the reader.

Part I. Experimental Setting.

Passing over the complex detail of experimental methods and the involved questions presented to the experimenter's judgment in the attempt to sift from among secondary or accidental aspects the main elements of the phenomena to which attention is to be directed, it will be sufficient here merely to notice certain salient features of the results, that either have been obtained or with appropriate experiments would be obtained if thermodynamics in its familiar form is valid. As to thermometry, it will be supposed that means have already been secured for identifying particular physical temperatures, and that the various temperatures are known to form a simple or linear ordinal series, in connection with which the terms "higher" and "lower" have their usual meaning. If data are to be transcribed with the aid of some numerical specification of temperature it will be understood that the physical order of rising temperature corresponds to the order of increasing values of the index variable t, without implying any initial physical interpretation for equality or comparison of intervals in different parts of the scale. In most cases it will be clear that t could be understood merely as some symbol of identification, not necessarily numerical.

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[1] Reech, Liouv. J., ser. 1, 18, 357–568, 1853.

[2] Larmor, Roy. Soc. Proc., 94, 326–339, 1918. Raveau, C. R., 167, 20–23, July 1, 1918.

Viewing then to begin with as merely tentative the notion that heat can be measured at all, one may ask what there is in the experimental processes developed and the results reached to show that heat possesses properties truly measurable, and such that the results of measurement can be described justly in terms of the standard language of equivalence and conservation. In the main the answer is that in all the types of comparison involved in thermal measurements there appears a certain unanimity in satisfying what may be called conditions of transitivity, or conditions of consistency as they become in the phrasing suggested by the point of view represented in the notion of equivalence; conditions as to sense or algebraic sign, and as to value or measure. Between these two kinds also there is a certain formal analogy that needs no attention here. It is because these conditions are satisfied that it is possible to formulate the meaning of a "quantity of heat" as something having value and sign. This formulation, found by experience to be both just to the facts and convenient in describing them, is best thought of as occurring in connection with observation of such variations in physical properties, supposedly due to thermal processes, as affect features already known to admit of algebraic representation, like length, volume, density, mass of a particular phase. Both kinds of conditions of course appear in a variety of forms, corresponding to the various kinds of bodies involved and the various kinds of thermal exchange between them.

In language made unconventional in order to avoid specific terms already adopted as technical, the conditions of sense may be described as referring to a distinction between some change called "waxing" and its opposite "waning," the terms bearing various meanings for various bodies or kinds of change. Suppose then in each single experiment of a set a certain pair of bodies in thermal exchange, all other bodies being found unaffected in certain prescribed respects. With A and B in exchange let A vary in a way called waxing by definition, then what B does may be called waning; in exchange with C suppose B waxes, then what C does is naturally called waning; no more definition being needed, there is content of fact in saying that with A and C in exchange if A waxes C wanes. In case n bodies were paired off in all possible ways there would be one arbitrary definition, $n - 1$ definitional pairings, and $(n - 1)(n - 2)/2$ pairings for test of fact. There are also in connection with processes that can be reversed items of fact to the effect that reversals of sense correspond in the two bodies of a pair. In view also of the number of ways in which the same body may be affected by thermal exchange according to the conditions imposed, it is clear how broad is the justification in fact for speaking of heat as something entering or leaving a body.

The matter of numerical value is closely similar. The magnitudes of corresponding changes in the bodies of a pair are found to be related in some way, and the observed relation for A and C can also be computed from the relations for A and B and for B and C by algebraic elimination. The character of the algebraic process depends on the choice of variables used in transcribing the measurements, the usual case being the multiplication of ratios. With any number of bodies in the set to be compared, a particular one may be chosen to be paired off with each of the others, either to decide on convenient definition of terms or by process of calibration to fix on a suitable choice of variables; then all other pairings give tests of fact. The consistency thus revealed is the ground for speaking of heat as something admitting quantitative estimate.

Whether every type of sequence, or in what sense any particular sequence, of thermal comparisons would satisfy these conditions need not be discussed here; some of the outstanding problems relate to the question whether other and independent quantitative aspects of heat remain to be discovered. The few special types to be mentioned here are actually of the consistent or conservative type, and prove to be sufficient as basis for the theory aimed at. In each case the meaning of the conditions of consistency in sense or sign may be passed over as obvious without comment; the conditions of consistency in value or magnitude will be sufficiently illustrated by the forms that they take in certain standard cases of the three types of comparison listed above.

Comparisons of the first type, of quantities of heat entering or leaving bodies at the same temperature, lead to the conclusion that all such can be compared among themselves and represented consistently by simple ratios; or in other words that for such single-temperature comparisons a quantity of heat can be considered a simple thing, capable of representation by a single number in terms of any chosen unit belonging to that temperature, regardless of the kind of body concerned or the kind of change produced in it according to collateral conditions imposed. This leads to the introduction of what may be called an empirical or arbitrary scale of heat, including a single specific unit for each temperature, but not implying anything as to the possibility of comparing two quantities of heat when the temperatures are different. An approximate example of such a scale is found in the system of differential calories, according to which the specific heat of water is by definition unity at every temperature. A better example for the purpose is to be found in strictly isothermal processes, especially changes of state, though latent heats of expansion would also serve. Through a suitable variation of pressure the latent heat of transition between liquid and vapor of a single substance could furnish such a scale for a considerable range of temperature.

Consider then a variety of substances such that the respective pressures can be adjusted so that the temperatures of transition agree. In the absence of such adjustment for any two particular substances A and B, suppose thermal exchange between them for a certain time leads to the transformation of the respective masses m_A, m_B, the transition being reversed in both bodies with reversal of sense of the temperature difference. With temperature difference approaching zero the ratio m_A/m_B of corresponding masses approaches a limiting value μ_{AB} to be understood as the ratio of masses transformed in the ideal limiting case of reversible or single-temperature conduction.

The main experimental facts of significance here are then: that the ratio μ_{AB}, depending on the temperature of transition and the nature of the substances, is independent of the absolute mass transformed, and that for the same temperature the various ratios for the various bodies of the set satisfy conditions of transitivity in the form $\mu_{AB}\mu_{BC} = \mu_{AC}$. This multiplication of ratios is of course not merely a mathematical identity, though conveniently given a mnemonic form as such, the meaning being that if m_A be the same in the experiments (AB) and (AC) and m_B the same in experiments (AB) and (BC) then as an experimental fact m_C will be the same in (AC) and (BC). The fact that the ratio of masses is characteristic suggests that a quantity of heat can be numerically indexed as proportional to the mass transformed of some particular substance.

The conditions of consistency in the compounding of the ratios furnish one instance of the universal consistency in the comparison of various temporary units of heat belonging to the same temperature, though in general the mathematical form varies. In such facts is found the experimental basis for the effective accuracy of the language of conservation, according to which for example one can speak of a quantity of heat going from A to C either directly or through B. In the sense illustrated it will now be supposed that the meaning of a quantity h of heat at a temperature t has been established. Certain familiar terms, like the differential specific heat dh/dt, become intelligible in connection with any definite choice of the scales of heat and temperature.

For the second type or multiple-temperature comparisons consider a set of bodies or reservoirs maintained at their respective various temperatures, any certain three of which, such as will be referred to, being labelled in descending order as t_1, t_2, t_3. The two distinct forms of conservation already alluded to will then appear in connection with thermal exchange between these bodies of the two kinds naturally named after Black and Carnot.

The first form, fundamental in classical calorimetry, occurs when the bodies communicate by pure irreversible conduction or radiation, or by the mediation of some mechanism such that the various quantities of heat are related in the same way. The second occurs when the bodies communicate by mediation of a reversible engine working in Carnot cycles between the two temperatures. In the first case suppose h_1 units of "t_1-heat" leave the first body and h_2 units of "t_2-heat" enter the second body; let $h_1/h_2 = b_{12}$. Similarly in the second case, the two quantities of heat being h_1' and h_2', passing when the engine is on the isothermals, put $h_1'/h_2' = c_{12}$.

Now calorimetry as ordinarily understood, or the measurement of heat by itself in Black's sense, rests largely on the experimental facts: that b_{12} depends on the two temperatures only save for its obvious variation according to the choice of the scale of heat; and that in terms of any one scale of heat the ratios for various pairs of bodies satisfy the condition $b_{12}b_{23} = b_{13}$. But according to standard thermodynamic theory it is indirectly understood to be a fact, and by suitable apparatus could doubtless be shown to be experimentally verifiable with considerable directness, that the ratios c_{12} and the like satisfy similar conditions. The fact that the b's are independent of the physical nature of the reservoirs and the character of the changes produced in them is paralleled by the fact of a similar independence on the part of the c's of the properties of the reservoirs and the nature of the mediating engine.

Thus in each case there is complete ground for concluding that there is a sense in which the heat is conserved in passing out of one reservoir and into the other. But it is also to be understood as found experimentally that $b_{12} < c_{12}$; accordingly the two types of conservation must be physically distinct.

Thus a quantity of heat h, entering or leaving a body at temperature t, exhibits two distinct measurable properties, thermal energy and entropy, whose measures q and s are related to the empirical measure of heat by equations of the form

$$q = hB(t), \qquad s = hC(t),$$

where the reducing factors $B(t)$, $C(t)$, appropriate to any particular scale of heat, are determined, except for an arbitrary constant factor in each depending on the choice of the units for q and s, by the conditions

$$B(t_2)/B(t_1) = b_{12}, \qquad C(t_2)/C(t_1) = c_{12},$$

which are the transcription of the equality of q_1 and q_2 in the Black process and of s_1 and s_2 in the Carnot process.

A change of the scale of heat can be represented by putting $h = h'\rho(t)$ where $\rho(t)$ is the ratio of the new to the old unit of t-heat; the corresponding new reducing factors are then $\rho(t)B(t)$ and $\rho(t)C(t)$, supposing the units of q and s not altered. In particular ρ could be taken as the reciprocal of either B or C, thus reducing the arbitrary scale of heat to the energy or entropy scale.

If by definition $T(t) = B(t)/C(t)$, then in view of the inequality $b_{12} < c_{12}$, which gives $B(t_1)/C(t_1) > B(t_2)/C(t_2)$ it follows that $T(t)$, the absolute or thermodynamic scale of temperature thus shown to exist, is a monotonic increasing function of the physical temperature, is independent of the choice of the scale of heat, and is determinate except for a constant factor depending on the units of q and s, whose values for the same quantity of heat are related by $q/s = T(t)$.

With the aid of this scale of temperature the two cases may then be epitomized as follows, the different equations in each set being equivalent but suggesting different interpretations:

$$B:\ q_1 = q_2;\ s_1T_1 = s_2T_2;\ (s_2 - s_1)/s_1 = (T_1 - T_2)/T_2;$$
$$(s_2 - s_1)/s_2 = (T_1 - T_2)/T_1;\ s_2 - s_1 = q_1(1/T_2 - 1/T_1);$$
$$C:\ s_1 = s_2;\ q_1/T_1 = q_2/T_2;\ (q_1 - q_2)/q_1 = (T_1 - T_2)/T_1;$$
$$(q_1 - q_2)/q_2 = (T_1 - T_2)/T_2;\ q_1 - q_2 = s_1(T_1 - T_2).$$

To each equation valid in one case there corresponds one valid in the other case obtained by interchanging q with s and T with $1/T$. For example there are two instances of the Carnot simile of the waterfall: in C the thermal energy $s(T_1 - T_2)$ is lost when entropy s falls through the temperature difference $T_1 - T_2$, while in B entropy $q(1/T_2 - 1/T_1)$ is generated when thermal energy q rises through the difference of the reciprocals of the temperatures. In each case the statement of the condition of conservation in terms of the non-conserved quantity gives a kind of calorimetric aspect of the familiar Carnot ratio and its dual. The universality of these ratios, as depending on the temperatures only, is a consequence of the universality of the ratios b and c mentioned above, which is what makes it possible to abstract the notions of thermal energy and entropy out of the properties of particular bodies.

Up to this point only heat entering or leaving a body has been spoken of. But the conservation of entropy in the Carnot process leads by a familiar reasoning to the conclusion that all reversible transformations connecting two states of the working substance must agree in the amount of entropy absorbed, this amount being the integral of the calorimetric expression for ds in terms of reversible changes of the temperature and other observable properties of the body. The existence of a function of state properly called the entropy of the body is thus recognized, in a purely calorimetric setting. The question is at once suggested whether a corresponding identification of the internal energy of the body is possible by thermal tests alone.

This calls attention to a certain feature of incompleteness in the parallel as thus far drawn, in that the Carnot process implies the mediation of a working substance, while in the primitive form of the Black process only the two reservoirs are concerned, unless one considers the optical ether as mediator. It is, however, possible to imitate the net result of the irreversible conduction or radiation through the mediation of a gas, for instance, working in a cycle of the Carnot type except that such a part of the expansion on the upper isothermal is "free" that the net work of the cycle is zero. This illustrates that the primary condition for Black conservation during communication of the two reservoirs is of course not that there shall be no mediator but merely that the ultimate changes shall affect only those reservoirs; in contrast with the Carnot cycle, where some change in other bodies accompanies the non-vanishing work done. Accordingly, any case where it is possible to use a mediating body in such a way that this condition is satisfied, and such that the quantities of heat absorbed can be identified in terms of the changes in that body itself, can be thought of, not only as exhibiting conservation of thermal energy, but as leading also to a calorimetric identification of changes in what is to be called the internal energy of the body. The simplest case is naturally that of a cycle passing through two states of the mediator, such as to connect those states in two different ways by a path consisting of an isothermal and an adiabatic, one path reversible, the other necessarily irreversible at least in part, and so adjusted that the net work of the cycle is zero. The experimental tests are then to be understood as verifying that the conditions of transitivity are satisfied, in fact with ratios identical with the *b*'s above. In such a case the identification of the internal energy function is strictly analogous to that of the entropy by a Carnot cycle. That the identification of the two functions of state in this way can rightly be described as purely calorimetric is clear from the fact that, although the conditions involve full work in one case and no work in the other, still no quantitative comparison of heat and work is implied. Just how far the range of such cases extends it is not easy to say, but it may be guessed to be not far from coextensive with that in which a really experimental meaning attaches to the quantities of heat involved.

A more fundamental lack in the parallel appears when the further experimental facts are included which complete the classical first law, here more naturally called the last law. These show that in the case of the Carnot cycle the $q_1 - q_2$ is proportional to the work done, the factor of proportionality depending only on the units, or in other words that the conditions of transitivity are satisfied when mechanical energy is included with thermal energy; and more generally that work done is equivalent in any case to decrease in internal energy. The Carnot ratio then assumes the form of the ratio of efficiency; and, conversely, as Callendar has pointed out, the principle of efficiency combined with Carnot conservation leads to the principle of equivalence for reversible processes.

There is on the other hand, however, no physical quantity yet identified whose variations compensate those of the recognized thermal entropy. The discovery of such would be expected to make it possible to state certain conditions on irreversible processes in terms of an extended kind of Carnot conservation, instead of in terms of inequalities. It might also help to give physical meaning to the measure of probability, in terms of which entropy is expressed in statistical theories; perhaps utilizing some large scale coefficient related to the action constant of recent physical theory much as the gas constant is related to the agitation constant in the molecular theory. Many things suggest that if such a quantity exists, its setting may involve the relation of matter and radiation.

A certain "instinct for conservation" has sometimes been looked upon as part of the natural equipment of the physical investigator, and the order of thought here sketched shows how fully successful in this subject is its guidance of the interpretation of experiment. With the exception of the single problematic instance mentioned—whose place so far as present knowledge goes is held by the inequalities that take such a unique place in physical theory—the basic features of thermodynamic phenomena are found in the four distinct types of transitivity listed, each admitting statement in terms of the language of conservation, which lead to the abstraction of the four corresponding fundamental notions: heat, thermal energy, entropy, thermo-mechanical energy. The corresponding arrangement suggested for the theory includes then five laws, four equations and one inequality. The experimental basis used is a blend of reversible and irreversible processes, and in particular the various experiments of Joule on equivalence deal mainly with irreversible processes of rather extreme type.

Since, however, mathematical developments in thermodynamics are so largely concerned with reversible processes it is natural to ask whether some basis could not be found for a theory of such processes by themselves, adequate for the complete theory so far as they are concerned. It will be shown in part II. that conservation of the first type only and Carnot's principle of efficiency, as formulated in terms of the empirical scale of heat, are together sufficient for such a basis.

Part II. The Scope of Carnot's Principle.

The actual course of development of thermodynamics has tended to obscure the real reach of both of Carnot's fundamental ideas. First, his conservation of caloric was rejected as erroneous instead of being recognized as distinct from that of Black and Joule; the irony of this appears when one notices that, for the processes he defines, a conservation essentially equivalent to his occurs as a theorem in the classical theory. Second, his principle of efficiency did not have its consequences developed independently, but rather it was merely superposed on the principle of equivalence, adding thus to the theory only the extra content it had to offer. From the very nature of his principle, however, in that it refers to a ratio of heat and work as related to two distinct temperatures, it would seem inevitable that it should lead to some kind of quantitative comparison of heat at different temperatures as well as of heat with work, independently of his postulate of conservation. In order to reveal the full scope of the principle therefore, and especially its independence of the ordinary principle of equivalence, it is natural to develop its consequences with only so much additional assumption as is required to yield the general notion of a quantity of heat, abstracted from the properties of particular bodies.

Accordingly, for the alternative theory now to be presented, relating to reversible processes only, the basis of assumption is the Carnot postulate of efficiency, that for reversible cycles of his type the ratio of work done to heat absorbed on the upper isothermal is a universal function of the two temperatures; where quantities of heat are to be understood as known to be measurable in the sense of an empirical scale of heat as above described. Thus of the various types of conservation mentioned in part I. only the first or single-temperature type is now assumed, this being apparently the mildest basis on which Carnot's principle has intelligible meaning as one of possible common application to all kinds of bodies. As thus understood it will be shown to yield the entire thermodynamics of reversible processes. The existence of thermal energy and of entropy proves to be deducible without reference to the experimental basis of irreversible phenomena on which the theory of energy ordinarily rests. The conservation during reversible conduction, which is assumed, is the type where conservation of energy and of entropy are not distinct, so that only the generic notion "quantity of t-heat" is needed.

The case to be for convenience considered first is that of a fluid as working substance, its thermodynamic properties under the limited calorimetry defined being embodied in differential expressions

$$(1) \qquad dh = kdt + ldv, \qquad dw = pdv,$$

for heat absorbed and work done during a differential change under equilibrium pressure, where the coefficients k, l, p are for each particular body certain definite functions of v and t. Now merely as a mathematical proposition there exists for any system of this kind, with only two independent variables, an integrating divisor ω such that dh/ω is an exact differential ds; the ω is of course not uniquely determined, but any particular choice gives a function s determinate except for an additive constant. The s chosen is to serve simply as parameter of the adiabatic lines, whose differential equation is $dh = 0$, and later naturally appears as some function of the entropy, the canonical choice of s being the entropy itself.

The reasoning can be carried out in terms of the original variables (v, t), but for brevity it is convenient to transform to (s, t) as new independent variables. These are independent except when the latent heat of expansion l vanishes, in which case a true Carnot cycle is not possible since then an adiabatic is also an isothermal; this is a physically unreal case which need not be pursued. It is also essentially general for the main purpose to suppose that the Jacobian $\partial(v, t)/\partial(s, t)$ is positive, so that the sense of equivalent cycles is the same in the (v, t) and (s, t) planes when the coördinates are taken in this order. Whatever the choice of the s, this transformation offers the convenience that a Carnot cycle is represented by a rectangle in the (s, t) plane. In terms of these new variables the heat and work take the forms

$$(2) \qquad dh = \omega ds, \qquad dw = a ds + b dt,$$

where ω, a, b are to be considered as functions of s and t; the explicit formulas connecting them with k, l, p are not needed. It will be clear also that the proof really applies to the general case of a two-variable system, since the same forms would result if the original expression for dw had a term in dt, and if the v and p were interpreted as any coördinate and corresponding generalized force.

If t_1 and t_2 be the higher and lower temperatures in the cycle Carnot's principle then imposes the condition

$$(3) \qquad w/h_1 = H(t_1, t_2),$$

where H is the same function for all bodies when the same scales of heat and temperature are used for all. A change of the scale of heat, represented by putting $h = h'\rho(t)$ and thus giving $k' = k/\rho$, $l' = l/\rho$, is allowed for by putting

$$H'(t_1, t_2) = \rho(t_1)H(t_1, t_2).$$

This is still universal in the sense intended, so that if the principle is valid for one scale of heat it is valid for all.

The case in hand, for the cycle represented by the rectangle $(s = s_2 \cdots s_1,\ t = t_2 \cdots t_1)$ has

$$(4) \qquad h_1 = \int_{s_2}^{s_1} \omega(s, t_1) ds, \qquad w = \int_{s_2}^{s_1} \int_{t_2}^{t_1} \varphi(s, t) ds dt,$$

in which

$$(5) \qquad \varphi(s, t) = \frac{\partial a}{\partial t} - \frac{\partial b}{\partial s} = \frac{\partial(v, p)}{\partial(s, t)},$$

where the area-integral for w is obtained by a known lemma from the initial perimeter integral. The fundamental condition is then that the relation

$$(6) \qquad \int_{s_2}^{s_1} \int_{t_2}^{t_1} \varphi(s, t) ds dt = H(t_1, t_2) \int_{s_2}^{s_1} \omega(s, t_1) ds$$

must be an identity in (s_1, s_2, t_1, t_2).

Consideration of a range infinitesimal in t suggests the introduction of the function

$$\psi(t) = \left[-\frac{\partial H(t, t')}{\partial t'}\right]_{t'=t}, \tag{7}$$

which is the familiar Carnot function adapted to an arbitrary scale of heat as well as of temperature. For a cycle infinitesimal in s only

$$\int_{t_2}^{t_1} \varphi(s, t)dt = \omega(s, t_1)H(t_1, t_2), \tag{8}$$

whence

$$\varphi(s, t_2) = -\omega(s, t_1)\frac{\partial H(t_1, t_2)}{\partial t_2}, \tag{9}$$

the limiting form of which for range infinitesimal in t also is

$$\varphi(s, t_1) = \omega(s, t_1)\psi(t_1), \tag{10}$$

so that division shows the ratio $\varphi(s, t_1)/\varphi(s, t_2)$ to be a function of t_1, t_2 only, say for abbreviation $f(t_1, t_2)$, identically in s, t_1, t_2. Because of the symmetry of notation it is no longer necessary to keep the condition $t_1 > t_2$. If then t_0 be any particular value of t this result is equivalent to

$$\varphi(s, t) = \varphi(s, t_0)f(t, t_0),$$

so that φ is built of two factors, each a function of one of the variables only, which for later convenience may be written as derivatives:

$$\varphi(s, t) = \sigma'(s)\tau'(t), \tag{11}$$

where the factors can be understood as positive so that σ and τ are monotonic increasing functions. Then (10) gives

$$\omega(s, t) = \sigma'(s)\theta(t), \tag{12}$$

where by definition

$$\theta(t) = \tau'(t)/\psi(t). \tag{13}$$

The differential forms in (2) and the relation (5) become

$$dh = \sigma'(s)\theta(t)ds, \qquad dw = ads + bdt, \tag{14}$$

$$\frac{\partial a}{\partial t} - \frac{\partial b}{\partial s} = \sigma'(s)\tau'(t),$$

the last being equivalent to

$$\frac{\partial}{\partial t}(a - \sigma'\tau) = \frac{\partial b}{\partial s},$$

which indicates the existence of a function u such that

$$a = \sigma'(s)\tau(t) - \frac{\partial u}{\partial s}, \qquad b = -\frac{\partial u}{\partial t}, \tag{15}$$

which make

$$(16)\qquad dw = \sigma'(s)\tau(t)ds - du = \frac{\tau(t)}{\theta(t)}dh - du.$$

This suggests the introduction of a particular scale of measurement of heat by the definition

$$(17)\qquad dq = \frac{\tau(t)}{\theta(t)}dh,$$

so that

$$(18)\qquad du = dq - dw, \qquad d\sigma = dh/\theta = dq/\tau,$$

where $d\sigma$, du are exact differentials, while dq in general would not be. In terms of the original differential forms (1) the result is then that there exist functions θ and τ of t only such that dh/θ and $\tau dh/\theta - dw$ are exact differentials. The dq and $d\sigma$ are the measures of heat on two special scales whose existence is implied in the Carnot principle.[1]

As to uniqueness, the mode of introduction of τ and θ shows that any two determinations for the same body or for different bodies are related by

$$\tau^* = \alpha\tau + \beta, \qquad \theta^* = \alpha\theta,$$

so that for the same dh:

$$dq^* = dq + \beta dh/\alpha\theta.$$

But since only the product $\sigma'\tau'$ is determined it is essentially general to hold α as unity, then the freedom left in the definition of dq can be used to arrange that equality in dh shall bring about equality in dq, by having $\beta = 0$; thus τ and θ may be understood as the same for all bodies.

In terms of the general and the two special scales of heat the efficiency ratio and the conditions of vanishing of the cyclic integrals of $d\sigma$ and du are

$$\frac{w}{h_1} = \frac{\tau_1 - \tau_2}{\theta_1}, \qquad \frac{w}{q_1} = \frac{\tau_1 - \tau_2}{\tau_1}, \qquad \frac{w}{\sigma_1} = \tau_1 - \tau_2;$$

$$\frac{h_1}{\theta_1} + \frac{h_2}{\theta_2} = 0, \qquad \frac{q_1}{\tau_1} + \frac{q_2}{\tau_2} = 0, \qquad \sigma_1 + \sigma_2 = 0;$$

$$w = \frac{h_1}{\theta_1/\tau_1} + \frac{h_2}{\theta_2/\tau_2} = q_1 + q_2 = \frac{\sigma_1}{1/\tau_1} + \frac{\sigma_2}{1/\tau_2};$$

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[1] This proof should be compared with the deduction given by Larmor, *loc. cit.*, especially p. 332. He begins with the more primitive form of the Carnot postulate, that in order to have work done heat must fall in temperature, and considers also irreversible processes. The factorization which introduces one of his temperature functions results from a functional equation transcribing the coupling of two Carnot cycles. His $g(\theta)$ and $\psi(\theta)$ correspond to the $\theta(t)$ and $\tau(t)$ above; Raveau similarly deduces the existence of two distinct scales or functions of temperature. As to the use of the term "universal," however, the distinction should be noted, that the θ in the present paper, although the same for all bodies, depends on the scale of heat, while τ is independent of that scale. It is naturally the latter only that is to be considered truly universal as a scale of temperature.

inspection of which shows for one thing that θ depends on the scale of heat while τ does not. Thus τ/θ, $1/\theta$, and τ correspond to $B(t)$, $C(t)$, and $T(t)$ of Part I., while q and σ are thermal energy and entropy.

The parity of setting of the two special scales of heat is noticeable here as in Part I. It is hard to see how either can be said flatly to give the true measure of heat, though Callendar's view as to the preëminent claim of the entropy scale seems fully as reasonable as the common exclusive use of the energy scale. The latter has of course the obvious advantage for measurement and computation that it gives a constant ratio of work to thermal equivalent and approximate constancy of ordinary specific heats, but for theoretical purposes at least the entropy scale offers some advantages. For example, if for k/θ and l/θ be put κ and λ, these being then temperature-capacity for entropy and latent entropy of expansion, the differential forms with temperature on the τ-scale are

$$d\sigma = \kappa d\tau + \lambda dv, \qquad dw = p dv,$$

and the conditions are that $d\sigma$ and $\tau d\sigma - dw$ be exact differentials. These conditions are equivalent to the adapted Clapeyron and Clausius equations

$$\lambda = \frac{\partial p}{\partial \tau}, \qquad \frac{\partial \kappa}{\partial v} = \frac{\partial^2 p}{\partial \tau^2},$$

which are so simply interpretable on the graph of the isochoric (p, τ) curve.

To extend the theory to a system with more than two coördinates it is convenient to use the τ-scale of temperature and the σ-scale of heat, as determined for two-variable systems. A system with n coördinates of configuration, say $v_1 \cdots v_n$, will then be characterized by the differential forms

$$d\sigma = \kappa d\tau + \Sigma \lambda_i dv_i, \qquad dw = \Sigma p_i dv_i.$$

The Carnot principle may then be understood to be applicable to every two-variable process which such a system can undergo, through the presence of possible or conceivable constraints. This is equivalent to the condition that if for $v_1 \cdots v_n$ be put any arbitrary functions of τ and a single other parameter v, then $d\sigma$ and $\tau d\sigma - dw$ must be exact differentials in these two variables. The conditions for this prove to be equivalent to those under which the same expressions are exact differentials in the $n + 1$ variables, regarded as independent. To show this it is enough to consider only the following types of cycle: (*a*) where only one of the variables v_i varies, (*b*) where only two are affected, these undergoing variations proportionate but in arbitrary ratio. With such cycles, starting at an arbitrary state of the system, the Carnot principle yields, from (*a*) the Clapeyron and Clausius equations for each of the variables separately, and from (*b*) the additional conditions

$$\frac{\partial p_i}{\partial v_j} = \frac{\partial p_j}{\partial v_i}, \qquad \frac{\partial \lambda_i}{\partial v_j} = \frac{\partial \lambda_j}{\partial v_i},$$

which indicate the conservation in an isothermal cycle of work and of heat separately. The scales θ and τ and the equations in (17) and (18) thus apply to these more general systems.

If for such a system the heat scale be still left arbitrary, the result is that dh/θ and $\tau dh/\theta - dw$ are exact differentials of functions of state σ and u, so that $dh = \theta d\sigma$, $dw = \tau d\sigma - du$. Now while a system with two variables only can have but two independent functions of state, one with more types of variation can have a corresponding number. But not more than two independent functions can have differentials linearly expressible in terms of the same two fundamental differential forms; in fact, in the case here, any function of state whose differential is a linear combination of dh and dw must be a function of σ and u. In this sense the theory of energy and entropy, and therefore the Carnot principle, yields the entire theory that can be based solely on dh and dw.

The University of Chicago,
March 18, 1919.

THE SIZE AND SHAPE OF THE ELECTRON.[1]

BY ARTHUR H. COMPTON.

SYNOPSIS.—Attention is called to two outstanding differences between experiment and the theory of scattering of high frequency radiation based upon the hypothesis of a sensibly point charge electron. In the first place, according to this theory the mass scattering coefficient should never fall below about .2, whereas the observed scattering coefficient for very hard X-rays and γ-rays falls as low as one fourth of this value. In the second place, if the electron is small compared with the wave-length of the incident rays, when a beam of γ-rays is passed through a thin plate of matter the intensity of the scattered rays on the two sides of the plate should be the same, whereas it is well known that the scattered radiation on the emergent side of the plate is more intense than that on the incident side.

It is pointed out that the hypothesis that the electron has a diameter comparable with the wave-length of the hard γ-rays will account qualitatively for these differences, in virtue of the phase difference between rays scattered by different parts of the electron. The scattering coefficient for different wave-lengths is calculated on the basis of three types of electron: (1) A rigid spherical shell of electricity, incapable of rotation; (2) a flexible spherical shell of electricity; (3) a thin flexible ring of electricity. All three types are found to account satisfactorily for the meager available data on the magnitude of the scattering coefficient for various wave-lengths. The rigid spherical electron is incapable of accounting for the difference between the emergent and the incident scattered radiation, while the flexible ring electron accounts more accurately for this difference than does the flexible spherical shell electron.

It is concluded that the diameter of the electron is comparable in magnitude with the wave-length of the shortest γ-rays. Using the best available values for the wave-length and the scattering by matter of hard X-rays and γ-rays, the radius of the electron is estimated as about 2×10^{-10} cm. Evidence is also found that the radius of the electron is the same in the different elements. In order to explain the fact that the incident scattered radiation is less intense than the emergent radiation, the electron must be subject to rotations as well as translations.

I. THE SCATTERING OF HIGH FREQUENCY RADIATION.

THE radius of the electron is usually calculated from its kinetic energy when in motion, taking this to be identical with its magnetic energy. According to the customary assumption that the charge on an electron is uniformly distributed over the surface of a sphere, the radius of the sphere as thus calculated is about 1×10^{-13} cm. There are, however, a number of phenomena in connection with the scattering and absorption of high frequency radiation by matter, which appear to be inexplicable according to classical electrodynamics if the dimensions of the electron are taken to be of this order of magnitude, whose explanation is obvious if the electron is assumed to be a flexible ring of electricity whose radius is comparable with the wave-length of short γ-rays. This paper is the first of a series of three, which will deal respectively with the scattering of high-frequency radiation, the absorption of high-frequency radiation, and the nature of the ultimate magnetic particle. The present discussion will deal with certain outstanding differences between experiment and the theory of scattering of high frequency radiation based upon the hypothesis of a sensibly point charge electron, and it will be shown that these differences may be explained on the basis of an electron of relatively large size. In order to preserve the directness of the argument, the details of the calculations will be reserved for the latter part of the paper.

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[1] A preliminary paper on this subject was read before the American Physical Society, December 28, 1917 (PHYS. REV., 11, 330, 1918). Cf. also J. Wash. Ac. Sci., 8, 1, 1918.

Part 1.

A. *The Scattering Coefficient of High Frequency Radiation.*

On the hypothesis that the electron is sensibly a point charge of electricity, Sir J. J. Thomson has shown[1] that the ratio of the energy of the electromagnetic radiation scattered by an isolated electron to the energy incident upon it is given by the expression

$$\frac{8\pi}{3}\frac{e^4}{m^2C^4}.$$

Here *e* and *m* are respectively the charge and mass of the electron, and *C* is the velocity of light. If the electrons in any substance act independently of each other, the scattering coefficient per unit mass of the substance will therefore be

$$\frac{\sigma}{\rho}=\frac{8\pi}{3}\frac{Ne^4}{m^2C^4}, \tag{1}$$

where σ is the ratio of the scattered to the incident energy per unit volume of the material, ρ is its density, and *N* is the number of electrons in unit mass of the substance.

Since this scattered energy is lost from the primary beam, the quantity σ/ρ represents also the part of the mass absorption coefficient which is due to scattering. As Barkla has pointed out,[2] there may be absorption due to other causes, such as the production of secondary beta or cathode rays, but this absorption due to scattering must always be present. Moreover, if the electrons in the absorbing material are grouped together in regions which are small compared with the wave-length of the incident beam, the electrons do not scatter independently. The rays scattered by the electrons in this case overlap in such a manner that a certain amount of "excess scattering" occurs. There is, however, no arrangement of the electrons which will result in less scattering than when they act independently. On the hypothesis of a point charge electron, it is possible for the scattering, and hence for the mass absorption coefficient, to be smaller than that predicted by Thomson's expression (1) only in case the electrons are held in position so firmly that their natural period of vibration is shorter than the period of the incident radiation.

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[1] J. J. Thomson, Conduction of Electricity through Gases, 2d ed., p. 325.

[2] C. G. Barkla and M. P. White, Phil. Mag., 34, 275, 1917.

In making the calculations from Thomson's theory, it may be assumed that the number of electrons in an atom which are effective in scattering the incident radiation is equal to the atomic number. This assumption is supported in the case of the lighter elements by the experiments of Barkla and Dunlop[1] when X-rays of ordinary hardness are used. It would seem possible that with the higher frequency γ-rays certain electrons might be effective in scattering which are too rigidly bound to scatter X-rays. Such an effect, however, would mean an increase rather than a decrease in the scattering for the shorter wave-lengths. That the atomic number is the number of effective electrons when γ-rays are used, is confirmed by the observations of Soddy and Russell[2] and of Ishino[3] to the effect that for the shortest rays the amount of energy scattered by atoms of the different elements is accurately proportional to their atomic numbers. This means that all the electrons outside of the nucleus are effective in producing absorption when hard γ-rays are used. If the electron is sensibly a point charge of electricity, the scattered energy should therefore be at least as great as the value assigned by equation (1).

Barkla and Dunlop[4] have shown that for a considerable range of wave-lengths of X-rays the mass scattering coefficients of the lighter elements are given accurately by equation (1) if the number of electrons in the atom is taken to be approximately half the atomic weight. For elements of high atomic weight, the scattering becomes greater than this value except for very short wave-lengths, indicating that the electrons are so closely packed that "excess scattering" occurs. For wave-lengths less than 2×10^{-9} cm., however, Barkla and White[5] have shown that the total mass absorption coefficient of the light elements is less than the value theoretically calculated for the mass scattering coefficient alone; and Soddy and Russell[6] have found that for the hard γ-rays from Radium

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[1] C. G. Barkla and J. G. Dunlop, Phil. Mag., 31, 222, 1916.

[2] Soddy and Russell, Phil. Mag. 18, 620, 1910; 19, 725, 1910.

[3] M. Ishino, Phil. Mag., 33, 140, 1917.

[4] C. G. Barkla and J. G. Dunlop, *loc. cit.*

[5] C. G. Barkla and M. P. White, *loc. cit.*, p. 277.

[6] Soddy and Russell, *loc. cit.*

C the absorption of substances of lower atomic weight than mercury is only a small fraction of that required by Thomson's expression. Direct measurements of the scattering of hard γ-rays confirm the conclusions based on absorption measurements. Thus it has been found[1] in the case of radiation of very high frequency that the mass scattering coefficient falls as low as one fourth of the value predicted by Thomson's theory.

As has just been pointed out, it is impossible, according to classical electrodynamics, to account for this low scattering and absorption of radiation of very high frequency by matter if the electron is taken to be sensibly a point charge of electricity. If, on the other hand, the electron is considered to have a radius comparable with the wave-length of the incident radiation, a qualitative explanation of the phenomenon of low scattering for short wave-lengths is obvious. The effect of this hypothesis is to make an appreciable phase difference between the rays scattered by different parts of the electron. Thus the radiation scattered from A, Fig. 1, traverses a longer path between S and P than does the ray scattered from the part of the electron at B. If the wave-length is many times the diameter of the electron, the phase difference between these two rays will be negligible, and the reduction in the intensity of the scattered beam will be inappreciable; if, however, the difference in the two paths is comparable with the wave-length of the incident radiation, the phase difference will be such that the intensity of the ray scattered to P will be much reduced. The assumption of a relatively large electron is therefore capable of explaining qualitatively the observed decrease in the scattering of electromagnetic radiation when the wave-length becomes very short.

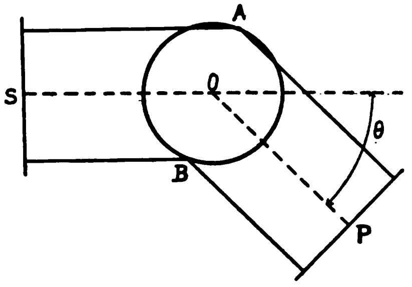

Fig. 1.

Calculation of the Scattering. I. *Rigid Spherical Electron.*—The exact manner in which the scattering will decrease with shorter wave-lengths will of course depend upon the form of electron considered. For example, taking the simplest case of the scattering due to a rigid, uniform, spherical shell of electricity, incapable of rotation, we find

$$\frac{\sigma}{\rho} = \frac{8\pi}{3}\,\frac{e^4 N}{m^2 C^4}\sin^4\left(\frac{2\pi a}{\lambda}\right)\Big/\left(\frac{2\pi a}{\lambda}\right)^4, \tag{2}$$

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[1] M. Ishino, *loc. cit.*

where a is the radius of the spherical shell and λ is the wave-length of the incident beam. The details of the derivation of this expression will be found in Part 2, Section 1. If $a = 10^{-13}$ cm., as usually assumed, this is practically identical with equation (1), even for radiation of the shortest known wave-length. The relative values of the scattering according to this expression are shown in curve *I*, Fig. 2, for different values of λ/a. In this diagram the value $\sigma/\sigma_0 = 1$ indicates the magnitude of the mass scattering if the electron were sensibly a point charge of electricity, and the calculated values are given in terms of this quantity.

2. *Flexible Spherical Electron.*—It would appear more reasonable to suppose that the spherical shell electron is subject to rotational as well as to translational displacements when traversed by a γ-ray. The scattering due to such an electron is difficult to calculate, but an approximate expression can be obtained if the electron is considered to be perfectly flexible, so that each part of it can be moved independently of the other parts. On this hypothesis it can be shown (cf. Part 2, Section 2) that the intensity of the beam scattered by an electron at an angle θ with an unpolarized beam of γ-rays is given by the expression,

$$(1)\qquad I_\theta = I\frac{e^4(1 + \cos^2\theta)}{2L^2m^2C^4}\left\{\sin^2\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)\Big/\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^2\right\}.$$

Here I is the intensity of the incident beam, L is the distance at which the intensity of the scattered beam is measured, and the other quantities have the same meaning as before. The mass absorption coefficient due to the scattering by such an electron is therefore,

$$(4)\qquad \frac{\sigma}{\rho} = 2\pi NL^2\int_0^\pi \frac{I_0}{I}\sin\theta d\theta.$$

This integral may be evaluated graphically or by expansion into a series (*cf. infra*, equation 17). The values of σ/σ_0 according to equation (4) are plotted in curve *II*, Fig. 2. The values for a rigid spherical electron which is subject to rotation should lie between curves *I* and *II* for the range of wave-lengths for which curve *II* is plotted.

3. *Ring Electron.*—According to electromagnetic theory it is obvious that the mass of an electron cannot be accounted for on the basis of a uniform distribution of electricity over the surface of a sphere of radius comparable with the wave-length of γ-rays. Much the same effect, so far as the scattering of high frequency radiation is concerned, results from the conception of the electron as a ring of electricity of relatively large diameter, similar in form to the "magneton" suggested by A. L. Parson.[1] It has been shown by Webster[2] and Davisson[3] that the assump-

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[1] A. L. Parson, Smithsonian Misc. Collections, Nov., 1915.

[2] D. L. Webster, PHYS. REV., 9, 484, 1917.

[3] Davisson, PHYS. REV., 9, 570, 1917.

tion of such an electron is compatible with the electromagnetic theory of mass.

The exact calculation of the scattering produced by a thin ring of electricity is difficult. A chief factor in the complexity of the problem is the fact that the effective electromagnetic mass of a short arc of the ring differs according as it is accelerated parallel to the tangent to the arc, parallel to the axis of the ring, or parallel to a radius of the ring. The ratios of the effective masses along these three axes depends moreover upon the speed with which the electricity in the ring is rotating. In order to make the problem manageable, the assumptions have been made that the mass of an arc element is the same in all directions, and that the velocity of the electricity in the ring is small compared with the velocity of light. On the basis of these assumptions the mass scattering coefficient for a flexible electronic ring is found to be

$$\frac{\sigma}{\rho}=\frac{8\pi}{3}\frac{Ne^4}{m^2C^4}\left\{1-a\left(\frac{a}{\lambda}\right)^2+b\left(\frac{a}{\lambda}\right)^4-c\left(\frac{a}{\lambda}\right)^6+\cdots\right\}, \tag{5}$$

where the coefficients $a, b, c, \cdots$ are constants which are evaluated below (cf. equation 21). The relative values of the scattering according to this expression are shown in curve *III*, Fig. 2. The scattering of γ-rays by a ring electron as thus calculated is an approximation which will doubtless correspond closely with the true value for relatively long waves, but which may differ appreciably for the shortest known radiation.

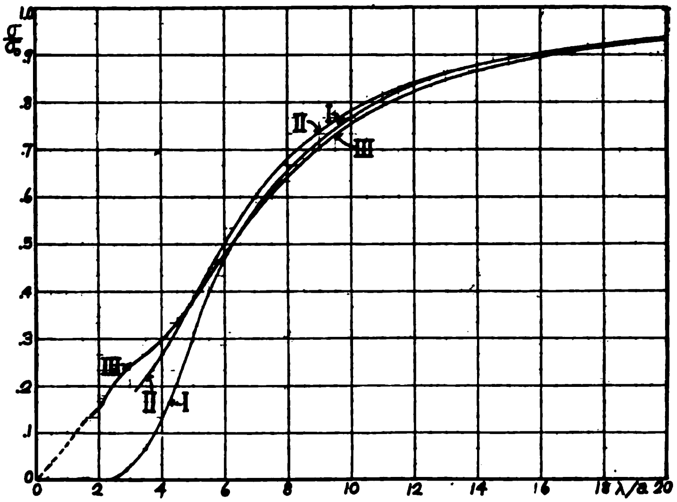

Fig. 2.

Unfortunately the experimental data are too meager to submit these formulæ to accurate quantitative test. There are, however, three points on the curve which have been established with some care. Barkla and Dunlop[1] have found that for relatively soft X-rays the light elements scatter according to equation (1), so that the part of the curves where λ/a is large is verified. Hull and Rice[2] have estimated from their absorption measurements that for X-rays and γ-rays whose wave-lengths are in the neighborhood of 0.15×10^{-8} cm. the value of σ/ρ for aluminium is 0.12. Taking the number of electrons in an aluminium atom to be 13, this gives for the relative scattering, 0.64. According to curve *I* this corresponds to an electronic radius of 1.9×10^{-10} cm. Curve *II* gives 2.0×10^{-10} cm., and curve *III*, 1.9×10^{-10} cm.[3] Ishino[4] finds that the value of σ/ρ for aluminium, using the hard γ-rays from radium C, is about 0.045, which means a value for the relative scattering of 0.24. The work of Rutherford and Andrade[5] shows that the principal part of the "homogeneous" radiations from radium C consists of a strong line $\lambda = 0.099$, and a weaker line $\lambda = .071$, $\times 10^{-8}$ cm. Both of these lines were prominent in Ishino's experiment, in which he filtered the γ-rays through a centimeter of lead. Rutherford has pointed out,[6] however, from a consideration of the velocities of the β-particles, that there must be a certain amount of radiation of much shorter wave-length. The existence of such extremely hard rays is confirmed by the fact that the absorption coefficient of the penetrating radiation of the atmosphere as determined at high altitudes is much smaller than that of the hard γ-rays from radium C, such as used by Ishino. The fact that it is impossible to detect these very short waves by crystal analysis, however, indicates that their effectiveness in the scattered beam is small compared with that of the two lines observed by Rutherford and Andrade. It seems reasonable, therefore, to take for the effective wave-length of the γ-rays used in Ishino's scattering experiments about $.08 \times 10^{-8}$ cm.[7] This gives for the value of the electronic radius, from curve *I*, 1.7, from curve *II*, 2.1 and from curve *III*, 2.7, $\times 10^{-10}$ cm.

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[1] C. G. Barkla and Dunlop, *loc. cit.*

[2] A. W. Hull and M. Rice, PHYS. REV., 8, 326, 1916.

[3] In the second part of this paper, by using a more accurate formula for the mass absorption coefficient, the data of Hull and Rice will be shown to lead to a value of $(1.85 \pm .04) \times 10^{-12}$ cm., if the electron is taken to be a ring.

[4] M. Ishino, *loc. cit.*, p. 141.

[5] Sir E. Rutherford and Andrade, Phil. Mag., 28, 263, 1916.

[6] Sir E. Rutherford, Phil. Mag., 34, 153, 1917.

[7] In the paper last referred to, Rutherford estimated the effective wave-length of Ishino's γ-rays to be much shorter than the value here used. His estimate is based upon measurements of the absorption of high frequency X-rays filtered by means of a lead filter. He calculated the frequency of the X-rays according to the relation $h\nu = eV$, taking V to be the maximum voltage applied to the tube. As is apparent from the work of Rutherford, Barnes and Richardson (Phil. Mag., 30, 339, 1915), this relation does not express the *effective* fre-

The value of σ/ρ given by Hull and Rice is a mean over a relatively large range of wave-lengths, and Barkla is of the opinion[1] that Ishino's value of the scattering of the γ-rays from radium C is appreciably in in error because of a too high estimate of the true absorption. Thus, though the experimental values of the electronic radius agree best on the basis of the flexible spherical shell electron, as represented in curve *II*, the accuracy of the experiments is by no means sufficient to distinguish between the three hypotheses.

The important thing to notice is that if the electrons had dimensions comparable with 10^{-13} cm., as usually assumed, the scattering should be represented by the upper line of Fig. 2 where $\sigma/\sigma_0 = 1.0$. The fact that experiment gives consistently lower values when short wave-lengths are used is sufficient proof that the electron is not sensibly a point charge of electricity. On the other hand, it is possible to account for this reduced scattering within the probable experimental error if the electron has a radius of 2×10^{-10} cm.

B. The Dissymmetry of the Scattering of Hard γ-rays on the Incident and Emergent Sides of a Plate.

A second difficulty which is found with Sir J. J. Thomson's simple theory is that it predicts that if a beam of X-rays is passed through a thin plate of matter, the intensity of the scattered rays on the two sides of the plate should be the same. Barkla and Ayers[2] have shown that, for rather hard X-rays and for those substances of low atomic weight whose mass scattering coefficients can be calculated accurately by equation (1), this second prediction of Thomson's theory is also valid. On the other hand, it is well known that both in the case of relatively soft X-rays and in the case of hard γ-rays the scattered radiation on the emergent side of the plate is more intense than that on the incident side.

When heavy atoms and long waves are used, the dissymmetry between the emergent and the incident scattered radiation is accompanied by an increase in the total scattered energy. For this reason the phenomenon is described by the term "excess scattering." It is satisfactorily accounted for[3] by the fact that the electrons in the heavy atoms do not act independently in scattering the longer wave-length X-rays, since

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quency of the filtered rays, especially when a lead filter is used. This doubtless accounts for the fact, which will be brought out in the following paper, that Rutherford's determinations of the absorption do not agree with those of Hull and Rice, who measured the absorption coefficient of homogeneous X-rays of known wave-length.

[1] C. G. Barkla and M. P. White, *loc. cit.*, p. 278.

[2] Barkla and Ayers, Phil. Mag., 21, 271, 1911.

[3] C. G. Darwin, Phil. Mag., 27, 329, 1914; D. L. Webster, Phil. Mag., 25, 234, 1913.

they are grouped so closely together that the rays scattered by the different electrons are in nearly the same phase. This has the effect of increasing the total scattering. But also, since the phase difference between the rays from the different electrons in an atom is less for the scattered rays which make small angles with the primary beam, there is greater reinforcement and hence greater intensity on the emergent than on the incident side of the scattering atom.

This explanation cannot be applied, however, to the case of the unsymmetrical scattering of very hard rays. This is clear for two reasons. In the first place, if an atom of medium weight is traversed by rays of increasing hardness, at first excess scattering occurs as described above; but as the wave-length becomes shorter the scattered radiation becomes nearly symmetrical until the scattered energy can be calculated according to Thomson's formula (1). The electrons now, therefore, are scattering independently, and must continue to do so for all shorter wave-lengths. Thus we see that the dissymmetry in the scattering which reappears as the wave-length becomes very short cannot be accounted for by the mutual action of the separate electrons. In the second place, the phenomenon of unsymmetrical scattering for very short waves is distinguished from the excess scattering which occurs with longer waves by the fact that in the former case the dissymmetry is accompanied not by an increase but by a decrease in the total scattering. If the phenomenon were due to the mutual action of the electrons, it would be accompanied by an increased total scattering, as before. It is thus evident that the unsymmetrical scattering of very short electromagnetic waves is due not to groups of electrons in the atoms, but to some property of the individual electrons.

The qualitative explanation of this phenomenon on the basis of our large electron hypothesis is at once apparent. Referring again to Fig. 1, it is obvious that if the diameter of the electron is comparable with the wave-length of the radiation, there will be an appreciable difference in phase between the rays scattered from different parts of the electron. Since this phase difference is greater for rays scattered at large than for those at small angles, the intensity of the incident radiation will be in the former case the more strongly reduced. In order to explain this phenomenon it is not sufficient, however, merely to assume that the electron is relatively large. For example, the hypothesis of the electron as a rigid spherical shell, incapable of rotation, though resulting in a reduced total scattering, would give rise to symmetrical scattering on the incident and emergent sides of a plate. To account for the observed dissymmetry, the further assumption must be made that the incident electromagnetic wave is capable of moving the different parts of the electron relatively to each other.

If the electron is sensibly a point charge of electricity, the intensity of the beam scattered by an electron at an angle θ with the incident beam is[1]

$$(6) \qquad I_\theta = I\frac{e^4(1+\cos^2\theta)}{2L^2m^2C^4}.$$

The corresponding expression for an electron in the form of a flexible spherical shell of electricity has already been given:

$$(3) \qquad I_\theta = I\frac{e^4(1+\cos^2\theta)}{2L^2m^2C^4}\left\{\sin^2\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^2\Big/\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^2\right\}.$$

For the ring type electron we obtain

$$(7) \quad I_\theta = I\frac{e^4(1+\cos^2\theta)}{2L^2m^2C^4}\left\{1-\alpha\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^2+\beta\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^4-\cdots\right\},$$

where the values of the constants α, β, γ, etc., are those determined below (equation 20). When a/λ remains small, the scattering according to both expressions (3) and (7) approaches the value for a point charge electron (6).

D. C. H. Florance[2] has determined experimentally the values of the relative intensity of the radiation scattered at different angles when the hard γ-rays from radium bromide traverse a plate of iron. His values are indicated in Fig. 3 by circles. Taking the effective wave-length of these rays to be $.09 \times 10^{-8}$ cm., and using in equations (3) and (7) $a = 2 \times 10^{-10}$ cm. as above estimated, the relative scattering at different angles may be calculated. The intensity of the radiation scattered at different angles by a point charge electron is indicated in Fig. 3 by the outer solid curve, that due to the spherical shell electron by the inner solid curve, and that from the ring form electron by the broken curve. Inasmuch as the γ-rays used by Florance were heterogenous, and as the softer rays are scattered relatively more strongly at larger angles, the agreement of the experimental values with either of the inner curves is as good as can be expected. The important point to be noticed is, however, that the experimental values are entirely out of harmony with

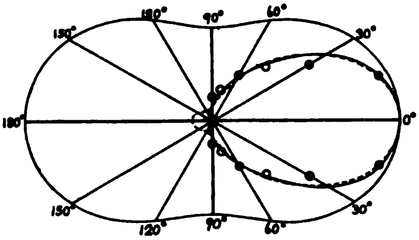

Fig. 3.

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[1] J. J. Thomson, Conduction of Electricity through Gases, 2d ed., p. 326.

[2] D. C. H. Florance, Phil. Mag., 20, 921, 1910.

what is to be expected if the electron is sensibly a point charge of electricity.

A better quantitative test of this explanation of the dissymmetry of scattered γ-radiation is afforded by determinations of the ratio of the total radiation scattered on the incident side of a plate struck by hard γ-rays to that scattered on the emergent side. The theoretical value of this ratio is

$$\frac{I_i}{I_e} = \int_{\pi/2}^{\pi} I_\theta \sin\theta d\theta \Big/ \int_0^{\pi/2} I_\theta \sin\theta d\theta. \tag{8}$$

The curves of Fig. 4 give the values of this ratio for different values of λ/a, the broken curve being calculated on the basis of the spherical electron.

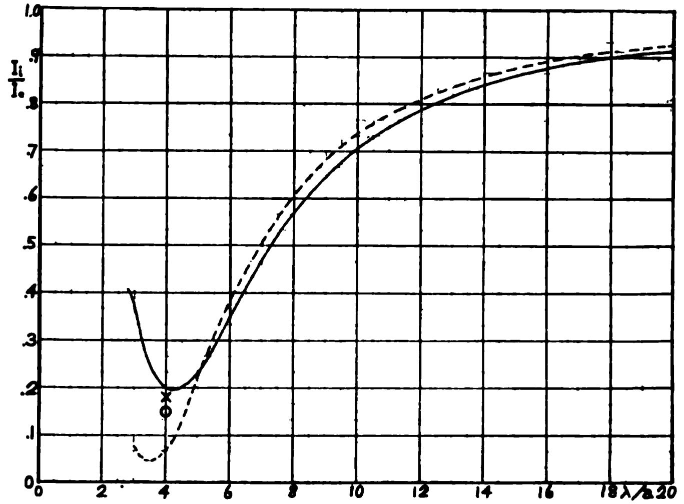

Fig. 4.

These curves doubtless explain at least in part the observation of Florance[1] that the "incident" scattered rays are softer than the "emergent" and the primary rays, since they show that the relative amount of rays scattered backward is much greater for soft or long wave-length γ-rays than for the harder radiation.

The ratio of the incident to the emergent radiation has been determined for the hard γ-rays from radium C by Madsen,[2] who found the value 18 per cent., and Ishino,[3] who found 15 per cent. Assuming for the effective wave-length in this case $\lambda = 0.08 \times 10^{-8}$ cm., and for the radius of the electron 2×10^{-10} cm., as estimated above, Ishino's datum for this ratio is represented by the open circle in Fig. 4 and Madsen's datum by the cross. It is possible that neither of the theoretical values agree within the probable experimental error with these determinations, but in view of the approximate method of calculating the scattering by a ring electron the difference is not serious. The fact that the predicted values are of the proper order of magnitude is strong evidence that the dissymmetry in the scattering of γ-rays by matter is due to the interference of the rays scattered by different parts of the electron. Thus not only must the electron have a size comparable with the wave-length of γ-rays, but it must also be subject to rotations or be sufficiently flexible for γ-rays to move its different parts relatively to each other.

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[1] D. C. H. Florance, Phil. Mag., 27, 225, 1914.

[2] Madsen, Phil. Mag., 17, 423, 1909.

[3] M. Ishino, *loc. cit.*, p. 138.

C. *Conclusions.*

As has been pointed out, therefore, according to classical electrodynamics the mass scattering coefficient for X-rays and γ-rays passing through matter should never fall below the value 0.18, as calculated on the basis of Thomson's theory, if an electron of the usual dimensions is postulated. Experiment shows, however, that for very high frequency radiation the scattering is much less than this. It is possible that certain assumptions regarding the conditions for scattering radiant energy, contrary to classical theory, might be made which would account for the observed low value of the scattering for very high frequencies. As long as the idea of the point charge electron is retained, however, no such assumptions can account for the observed dissymmetry between the incident and the emergent scattered radiations. Unless the theory that X-rays and γ-rays consist of waves or pulses is abandoned, the only possible explanation of this dissymmetry would seem to be that the scattering particles have dimensions comparable with the wave-length of the rays which they scatter. Since the scattering particles have been shown to be the electrons, the statement may therefore be made with confidence that *the diameter of the electron is comparable in magnitude with the wave-length of the shortest γ-rays.*

According to the best available values for the wave-length and the scattering by matter of hard X-rays and γ-rays, *the radius of the electron is about 2×10^{-10} cm.*

The fact that the scattering of hard γ-rays by atoms of the different elements is proportional to the atomic number shows that if the number of the electrons in an atom which are effective in scattering is equal to the atomic number, *the radius of the electron is the same in the different atoms.* This is clear, since if the electron had smaller dimensions in the atoms of one element, the scattering coefficient of this element would not decrease as rapidly for shorter wave-lengths, and the scattering by the different atoms would not be proportional to the number of electrons in the atoms.

In order to explain the fact that the emergent scattered radiation is more intense than the incident radiation, it is necessary to assume further that the different parts of the charge of the electron can possess certain motions independently of each other. That is, *the electron is subject to rotations as well as translations.*

PART 2.

1. *To calculate the energy scattered by a rigid spherical shell electron, incapable of rotation, whose diameter is comparable in magnitude with the wave-length of the incident radiation.*

Let us first derive an expression for the acceleration to which such an electron is subject when traversed by an electromagnetic wave, and then determine the energy scattered by integrating the intensity of the beam due to this acceleration over the surface of a sphere drawn with the electron at the center.

In Fig. 5, let us suppose that the γ-ray traverses the electron along the axis Z. We shall let A represent the amplitude of the incident wave, X its electric intensity at the plane z, and λ its wave-length. The radius of the electron we shall call a, and η will represent the charge on the surface of the electron between two planes z_1 and z_2 placed unit distance apart. As the electricity is by hypothesis distributed uniformly over the surface of the sphere, η is constant between $z = -a$ and $z = +a$, and the total charge on the electron is $e = 2a\eta$. The electric intensity at the plane z at any instant may be expressed by the relation,

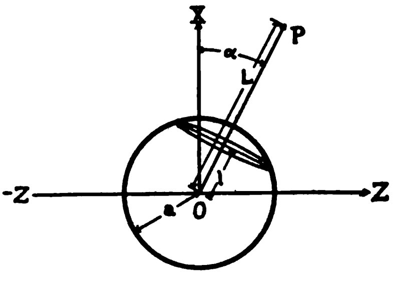

Fig. 5.

$$X = A\cos 2\pi\left(\frac{\delta - z}{\lambda}\right),$$

where $2\pi\delta/\lambda$ is the phase angle at $z = 0$ at that instant. The total force acting on the electron at this instant is therefore,

$$F = \int_{-a}^{a} A\cos 2\pi\left(\frac{\delta - z}{\lambda}\right)\eta dz$$

$$= \frac{A\eta\lambda}{\pi}\cos 2\pi\frac{\delta}{\lambda}\sin 2\pi\frac{a}{\lambda},$$

and the acceleration of the electron is

$$\ddot{x} = \frac{A\eta\lambda}{\pi m}\cos\frac{2\pi\delta}{\lambda}\sin\frac{2\pi a}{\lambda}. \tag{9}$$

Let us now calculate the intensity of the scattered ray at a distance L along the line OP which makes an angle α with the direction OX of the acceleration. According to classical theory the electric intensity at P due to a point charge electron at O subject to this acceleration would be

$$\frac{e\ddot{x}\sin\alpha}{LC^2} \tag{10}$$

where C is the velocity of light. Replacing e by ηdl and $\ddot{x}$ by its value as defined above, and integrating over the electron along the axis L from $l = -a$ to $1 = +a$, the electric intensity of the scattered beam at L, α becomes:

$$\begin{aligned}R_{L,\alpha} &= \frac{\sin\alpha}{LC^2}\int_{-a}^{a}\frac{A\eta\lambda}{\pi m}\sin 2\pi\frac{a}{\lambda}\cos\frac{2\pi}{\lambda}(\delta - L + l)\eta dl\\ &= \frac{\eta^2\lambda A}{\pi m LC^2}\sin\alpha\sin 2\pi\frac{a}{\lambda}\int_{-a}^{a}\cos\frac{2\pi}{\lambda}(\delta - L + l)dl\\ &= \frac{\eta^2\lambda^2 A}{\pi^2 LmC^2}\sin^2 2\pi\frac{a}{\lambda}\sin\alpha\cos 2\pi\frac{\delta - L}{\lambda}.\end{aligned}$$

The amplitude of the electric vector of the scattered wave at the point L, α is therefore

$$A_{L,\alpha} = \frac{\eta^2\lambda^2 A}{\pi^2 LmC^2}\sin^2 2\pi\frac{a}{\lambda}\sin\alpha,$$

or substituting for η its value $e/2a$,

$$A_{L,\alpha} = \frac{Ae^2}{LmC^2}\frac{\sin^2(2\pi a/\lambda)}{(2\pi a/\lambda)^2}\cdot\sin\alpha.$$

The intensity of the radiation at this point is

$$I_{L,\alpha} = cA_{L,\alpha}{}^2 = \frac{cA^2e^4}{L^2m^2C^4}\frac{\sin^4(2\pi a/\lambda)}{(2\pi a/\lambda)^4}\sin^2\alpha,$$

so that the total energy scattered by the electron is

$$\begin{aligned}E_s &= \int_0^{\pi} I_{L,\alpha}\cdot 2\pi L\sin\alpha\cdot Ld\alpha\\ &= \frac{2\pi cA^2e^4}{m^2C^4}\cdot\frac{\sin^4(2\pi a/\lambda)}{(2\pi a/\lambda)^4}\int_0^{\pi}\sin^3\alpha d\alpha\\ &= \frac{8\pi cA^2e^4}{3m^2C^4}\cdot\frac{\sin^4(2\pi a/\lambda)}{(2\pi a/\lambda)^4}.\end{aligned}$$

The energy incident on unit area at the electron is, however, $I = cA^2$, so that the fraction of the incident energy scattered by the electron is

$$E_s/I = \frac{8\pi e^4}{3m^2C^4}\frac{\sin^4(2\pi a/\lambda)}{(2\pi a/\lambda)^4}. \tag{11}$$

When a is small compared with λ this becomes

$$\frac{e^4}{m^2C^4}, \tag{12}$$

which is identical with the value given by Thomson[1] for a sensibly point charge electron.

If there are N electrons per unit mass of any substance, the mass scattering coefficient of the substance is therefore

$$\frac{\sigma}{\rho} = \frac{8\pi}{3}\frac{e^4N}{m^2C^4}\frac{\sin^4(2\pi a/\lambda)}{(2\pi a/\lambda)^4}, \tag{13}$$

where σ is the scattering coefficient per unit volume and ρ is the density of the scattering material.

2. *To calculate the energy scattered by an electron in the form of a flexible spherical shell of electricity.*

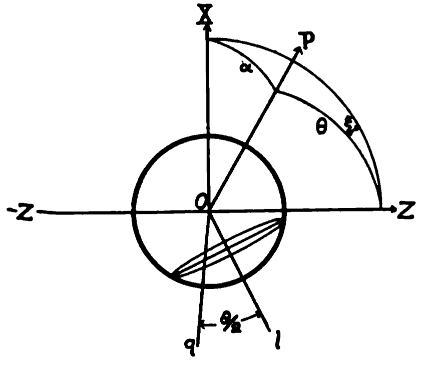

Fig. 6.

We shall treat this problem as if the mass of an element of the spherical shell were independent of the rest of the electron, being equal to $dm = m\cdot ds/s$, where s is the area of the surface of the electron, and m is its mass. As has been pointed out in part I. of this paper, the electro-magnetic mass of a spherical electron of the size here considered would be negligible. This form of electron is therefore only a convenient hypothesis to use in calculating the general effect on the scattering to be expected with any form of electron when the wave-length of the incident radiation approaches its largest dimensions.

Let us suppose that the γ-ray strikes the electron when moving in the direction $-ZOZ$, and determine the intensity of the beam scattered rection OP. As shown in Fig. 6, OP makes the angle θ with the ay, and the angle α with the direction OX of the electric vector ident ray. The plane POZ is inclined at the dihedral angle ξ

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[1] omson, *loc. cit.*

with the plane XOZ. We shall draw also the lines Ol and Oq in the plane POZ, line Oq being perpendicular to OZ, and Ol at an angle $\theta/2$ with Oq. The line Ol therefore bisects the external angle $-ZOP$. If now we consider the beam scattered in the direction OP, all the rays scattered from the element of the electron included between the planes l and $l + dl$ are in the same phase at P.

The amplitude of the electric vector at P due to the rays scattered by this element is, in accordance with expression (10),

$$\eta dl \cdot A \frac{\eta}{\mu} \cdot \sin \alpha / LC^2,$$

where η is the charge and μ the mass of the electron per unit distance along Ol, A is the amplitude of the electric vector of the incident beam, and as before L is the distance OP. If $2\pi\delta/\lambda$ is the phase of the ray at P scattered from the element of the electron at $l = 0$ at a given instant, the electric intensity at that instant of the ray scattered from any element to P is

$$\frac{A\eta^2}{\mu} \frac{dl}{LC^2} \sin \alpha \cdot \cos \frac{2\pi}{\lambda}\left(\delta - 2l \sin \frac{\theta}{2}\right),$$

and the electric intensity due to the whole electron is

$$\begin{aligned} R_{L,\alpha,\theta}, &= \frac{A\eta^2}{\mu} \frac{\sin \alpha}{LC^2} \int_{-a}^{a} \cos \frac{2\pi}{\lambda}\left(\delta - 2l \sin \frac{\theta}{2}\right) dl \\ &= \frac{A\eta^2\lambda}{2\pi\mu LC^2} \cdot \frac{\sin \alpha}{\sin \theta/2} \cdot \sin\left(\frac{4\pi a}{\lambda} \sin \frac{\theta}{2}\right) \cos \frac{2\pi\delta}{\lambda}. \end{aligned}$$

Here, as before, a represents the radius of the electron. The amplitude is obviously the value of this quantity when $\cos (2\pi\delta/\lambda) = 1$, and the intensity of the ray scattered ot P by the electron is therefore

$$\begin{aligned} I_{L,\alpha,\theta} &= cA_{L,\alpha,\theta}^2 \\ &= \frac{cA^2\eta^4\lambda^2}{4\pi^2\mu^2L^2C^4} \cdot \frac{\sin^2 \alpha}{\sin^2 \theta/2} \sin^2\left(\frac{4\pi a}{\lambda} \sin \frac{\theta}{2}\right). \end{aligned}$$

Since $2a\eta = e$, $2a\mu = m$ and $cA^2 =$ the intensity I of the incident beam, the first factor of this expression becomes

$$\frac{Ie^4}{m^2L^2C^4}\left(\frac{\lambda}{4\pi a}\right)^2.$$

When α is expressed in terms of θ and ξ we obtain

$$\cos \alpha = \sin \theta \cos \xi,$$

i. e.,

$$\sin \alpha = \sqrt{1 - \sin^2 \theta \cos^2 \xi}.$$

We may therefore write:

$$I_{L,\xi,\theta} = \frac{Ie^4}{m^2L^2C^4}\left(\frac{\lambda}{4\pi a}\right)^2 \sin^2\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)\frac{1-\sin^2\theta\cos^2\xi}{\sin^2(\theta/2)}.$$

The intensity of the beam scattered by an electron at an angle θ with an unpolarized incident beam is the average of this quantity for all values of ξ, or

$$\begin{aligned} I_{L,\theta} &= \frac{1}{2\pi}\int_0^{2\pi} I_{L,\xi,\theta}\,d\xi \\ &= \frac{Ie^4}{2m^2L^2C^4}\left(\frac{\lambda}{4\pi a}\right)^2 \sin^2\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)\frac{(\cos^2\theta+1)}{\sin^2(\theta/2)}. \end{aligned} \tag{14}$$

The total energy scattered in unit time by the electron is given by the quantity

$$\begin{aligned} E_\theta &= \int_0^\pi I_{L,\theta}\cdot 2\pi L^2 \sin\theta d\theta \\ &= \frac{\pi I e^4}{m^2C^4}\left(\frac{\lambda}{4\pi a}\right)^2\cdot\int_0^\pi \sin^2\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)\frac{(\cos^2\theta+1)}{\sin^2(\theta/2)}\sin\theta d\theta. \end{aligned} \tag{15}$$

This integral may be evaluated either graphically or by means of a series. To integrate by series, substitute $\theta/2 = x$ and $4\pi a/\lambda = b$. The integral factor then becomes:

$$\int_0^{\pi/2}\sin^2(b\sin x)\frac{4\sin^4 x - 4\sin^2 x + 2}{\sin^2 x}\cdot 2\sin x\cos x\cdot 2\,dx.$$

Writing $b\cdot\sin x = z$, this reduces to

$$8\int_0^b \sin^2 z\left(\frac{2z^3}{b^4}-\frac{2z}{b^2}+\frac{1}{z}\right)dz.$$

If $\sin^2 z$ is expanded into the series

$$\sin^2 z = z^2 - \left(\frac{1}{3!}+\frac{1}{3!}\right)z^4 + \left(\frac{1}{5!}+\frac{1}{3!3!}+\frac{1}{5!}\right)z^2 - \cdots,$$

each term may be integrated separately, the result being,

$$8b^2\{\alpha - \beta b^2 + \gamma b^4 - \cdots\},$$

where

$$\alpha = \frac{1}{3} \qquad = .33333,$$

$$\beta = \left(\frac{1}{4}-\frac{1}{3}+\frac{1}{4}\right)\left(\frac{1}{3!}+\frac{1}{3!}\right) \qquad = .05556,$$

$$\gamma = \left(\frac{1}{6}-\frac{1}{4}+\frac{1}{5}\right)\left(\frac{1}{5!}+\frac{1}{3!3!}+\frac{1}{5!}\right) \qquad = .00519,\ 5,$$

$$\delta = \left(\frac{1}{8}-\frac{1}{5}+\frac{1}{6}\right)\left(\frac{1}{7!}+\frac{1}{5!3!}+\frac{1}{3!5!}+\frac{1}{7!}\right) = .00029,\ 10,$$

$\epsilon = .00001,\ 076$ $\eta = .00000,\ 00053,\ 5,$

$\zeta = .00000,\ 0280$ $\theta = .00000,\ 00000,\ 77.$

The total energy scattered by the electron in unit time is therefore

$$E_s = \frac{8\pi I e^4}{m^2 C^4}\left\{\alpha - \beta\left(\frac{4\pi a}{\lambda}\right)^2 + \gamma\left(\frac{4\pi a}{\lambda}\right)^4 - \cdots\right\}. \tag{16}$$

When the wave-length is large compared with the radius of the electron, all terms after the first are negligible, in which case

$$(E_s/I)_{a/\lambda=0} = \frac{8\pi}{3}\frac{e^4}{m^2C^4},$$

as it should. Writing as before N as the number of electrons per unit mass, σ as the scattering coefficient per unit volume and ρ as the density, the mass scattering coefficient of a substance composed of flexible spherical electrons is

$$\frac{\sigma}{\rho} = \frac{8\pi e^4 N}{m^2 C^4}\left\{\alpha - \beta\left(\frac{4\pi a}{\lambda}\right)^2 + \gamma\left(\frac{4\pi a}{\lambda}\right)^4 - \cdots\right\}. \tag{17}$$

3. *To calculate the energy scattered by an electron in the form of a thin, flexible, circular ring of electricity.*

In order to account for the electromagnetic mass of a ring electron, Webster[1] and Davisson[2] have shown that the ring must be very thin compared with its diameter. As a result of this fact, the inertia of any element of the ring is practically dependent only upon those parts of the ring immediately adjacent to it. Unless the wave-length is much smaller than the diameter of the electron, therefore, it is permissible to treat the mass of an element of the electronic ring as having a definite value.

Difficulties arise in this calculation, however, from the fact that the electromagnetic mass of an element differs according as the element is accelerated perpendicular or parallel to the tangent to the electronic ring at that point. The effective perpendicular mass depends also upon the speed with which the ring of electricity is rotating. For purposes of calculation I have assumed that the speed with which the electricity in the ring is moving is small compared with the velocity of light, and also that the mass of an element of the electron is independent of the direction of the acceleration. Admittedly the latter assumption makes the calculated value of the scattering only approximate, but it is probable that except possibly for the hardest γ-rays the approximation is close. I have further assumed, as in the case of the sphere, that the ring electron is flexible, *i. e.*, that the different parts of the charge are free to move relatively to each other. As was pointed out when the flexible sphere was considered, for comparatively long waves such an electron will scatter in practically the same manner as will a rigid electron which is free to rotate about any axis; for very short waves, however, the scattering by the two types of electron will not be exactly the same. The expression derived below for the scattering by a ring electron may therefore be relied upon for any except very short γ-rays.

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[1] D. L. Webster, *loc. cit.*

[2] Davisson, *loc. cit.*

In Fig. 7, imagine a beam of γ-rays going in the direction $-ZOZ$, and being scattered by an electron of radius a, represented by the heavy ring. Let us first determine the energy scattered in the direction OP, at an angle θ with OZ. As is evident from the figure, the plane of the electronic ring makes a dihedral angle β with the plane ZOP, and the line of intersection of these two planes makes an angle α with the line OR bisecting the external angle $-ZOP$.

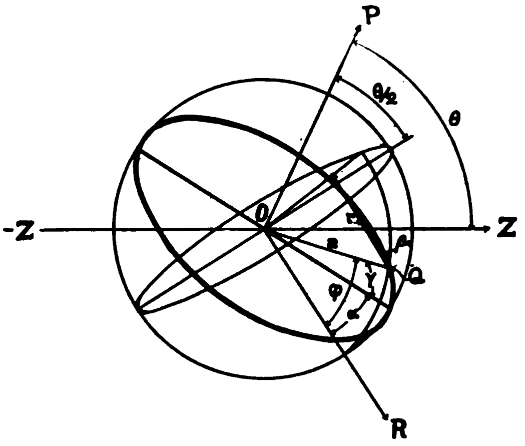

Fig. 7.

Consider first the energy scattered toward P due to the component of the electric vector which is perpendicular to the plane of the paper. If L is the distance, in the direction OP, at which the scattered radiation is evaluated, the electric displacement due to an element of the electron at $Q(a, \alpha, \beta, \gamma)$ is, in accordance with expression (10),

$$\frac{\eta a d\gamma \cdot A_1 \frac{\eta}{\mu}}{LC^2} \cdot \cos\frac{2\pi}{\lambda}\left(\delta - 2r\sin\frac{\theta}{2}\right).$$

In this statement, η is the charge and μ the mass per unit length of the circumference of the ring, A_1 is the amplitude of the electric vector perpendicular to the plane of the paper, r is the distance from Q to a plane drawn through O perpendicular to OR, and $2\pi\delta/\lambda$ is the phase angle of a ray scattered to P from this plane.

The displacement at P due to the whole electron is therefore

$$\frac{A_1\eta^2 a}{\mu L C^2}\int_0^{2\pi}\cos\frac{2\pi}{\lambda}\left(\delta - 2r\sin\frac{\theta}{2}\right)d\gamma.$$

This quantity is obviously a maximum when $\delta = 0$, so the amplitude at P due to the rays scattered by the whole electron is

$$\frac{A_1\eta^2 a}{\mu L C^2}\int_0^{2\pi}\cos\frac{4\pi}{\lambda}\left(r\sin\frac{\theta}{2}\right)d\gamma,$$

and the corresponding intensity is

$$(18)\qquad \frac{cA_1^2\eta^4 a^2}{\mu^2 L^2 C^4}\left\{\int_0^{2\pi}\cos\frac{4\pi}{\lambda}\left(r\sin\frac{\theta}{2}\right)d\gamma\right\}^2.$$

This expression represents the intensity at P due to an electron with the particular orientation defined by the values of α and β. The probable intensity at P due to polarized rays scattered by an electron at O is the average of this quantity for all values of α and β; *i. e.*,

$$\frac{cA_1^2\eta^4 a^2}{\mu^2 L^2 C^4}\int_0^{\pi}\frac{d\alpha}{\pi}\int_0^{\pi}\frac{d\beta}{\pi}\left\{\int_0^{2\pi}\cos\frac{4\pi}{\lambda}\left(r\sin\frac{\theta}{2}\right)d\gamma\right\}^2.$$

In a similar manner, the probable intensity at P due to the component of the incident electric vector which is parallel with the plane of the paper is

$$\frac{cA_2^2\eta^4 a^2\cos^2\theta}{\mu^2 L^2 C^4}\int_0^{\pi}\frac{d\alpha}{\pi}\int_0^{\pi}\frac{d\beta}{\pi}\left\{\int_0^{2\pi}\cos\frac{4\pi}{\lambda}\left(r\sin\frac{\theta}{2}\right)d\gamma\right\}^2,$$

where A_2 is the component of the incident amplitude parallel to the plane of the paper. Since on the average $A_1 = A_2$, and since the intensity of the unpolarized incident beam is $I = c(A_1^2 + A_2^2)$, we may write as the intensity of the beam scattered to P by an electron at O traversed by an unpolarized γ-ray,

$$I_{L,\theta} = \frac{I\eta^4 a^2}{\pi^2\mu^2 L^2 C^4}\,\frac{(1+\cos^2\theta)}{2}\int_0^{\pi}d\alpha\int_0^{\pi}d\beta\left\{\int_0^{2\pi}\cos\frac{4\pi}{\lambda}(r\sin\theta/2)d\lambda\right\}^2.$$

Remembering that $2\pi a\eta = e$ and $\eta/\mu = e/m$, the intensity of the beam scattered at the angle θ by an electron is

$$(19)\quad I_{L,\theta} = \frac{Ie^4(1+\cos^2\theta)}{8\pi^4 m^2 L^2 C^4}\int_0^{\pi}d\alpha\int_0^{\pi}d\beta\left\{\int_0^{2\pi}\cos\frac{4\pi}{\lambda}(r\sin\theta/2)d\gamma\right\}^2.$$

To evaluate this expression it is necessary to write r in terms of α, β and γ. In Fig. 7,

$$r = \alpha\cos\varphi$$

and

$$\cos\varphi = \cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\gamma.$$

Thus

$$r = a\,(\cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\gamma).$$

The first integral of equation (19) then becomes

$$F_1 = \int_0^{2\pi}\cos\frac{4\pi a}{\lambda}\left\{\sin\frac{\theta}{2}\,(\cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\gamma)\right\}d\gamma.$$

Substitute

$$k = \frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\cos\alpha,$$

and

$$l = \frac{4\pi a}{\lambda}\sin\theta/2\,\sin\alpha\cos\beta.$$

Then

$$F_1 = \int_0^{2\pi}\cos(k\cos\gamma - l\sin\gamma)d\gamma.$$

We may write

$$k\cos\gamma - l\sin\gamma = m\sin(\gamma+\Delta),$$

where m is the maximum value of $k\cos\gamma - l\sin\gamma$, *i. e.*,

$$\begin{aligned} m &= \sqrt{k^2+l^2} \\ &= \frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\sqrt{1-\sin^2\alpha\sin^2\beta}, \end{aligned}$$

and Δ is the appropriate phase angle. With this substitution,

$$F_1 = \int_0^{2\pi}\cos\{m\sin(\gamma+\Delta)\}d\gamma.$$

Since the integration extends from 0 to 2π, the value of Δ is immaterial, and may therefore be put to equal zero. The integral then becomes,

$$\begin{aligned} F_1 &= \int_0^{2\pi}\cos(m\sin\gamma)\,d\gamma \\ &= 2\pi\cdot\frac{1}{\pi}\int_0^{\pi}\cos(m\sin\gamma)\,d\gamma \\ &= 2\pi\cdot J_0(m), \end{aligned}$$

where

$$J_0(m) = 1 - \frac{m^2}{2^2} + \frac{m^4}{2^2\cdot4^2} - \frac{m^6}{2^2\cdot4^2\cdot6^2} + \cdots,$$

i. e., Bessel's J function of the zero order. Thus

$$F_1 = 2\pi J_0\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\sqrt{1-\sin^2\alpha\sin^2\beta}\right).$$

The second integral of expression (19) may be written

$$F_2 = \int_0^\pi F_1^2 d\beta = \int_0^\pi \left\{ 2\pi J_0\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\sqrt{1-\sin^2\alpha\sin^2\beta}\right)\right\}^2 d\beta.$$

By substituting

$$\frac{4\pi a}{\lambda}\sin\theta/2 = k$$

and $\sin\alpha = l$ this is reduced to the form

$$F_2 = 4\pi^2\int_0^\pi J_0^2(k\sqrt{1-l^2\sin^2\beta})d\beta.$$

The integral can be evaluated by expansion into a series of the form

$$J_0^2(x) = 1 - Ax^2 + Bx^4 - Cx^6 + \cdots,$$

and integrating term by term. In this series

$$A = \frac{1}{2^2} + \frac{1}{2^2},$$

$$B = \frac{1}{2^2\cdot 4^2} + \frac{1}{2^2\cdot 2^2} + \frac{1}{2^2\cdot 4^2},$$

$$C = \frac{1}{2^2\cdot 4^2\cdot 6^2} + \frac{1}{2^2\cdot 4^2\cdot 2^2} + \frac{1}{2^2\cdot 2^2\cdot 4^2} + \frac{1}{2^2\cdot 4^2\cdot 6^2},$$

$$D = \frac{1}{2^2\cdot 4^2\cdot 6^2\cdot 8^2} + \frac{1}{2^2\cdot 4^2\cdot 6^2\cdot 2^2} + \frac{1}{2^2\cdot 4^2\cdot 2^2\cdot 4^2} + \frac{1}{2^2\cdot 2^2\cdot 4^2\cdot 6^2} + \frac{1}{2^2\cdot 4^2\cdot 6^2\cdot 8^2},$$

etc.

Performing the integration we obtain

$$F_2 = 4\pi^2\{M + Nl + Ol^2 + Pl^3 + \cdots\},$$

where

$$M = J_0^2(k),$$

$$N = \tfrac{1}{2}(Ak^2 - 2Bk^4 + 3Ck^6 - \cdots),$$

$$O = \tfrac{1}{2}\cdot\tfrac{3}{4}(Bk^4 - 3Ck^6 + 6Dk^8 - 10Ek^{10} + \cdots),$$

$$P = \tfrac{1}{2}\cdot\tfrac{3}{4}\cdot\tfrac{5}{6}(Ck^6 - 4Dk^8 + 10Ek^{10} - 20Fk^2 + \cdots),$$

etc.

The third integral is

$$F^3 = \int_0^\pi F_2 d\alpha = 4\pi^3\int_0^\pi (M + N\sin^2\alpha + 0\sin^4\alpha + \cdots)d\alpha$$

$$= 4\pi^4\left(M + \tfrac{1}{2}N + \tfrac{1}{2}\cdot\tfrac{3}{4}O + \tfrac{1}{2}\cdot\tfrac{3}{4}\cdot\tfrac{5}{6}P + \cdots\right).$$

Substituting the above values of M, N, O, etc., this may be written

$$F^3 = 4\pi^4(1 - \alpha k^2 + \beta k^4 - \gamma k^6 + \cdots),$$

where

$$\alpha = (1 - \tfrac{1}{4})A \qquad \doteq .37500,$$

$$\beta = (1 - \tfrac{2}{4} + \tfrac{1}{4}\cdot\tfrac{9}{16})B \qquad = .06006,\ 0,$$

$$\gamma = (1 - \tfrac{3}{4} + 3\cdot\tfrac{1}{4}\cdot\tfrac{9}{16} - \tfrac{1}{4}\cdot\tfrac{9}{16}\cdot\tfrac{25}{36})C \qquad = .00498,\ 48,$$

$$\delta = (1 - \tfrac{4}{4} + 6\cdot\tfrac{1}{4}\cdot\tfrac{9}{16} - 4\cdot\tfrac{1}{4}\cdot\tfrac{9}{16}\cdot\tfrac{25}{36} + \tfrac{1}{4}\cdot\tfrac{9}{16}\cdot\tfrac{25}{36}\cdot\tfrac{49}{64})D \qquad = .00025,\ 060,$$

$$\epsilon = (1 - \tfrac{5}{4} + 10\cdot\tfrac{1}{4}\cdot\tfrac{9}{16} - 10\cdot\tfrac{1}{4}\cdot\tfrac{9}{16}\cdot\tfrac{25}{36} + 5\cdot\tfrac{1}{4}\cdot\tfrac{9}{16}\cdot\tfrac{25}{36}\cdot\tfrac{48}{64}$$

$$- \tfrac{1}{4}\cdot\tfrac{9}{16}\cdot\tfrac{25}{36}\cdot\tfrac{49}{64}\cdot\tfrac{81}{100})E = .00000,\ 84241,$$

$$\zeta = .00000,\ 02023,\ 6,$$

$$\eta = .00000,\ 00036,\ 51,$$

$$\theta = .00000,\ 00000,\ 5056.$$

The scattering at the angle θ is therefore

$$I_{L,\theta} = \frac{Ie^4(1 + \cos^2\theta)}{2m^2L^2C^4}\left\{1 - \alpha\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^2 + \beta\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^4 - \cdots\right\}. \tag{20}$$

The rate at which energy is scattered by an electron is obtained by integrating this expression over the surface of a sphere of radius L. That is

$$E_\theta = \int_0^\pi I_{L,\theta}\cdot 2\pi L^2 \sin\theta d\theta,$$

and since

$$\frac{\sigma}{\rho} = \frac{N}{I}E_\theta,$$

$$\frac{\sigma}{\rho} = \frac{Ne^4}{2m^2C^4}\int_0^\pi (1 + \cos^2\theta)\sin\theta\left\{1 - \alpha\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^2 + \beta\left(\frac{4\pi a}{\lambda}\sin\frac{\theta}{2}\right)^4 - \cdots\right\}d\theta.$$

If for $\sin^2(\theta/2)$ we substitute $1/2(1 - \cos\theta)$, this expression is immediately integrable, and we obtain for the mass scattering coefficient,

$$\frac{\sigma}{\rho} = \frac{8\pi}{3}\frac{Ne^4}{m^2C^4}\left(1 - a\left(\frac{a}{\lambda}\right)^2 + b\left(\frac{a}{\lambda}\right)^4 - c\left(\frac{a}{\lambda}\right)^6 + \cdots\right), \tag{21}$$

where

$$a = \tfrac{3}{4}(8\pi^2)(1 + \tfrac{1}{3})\alpha \qquad = 29.60881,$$

$$b = \tfrac{3}{4}(8\pi^2)^2(1 + \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{5})\beta \qquad = 524.1827,$$

$$c = \tfrac{3}{4}(8\pi^2)^3(1 + \tfrac{1}{3} + \tfrac{3}{3} + \tfrac{3}{5})\gamma \qquad = 5{,}397.801,$$

$$d = \tfrac{3}{4}(8\pi^2)^4(1 + \tfrac{1}{3} + \tfrac{6}{3} + \tfrac{6}{5} + \tfrac{1}{5} + \tfrac{1}{7})\delta \qquad = 35{,}619.04,$$

$$e = \tfrac{3}{4}(8\pi^2)^5(1 + \tfrac{1}{3} + \tfrac{10}{3} + \tfrac{10}{5} + \tfrac{5}{5} + \tfrac{5}{7})\epsilon \qquad = 162{,}501.7,$$

$$f = \frac{3}{4}(8\pi^2)^6\left(1 + \frac{1}{3} + \frac{15}{8} + \frac{15}{5} + \frac{15}{6} + \frac{15}{7} + \frac{1}{7} + \frac{1}{9}\right)\zeta = 541,970.2,$$

$g = 1,377,792,$

$h = 2,757,220,$

$i = 4,455,520,$

$j = 5,935,500,$

k[1] $= 6,632,700.$

l[1] $= 6,311,200,$

m[1] $= 5,182,000,$

n[1] $= 3,717,000,$

o[1] $= 2,356,000,$

p[1] $= 1,334,000,$

q[1] $= 682,000,$

r[1] $= 318,000,$

s[1] $= 136,000,$

t[1] $= 54,000.$

The right hand member of this equation is convergent for all values of a/λ, but the convergence is very slow when λ approaches equality with a. The values of σ/ρ calculated according to this expression for different values of λ/a are shown in Fig. 2 by the solid part of curve *III*.

Elementary considerations suffice to determine the manner in which the scattering by a ring electron depends upon the wave-length when the frequency is very high. If we consider waves shorter than the diameter of the electron, it is apparent that the length of the arc of the ring which may be considered to vibrate as a unit due to the action of the incident beam will be proportional to the wave-length of the incident rays. The amplitude of the beam scattered by such a unit will therefore, by equation 10, be proportional to λe, and the intensity to $\lambda^2 e^2$. Since, however, the total number of such units in each electron will be inversely proportional to the wave-length, the intensity of the beam scattered by the whole electron will be proportional to $\lambda^2 e^2/\lambda$, *i. e.*, proportional to the wave-length. The solid part of curve *III* (Fig. 2) shows, as we should expect, that this relation holds approximately even for waves considerably longer than the diameter of the electron. I have therefore extrapolated curve *III* according to this law for values of λ/a too small for the practical application of formula 21, indicating these approximate values by the broken part of the curve.

Research Laboratory,
Westinghouse Lamp Company,
March 17, 1919.

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[1] These values were determined by an approximation formula.

A FURNACE TEMPERATURE REGULATOR.

By Walter P. White and Leason H. Adams.

Synopsis.—By making the heating coil of an electric furnace one arm of a wheatstone bridge, and combining this with a galvanometer regulator, thus keeping constant the resistance of the coil, we can, regardless of variations in the current supply, and with no attention, maintain constant the temperature of furnaces not too directly influenced by the temperature of the room, or where the surrounding air is kept constant. The power available in this regulator is relatively very great indeed; nothing has to be inserted within the furnace cavity, and the lag is practically nothing; the regulator is often almost at its best under conditions most unfavorable to other regulators. It has held a small furnace constant to 0.1° for hours at temperatures from 500° to 1400°.

THIS paper describes a temperature regulator for electric furnaces, which has extraordinary sensitiveness and is effective under extreme conditions.

We had already succeeded in automatically regulating to about 5° at 1400° by letting a thermocouple recorder make electric contacts which actuated a polarized relay controlling a larger magnet which controlled the current. But we wished to regulate to about 0.1°, or 50 times as close, and at the same time to change the heating current by 10 per cent., in order to minimize the amount of attention required from the observer. This change in the heating current was found to change the furnace temperature so rapidly that to prevent fluctuations of 0.1° an adjustment nearly once a second was needed. Since the sensitiveness of a galvanometer falls off rapidly as the period is shortened, this combination of sensitiveness and speed required hundreds of times the energy that could be obtained from a platinum thermocouple.

This energy was obtained by drawing on the heating current of the furnace. The platinum furnace coil was made one arm of a wheatstone bridge, Fig. 1, so that the regulation was really by a resistance thermometer with a current of from 12 to 18 amperes. This method of regulating by controlling the coil temperature has, besides the great supply of energy, two other advantages. It avoids all necessity of introducing appliances into the furnace cavity, and it eliminates entirely the lag between heater and regulator, since the two are now identical.[1]

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[1] Any device employing a wheatstone bridge has also the advantage of being more easily made to use the galvanometer as a null instrument—a thing necessary in work of high precision.

It has one important disadvantage; the temperature of the furnace is nearly always somewhat influenced by that of the room, so it may vary slightly, even when the coil temperature is maintained constant.

These peculiarities determine the most favorable conditions for this type of regulator. For furnaces whose small cavities render undesirable the introduction into them of any apparatus; for those whose small mass causes a quick response to current changes, and so calls for absence of lag, and for quickness, that is, sensitiveness, in the regulator, this type may have superior advantages. These conditions are more apt to occur in work at relatively high temperatures, and such temperatures also impair the efficiency of this regulator less than that of some others. Where the furnace is more capacious, or more massive, and hence adapted to more sluggish regulation, there is less need for the present regulator, and if the ends of the furnace contribute a considerable portion of its surface other regulators may even be more satisfactory.

Adjustments or tests of these regulators are very tedious if the furnace is large, and doubly so, of course, if the room temperature requires exact manipulation for the test. The approximate value of the effect of room temperature on one furnace was rather rapidly determined, as follows: First the constant rate, V, at which the furnace cooled with heating shut off, was observed, as well as φ, the thermal head, or temperature difference causing heat flow; that is, the difference between furnace and room. Then by Newton's Law of Cooling, we have $K_1 = V/\varphi$, where K_1 is the thermal leakage modulus of the furnace to the room. Second, the way was observed in which the furnace temperature changed when, after this had become constant, the coil was suddenly brought to a different temperature and held there. Then K_2, the modulus of the coil to the furnace, is obtained by applying the formula

$$\theta = \theta_s + (\theta_0 - \theta_s)e^{-K_2 t},$$

where θ is furnace temperature; and θ_0 and θ_s are its initial and final values, with θ_s also the second value of the coil temperature. If we then put θ_1, θ_2, θ_3, for the temperatures of coil, furnace, and room respectively, it is clear that the furnace in equilibrium will reach a temperature such that the heat coming to it from the coil equals that which it loses to the air, or that

$$K_2(\theta_1 - \theta_2) = K_1(\theta_2 - \theta_3);$$

that is

$$\theta_2 = \frac{K_1\theta_3 + K_2\theta_1}{K_1 + K_2},$$

so, as may be seen by simple differentiation, a given change in the room (θ_3) produces $K_1/(K_1 + K_2)$ times as much change in the furnace (θ_2).

This calculation gives too high results, since when the furnace is heated no heat escapes from it by way of the heating wire, while it does so escape during the determination of K_1. It is evident that if the heating coil is, instead of a wire, a flat strip of metal practically covering all the surface of the furnace, there would be almost no effect from the room. By combining with the heater wire a wire of copper or nickel exposed to the air a large amount of compensation can be obtained for the room effect. If the room is cooler the exposed wire is cooler; in order to make the combined resistance the same the heater wire must run warmer than usual; and thus it overcomes the cooling effect of the room. The difficulty with such an arrangement is, first, the time required to adjust for correct compensation, and second, the effect of lag, which tends to throw the balance out, except for slow changes.

In the furnace which we tested for K_1 and K_2 the indications were satisfactory, but the end effects, which were complicated by separately controlled coils, proved to be a disturbing factor. In our high temperature platinum-wound furnace, where the ends were small, and the coil adhered closely to the chamber wall, no effect from the room temperature was ever noticed, and the furnace, after once reaching equilibrium, usually held to 0.1° for several hours, even at 1400°. The room, however, seldom varied over 5°.

Since an account of this regulator was first presented in public, by Mr. Adams (at the Washington meeting of the Physical Society, April, 1913) several experimenters have used the principle of a heater-thermometer-regulator in oil baths. This is an appropriate use for it, for if the wire is in contact with the oil K_2 is very large, and the room temperature disturbance is therefore small, especially if K_1 can be kept small by covering the top.

As Fig. 1 indicates, some mechanical arrangement is necessary to cause the current-regulating switches to move according to the changes in the position of the galvanometer boom. This can very readily be accomplished in one of several fairly familiar ways. The easiest, perhaps, is the one suggested in the first paragraph of this paper, namely, having the boom pressed up or down at intervals so as to either strike or miss a contact maker (or either one of a pair) which controls a suitable relay. A light motor-driven mechanism pressing the boom every second or so is then the only special machinery required. The Leeds and Northrup Co. furnish a stock instrument which operates in this way. They also offer a form in which the switches are moved by the motor, without magnets. This is a little tardy in its action, since the switch setting for a given pressing of the boom is not made till the next pressing occurs.

But if one of the ratcheting rheostats now available is used, the single step can be made smaller and the period does not need to be so short. If we had planned to use such a rheostat our own time requirement would not have been so exacting as it was. Our own mechanical arrangement[1] utilizes the power and certainty of the motor to operate the switch, but gives nearly the promptness of a magnet, so that a single switch is enough for the regulation. The change in switch setting due to one pressing of the boom produces a large effect on the galvanometer before the next pressing occurs. This instrument is not represented because Mr. Shaw now thinks he could get as great efficiency with a simpler design, and no such design has been completely worked out.

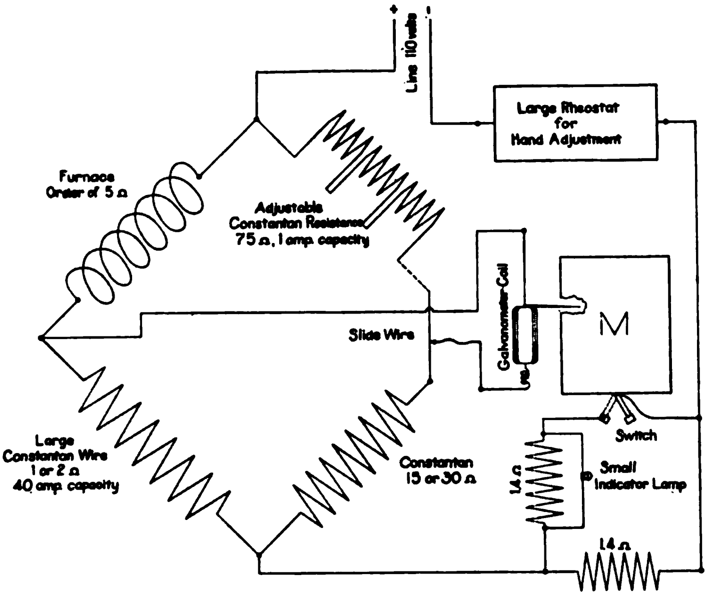
+
-
Line 110 volts
Large Rheostat for Hand Adjustment
Furnace Order of 5 Ω
Adjustable Constantan Resistance 75 Ω, 1 amp. capacity
Galvanometer-Coil
M
Slide Wire
Switch
Large Constantan Wire 1 or 2 Ω 40 amp. capacity
Constantan 15 or 30 Ω
14 Ω
Small Indicator Lamp
14 Ω

Fig. 1.

Our galvanometer coil was an old wooden-spooled affair, of the type much used in the nineties, provided with a more modern magnet. With a 1-second period, auxiliary oil damping was needed. The boom was 5 cm. long, and a shift of 0.1 mm. at its end was enough to cause a change of the switch setting. This shift was produced by 24 microvolts across terminals. For 14 amperes (about the average current) 24 microvolts corresponded to a change of 0.004°, so the possibilities of the arrangement in the way of sensitiveness are, as was to be expected, in excess of any conceivable requirement. Indeed, it is clear that since the sensitiveness of an underdamped galvanometer varies as the square of the period, for a period of 3 seconds or more this galvanometer would adjust to 0.1° with a silver-constantan couple (5 microvolts per 0.1°), or with a smaller resistance thermometer. This is, however, an arrangement for lower temperatures and larger furnaces, and is one which abandons the special feature which this paper is written to describe.

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[1] Designed by Mr. C. M. Shaw, Master Mechanic of the Laboratory.

Since this special feature, namely, operation by means of the furnace coil itself, calls for resistance variation with temperature, it is not so well adapted to nichrome coils, whose temperature coefficient is very small. Nevertheless, our own arrangement is apparently sensitive enough to work with them, especially where the period can be lengthened a little. We actually used, at 500°, a nickel manganese alloy of high coefficient, but do not feel justified in recommending it as satisfactorily free from liability to injury by oxidation, even at that temperature. Satisfactory alloys may be known, however, whose temperature coefficient is higher than that of ordinary nichrome, and there seems to be no doubt that such can be made. The loss of power in the other three arms of the wheatstone bridge can generally be made negligible if the galvanometer sensitiveness is comparable to ours. The device, in its present form, evidently is not applicable to alternating currents.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C.,
March, 1919.

THE VARIATION IN SOUND INTENSITY OF RESONATORS AND ORGAN PIPES WITH BLOWING PRESSURE.

By Beryl F. Love and Margaret K. Dawson.

Synopsis.—Quantitative measurements were taken to ascertain the relation between the sound intensity of wind-blown instruments and the blowing pressure. The intensities of sound emitted by a tone-variator resonator and an organ pipe were measured by a Rayleigh resonator while the blowing pressure was noted by means of a water manometer. Assumptions concerning the constancy of the pitch of the sound and permancy of the sound pattern appeared justified.

The measurements showed that the intensity of the sound varied directly with the blowing pressure for the tone-variator and gave nearly a direct relation for the organ pipe. A discussion of the theory indicated that only part of the energy of the air pressure was converted into sound.

That the intensity of sound emitted by an organ pipe becomes greater when blown more strongly is a fact easily perceived by the ear; but a quantitative estimation of the comparative intensities is not so readily determined because the ear is not trustworthy for such measurements. The present investigation was undertaken to secure quantitative data on the variation of sound intensity of organ pipes and resonators when the blowing pressure was increased. The apparatus is pictured in Figs. 1 and 2.

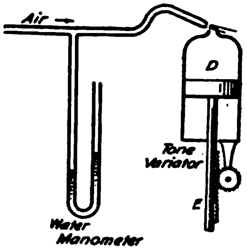

Fig. 1.

Showing arrangement for generating sound from a tone variator by means of compressed air.

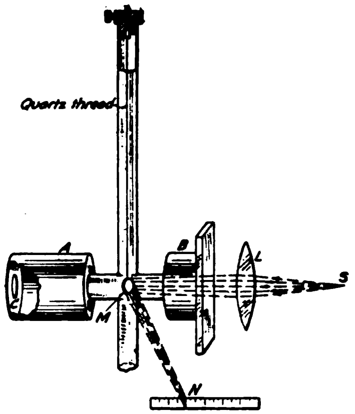

Fig. 2.

Rayleigh resonator used for measuring intensity of sound.

The sound was generated by blowing compressed air across the mouth of a "tone variator" resonator,[1] the pressure of the air being measured by a water manometer. By adjusting the piston of the tone variator, a range of tones of varying pitch could be obtained. The intensity of the sound was measured by means of a modified Rayleigh resonator[2] shown in Fig. 2. In this instrument, a glass disc *M*, suspended by a quartz thread, was placed in the intercommunicating channel of a double resonator *AB*. When properly tuned to the external sound, a resonance resulted causing the disc to rotate through a small angle, the amount of rotation being noted by the deflection of a spot of light on the scale *N* after reflection from the disc. The deflections on the scale are proportional within a small per cent. error to the intensities of the sound.[3] The resonator was tuned by adjusting the length, and consequently the volume, of *B*, and by varying the size of the opening of the iris diaphragm at *C*.[4]

The procedure in taking measurements was as follows: The tone variator was blown by a steady air pressure and then adjusted in pitch until it gave a maximum deflection of the Rayleigh resonator. The air pressure was increased successively by small amounts and the rise in pitch[5] of the tone variator reduced each time to give a maximum deflection of the resonator. This was continued until the sound showed distortion due to overtones or other causes. Measurements taken in this way gave simultaneous values of blowing pressure and resultant sound intensity. To avoid disturbance of the standing waves of sound in the room, the observer was stationed in a closed compartment where observations could be taken through a window and the tone variator adjusted by means of a long steel rod.

It was assumed in this procedure that the intensities measured were comparable provided the interference pattern of the sound in the room remained constant. That this condition was fulfilled is explained in the following way. When a sound is generated, waves spread out from the source and, by successive reflections from the boundaries of the room, set up interferences that give stationary maxima and minima of sound. The Rayleigh resonator will give a maximum deflection when the opening of the iris diaphragm is at a node.[6] The response is less for other positions and becomes zero if the opening coincides with a loop. If a new arrangement of maxima and minima is set up, as would be the case if the pitch of the sound were changed, or if any object in the room were moved to a new position, the deflection of the resonator would probably change even though the intensity of the source of sound remained unaltered. If the pitch and arrangement of articles in the room remained constant as described, it was assumed that the interference pattern would not change when the pressure of the air blowing the tone variator increased but that the intensity of sound at each point in the room would increase in proportion to the pressure. This assumption receives support from some earlier work[1] where sound reflected into a room from successively increased layers of material gave corresponding deflections of a Rayleigh resonator what were directly proportional to the thicknesses of the layers.

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[1] L. W. Stern, Zeitsch. f. Psychologie u. Physiologie der Sinnesorgane, Vol. 30, p. 422, 1902.

[2] Rayleigh, Phil. Mag., Vol. 14, p. 186, 1882.

[3] Watson, PHYS. REV., Vol. 7, p. 125, 1916.

[4] Rayleigh, Theory of Sound, Vol. 2, Sec. 310.

[5] Rayleigh, Phil. Mag., III., p. 462, 1877; XIII., p. 340, 1882. See also article by Mr. A. Mallock, Proc. of Royal Society, Ser. A, Vol. 95, p. 99, 1918. Describes experimental work leading to simple explanation of change of pitch of organ pipes with pressure.

[6] Rayleigh, Theory of Sound, Vol. II., Sec. 269a.

It was further assumed that the pitch of the tone variator had the same value for each measurement if adjusted to give a maximum deflection of the Rayleigh resonator. This assumption was verified in later experiments by counting the number of beats made by the tone variator with a tuning fork for each measurement and finding them constant within experimental error. It was found that the adjustment for constant pitch could be made more easily by getting the maximum deflection of the resonator than by adjusting the tone variator to get the proper number of beats.

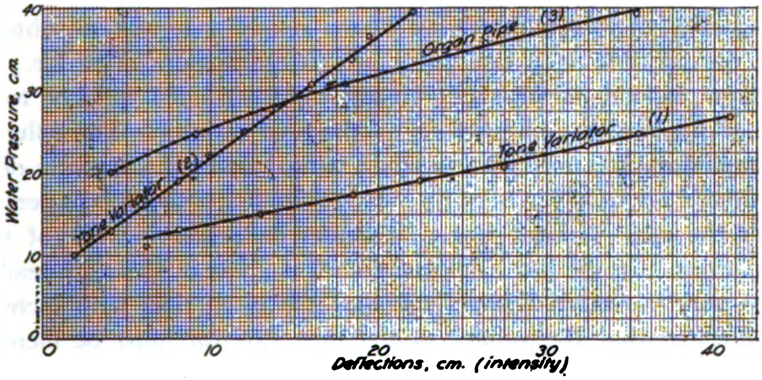

Fig. 3.

Curves showing the variation of sound intensity with blowing pressure for a tone variator and organ pipe.

The results are given graphically in Fig. 3. Curve 1 shows the variation of the intensity of a tone variator with the blowing pressure. Since this appears to be a straight line, it indicates that the sound intensity varies directly with the pressure. Curve 2 gives data taken for the same tone variator with a rearrangement of apparatus. The data for Curve 2 are given in the following table.

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[1] Watson, Physical Review, Vol. VII., p. 287, 1916.

Water Pressure.	Resonator Deflections.	Beats with Tuning Fork.	Reading of Tone Variator.
7.2 cm.	Readings uncertain		
10.0	1.9±0.1 cm.	3.08 per sec.	395
13.0	4.1	3.49	391
16.1	6.2	3.26	389
19.0	8.1	3.31	387
22.2	9.9	3.51	385
25.0	12.0±0.5	3.19	383
27.9	13.9	3.00	382
30.0	16.0	3.26	380
33.7	18.4	3.26	380
36.6	19.4	3.31	378
39.5	22.0±1.0	3.04	375

Inspection of the results shows that a steady tone was not obtained until the pressure increased to about 10 centimeters of water. The resonator deflections were not steady but responded quickly to the fluctuations in pitch and intensity of the tone variator. These fluctuations are due most probably to small variations in the pressure of the compressed air used for blowing. It required considerable patience to adjust the tone variator for a maximum deflection in view of these fluctuations. In the last column a record is given of the scale readings of the tone variator. They show that the pitch rises with increased pressure and that the volume of the tone variator must be increased to reduce the pitch to a fixed value. They indicate also the necessity of calibrating such instruments for a stated, constant blowing pressure. Curve 3 in Fig. 3 shows the results for an organ pipe giving a note of about 320 vibrations per second. A movable piston allowed an adjustment of pitch, but some difficulty was experienced in getting the pipe to sound properly. The smooth curve in this case indicates that the intensity of the sound varied regularly with the blowing pressure but not in direct proportion as shown for the tone variator.

These observed results are not what would be expected from the theory of the subject. Rayleigh[1] has shown that the energy per second in blowing a whistle is proportional to the pressure p and the volume v of the air blown per second. If all of this energy went into the production of sound, then the intensity of sound, I, could be written: $I = kpv$, where k is a constant. If it is assumed that the volume increases directly proportional to p, I would become: $I = k_1p^2$. Since, however, it was observed that $I = k_2p$, the conclusion is drawn that not all the energy of the air goes into sound but only that portion obtained as if v were constant.

In conclusion, aside from the interesting fact shown by the results, it would appear of some practical advantage to know that the intensity of sound of a resonator or organ pipe varies almost directly with the blowing pressure.

The authors desire to acknowledge the guidance and assistance of Dr. F. R. Watson during the course of the investigation.

Laboratory of Physics,
University of Illinois,
March, 1919.

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[1] Proc. of Royal Society, XXVI., p. 248, 1877.

THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF ERBIUM OXIDE.

BY W. S. MALLORY.

SYNOPSIS.—In the experiments described in this paper the statements of previous observers to the effect that the selective emission of erbium oxide is not due to volatilization but is a radiation from the solid itself are corroborated. The temperatures at which the bright bands appear, reach their maximum and disappear are measured. By detailed spectrophotometric measurements it is shown that at 1000° C. both the red and green groups of bands greatly exceed in intensity the radiation of similar portions of the spectrum of a black body at the same temperature.

INTRODUCTION.[1]

IT was discovered in 1864 by Bunsen[2] that didymium, when heated to incandescence did not give the continuous spectrum commonly seen when solids are heated, but a spectrum consisting of bright bands, superimposed upon a relatively weak continuous spectrum. The same phenomenon was discovered in the case of erbium by Bahr[3] in 1865. Neodymium, holmium and perhaps a few other rare earths have been found to give the banded emission spectra, although in a lesser degree. Another peculiarity in the case of these substances lies in the fact that their absorption spectra consist of a great number of very narrow bands. In some cases these absorption bands may very properly be spoken of as lines. The narrow banded absorption spectra are characteristic of these rare earths, both in solution and in the solid condition.

The present investigation was undertaken in part to determine, if possible, whether the peculiar emission spectrum of erbium oxide, when heated to incandescence, is a purely thermal radiation, as indicated by the excellent agreement, at the same temperature, in the position of the emission and the absorption bands, or whether it might not be a case of luminescence, excited perhaps by the short wave-lengths of the flame used for heating the specimen under observation.

Another purpose was to determine for different wave-lengths the relation between the intensity of the radiation from the oxide, and the intensity of the radiation from a black body at the same temperature.

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[1] A good summary of the work that has been done on the spectra of the rare earths may found in an article in the Astrophysical Journal of September, 1907. "The Absorption d Emission Spectra of Neodymium and Erbium Compounds," by J. A. Anderson.

[2] Bunsen, Liebig's Annalen, CXXXI, 255, 1864.

[3] Bahr, Liebig's Annalen, CXXXV, 376, 1865.

I later made an approximate determination of the temperature at which the radiation in the region of the green band of the emission spectrum had the same intensity as the corresponding radiation from a black body at 1000° C.

Some Preliminary Observations.

In preparation for the investigation it seemed best to repeat a number of experiments upon which others have reported. The results were briefly as follows:

1. *The Emission Spectrum not a Flame Spectrum.*—Some erbium oxide powder was placed upon a small piece of quartz and heated in the Bunsen flame. The powder was directly opposite the middle of the slit of a spectrometer. Sodium in the flame caused a yellow line to appear the entire length of the slit. Lithium added to the flame caused a red line to show the entire length of the slit. The erbium spectrum did not extend above the solid; there was nothing like its spectrum coming from the gas immediately above the solid. This indicated that the emission was from the solid and not from the vapor near the solid.

2. *Identity of Emission from the Oxide and from a Phosphate Bead.*—There has been some difference of opinion as to whether the emission is the same from the oxide as from the phosphate. To determine this a small portion of powdered oxide was intensely heated in the oxy-hydrogen flame and finally fused down into a cream-colored bead, without the use of any flux. I mounted this bead before the slit of the collimator and immediately above it another sample in which a syrupy phosphorus pentoxide had been used as flux. The two specimens were in the same flame and seen at the same time, one above the other. The two spectra were identical; so far as I was able to judge the red band appeared in the same place and in the same manner. The green lines appeared identical in the two spectra. In one trial it seemed as though the green bands stood out a little more distinctly in the case of the specimen prepared with the flux. This appearance was not characteristic and was not very marked in the one case that was noted.

3. *Relation between Emission and Absorption.*—It has been recognized, almost from the first, that there is a very close agreement in both the positions and intensities of the emission and absorption bands. J. A. Anderson,[1] in the summary of his investigations, concluded that the

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[1] Anderson, Astrophysical Journal, XXVI., p. 93, 1907.

emission is due to the oxide itself, and that it corresponds exactly to its absorption at the same temperature. The absorption of the solid changes with temperature, the change consisting in a widening of the bands, and in some cases, a shift toward the red, with an increase of temperature. Anderson further concluded that the absorption of the dry compounds is different for different compounds of the same element.

In the case of the oxide under consideration in the present paper the absorption bands at room temperature and the emission bands during incandescence were located visually with a spectrometer. To obtain the absorption spectrum the fused oxide was placed before the slit of the instrument and the light of a carbon arc was focused upon it. As will be seen from the following table and from the spectrum map at the bottom of Fig. 2 the absorption bands occur in six groups each group corresponding approximately in position to one of the broader emission bands. The single band at .436 μ may perhaps be the only visible member of still another group which may coincide with a feeble emission band which escaped notice.

TABLE I.

Absorption Bands at + 20° and Emission Bands at 1000° observed in a specimen of commercial Erbium Oxide.

Group.	Absorption.		Emission.	Group.	Absorption.		Emission.
	μ.	$1/\mu \times 10^3$.	μ.		μ.	$1/\mu \times 10^3$.	
1	.4180	2392	.4100		.5198	1924	.5135
(unresolved)	to	to	to	4	.5222	1915	to
	.4223	2368	.4250		.5260	1901	
					.5306	1885	.5360
—	.4361	2293	—		.5407	1849	.5360
					.5417	1846	
	.4534	2206	.4500		.5438	1839	to
2	.4571	2188		5	.5461	1831	
	.4611	2169	to		.5474	1827	
	.4655	2148	.4685		.5562	1798	.5555
	.4916	2035	.4885		.6400	1562	.6400
	.4936	2188	to		.6521	1534	
3	.4971	2169			.6532	1531	to
	.4978	2148	.5015		.6551	1526	
					.6656	1502	
					.6751	1481	.6750

4. *Fluorescence and Emission.*—The oxide under investigation was excited to fluorescence by cathode rays in a vacuum tube. The spectrum, as had been noted by earlier students of the rare earths, was distinctly different from the spectrum obtained when the same specimen was heated to incandescence. A broad red band began at wave-length .6065 μ and merged into a bright yellow band at .5880 which extended to .5730. A green band began at .5605 and ended at .5450. A dark space followed and then another green band between .5380 and .5020. There was brightness again beginning at about .4950 but this gradually became less intense and ended at about .4730. These broad bands are likewise indicated in Fig. 2 for convenience of comparison with the regions of emission and absorption. The determination of the relations between these spectra lies beyond the scope of the present paper.

5. *Development of Selective Emission with increasing Temperatures.*—In order to get a general idea as to the growth of the emission spectrum with increasing temperatures, I placed some of the erbium oxide powder between two thin plates of clear quartz. In the midst of the powder I placed one of the junctions of a thermocouple made of Pt, Pt-Rh, wires. The couple was joined through the proper resistance with a Paul unipivot pyrometer, reading approximately in degrees centigrade. The appearance at different temperatures may be described as follows:

600° C. A broad faint haze in the yellow green, ending rather abruptly on the side toward the blue. No red discernible.

640° C. Red band clearly seen and more light in the green, but it ended abruptly as before.

680° C. Sufficient light so that the slit could be narrowed. The green bands began to stand out from the green haze, which served as a background.

770° C. A faint suggestion of another band in the blue.

800° C. The red continued to stand out well, although there was a faint continuous spectrum over the whole field.

930° C. Two faint bands seen beyond the green. In the red was a dark portion, as though there were two bands, separated from one another.

1000° C. No further question about there being varying degrees of intensity in the red. The continuous spectrum was more pronounced, yet all the bands showed well.

1120° C. The band appearing in the blue seemed to be of two parts, while in the violet there seemed to be a very dim band made up of three parts. At this temperature the continuous spectrum began to mask the bands.

1220° C. The red had spread into a much broader band. In the extreme violet another band could be dimly seen. The boundaries of the green bands could be seen distinctly. The other bands appeared as broad patches, slightly more intense than the continuous spectrum which had pretty completely masked them.

6. *Emission not Due to Excitation by Radiation of Short Wave-Lengths.*— In the article by J. A. Anderson it was suggested that the simplest and best way to settle the question as to whether the radiation is due to the oxide itself or to a surface layer of gas, formed by the action of the flame, would be to heat the oxide without using the flame. In order to make this test, and at the same time prevent the excitation of fluorescence by any ultra-violet radiation from the flame itself, I made a nickel box about 2 cm. in diameter and 7.5 cm. in depth. This box was heated by a blast lamp flame playing against the bottom and sides. The specimen to be examined was mounted at the end of a bent quartz rod, in such a manner that it could be quickly swung up out of the box and before the slit of the collimator. When heated in this manner the red and the green bands were seen just the same as when the specimen was heated by the flame directly. The nickel box was not white hot, and the bands persisted for several seconds after cooling had begun, due to removal from the box. Since performing the experiment with the nickel box I have heated specimens in the electric furnace to be described in a subsequent paragraph and have found the spectrum to be the same as when heating directly with the flame. This is taken to establish that the banded emission spectrum is not due in any way to peculiar properties of the flame but that it is a thermal effect and independent of the manner of heating.

THE DISTRIBUTION OF ENERGY IN THE EMISSION SPECTRUM.

The main purpose of the present investigation, as has been indicated in the opening paragraph of this paper, was to determine whether the emission in the bright bands exceeds that in the corresponding regions in the spectrum of a black body at the same temperature. If so, the well-known corollary to Kirchhoff's law; that no body can emit more radiation of a given wave-length than does a black body at the same temperature is inapplicable to these highly selective radiators.

The instrument employed was a spectrophotometer designed for the study of luminescence spectra.[1] It consists of Hilger spectrometer of the constant deviation type to which a second collimator is added.

[1] Nichols, Proc. Am. Philos. Soc., LVI., p. 262.

A Lummer-Brodhun cube (*B*, Fig. 1) introduces light from the two collimators C_1 and C_2 into the prism *P* and the observing telescope *T*.

The comparison lamp *L* is a ten-volt straight filament lamp moving along a photometer track in line with collimator C_2. A carrier for a small electric furnace *F* and the erbium oxide *O*, moves along another track which is perpendicular to the axis of C_1. It is thus possible to compare the radiation from either the interior of the furnace, or the oxide, with the radiation from the comparison lamp by a method of substitution. If there is match of the two fields when the furnace is before the slit, and this match is destroyed when the oxide has replaced the furnace, there can be no doubt as to which gives the stronger radiation for that particular part of the spectrum.

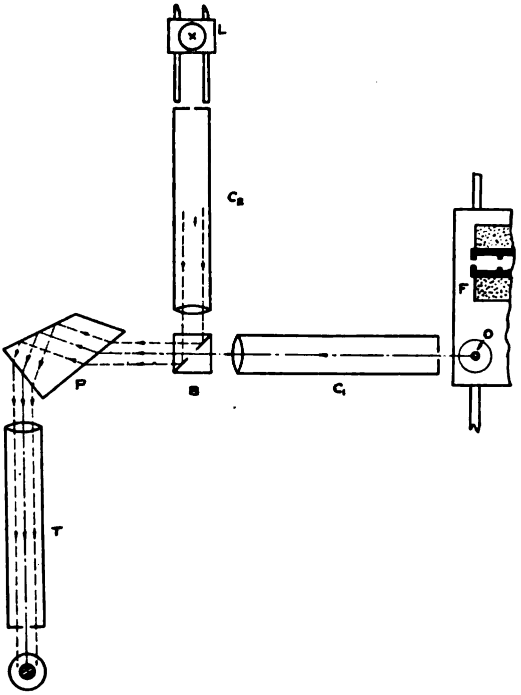

Fig. 1.

Erbium oxide powder was mounted between two thin pieces of clear fused quartz, thus presenting a matte surface to the slit. This surface was about 1 cm. long and 0.6 cm. broad at the top, with an average thickness of 0.15 cm. A colorless hydrogen flame played around the quartz holder. When heated the powder would glow while the surrounding border of clear quartz would remain transparent and nearly non-luminous. A slender quartz rod, fused to the bottom of the holder, supported the arrangement. I made two thermo-junctions out of platinum and platinum-rhodium wire. This wire had been carefully tested and found to give no E.M.F. due to lack of homogeneity. One of these couples was used to determine the temperature of the powder, a junction being placed in the midst of the powder. One junction of the other couple was immediately in front of the partition in the middle of the electric furnace the inner construction of which was similar to the well-known "Reichanstalt" black body. The tube was 45 cm. in length and had an internal diameter of 3 cm. I sighted against a disc placed in the middle of the tube. A number of diaphragms in the tube served to prevent air currents.

When heated it was impossible to make out the boundary of the disc, or the wires of the thermo-junction. Both thermocouples were connected through switches to a Wolff potentiometer, and it was also possible to join the couples in opposition through a Sullivan galvanometer of such a sensibility that a temperature difference of one degree, centigrade, was easily detected by the movement of the reflected spot of light. Since taking the data for the curves that appear in this paper, the junctions have been compared and found to agree to within 0.00002 volt throughout the temperature range, 700° C. to 1046° C. The temperatures as gotten from the potentiometer readings are probably not in error by more than two degrees at any time.

I tested for the uniformity of temperature over the surface of the oxide by means of a Morse thermal gage. This method while not very delicate was fairly conclusive, so far as the detection of considerable temperature differences is concerned. In the exploration of one specimen, for example, that had been used in several runs, the indicated temperature variations were from 827° C. to 840°, or a variation of 13°. Readings taken at different points varied no more however than when the successive settings were all made upon the same point. For both cases the average departure of the temperatures from the mean was about 1.8 per cent.

The use of clear quartz as a mounting in these experiments is essential. When heated in the flame up to 1200° C. it remains so nearly non-luminous that its visible spectrum is of a low order of intensity. Other advantages of the quartz are that it does not soften at the temperatures where the experiments were being carried on, and that at a high temperature it is a good conductor of heat. It is essential that the quartz be free of air bubbles, for although the clear quartz when heated emits little or no visible radiation, if it contains spots made up of an emulsion of quartz and air, these spots emit strongly, and this is true even when the spots are at some distance from the heated portion of the tube or rod.

When making observations the two couples were commonly joined in opposition through the galvanometer. It was possible to keep the temperatures nearly constant. No readings were taken when the two temperatures differed by as much as ten degrees. In case of a lesser difference the readings were not used in the plotting of the curves except when the furnace temperature was the higher of the two. The furnace and the oxide were alternately placed before the slit. Three readings were made for each, at each wave-length. When working near the end of the visible spectrum it was sometimes difficult to make an exact setting but when the shift was made it was possible to tell whether the furnace or the oxide gave the more brilliant field.

In the spectrophotometric determinations the two collimator slits were set at equal width once for all, the slit-widths being as small as the intensity of the weaker regions of the spectrum would permit.

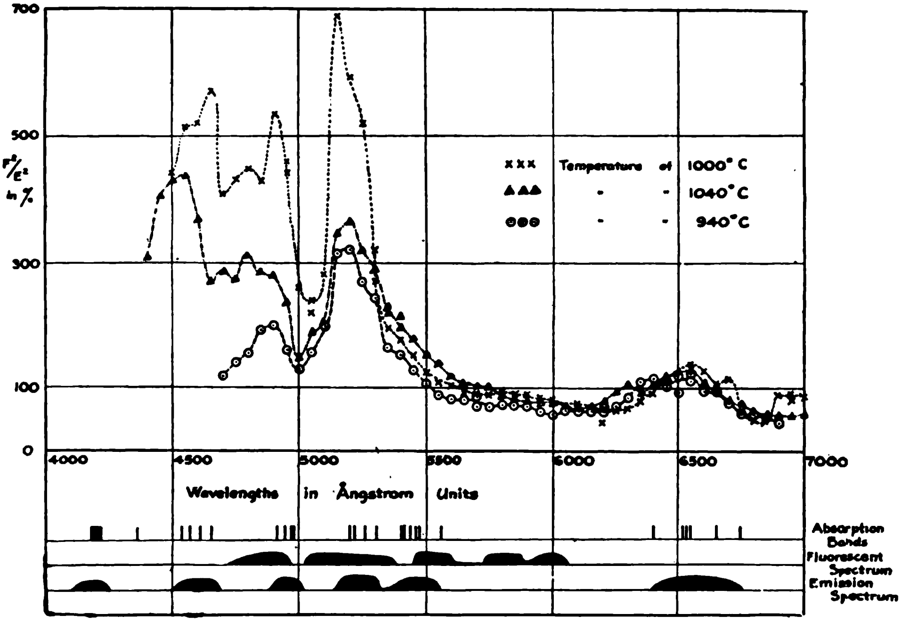

Fig. 2.

The carriage upon which the furnace and the incandescent oxide were mounted was brought to the position where light from the furnace entered collimator C_1. The place on the photometer bar where the light from the comparison lamp matched that from the furnace was determined when the thermocouples indicated equality of temperature between furnace and oxide. The carriage was then immediately shifted so as to bring the oxide before the slit and the position of the lamp readjusted to a match. If the balance of temperatures was still satisfactory the readings on the photometer bar were accepted. Readings were made at intervals of 50 Ångström units.

TABLE II.

Emission of Erbium Oxide referred to that of a Black Body at the same Temperature.

Wave-length.	Relative Intensity.			Wave-length.	Relative Intensity.		
	940°.	1000°.	1040°.		940°.	1000°.	1040°.
.4400μ	—	—	3.06				
.4450	—	—	4.06	5750μ	0.70	0.85	0.98
.4500	—	4.42	4.29	5800	0.72	0.92	0.88
.4550	—	5.14	4.39	5850	0.70	0.90	0.84
.4600	—	5.20	3.68	5900	0.70	0.82	0.80
.4650	—	5.72	2.48	5950	0.62	0.82	0.80
.4700	1.20	4.08	2.86	6000	0.58	0.80	0.78
.4750	1.40	4.30	2.74	6050	0.62	0.72	0.70
.4800	1.55	4.50	3.11	6100	0.59	0.73	0.65
.4850	1.92	4.28	2.85	6150	0.60	0.70	0.70
.4900	2.00	5.35	2.79	6200	0.62	0.62	0.79
.4950	1.60	4.50	2.35	6250	0.65	0.71	0.94
.5000	1.34	2.60	1.48	6300	0.82	0.64	1.00
.5050	1.58	2.30	1.92	6350	1.05	0.82	0.92
.5100	2.10	2.80	2.12	6400	1.12	0.96	1.08
.5150	3.14	6.88	3.46	6450	1.08	1.15	1.18
.5200	3.20	5.90	3.65	6500	1.12	1.26	1.22
.5250	2.68	5.90	3.19	6550	1.12	1.36	1.28
.5300	2.41	2.68	2.68	6600	0.90	1.28	1.06
.5350	1.62	1.94	2.24	6650	0.92	1.02	0.92
.5400	1.51	1.79	2.04	6700	0.72	1.14	0.75
.5450	1.26	1.52	1.79	6750	0.57	0.62	0.72
.5500	1.04	1.22	1.44	6800	0.52	0.56	0.62
.5550	0.88	1.09	1.39	6850	0.48	0.44	0.58
.5600	0.80	1.02	1.18	6900	0.42	0.90	0.57
.5650	0.80	0.95	1.05	6950	—	0.87	0.56
.5700	0.75	0.85	1.00	7000	—	0.84	0.58

In this method of substitution if F is the distance on the photometer bar from the slit C_2 to the filament of the comparison lamp, when the fields are equally bright and the furnace is before the slit C_1 while E represents the corresponding distance, with the oxide before slit C; F^2/E^2 gives the ratio of emission of the oxide and the black body for the temperature in question and the wave-length under observation.

The results of a typical experiment, one of many, are given in Table II. and graphically in the curves in Fig. 2. The measurements were made at 940°, 1000° and 1040° C. In the curves values of F^2/E^2, which give the relative intensities, are plotted as ordinates and wave-lengths as abscissæ. The results are quite different from what I had expected to find, it having seemed probable that the curves might approach a radiation approximately equal to that from a black body for the wave-lengths corresponding to the strong emission bands.

Below the curves showing the emission of the oxide when compared to that of the black body at the same temperature are shown the three spectra mentioned in an earlier paragraph. They were obtained from a sample of the erbium oxide prepared with a flux. The first one shows the absorption when the substance is placed before the slit and the light from a carbon arc is focused upon it. The second shows the fluorescent spectrum when it is excited by cathode rays in a vacuum tube. The third shows the emission spectrum when the same specimen is heated in the oxyhydrogen flame. The close agreement of the positions of the bands in the emission and the absorption spectra can easily be seen.

The range of temperatures employed in the spectrophotometric examination was so small, when we consider the necessarily somewhat uncertain temperature control in the use of the hydrogen flame that relative positions of the curves for 1000° and 1040° may perhaps be open to question. The evidence however seems strongly to favor the existence of a critical temperature of about 1000°.

To check the foregoing temperature determinations, the following modification in the method of mounting was tried. A piece of clear quartz tubing, 3 mm. internal diameter, was sealed at the lower end. Powder was tamped into the tube to a depth of 2.5 cm. The thermojunction was placed in the midst of the powder, opposite the middle of the slit. This tube was rotated about a vertical axis, thus heating the powder from all sides. Small quartz tubes enclosed the lead wires which at the upper end made contact with two rings of mercury, and through these rings the connection was made to the potentiometer and galvanometer, as before. The flame in this case rises about the tube containing the wires, which are heated to incandescence for two or three centimeters above the top of the powder.

With this new arrangement there is no doubt as to which of the two sources gives the more intense radiation. I have not had time to make complete runs with this set up of apparatus but I have examined the spectrum at those wave-lengths where the radiation is most intense and have found the effect to be of the same order of magnitude as shown in the curves.

A further check was obtained by determining the temperature at which the emission bands of the oxide would give the same radiation as the black body at 1000° C. In this experiment the furnace was kept at that temperature and readings were taken for several temperatures

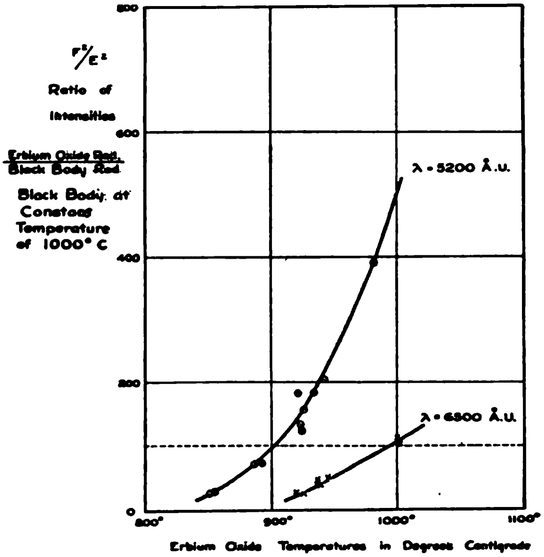

Fig. 3.

of the oxide. Observations were made in only two portions of the spectrum; in the red at wave-length .6500 μ and in the green at .5200 μ.

The results, which are plotted in Fig. 3 do not correspond quantitatively in all respects with the spectrophotometric determinations. That is because we happen to have to do in this experiment with a less intense selective radiator. Preparations of erbium oxide vary very widely in this respect. The radiating power, for example, is greatly lessened when the oxide is fused down in the electric arc. The bands are visible in such cases but at no wave-length does the radiation equal that from the furnace at the same temperature. Although the sample used in the test just described was obviously less active than that previously employed in the exploration of the spectrum, the green band attained at 900° a brightness equal to the corresponding region in the spectrum of the black body at 1000° and at 980° was four times as intense. The red band at 1000°, however, barely exceeded the black body radiation at that temperature.

SUMMARY.

(*a*) As a result of this investigation it seems certain that the peculiar banded emission spectrum of erbium oxide is due to the oxide itself and requires nothing more than a sufficiently high temperature to bring it out. The manner of obtaining this temperature seems to be of no importance.

(*b*) The emission bands at 1000°, especially in the green, are found to radiate much more strongly than a black body at the same temperature.

(*c*) The temperature at which the emission of the green bands equals that of a black body at 1000° is about 900°. This difference of temperature is much greater than that which could possibly have existed in the comparisons which were made under conditions where approximate equality of temperature was indicated.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
May, 1918.

EXPLANATORY NOTE.

Dr. Mallory's investigation was completed in the spring of 1918. The author, who had just been appointed to a professorship at Oberlin University, died suddenly of pneumonia, following an attack of influenza, before his paper was entirely ready for the press. I have completed it from his manuscript, retaining as far as possible the original form and point of view. His work, which was intended to be merely preliminary to a more general study of the emission of the rare earths, in its relation to purely thermal radiation and to luminescence, was interrupted by the untimely close of a career full of promise. It is however being continued by others who will endeavor to build upon the foundation which he laid, in loving recognition of the labors of a departed colleague.

If we accept a recent definition of luminescence, as radiation in excess of that which a body would emit by virtue of its temperature alone, and the well-known doctrine that thermal radiation of a given wave-length can never exceed that of a black body, the type studied by Mallory must clearly be designated as luminescence.

The make up of the spectrum as shown in Fig. 3 of his paper is strongly suggestive of luminescence. The complex group of overlapping bands in the blue which is suddenly developed as the temperature of 1000° is approached is very like those in the spectra of the phosphorescent sulphides as revealed by similar spectrophotometric measurements.[1] This phenomenon as it occurs in Erbium oxide, moreover, is strikingly similar to that described long since in the case of the oxides of calcium,[1] magnesium and zinc[2] where strong selective radiation in the blue of the spectrum appears at about the same temperatures. It is true that in these cases the effect is temporary, whereas in erbium oxide it appears to be permanent; but it may well be that in the latter instance the fatigue is so slow that it has not as yet been detected. Finally the rare earths are in general fluorescent and phosphorescent at ordinary temperatures and they are as subtly dependent in respect to their luminescent activity, upon mechanical and thermal treatment and crystalline condition as the sulphides and certain other phosphorescent materials. Thus different observers are likely to reach apparently discordant results according to the methods by which their materials are prepared. Taken all in all the evidence thus far appears to favor luminescence, superimposed upon ordinary thermal radiation, as an explanation of the phenomena described in Dr. Mallory's paper.—E. L. NICHOLS.

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[1] Nichols, Proc. Am. Philos. Soc., LVI., 258, 1917.

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[1] Nichols and Crehore, PHYS. REV. (1), I., 161, 1894.

[2] Nichols and Snow, Phil. Mag. (5), XXXIII., 19, 1891.

ON THE RELATION BETWEEN THE K SERIES AND THE L SERIES OF X-RAYS.[1]

By William Duane and Takeo Shimizu.

Synopsis.—The object has been to measure both the emission and the absorption wave-lengths in the K and L-series characteristic of a chemical element (tungsten), using the same X-ray spectrometer throughout, to see if general relations among them could be found. Graphs have been drawn representing the ionization currents as functions of the crystal table angle. The peaks in these graphs corresponding to the lines α_1 (strong) and α_2 (medium) appear completely separated from each other, although the difference between their wave-lengths amounts to only 2.2 per cent. There is also faint evidence of a third (very weak) line, α_3.

A sharp drop in the absorption graph occurs near the γ line in the K-series corresponding to the K-critical absorption wave-length, and three sharp drops (a strong, a medium and a weak one occur near the L-series, indicating three critical absorption wave-length in that series. The critical absorption frequencies calculated from these graphs are, for the K-series: $\nu = 1.680 \times 10^{19}$; and for the L-series, $\nu_1 = .2438 \times 10^{19}$ (strong), $\nu_2 = .2773 \times 10^{19}$ (medium), $\nu_3 = .2917 \times 10^{19}$ (weak). The frequencies calculated from the peaks representing the emission lines are $\nu_{a_1} = 1.437 \times 10^{12}$ (strong), $\nu_{a_2} = 1.405 \times 10^{19}$ (medium) $\nu_{a_3} = 1.39 \times 10^{19}$ (?) (weak). The precision of measurement is such that the probable error amounts to less than 1/5 per cent.

It appears from these data that to within 1/5 per cent. $\nu - \nu_1 = \nu_{a_1}$ (strong), $\nu - \nu_2 = \nu_{a_2}$ (medium) and $\nu - \nu_3 = \nu_{a_3}$ (weak); in other words *each K α emission line frequency equals the difference between the K critical absorption frequency and one of the L critical absorption frequencies* to within the limits of experimental error. If this law, that *an emission frequency equals the difference between two critical absorption frequencies* applies to all emission lines we must assume critical absorption frequencies in the M series, etc. Two critical absorption frequencies lying between $\nu = 4 \times 10^{17}$ and $\nu = 5 \times 10^{17}$ taken together with those mentioned above will explain the presence of the β line in the K-series and six of the emission lines in the L series, etc.

1. In the analysis of X-rays about sixteen lines have been found to belong to the characteristic spectrum of each chemical element. Usually, although illogically, these have been divided into two series, the K series and the L series. The K series contains at least four lines, known as the α_1, α_2, β and γ lines, of which the γ line has the highest frequency. These four lines belong together in the sense that none of them can be produced by itself. As we increase the voltage V applied to the X-ray tube none of these lines appear in the spectrum until the voltage V reaches the value given by the quantum equation

$$Ve = h\nu, \tag{1}$$

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[1] A paper read at the New York meeting of the Physical Society on March 1, 1919.

where e is the charge on the electron h Plank's action constant and ν, a frequency at least as high, and possibly slightly higher than that of the γ line. When the critical voltage has been reached, all four lines appear together; and their intensities increase in approximately the same ratio, if the voltage is raised still higher.[1]

The above is not true of the L series as a whole. In this series there are at least two and probably three groups of lines, the lines in each group being connected together in much the same way as are the lines in the K series.[2]

In the analysis of the X-ray absorption spectra of the chemical elements there appears to be only one critical absorption frequency connected with the K series. This frequency lies close to and probably slightly above that of the γ line. Within the limits of experimental error it equals the frequency that must be substituted in the quantum equation (1) to give the energy of the electron required to produce the K series.

In the absorption spectra of uranium, thorium, bismuth and gold De Broglie found three critical absorption frequencies characteristic of each element and in the neighborhood of its L series. The writers have found three such critical frequencies in the absorption spectra of both lead and tungsten. Doubtless these three absorption frequencies belong respectively to the three distinct groups of lines in the L series.

It appears, therefore, from purely experimental evidence that we may divide the known lines in the X-ray spectra of a chemical element into four groups, one in the K series and three in the L series, each group containing a critical absorption frequency and about four emission lines. The lines in each group are connected together in some way, but the relation between their frequencies is not given by the equation

$$\nu = \nu_0\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)$$

in which ν_0 is a constant, and τ_1 and τ_2 are small whole numbers.

The question now arises: Is there any relation between the various groups of lines? Some years ago Kossel suggested that the difference between the frequencies of the α and β lines in the K series might be equal to the frequency of one of the strong lines in the L series, and considerable discussion on this point has appeared in the literature expecially in connection with the theories of the mechanism of X-ray emission spectra and of the structure of the atom. It is evident, however, that this relation cannot be a general relation, for there are many more lines in the L series than in the K series.

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[1] See D. L. Webster, PHYS. REV., June, 1916, p. 599, and June, 1917, p. 571.

The authors have undertaken the research reported in this paper for the purpose of measuring both the absorption and the emission frequencies with the same apparatus, to see if a more general relation between the frequencies can be found.

We have used the X-ray spectrometer previously described,[1] with a calcite crystal, the formula for the wave-length λ being

$$\lambda = 2a \sin \theta = 6.056 \times \sin \theta \times 10^{-8} \text{ cm}.$$

where θ is the grazing angle of incidence. The X-rays passed through two narrow slits in thick lead blocks before striking the crystal thus obviating a large correction for the absorption of the X-rays in the calcite, as previously explained (*l. c.*). The X-ray tubes were of the Coolidge type, one of them having a tungsten and the other a molybdenum target.[2] The currents passing through these came from a high tension transformer with a system of condensers and kenotrons attached to it for producing approximately a constant difference of potential.

The curves in Fig. 1 represent the K lines of tungsten in both the first and second order spectra. The constant difference of potential applied to the tube amounted to about 100,000 volts in this experiment. The α lines were measured on both sides of the zero line of the spectrometer thus obviating a direct determination of the zero point. A correction of about 45″ of arc must be subtracted from the double grazing angle on account of the excentricity of the spectrometer scale.

There appears to be a somewhat better definition of the lines on the right hand side of the zero line than on the left. Doubtless this is due to a slight lack of uniformity in the crystal structure.

The two α lines on the right hand side appear to be completely separated from each other, even in the first order spectrum. This gives an idea of the precision with which the wave-lengths can be estimated, the difference between the two α wave-lengths being about 2.2 per cent. The position of each peak can be estimated to within 10″ of arc. Since the double grazing angles are about 4°, this means that the error of measurement is less than 1 part in 1,000, unless the same error is made on both sides of the zero line.

The peaks corresponding to α_2 are somewhat unsymmetrical, the slopes on the long wave-length sides being slightly more extended than on the short wave-length sides. This lack of symmetry suggests the presence of an additional line α_3 (?) close to and on the long wave-length side of α_2. Its wave-length should be about $.214 \times 10^{-8}$ cm. To the left of the β

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[1] Phys. Rev., December, 1917, p. 624.

[2] Dr. W. D. Coolidge kindly sent us these tubes.

line appears a sharp drop in the curve. This marks the position of the critical absorption wave-length in the K series. The angular breadth of the drop about equals that of one of the peaks, namely 2′ of arc, and the angle corresponding to the center of the drop was used in calculating the critical absorption wave-length.

In this experiment the rays did not pass through an absorbing screen of tungsten outside of the X-ray tube, and the reason why the absorption drop appears in the curve is that we placed the X-ray tube in such a position that the rays passing through the spectrometer slits came from the target in a direction almost parallel to its surface. Under these conditions, since some of the rays are produced slightly below the surface of the target, part of them is absorbed by the surface layers of the target and by any irregularities there may be on the surface.

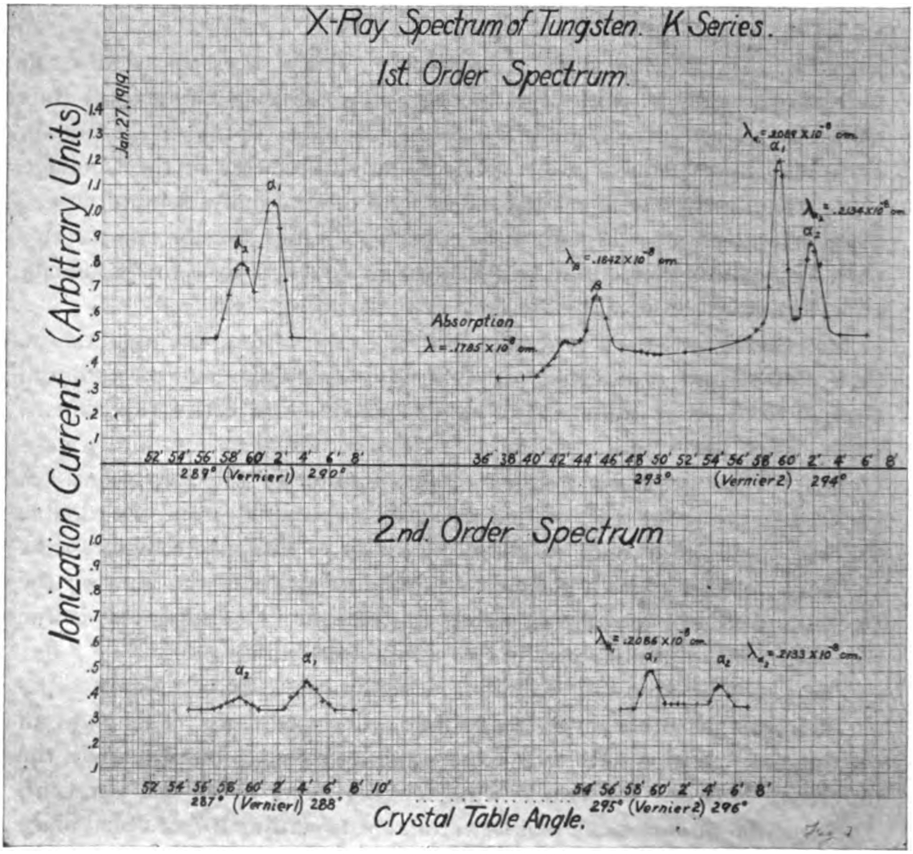

Fig. 1.

We have also measured the critical absorption wave-length in the K series using an X-ray tube with a molybdenum target, and placing a layer of sodium tungstate between the tube and the spectrometer as an absorbing screen. In this way we obtained the following values for the wave-length: $\lambda = .1786 \times 10^{-8}$ cm. in the first order spectrum, and $\lambda = .1784 \times 10^{-8}$ cm. in the second order spectrum.

The critical absorption wave-lengths in the L-series of tungsten were measured as follows:

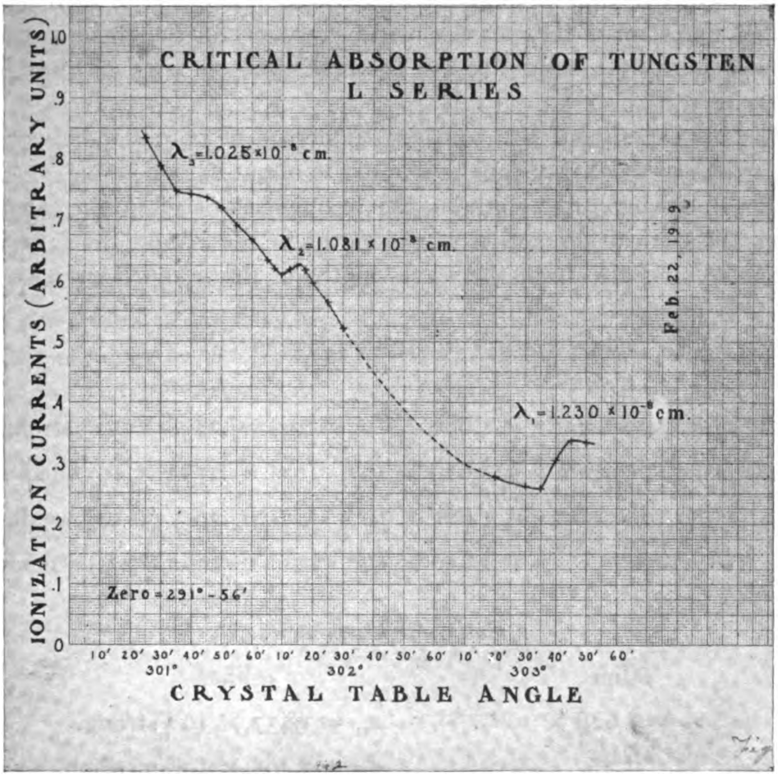

Fig. 2.

The curve in Fig. 2 represents the current in the spectrometer's ionization chamber as a function of the readings of one of the verniers attached to the crystal table in an experiment in which the X-rays came from a molybdenum target and passed through a *thin* layer of sodium tungstate before reaching the spectrometer slits. The three breaks in the curve correspond to the three critical absorption wave-lengths of tungsten that lie in the neighborhood of its L series of emission lines.

We determined the zero on the scale by taking readings on both sides of the zero line. The value of the zero point given in the figure contains the correction for eccentricity.

The amount of X-ray energy that passes through the glass walls of the X-ray tube in the region corresponding to these long waves is quite small, and we, therefore, had to widen the spectrometer slits somewhat in order to measure the ionization currents. As a consequence the angular breadth of the drops in the curve have increased to four or five times the breadth of the drop in the K series curves. This has not increased the percentage error of the measurements, however, for the grazing angles of incidence in the L series are five to seven times as large as those in the K series.

In this experiment the constant difference of potential between the electrodes of the X-ray tube amounted to about 24,000 volts.

The following table contains the frequencies of vibration calculated from the critical absorption wave-lengths in the K and the L series of tungsten, and also from the wave-lengths of the α and β emission lines of its K series.

Critical Absorption Frequencies.

K Series	*L Series*
1st order spectrum $\nu = 1.680 \times 10^{19}$	$\nu_1 = .2438 \times 10^{19}$ strong,
1st order spectrum $\nu = 1.679 \times 10^{19}$	$\nu_2 = .2773 \times 10^{19}$ medium,
2d order spectrum $\nu = 1.681 \times 10^{19}$	$\nu_3 = .2917 \times 10^{19}$ weak.
Average $\nu = 1.680 \times 10^{19}$	

Emission Frequencies.

β-line	α-lines
$\nu_\beta = 1.628 \times 10^{19}$	$\nu_{\alpha_1} = 1.437 \times 10^{19}$ strong.
	$\nu_{\alpha_2} = 1.405 \times 10^{19}$ medium.
	$\nu_{\alpha_3} = 1.39 \times 10^{19}$ (?) weak.

In searching for a general relation between the emission frequencies and the critical absorption frequencies the idea immediately presents itself that an emission frequency may be the difference between two absorption frequencies. The fact that the L series contains three critical absorption frequencies, (a strong, a medium and a weak one), and that there are at least two α emission lines (a strong and a medium one), and possibly a third weak one, lends weight to the idea. From the frequencies contained in the above table we have

$\nu - \nu_1 = 1.436 \times 10^{19}$ (strong), $\nu_{\alpha_1} = 1.437 \times 10^{19}$ (strong),
$\nu - \nu_2 = 1.403 \times 10^{19}$ (medium), $\nu_{\alpha_2} = 1.405 \times 10^{19}$ (medium),
$\nu - \nu_3 = 1.388 \times 10^{19}$ (weak). $\nu_{\alpha_3} = 1.39 \times 10^{19}$ (?) (weak).

It appears from this data, that as a matter of fact *each α line frequency equals the difference between the* K *critical absorption frequency and one of the* L *critical absorption frequencies* to within the limits of experimental error.

If this law applies to all emission lines we must assume the existence of one and perhaps more critical absorption wave-lengths lying between $\lambda = 6 \times 10^{-8}$ cm. and $\lambda' = 7.5 \times 10^{-8}$ cm. (the M series) in order to explain the presence of the β-line in the K series. Two critical absorption wave-lengths lying in this M region taken together with the three critical absorption wave-lengths in the L series explain, according to the law, the presence of 6 of the lines in the L emission series.

The γ line in the K series and several lines in the L series lie very close to the critical absorption wave-lengths, and in order to explain them we must assume the existence of other critical absorption wave-lengths in the suspected N series, etc.

In conclusion we call attention to the fact that the equations which have been suggested to represent the lines in ordinary spectrum analysis are usually in the form of the difference between two functions, and also that X-ray critical absorption is not the same as ordinary absorption which obeys Kirchhoff's law.

HARVARD UNIVERSITY,
March, 1919.

THE VELOCITY OF SOUND AND THE RATIO OF THE SPECIFIC HEATS FOR AIR.

BY T. C. HEBB.

SYNOPSIS.—The ratio of the specific heats for air obtained from the author's determination of the velocity of sound does not seem to be as high as that obtained by different observers using the Lummer-Pringhseim method. This was thought at first to be due to experimental errors in the determination of the velocity of sound. A redetermination gives, however, a value very close to the previously determined value.

The cause of the discrepancy is found, however, to be due to the method of obtaining the velocity of sound for dry ir at 0° C. from observations made in air containing moisture. The usual transformation leaves out the term $\sqrt{\gamma_0/\gamma_m}$ where γ_0 is for dry air at 0° C. and γ_m is for air containing a certain per cent. of moisture. The term is small but not negligible. It can be calculated with sufficient accuracy.

The corrected values of the velocity of sound can then be used to determine γ for dry air at 0° C. The results obtained are very satisfactory.

SOME years ago I made a determination of the velocity of sound by a new method which seemed to give very accurate results.[1] At that time I was not interested in the ratio of the specific heats, and, hence, made no effort to calculate its value. The value of the velocity obtained at that time, however, has since been used by Moody[2] in his paper on the ratio of the specific heats; but he failed to correct for the fact that the ideal gas laws were used in its calculation. Even if this correction be applied the value of γ thus obtained is still lower than the value obtained by different observers using the Lummer and Pringsheim method. This is due to the fact that I failed to apply a correction which is necessary in obtaining the velocity of sound for dry air at 0° C. from observation made in moist air.

Now both of these corrections can be calculated, and, hence, it is of interest to see what value of γ is obtained by applying them. This paper contains an application of these corrections and also the account of a new determination of the velocity of sound. This determination was made two years ago and before I saw what was the explanation for the difference in value between γ as obtained by my velocity of sound and that obtained by the Lummer-Pringsheim method. I may forecast the results of the experiment by saying that the value of the velocity obtained is practically the same as that obtained in 1904.

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[1] PHYS. REV., Vol. XX., No. 2, Feb., 1905.

[2] PHYS. REV., Vol. 34, No. 4, p. 275, 1912.

1. VELOCITY OF SOUND.

The method used was the same as that used in my previous determination. The same apparatus was used which had been carefully preserved at Ryerson Physical Laboratory, University of Chicago, where the experiment was done. The method was of an interference type. Two coaxial paraboloids—5 ft. in diameter—had each a telephone transmitter. In one paraboloid the transmitter was placed in the focus, and in the other it was placed near the focus. Each transmitter was in series with a battery and one primary of an induction coil containing two primaries and one secondary. A telephone receiver was in series with the secondary of the induction coil. A source of sound was placed in the focus of the paraboloid, having a transmitter near its focus. This paraboloid was always kept stationary. If the other was moved, always keeping it coaxial with the stationary one, the sound in the receiver of the telephone passed through maximum and minimum values. By this means it was possible to measure the wave-length of the particular note used. And, as a single setting could be made with considerable accuracy, and, as it was possible to move the one paraboloid through a distance of 80–100 feet, it was possible to obtain the wave-length with an accuracy of about one tenth of one per cent., as was shown in my previous paper.

A whistle blown with a constant air pressure was used as a source of sound. This, although presenting difficulties, was the most constant source of frequency, with sufficient intensity, available. To insure constancy when observations were made, the whistle was kept in unison with a tuning fork of known frequency.

The experiment was performed indoors in a basement hall, and, hence, there were no wind disturbances. The temperature was determined by eleven thermometers spaced along the distance over which the observations were made. The dew point was determined with an Alluard hygrometer.

In my previous experiment I used only one frequency, owing to the fact that I had only one tuning fork of sufficiently high pitch. In this experiment, however, I had at my disposal a set of standard tuning forks. These forks, with the exception of the 4,096 d.v., had been kept in good condition. The one marked 4,096 d.v. had apparently been hit and dented with a sharp metallic hammer. On comparing the forks among themselves it was found that it was impossible to get beats due to combination tones, excepting in the case of the 4,096 fork, which was found to have a frequency of 4,099.4 relative to the 2,048 d.v. fork. It was later found, however, that this fork could not be used as its frequency was too great.

In order to be sure that the absolute pitches of the forks were as marked, three of them were tested in the following manner. First, a fork with a frequency of about 100 was compared with a standard clock; and, second, the forks to be tested were compared with the standardized tuning fork.

In order to get the frequency of this intermediate fork, a stylus was attached to one prong so that it could make a trace on a smoked drum. By this means the period was found, (1) by direct comparison with the clock, and, (2) by comparison with a pendulum which had been compared with the clock. As the comparison took a great deal of time, the tuning fork was kept in vibration by means of an electromagnet between its prongs. This electromagnet was operated once a second from the clock circuit. This method resulted in an occasional phase change for which, however, correction was made. The results of five determinations are given in the following table:

TABLE I.

Method.	Time, Seconds.	t.	N_t.	N_{21}.
1. Clock............	120	23° C.	99.817	99.839
2. "	120	21° C.	99.848	99.848
3. "	120	21° C.	99.812	99.812
4. Pendulum.........	106	32.2° C.	99.815	99.839
5. "	80	23.2° C.	99.800	99.824
			Mean.......	99.833

The probable error of the mean is .004.

To find the frequency of the forks to be directly used in the experiment, a high-frequency generator was used. On its shaft was fitted a switch which operated once for every revolution of the generator. This actuated an electromagnet which recorded on a revolving drum with the trace of the standardized fork alongside of it. The idea entertained at first was to get the note produced by a telephone receiver in series with the generator, in unison with that of the fork whose pitch was to be determined, and, then, calculate its pitch from the speed of the generator as obtained from the tracings on the drum. This did not succeed, as it was found to be impossible to keep the generator speed sufficiently constant to keep the two notes in unison. It was found, however, that it was possible to keep the generator-telephone note out of unison with the tuning fork just enough to enable the number of beats in a certain time to be counted. Then, if the precaution was taken to be sure which frequency was the higher, it was possible to determine the relative frequencies. Table II. will make this clearer.

TABLE II.

Revolutions of Generator from Drum Record.	Corresponding Beats.	No. of Revolutions to Make Unison.	No. of Waves of Standard Fork.	Waves per Revolution.
279	30	278.34	869.0	3.1221
257	20	256.34	800.0	3.1212
214	20	213.34	665.4	3.1190
300	20	299.34	934.1	3.1205
219	20	218.34	680.97	3.1188
246	20	245.34	765.65	3.1207
			Mean..........	3.1204

In this case the generator-telephone note was higher than that of the tuning fork, and the rotor of the generator had 30 teeth. These results were obtained with the fork marked 960 d.v. It will be noticed that the number of beats is always the same. By the use of a switch in the circuit which recorded the revolutions of the generator it was possible to begin the record at any instant. Starting with the close of the switch 20 beats were counted and then the switch was opened.

Two other such determinations as given in Table II. gave means as follows:

$$\text{Means}_2 = 3.1199,$$
$$\text{Means}_3 = 3.1208.$$

The mean of these three means is 3.1204. The frequency of the 960 d.v. tuning fork, therefore, at 21° C. was:

$$N = \frac{30 \times 99.833}{3.1204} = 959.84.$$

As a check on these results the frequencies of the 1,024 d.v. and the 2,048 d.v. forks were determined in the same manner. The three means obtained for the 1,024 d.v. fork were:

Mean_1..........2.9250
Mean_2..........2.9247
Mean_3..........2.9253

Mean.......... 2.9250.

The frequency of the 1024 d.v. fork was, therefore, at 21° C.:

$$N = \frac{30 \times 99.833}{2.9250} = 1023.93.$$

As only a single set of observations was made for the 2048 d.v. fork I include it in Table III. For this case the rotor of the generator contained 60 teeth.

TABLE III.

Revolutions of Generator Taken from Drum Record.	Corresponding Beats.	Number of Revolutions to Make Unison.	Number of Waves of Standard Fork.	Waves Per Revolution.
313	0	313.0	915.7	2.9256
457	50	456.17	1334.1	2.9246
491	30	491.5	1438.0	2.9257
213	20	212.67	621.8	2.9238
306	30	305.5	893.5	2.9247
667	20	666.67	1949.9	2.9248
442	50	441.17	1290.0	2.9240
			Mean..........	2.9247

Hence the frequency of the 2,048 d.v. fork was at 21° C.:

$$N = \frac{60 \times 99.833}{2.9247} = 2048.02.$$

These results show that the 960 d.v. fork was in error about .02 per cent.; the 1,024 d.v. fork about .01 per cent.; and the 2,048 d.v. fork was about correct. As the previous experiments had shown that the forks were consistent among themselves it was assumed from these results that all the forks were accurate at 20° C. It should be observed that all through this paper the following formula has been used:

$$N_t = N_0(1 - .00011t),$$

where N_t is the frequency of a fork at t° C. and N_0 is its frequency at 0° C.

With the frequencies known and the wave-length determined as indicated above, it was easy to get the corresponding velocity. The following table gives the results of my observations on page 78.

It will be noticed that only two results are given for the 3,072 frequency. No attempt was made to use it on August 11, and it was impossible to get any results with it on August 21. This was due to the fact that the reflected wave caused a change in the frequency of the whistle just as soon as the one paraboloid was moved. This caused trouble for all frequencies but the effect increased with the frequency. The trouble can be obviated by adjusting the air pressure supplied to the whistle. This is a long and tedious process and was not followed out on August 21. It was this reflected wave which made it impossible to use the 4,096 frequency.

An examination of the table reveals the fact that the mean velocity

Date.	Number of Waves.	Distance in Inches.	Temperature °C.	Dew Point °C.	Pressure Mm.	λ_t in Inches.	N_{20}.	V_t in Inches.	V_0 (Meters.)
Aug. 11	90	957.0	22.1	14	746.9	10.633	1,280	13,610	331.43
" 15	91	968.4	22.1	16	746.6	10.642	"	13,621	331.69
" 19	91	968.1	22.1	16.5	743.4	10.638	"	13,617	331.43
" 21	91	969.1	22.4	18.5	741.8	10.649	"	13,631	331.51
								Mean .	331.51
Aug. 11	108	957.5	22.1	14	746.9	8.866	1,536	13,618	331.62
" 15	109	964.9	22.1	16	746.6	8.8521	"	13,597	331.11
" 19	109	965.5	22.1	16.5	743.4	8.8582	"	13,606	331.16
" 21	109	966.4	22.4	18.5	741.8	8.8660	"	13,618	331.19
								Mean .	331.27
Aug. 11	144	956.3	22.1	14	746.9	6.6406	2,048	13,600	331.19
" 15	146	970.9	22.1	16	746.6	6.6499	"	13,619	331.64
" 19	145	963.5	22.1	16.5	743.4	6.6448	"	13,609	331.23
" 21	146	971.0	22.4	18.5	641.8	6.6507	"	13,621	331.26
								Mean .	331.33
Aug. 15	219	970.0	22.1	16	746.6	4.4292	3,072	13,606	331.33
" 19	218	967.0	22.1	16.5	743.4	4.4358	3,072	13,627	331.68
								Mean .	331.51

for a frequency of 1,280 is somewhat higher than the mean in the cases of the 1,536 and 2,048 frequencies. Frequencies of 960 and 1,024 were also tried, but it was found that they gave still higher values. This agrees with what was found in the previous experiment and appears to be related to the relative size of reflector and wave-length. Therefore, the results with the 1,280 frequency have not been used, although their addition does not affect the results seriously. The mean value of the velocity for the 1,536, 2,048 and 3,072 frequencies is 331.34 m. per second. This has to be corrected for the change in pitch due to the fact that the tuning forks were used at about 22.2° C., whereas they were correct at 20° C. The corrected value is, therefore, 331.26 m. per second, with a probable error of .05. This value is very close to the value obtained in my previous experiment, viz., 331.29. When, however, the correction given in the next paragraph is applied the agreement is not so close.

In calculating the results of Table IV. the relation

$$V_0 = V_m\sqrt{\frac{\rho_m}{\rho_0}}$$

was used where V_0 is the velocity in dry air at 0° C., V_m is the velocity as determined by the experiment, ρ_m is the density of the air at the time of the experiment and ρ_0 is the density of the air at 0° C. and at a pressure equal to that at the time of the experiment. Now this formula assumes that the ratio of the specific heats for dry air is the same as that for moist air, which is not correct. The correct transformation should be obtained in this manner:

$$V_0 = \sqrt{\frac{p\gamma_0}{\rho_0}} \quad \text{and} \quad V_m = \sqrt{\frac{p\gamma_m}{\rho_m}},$$

where γ_m is the ratio of the specific heats for air at the time of the experiment and γ_0 is the ratio for dry air at 0° C.

Hence

$$V_0 = V_m\sqrt{\frac{\rho_m}{\rho_0}}\cdot\sqrt{\frac{\gamma_0}{\gamma_m}},$$

i. e., the results given in Table IV. should be multiplied by $\sqrt{\gamma_0/\gamma_m}$ in order to give an accurate result. As γ_0/γ_m is nearly equal to one we can calculate its value with sufficient exactness.

In order to do that consider a unit mass of a mixture of three gases—mono-, di- and triatomic. The three gases referred to are argon, oxygen and nitrogen as one gas, and water vapor.

Then $C_v = C_{v_1}m_1 + C_{v_2}m_2 + C_{v_3}m_3$, where the C_v's are the specific heats at constant volume of the mixture and individual gases respectively and m_1, m_2 and m_3 are the masses of the ingredients.

But $C_p - C_v = R$ for the mixture

$$\therefore\ \gamma - 1 = \frac{R}{C_v}.$$

Where γ is the ratio of the specific heats for the mixture

$$\therefore\ \gamma - 1 = \frac{R}{C_{v_1}m_1 + C_{v_2}m_2 + C_{v_3}m_3}$$

$$= \frac{R}{\dfrac{R_1}{\gamma_1 - 1}m_1 + \dfrac{R_2}{\gamma_2 - 1}m_2 + \dfrac{R_3}{\gamma_3 - 1}m_3},$$

where γ_1, γ_2 and γ_3 are the ratios of the specific heats for the three gases and R_1, R_2 and R_3 are their gas constants;

$$\therefore\ \gamma - 1 = \frac{1}{\dfrac{1}{\gamma_1 - 1}\cdot\dfrac{m_1R_1}{R} + \dfrac{1}{\gamma_2 - 1}\cdot\dfrac{m_2R_2}{R} + \dfrac{1}{\gamma_3 - 1}\cdot\dfrac{m_3R_3}{R}}$$

But

$$\frac{m_1R_1}{R} = \frac{p_1}{p}, \quad \frac{m_2R_2}{R} = \frac{p_2}{p} \quad \text{and} \quad \frac{m_3R_3}{R} = \frac{p_3}{p},$$

where p, p_1, p_2 and p_3 are the pressures of the mixture and the component gases respectively.

Therefore

$$\gamma = 1 + \frac{1}{\frac{1}{\gamma_1 - 1}\cdot\frac{p_1}{p} + \frac{1}{\gamma_2 - 1}\,\frac{p_2}{p} + \frac{1}{\gamma_3 - 1}\,\frac{p_3}{p}}.$$

It will be sufficiently accurate to use the theoretical values of γ_1, γ_2 and γ_3, *i. e.*, $= \gamma_1 = 1\frac{2}{3}$, $\gamma_2 = 1.4$ and $\gamma_3 = 1\frac{1}{3}$;

$$\therefore \quad \gamma = 1 + \frac{1}{3/2(p_1/p) + 2.5(p_2/p) + 3(p_3/p)}.$$

The per cent. of the gases at the time of my experiment were .95 for argon, 1.85 for water vapor and 97.2 for oxygen and nitrogen.

$$\therefore \quad \gamma_m = 1 + \frac{1}{3/2 \times .0095 + 2.5 \times .972 + 3 \times .0185},$$

where γ_m is the ratio of the specific heats for air with 1.85 per cent. of water vapor in it.

$$\therefore \quad \gamma_m = 1.4000.$$

If $p_3 = 0$, *i. e.*, if the air is dry

$$\gamma_0 = 1 + \frac{1}{3/2 \times .0095 + 2.5 \times .9905}$$
$$= 1.4015.$$

Hence

$$\sqrt{\frac{\gamma_0}{\gamma_m}} = \sqrt{\frac{1.4015}{1.4000}} = 1.00053.$$

Hence the corrected velocity of sound for dry air at 0° C. as obtained from these experiments is

$$= 331.26 \times 1.00053 = 331.44 \text{ m.p.s.}$$

It is of interest to correct my previous value of the velocity of sound. The value obtained was 331.29 m. per second. Now the average value of the water vapor for that experiment was 1.03 per cent.

Hence

$$\gamma_m = 1 + \frac{1}{3/2 \times .0095 + 2.5 \times .9802 + 3 \times .0103}$$
$$= 1.4007;$$

$$\therefore \quad \sqrt{\frac{\gamma_0}{\gamma_m}} = \sqrt{\frac{1.4015}{1.4007}} = 1.000285;$$

$$V_0 = 331.29 \times 1.000285$$
$$= 331.38 \text{ m. per second.}$$

If we give these two values of the velocity of sound equal weights we get for the mean:

$$V_0 = 331.41,$$

which appears to me cannot have an error of more than 2 or 3 centimeters.

2. The Ratio of the Specific Heats.

To calculate the ratio of the specific heats it is necessary to apply a correction to the formula $v = \sqrt{p\gamma/\rho}$ in order to take into account the deviation of air from the ideal gas law. This correction may be obtained in the following manner: An infinitely small adiabatic change may be expressed by the following equation—

$$C_p\,dT_p + C_v\,dT_v = 0,$$

where dT_p and dT_v are the changes in temperature when the pressure and volume are constant respectively. Their values can be obtained from the ideal gas law: $pv = RT$ and hence

$$Cp\cdot pdv + C_v vdp = 0.$$

In this equation C_p and C_v are the specific heats for an ideal gas. If, however, we determine the dT's from Van der Wall's equation then C_p and C_v will be the specific heats of the gas under consideration in so far as Van der Wall's equation represents the characteristics of that gas. Thus:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT;$$

hence

$$dT_p = \left[\frac{p - \frac{a}{v^2} + \frac{2ab}{v^3}}{R}\right] dv$$

and

$$dT_v = \frac{v - b}{R}\,dp.$$

$$\therefore\quad C_p\frac{p - \frac{a}{v^2} + \frac{2ab}{v^3}}{R}\,dv + C_v\frac{v - b}{R}\,dp = 0.$$

$$\therefore\quad \gamma\left[\frac{p - \frac{a}{v^2p} + \frac{2ab}{v^3p}}{1 - \frac{b}{v}}\right] \times pdv + vdp = 0.$$

Now the term in the brackets is small and hence may be considered as a constant.

$\therefore$ pv^{n} = a constant is the equation of the adiabatic for this gas, where

$$\gamma_1 = \gamma\left[\frac{p - \dfrac{a}{v^2p} + \dfrac{2ab}{v^3p}}{1 - \dfrac{b}{v}}\right].$$

As the term $2ab/v^3p$ is negligible in comparison with the rest of the expression under the brackets we get

$$\gamma_1 = \gamma\left[\frac{1 - \dfrac{a}{v^2p}}{1 - \dfrac{b}{v}}\right].$$

Using the critical constants for air, viz., $T_c = -140°$ C. and $P_c = 39$ atmospheres we get for the expression in the brackets at 0° C. and 76 cm. pressure the fraction .99896.

$$\therefore\quad \gamma = \gamma_1 \times 1.00104.$$

Now γ_1 is obtained from the velocity of sound thus:

$$\gamma_1 = \frac{Vo^2\rho_0}{P_{760}} = \frac{331.44^2 \times .0012928}{760 \times 13.5955 \times 980.616}$$

$$= 1.4017.$$

$$\therefore\quad \gamma = 1.4017 \times 1.00104$$

$$= 1.4031.$$

This is the value from the velocity of sound reported in this paper.

The value of γ obtained by using my previous value of V_0 as corrected in this paper, viz., 331.38 is

$$\gamma = 1.00104 \times \frac{331.38^2 \times .0012928}{760 \times 13.5955 \times 980.616}$$

$$= 1.00104 \times 1.4012$$

$$= 1.4026.$$

This value differs considerably from the value Moody obtained from my velocity of sound.

These two values of γ, 1.4031 and 1.4026, agree very well with the values obtained in recent years by the Lummer & Pringsheim method. This is shown in the following table, part of which is taken from Miss Shield's paper on the ratio of the specific heats of hydrogen.[1]

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[1] Phys. Rev., Ser. 2, Vol. X., No. 5, p. 525, 1917.

TABLE V.

Observer.	Method.	
Lummer & Pringsheim	L. & P.	1.4025
Moody	"	1.4003
Partington	"	1.4034
Shields	"	1.4029
Hebb (1904)	Velocity of Sound	1.4026
Hebb (1919)	"	1.4031

The average of the two values of γ presented in this paper is 1.40285 and as I am of the opinion that the average of the two velocities of sound presented is accurate to 2 or 3 parts in 33,000, I am also of the opinion that this value of γ must be accurate to 2 or 3 parts in 14,000. This value, 1.40285, is almost identical with what one would expect from a theoretical standpoint. In the section on the velocity of sound it was shown that a gas consisting of .95 per cent. of argon and 99.05 per cent. of oxygen and nitrogen would have a ratio of the specific heats equal to 1.4015 considered as an ideal gas. If there be added to this the correction due to its deviation from an ideal gas, viz., .0014, there results the value 1.4029.

I wish to thank the staff of Ryerson Physical Laboratory for showing me every courtesy during the time I spent at the laboratory doing the experimental part of this work.

UNIVERSITY OF BRITISH COLUMBIA,
VANCOUVER, B. C.

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THE

PHYSICAL REVIEW.

A SENSITIVE MODIFICATION OF THE QUADRANT ELECTROMETER: ITS THEORY AND USE.

BY A. H. COMPTON AND K. T. COMPTON.

SYNOPSIS.

New Quadrant electrometer of high sensitivity, with tilted needle and movable quadrants. *Principle.* If the needle is given a slight tilt about its long axis and one pair of quadrants is raised or lowered a small distance with respect to the other pair, electrical control forces are set up which add to or subtract from the restoring torque from the suspension. *The theory* of these forces is discussed, the equation giving the sensitiveness in terms of the potential of the needle and the geometrical arrangement of the quadrants is derived and the method of utilizing electrostatic control to the best advantage is described. *Advantages:* High sensitivity, nearly independent of the deflection, and quick adjustment of sensitivity through a great range. *Results.* By using a small needle, 4.5 mm. in radius, with a slight tilt, sensitivities as high as 60,000 mm. per volt have been obtained. Adjustments are difficult at these extreme sensitivities, but the electrometer can usually be used up to 15,000 mm. per volt without undue trouble. Practical suggestions are given regarding needles, suspensions and mirrors.

Quadrant electrometer: needles of mica or aluminum. Technique.

Quadrant electrometer sensitivity; summary of elementary theory.

1. INTRODUCTION.

IF a quadrant electrometer is to be used for the quick and accurate measurement of small electric charges or small potential differences it should be very sensitive, it should be as nearly as possible equally sensitive over all parts of the scale, and the moving parts should come quickly to rest when a measurement is being made. If we consider certain possible means of securing high sensitiveness, we shall see that this may be obtained without unduly sacrificing either the uniformity of the scale deflections or the rapidity with which readings may be taken.

According to the simple theory of the quadrant electrometer, if the needle is at a potential V and the two pairs of quadrants are at potentials

v_1 and v_2 respectively, the angle through which the needle is turned is given by

$$k_2\theta = k_1(v_2 - v_1)\left(V - \frac{v_2 + v_1}{2}\right),$$

where $k_2\theta$ is the torque exerted on the needle by the suspending fibre when twisted through an angle θ and $k_1 = R^2/\pi h$, where R is the radius of the needle and h is the distance from the lower to the upper side of the quadrants. In practice, the term $(v_2 + v_1)/2$ may nearly always be neglected in comparison with V, so that the sensitiveness is directly proportional to the potential of the needle. As long as V remains comparatively small this expression is found to be accurate, but when the potential of the needle is large, another term, which is approximately proportional to $V^2\theta$, becomes of prominence. We have then[1]

$$k_2\theta = k_1 V(v_2 - v_1) - k_3 V^2\theta,$$

and the sensitiveness is

$$S = \frac{\theta}{v_2 - v_1} = \frac{k_1 V}{k_2 + k_3 V^2}. \qquad (1)$$

With a given suspension the quantity k_2 is constant, but it is possible to vary k_3 over a large range of positive and negative values by changing the geometrical arrangement of the electrometer needle and quadrants. When k_3 is positive, the needle is held in more stable equilibrium, and there is said to be a "positive electrostatic control"; when k_3 is negative, the needle is held less firmly in position, and there is said to be a "negative electrostatic control." It is evident that, by adjusting the quantity $k_3 V^2$ until it is opposite in sign and nearly equal to k_2, any desired degree of sensitiveness can be attained.

If, as is usually the case in practice, the two pairs of quadrants are not connected directly to a battery, but one pair is connected to an insulated system whose gain of charge is to be measured, there is another factor, called the "inductional electrostatic control," which has to be considered. This control is due to the difference of potential between the quadrants which is set up when the electrometer needle moves from one into the other. The torque produced by this effect is always in a direction to resist the motion of the needle, and may be written as $- k_4 \cdot V^2/C \cdot \theta$, where the constant k_4 depends on the construction of the electrometer and C is the capacity of the system connected with the insulated quadrants.[1] The sensitiveness in this case is

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[1] *Cf., e. g.*, R. Beatty, Electrician, 65, p. 729, 1910 and 69, p. 233, 1912; or Makower and Geiger, "Practical Measurements in Radioactivity," p. 6.

$$S = \frac{\theta}{v_2 - v_1} = \frac{k_1 V}{k_2 + k_3 V^2 + k_4 \dfrac{V^2}{C}}, \tag{2}$$

where v_2 and v_1 refer now to the potentials of the quadrants before the needle moves from its zero position,—or their potentials if the needle were prevented from turning. Thus the sensitiveness when one pair of quadrants is insulated is always less than when both are at definite potentials. It is still possible to adjust k_3 in such a manner that the needle borders on instability and the sensitiveness becomes very great, but there is now the difficulty that when the insulated quadrants are grounded the term $k_4 \cdot V^2/C$ vanishes and, if $k_3 V^2$ is greater than k_2, the needle becomes unstable. This difficulty need not be serious, however, if it is possible to adjust k_3 quickly over a sufficiently large range. In any case, the sensitivity is increased by the introduction of the "negative electrostatic control" term $k_3 V^2$.

In order to use a quadrant electrometer at its maximum sensitiveness it is therefore essential to be able to vary the electrostatic control readily over wide limits. A number of practical means of obtaining an adjustable electrostatic control have been discussed by Beatty,[2] and one of these has been used by Parson[3] in his highly sensitive electrometer. When the needle of Parson's electrometer is in its zero position it rests over a slit, whose width can be varied at will. The slit has the effect of repelling the needle, so that by widening the slit the needle may be made to approach a condition of instability. The trouble with this arrangement is that the force on the needle, due to the slit, is not proportional to the angle through which the needle has turned, and the sensitiveness therefore varies over different parts of the scale. Thus, while the electrometer is well adapted to "detection" and use in measurements by a compensation or "null" method, it is not so suitable to use in the usual work where deflections are measured.

We have found that an electrostatic control may be obtained by slightly tilting the electrometer needle about its long axis, and at the same time moving one of the quadrants a little above or below the plane of the other quadrants. This type of control possesses the important advantage of giving nearly constant sensitiveness over a large range of deflections and of permitting quick adjustment to any desired amount of control, either positive or negative.

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[1] J. J. Thomson, Phil. Mag., 46, p. 536, 1898; Makower and Geiger, *loc. cit.*

[2] *Loc. cit.*

[3] A. L. Parson, Phys. Rev., VI., p. 390, 1915.

2. THEORY.

Let one pair of quadrants be displaced vertically a distance δ with respect of the other. The needle is suspended with its center at a distance p above the horizontal plane of symmetry and is tilted about its long axis so that its slope with respect to the horizontal plane is s. Evidently, as the needle deflects through an angle θ, it approaches the sides a and c (Fig. 1) and recedes from the sides b and d of the quadrants, the effect being to change the capacity of the system and therefore to give rise to forces tending to produce or resist further deflection. The magnitude of these forces, which combine to produce the electrostatic control, we shall proceed to calculate.

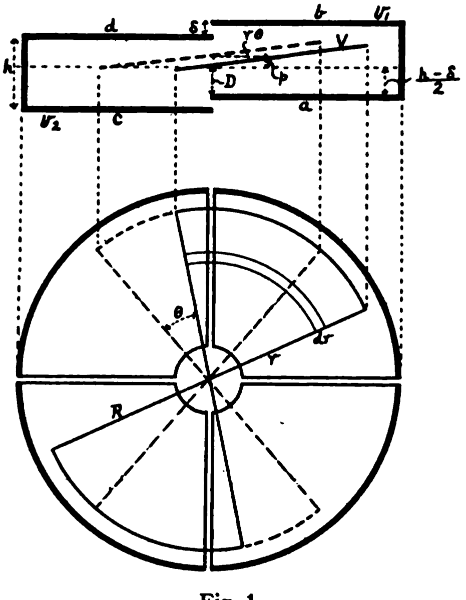

Fig. 1.

Consider first that portion of the needle lying between distances r and $r + dr$ from its center. The distance D between this element and the edge of the side a is

$$D = \frac{h - \delta}{2} + p - sr\theta,$$

where s is the slope $dD/rd\theta$ of the needle and θ is the angle of deflection of the needle from its zero position. The change in the charge induced on the side a by that part of the needle between r and $r + dr$ in turning through an additional infinitesimal angle $d\theta$ is $dq = \sigma r d\theta dr$. The surface density σ is, by Coulomb's law, $\sigma = (V - v_1)/4\pi D$, where V is the potential of the needle and v_1 that of the side a of the quadrant.

Thus

$$\frac{dq}{d\theta} = \frac{V - v_1}{4\pi\left(\frac{h}{2} - \frac{\delta}{2} + p - sr\theta\right)} r\,dr.$$

The change dW in the potential energy of this part of the system is $dW = \frac{1}{2}(V - v_1)dq$. Thus

$$dT_a = \frac{dW}{d\theta} = \frac{1}{2}(V - v_1)\frac{dq}{d\theta}$$

is the torque exerted on the portion of the needle considered by the side a. The total torque exerted on the needle by the side a is therefore

$$T_a = \int_0^R \frac{(V - v_1)^2 r\,dr}{4\pi(h - \delta + 2p - 2sr\theta)},$$

which, when integrated, takes the form of the rapidly converging series

$$T_a = \frac{R^2(V - v_1)^2}{16\pi}\left[\frac{4}{2}\frac{1}{(h - \delta + 2p)} + \frac{8}{3}\frac{Rs\theta}{(h - \delta + 2p)^2} + \frac{16}{4}\frac{R^2s^2\theta^2}{(h - \delta + 2p)^3} + \cdots\right].$$

The torque due to the other faces may be similarly calculated, whence the total torque due to both ends of the needle is

$$T = 2(T_a + T_b + T_c + T_d),$$

which may be written in the most convenient form by grouping separately the coefficients of different powers of θ. Neglecting the term in $(v_2 + v_1)/2$ in comparison with those in V^2 and $V(v_2 - v_1)$, we have

$$\begin{aligned} T = \frac{R^2}{\pi h}\Bigg\{ & V(v_2 - v_1)\left[\left(1 + \frac{\delta^2}{h^2} + 4\frac{p^2}{h^2} + \cdots\right) + \cdots\right] \\ & + V^2\Bigg[-\frac{4\delta p}{h^2}\left(1 + 2\frac{\delta^2}{h^2} + 8\frac{p^2}{h^2} + \cdots\right) \\ & + \frac{8Rs\delta}{3h^2}\left(1 + 2\frac{\delta^2}{h^2} + 8\frac{p^2}{h^2} + \cdots\right)\theta \\ & + \frac{64R^3s^3\delta}{5h^4}\left(1 + 5\frac{\delta^2}{h^2} + 20\frac{p^2}{h^2} + \cdots\right)\theta^3 + \cdots\Bigg]\Bigg\}, \end{aligned}$$

which may be written

$$T = k_1 V(v_2 - v_1) - k' V^2 \delta p - k_3 V^2 \theta - k_5 V^2 \theta^3.$$

This equation for torque due to electrical forces includes all terms whose influence on the action of an electrometer of the type discussed may be detected. The values of the important constants k_3 and k_5 may be taken to be

$$k_3 = -\frac{8R^3s\delta}{3\pi h^3} \quad \text{and} \quad k_5 = -\frac{64R^5s^3\delta}{5\pi h^5}, \tag{3}$$

and it is to be noted that both must have the same sign.

In equilibrium, this torque must be balanced by that due to the suspension, *i. e.*,

$$k_2\theta = k_1V(v_2 - v_1) - k'V^2\delta p - k_3V^2\theta - k_5V^2\theta^3. \tag{4}$$

The term $-k'V^2\delta p$ is of interest because it shows the cause of the deflection from the zero position generally observed when the needle is charged while the quadrants remain grounded. Other possible causes are contact difference of potential between the quadrants due to solder or imperfect "finish" and a tilt of the needle about its short axis,—this being equivalent, as far as the equation is concerned, to a change of θ. With care in construction these latter causes are not effective. In order to use the electrometer, this zero deflection must be prevented, which can be done by reducing either p or δ to zero. It is advantageous to adjust p rather than δ to zero, for by so doing the values of the constants k_3 and k_5 are not appreciably affected. If the zero shift is thus eliminated for one potential V of the needle, it should not appear for any other needle potential unless one or both of the other causes of zero shift are effective. Therefore, when the height of the center of the needle is adjusted to its position ($p = 0$) of symmetry between the quadrants, as judged by the absence of zero shift, the term $k'V^2\delta p$ of equation (3) vanishes and we obtain for the sensitiveness

$$S = \frac{\theta}{v_2 - v_1} = \frac{k_1V}{k_2 + k_3V^2 + k_5V^2\theta^2}. \tag{5}$$

If δ and s are of the same sign, as in Fig. 1, k_3 and k_5 are negative and the sensitiveness is increased, *i. e.*, there is a negative electrostatic control. Similarly if δ and s are of opposite sign, the electrostatic control is positive. Thus, with a given inclination of the needle, the control can be varied through wide limits by moving one pair of quadrants up and down. In practice we have used three of the quadrants in their position of symmetry, in which case the constants k_3 and k_5 have just half the values assigned in equations (3). With negative control, it is theoretically possible to obtain any sensitiveness between zero and infinity, while with positive control there is, for any given value of the constants, a maximum sensitiveness given by placing the derivative of S with respect to V equal to zero. If we neglect the term in k_5, which is very small and does not take a value until the needle has deflected, we obtain for the maximum sensitiveness under positive control

$$\begin{aligned} {}_{+}S_{\max} &= \frac{k_1}{2k_2}\sqrt{\frac{k_2}{k_3}} \\ \text{when} \qquad & \\ V_m &= \sqrt{\frac{k_2}{k_3}}. \end{aligned} \tag{6}$$

Practical difficulties of a mechanical nature prevent the attainment of infinite sensitiveness under negative control. These difficulties will be discussed later.

When the electrometer is thus made very sensitive a relatively long time is required for the needle to come to its position of equilibrium. While the needle is in motion, the air resistance is proportional to the linear velocity of the needle, to its area, and approximately inversely to the distance between the needle and the quadrants. The torque due to air damping may therefore be written $k_6 \cdot R^4/h \cdot d\theta/dt$, where $d\theta/dt$ is the angular velocity of the needle. If I is the moment of inertia of the needle, the equation of its motion is therefore

$$k_1V(v_2 - v_1) - (k_2 + k_3V^2 + k_5V^2\theta^2)\theta - k_6\frac{R^4}{h}\frac{d\theta}{dt} = I\frac{d^2\theta}{dt^2}.$$

By using needles made of thin mica sputtered with platinum (suggested to us by Professor Pegram of Columbia University) or made of thin aluminium leaf, the moment of inertia term may be made negligibly small in comparison with the other terms. In this case the time T required for the needle to return to zero after a deflection is evidently approximately proportional to

$$\begin{aligned} \frac{\theta}{\frac{d\theta}{dt}} &= \frac{k_6R^4}{h(k_2 + k_3V^2 + k_5V^2\theta^2)} \\ &= S\frac{k_6R^4}{hk_1V} = \frac{\pi k_6R^2S}{V}, \end{aligned}$$

by definition of the constant k_1. Thus

$$\begin{aligned} T &\sim S\frac{R^2}{V} \\ \text{and} \qquad & \\ S &\sim T\frac{V}{R^2}. \end{aligned} \tag{7}$$

Equations (7) illustrate the well-known fact that the most advantageous combination of high sensitiveness with short period may be attained with small needles. The lower limit to which the needle may profitably be reduced is limited only by the fact that the needle must turn a mirror of sufficient size to give a well-defined spot of light on the scale. By taking into account the resolving power of the mirror, it can easily be shown that the minimum potential difference which can be detected is proportional to $1/R$ instead of to $1/R^2$, if dimensions of needle and mirror are maintained in a constant ratio. For detecting charge, there is further advantage in reducing the size of the needle and quadrants because the capacity of the electrometer is roughly proportional to the linear dimensions of the needle and quadrants.

3. DESIGN.

Equations (5) and (7) give us information essential to the design of an electrometer to operate in the most satisfactory manner. We have used the electrometer in two sizes, one with a needle of 7 mm. radius and the other with a needle of 4.5 mm. radius. The former is rather easier to operate and adjust, while the latter gives greater sensitiveness for a given period. For very delicate work the size can be profitably reduced still further, and we understand that this has been done by Professor Pegram.

As to the use of the electrostatic control, there are a number of factors to be considered. The presence of the term $k_6 V^2\theta^2$ in equation (5) shows that the sensitiveness is not the same for all values of θ, *i. e.*, over all parts of the scale, though this term is not important except at very high sensitivities. It always tends to increase the control, whether positive or negative, as the electrometer deflects from the zero position. Thus, with positive control the sensitiveness decreases as the needle deflects from zero, and the needle cannot be put into unstable equilibrium. With negative control the sensitiveness increases as the needle deflects from zero and, if the deflection is too large, the needle may move past the point of instability and turn through an angle of 90° to a new position of equilibrium under positive control. Thus the size of the scale deflection which can be used decreases as the sensitiveness increases. This is a disadvantage, but it need not be serious if the design is guided by the following considerations.

Since these difficulties are due to the term in k_6, they are reduced to a minimum by reducing k_6 in relation to k_3. By equations (3) it is seen that the ratio

$$\frac{k_6}{k_3} = \frac{24R^2s^2}{h^2}.$$

It is therefore advantageous to make the tilt s of the needle small and the height h of the quadrants large. In other words, the desired amount of electrostatic control, given by the term k_3V^2, may be most advantageously obtained with a small tilt s of the needle and a relatively large displacement δ of the quadrant. In an actual case in which k_3 was about ten times larger than k_5, the action of the electrometer was apparently independent of the k_5 term for sensitivities below 15,000 mm. per volt, was not seriously disturbed up to 30,000 mm. per volt for deflections less than 100 mm., while at 60,000 mm. per volt the influence of this term was so serious that the electrometer could only be used as a "null" instrument.

Furthermore it is doubly advantageous to have both k_3 and k_5 small, so that the high sensitiveness arises largely from the term $k_1/k_2 \cdot V$. In the first place the high sensitiveness is then reached with little disturbance from the $k_5V^2\theta^2$ term and in the second place the greatest sensitiveness in relation to the period is obtained owing to the larger value of V attainable, as shown by equation (7). It is better, therefore, to use a suspending fiber whose constant k_2 is sufficiently small to permit the attainment of fairly high sensitiveness without electrostatic control and then to attain very high sensitiveness by applying a small negative control than to neutralize the torque of a strong fiber by a very strong control.

It is best, therefore, to use a fairly delicate suspension, to give the needle only a slight tilt and to displace the quadrants only enough to make the electrostatic control important for relatively high potentials of the needle. For example, we usually use a sputtered quartz suspension of such size that the little hook cemented on its end will oscillate with a period between 0.5 sec. and 1.0 sec. The needle is given just enough tilt to be detected by inspection. The movable quadrant is displaced about 0.15 mm. in the direction to produce negative control. Under these conditions extreme sensitivities are reached with needle potentials of 75 or 100 volts. If the electrometer is to be used at high sensitivity for work where the quadrants are to be insulated for considerable periods of time, as in getting a continuous photographic record of an X-ray spectrum, it is better to use a very fine suspension and slight positive control, for the zero position of the needle is then more stable.

The remaining features in the design of an electrometer of this type are obvious: a convenient means of adjusting the height of a quadrant from outside of the electrometer case and a means of adjusting the height of the needle while the potential source is connected with it. Both adjustments must be delicate and free from "lost motion."

4. TYPICAL RESULTS.

Figs. 2 and 3 show the action of the electrometer with various degrees of positive and negative control, respectively. Each represents results obtained with a given suspension and a given tilt of the needle—the various degrees of control being obtained by adjusting the moveable quadrant. If equation (1) is written in the form

$$S = \frac{aV}{1 - bV^2}, \tag{8}$$

the constant b determines the "electrostatic control."

Curves *1*, *2*, *3* and *4* are all coincident at the origin where, obviously, the "control" term vanishes. The straight dotted line represents a condition of zero electrostatic control in which the sensitiveness is strictly proportional to the potential of the needle. This condition was not quite reached in the measurements shown in curve *1*. Curve *2* was obtained by raising the moveable quadrant 0.08 mm., *i. e.*, a quarter turn of the adjusting screw, and readjusting the height of the needle. Curves *3* and *4* indicate further increased amounts of "control." The numbers above the curves represent the period of swing, being the

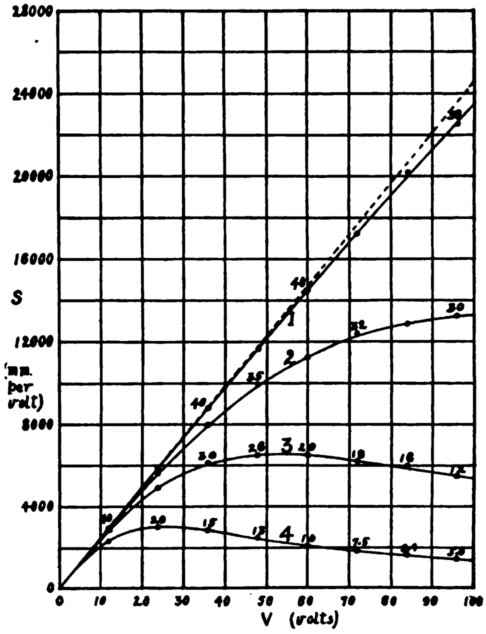

Fig. 2.

time required to come to within 1 mm. of the resting point after a 50-mm. deflection. It is seen that the period is independent of the sensitiveness when there is no electrostatic control, but decreases as the control increases.

Curves *5*, *6*, *7*, *8*, *9* represent various degrees of "negative control," using a stronger suspension than was used in the experiments on "positive control." In this case curve *5* represents zero control, curve *6* the control obtained by lowering the moveable quadrant 0.08 mm., etc. In each case there is a critical potential V_c above which the needle cannot be kept in equilibrium and near which the sensitiveness is very high. This unstable condition is approached very rapidly when the control is strong as shown in curve *9*, where an increase of one fourth volt in the needle potential 24 volts would double the sensitivity. In such a condition the needle is very hard to adjust and is sensitive to movements of the support. For this reason also it is best in practice to use relatively small electrostatic control and relatively high needle potentials, as in curve *6*. The highest sensitiveness at which the zero point was sufficiently steady to permit measurements to be made is indicated by a circle in each case. The figures below the curves give the times required to make half the

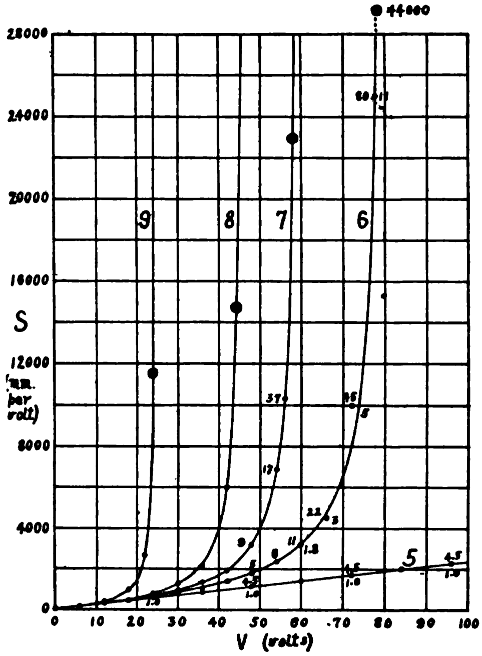

Fig. 3.

deflection, and show that most of the time of deflection is taken up with the final slow drift.

In both sets of curves the solid curves represent graphs of equation (8), with values of the constants indicated in Table I. The constants were

TABLE I.

Curve.	a.	b.	V_c, Volts.	Curve.	a.	b.	V_c, Volts.
1	245.0	0.000004		6	23.7	– 0.000152	81.2
2	245.0	0.000085		7	23.7	– 0.000279	59.7
3	245.0	0.000355		8	23.7	– 0.000475	45.8
4	245.0	0.001650		9	23.7	– 0.001650	24.6
5	23.7	0.000000	∞				

chosen to give best agreement with the experimental measurements, which are recorded as dots. The needle was of 4.5-mm. radius, in quadrants of 6.5 mm. radius. The capacity of the instrument was 10.4 cm. Exactly similar curves were obtained with a larger instrument with a needle of 7-mm. radius and a capacity of 12.5 cm., except that the periods of swing were somewhat longer.

5. PRACTICAL HINTS.

Needles.—We have used mica and aluminium leaf needles, the former being more difficult to make but more satisfactory in operation because of greater rigidity and less air-damping. The mica needles are cut from the thinnest mica by scratching with a sharp needle around the outline of a brass template, cut to the desired shape and pressed firmly on the mica sheet. The fine wire stem is fastened to the needle with sealing wax and the whole system rendered conducting by sputtering beneath a platinum cathode *in vacuo*. While sputtering, the stem must be supported to prevent its falling over if the wax is softened by the discharge. An alternative procedure is to sputter the needle alone, then mount the stem with a touch of sealing wax, and make conducting contact across the wax by a tiny strip of gold leaf, cemented with india ink. The aluminium needles are easily cut from thin leaf held between a metal template and a flat piece of rubber, such as tire or shoe sole rubber, while a sharp-pointed knife blade is passed around the outline. The stem is mounted with soft wax, and is pressed against the needle so as to insure contact. This mounting and the adjustment of the "tilt" is facilitated by the use of a flat piece of metal which may be heated from underneath and against which the needle, held by the stem in a pair of tweezers, may be pressed. If the needle is irregularly warped, difficulties may be encountered in making adjustments and securing uniform scale deflections.

Suspensions.—Quartz fibers, 1 to 2 cm. long are mounted on platinum or silver hooks with sealing wax, and sputtered to obtain a conducting platinum coat. For use with "negative control," these fibers should be the finest ones that can be drawn in the oxy-gas flame by the bow and arrow method, or the coarsest ones that can be drawn out by the flame itself. For use with "positive control," the very finest fibers may be used, the limit being imposed only by the strength requisite for the support of the needle and mirror. The conductivity of the sputtered coat gradually diminishes and the suspensions usually require re-sputtering after about a year of service. If suspensions break, it is usually at the point of joining to the hook. They may then be easily repaired by dipping the end of the hook several times into india ink and then touching with the wet end the free end of the suspension. They are held together at once, and after drying for two or three minutes the suspension is apparently as strong as ever, and retains its conducting properties.

Mirrors.—When working at high sensitivity, the instrument is so much over damped that the moment of inertia may be made quite large without appreciably increasing the period. Thus the mirrors may be larger than might be supposed. We use mirrors with as much as 10 sq. mm. area on the 4.5 and 7.0 mm. instruments.

Ease of Adjustment.—The most serious sources of difficulty appear to be irregularities in the needle or quadrants and contact difference of potential between quadrants. Thus with some instruments it is easy to reach 10,000 or 15,000 mm. per volt sensitivity and in others difficult to reach 10,000 mm. per volt, due to accidental differences in construction. Among the seven or eight instruments of this type in use in our laboratories we have never had difficulty in obtaining 5,000 mm. per volt at the first trial, and usually count on being able to work between 5,000 and 10,000 mm. per volt without trouble. At higher sensitivities more trouble is experienced with zero drift. Difficulty due to contact difference of potential between quadrants may be identified by the failure to secure identical deflections from the zero, with quadrants earthed, when the sign of the potential of the needle is reversed, for this effect is proportional to the first power of the needle potential while all effects due to lack of symmetry depend on the square of the potential and are therefore independent of its sign. If such contact difference of potential is found to be troublesome, it may be removed by cleaning or compensated by a small potential permanently applied to the "earthed" quadrants.

By taking the precautions suggested in this paper we have been able to employ to advantage sensitiveness as high as 50,000 mm. per volt, at 1 meter scale distance.

Valuable suggestions in the design of the electrometer were made by Professor H. L. Cooke, to whom we are greatly indebted.

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TEMPERATURE DISTRIBUTION IN SOLIDS DURING HEATING OR COOLING.

By E. D. Williamson and L. H. Adams.

Synopsis.

Temperature distribution in solids; surface heated at uniform rate. Equations are derived for the following typical shapes: (1) Rectangular parallelopiped. (1a) Long rectangular rod. (1b) Very thin slab. (2) Cylinder. (2a) Long cylindrical rod. (3) Sphere. (4a) Cylindrical tube heated only outside. (4b) Cylindrical tube heated both inside and outside. (5) Spherical shell heated only outside. Results calculated from these equations are tabulated and in some cases shown graphically. These numerical results may be readily applied to the case of similar solids of any size and diffusivity.

Temperature distribution in solids; surface suddenly cooled or heated. Equations are derived for the following shapes: (1) Rectangular parallelopiped. (1a) Long rectangular rod. (1b) Very thin slab. (2) Cylinder. (2a) Long cylindrical rod. (3) Sphere. The distribution inside a sphere at various instants is computed and also the temperature at the center of a slab, square bar, cube, long cylinder, and sphere as a function of the time. These numerical results may be readily applied to the case of similar solids of any size and diffusivity.

Thermal diffusivity; method of measurement involving determination of temperature-time relation at the center of a symmetrical solid whose surface is heated either at a uniform rate or very suddenly. The convenience of this method is pointed out, but practical details are not considered. The equations given are in convenient form for such uses.

IN deciding on the best methods of carrying out various operations in the manufacture of optical glass we found it necessary to have some idea of the temperature gradients in the pieces during heat treatment. While great precision in absolute magnitudes is generally of minor importance in such cases, the only way to gain insight into the question of the variation of the temperature differences with the shape and dimensions of the blocks and the method of heating is actually to work out numerical examples.

While the authors' main interest at the time was in the application to glass manufacture, the equations are perfectly general, as are also all the qualitative deductions made. The numerical computations can be applied to other cases with a very little manipulation owing to the fact that the only physical constant used (κ, the so-called diffusivity constant), occurs, at least when it occurs in any complicated term, multiplied by the time. Hence the values of these terms for different substances are the same at equal values of the product of the time by the diffusivity constant.

The formulas given also indicate methods for the accurate determination of the heat conductivity of solids. For instance, anticipating a little, one of the last equations is for calculating the temperature at the center of a cylinder which is initially at a uniform temperature and is plunged into a well-stirred constant temperature bath. The temperature would not be appreciably disturbed by the presence of a small axial hole as the temperature gradient across the axis is zero. A thermocouple may therefore be introduced and the temperature measurements made which serve to compute the constant required.

The ordinary text-books[1] dealing with heat conduction indicate the necessary mathematical transformations for the discussion of the problems, but there is a lack of actual detailed results available for practical reference. In the following pages will be found a synopsis of what we have found of actual use to us.

Solids of the following shapes are considered: (1) Brick (rectangular parallelopiped). (1*a*) Rod of rectangular section, or brick with two opposite faces insulated. (1*b*) Infinite slab, thin plate, or brick with two pairs of opposite faces insulated. (2) Cylinder. (2*a*) Rod of cylindrical section or cylinder with the flat ends thermally insulated. (3) Sphere.

Two modes of heating are considered: (A) Heating of the surface at a uniform rate. (B) Sudden change of surface temperature, as, for instance, by plunging the block into a constant temperature bath.

In both cases the initial temperature is taken as being uniform throughout.

MATHEMATICAL DISCUSSION.

Nomenclature.

θ° = difference, from the initial temperature, of the point (x, y, z) at time t seconds from start.

κ = diffusivity constant in cm.^{2} per second.

h = number of degrees that the surface changes per second.

t = time in seconds.

The forms[2] of the differential equations for heat conduction suggest

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[1] *E. g.*, Ingersoll and Zobel, Mathematical Theory of Heat Conduction, Ginn & Co. Byerly, Fourier's Series and Spherical Harmonics, Ginn & Co. Carslaw, Introduction to Fourier's Series and Integrals, Macmillan Co.

[2] Case (1)
$$\frac{\partial \theta}{\partial t} = \kappa \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \qquad (1)$$

Case (2)
$$\frac{\partial \theta}{\partial t} = \kappa \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x} \frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial z^2} \right). \qquad [x \text{ radial}, z \text{ axial}] \qquad (2)$$

Case (3)
$$\frac{\partial (x\theta)}{\partial t} = \kappa \frac{\partial^2 (x\theta)}{\partial x^2} \qquad [x \text{ radial}] \qquad (3)$$

that a likely solution may be made up of terms which are the product of an exponential function of the time and a trigonometrical function of the coördinates except in case (2) which is likely to require Bessel's functions. The values of such functions are, however, tabulated in readily accessible form and therefore it is a simple matter to compute particular examples when the equations are known.

EQUATIONS FOR CASE OF LINEAR HEATING.

In the case of linear heating of the surface, if the suggested form apply, the time term must be of the shape $(1 - e^{-\alpha t})$ since θ must vanish for $t = 0$ and must approach a limiting steady state, as t increases, in which all parts of the body change uniformly h degrees per second.

Case 1.—Brick shape (rectangular parallelopiped).—In the discussion of this it is simplest to take the origin of coördinates[3] at the corner of the block. Let $2a$, $2b$, $2c$ be the dimensions of the block. The conditions give $\theta = 0$ when $t = 0$ and $\theta = ht$ over the surface ($x = 0$, $x = 2a$, $y = 0$, $y = 2b$, $z = 0$, $z = 2c$). The suggested form of solution is therefore

$$\theta = ht + \Sigma f(m, n, p) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \sin\frac{p\pi z}{c} (1 - e^{-\alpha t}).$$

where m, n and p are any integers. However many or few terms are included under the Σ sign, the whole expression satisfies the above-mentioned conditions. It remains to be seen whether values can be found for α and $f(m, n, p)$ so that the combination of terms satisfies the differential equation

$$\frac{\partial\theta}{\partial t} = \kappa\Sigma\frac{\partial^2\theta}{\partial x^2}.$$

Equating the values of these differentials leads to

$$h + \Sigma\alpha e^{-\alpha t} f(m, n, p) \sin\frac{m\pi x}{2a} \sin\frac{n\pi y}{2b} \sin\frac{p\pi z}{2c}$$

$$\equiv -\kappa\Sigma f(m, n, p)\left(\frac{m^2\pi^2}{4a^2} + \frac{n^2\pi^2}{4b^2} + \frac{p^2\pi^2}{4c^2}\right)\sin\frac{m\pi x}{2a} \sin\frac{n\pi y}{2b} \sin\frac{p\pi z}{2c} (1 - e^{-\alpha t}).$$

On either side of this identity we have two parts, one containing $e^{-\alpha t}$

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[3] If it be desired to change the origin to the center of the block the only change necessary is to put $X + a$ for x, $Y + b$ for y and $Z + c$ for z, and note that

$$\frac{\sin (2m + 1)\pi(X + a)}{2a} = (-1)^{m+1} \cos\frac{(2m + 1)\pi X}{a}.$$

and the other not. Equating these separately yields

$$\Sigma \alpha e^{-\alpha t} f(m, n, p) \sin\frac{m\pi x}{2a} \sin\frac{n\pi y}{2b} \sin\frac{p\pi z}{2c}$$

$$= \kappa \Sigma f(m, n, p)\left(\frac{m^2\pi^2}{4a^2} + \frac{n^2\pi^2}{4b^2} + \frac{p^2\pi^2}{4c^2}\right) \sin\frac{m\pi x}{2a} \sin\frac{n\pi y}{2b} \sin\frac{p\pi z}{2c} e^{-\alpha t}, \tag{4}$$

and

$$h = -\kappa \Sigma f(m, n, p)\left(\frac{m^2\pi^2}{4a^2} + \frac{n^2\pi^2}{4b^2} + \frac{p^2\pi^2}{4c^2}\right) \sin\frac{m\pi x}{2a} \sin\frac{n\pi y}{2b} \sin\frac{p\pi z}{2c}. \tag{5}$$

(4) yields

$$\alpha = \kappa\left(\frac{m^2\pi^2}{4a^2} + \frac{n^2\pi^2}{4b^2} + \frac{p^2\pi^2}{4c^2}\right). \tag{6}$$

But to evaluate (5) we need to make use of the trigonometric series

$$\frac{\pi}{4} = \sin\frac{\pi x}{2a} + \frac{1}{3}\sin\frac{3\pi x}{2a} + \frac{1}{5}\sin\frac{5\pi x}{2a} +, \text{ etc.}$$

Multiplying together three such series for x, y and z yields

$$\frac{\pi^3}{64} = \Sigma \frac{\sin\frac{m\pi x}{2a} \sin\frac{n\pi y}{2b} \sin\frac{p\pi z}{2c}}{mnp}.$$

where m, n and p take the values of every odd integer. Equation (5) reduces to this if $f(m, n, p)$ be put equal to

$$\frac{64h}{\pi^3} \cdot \frac{1}{mnp\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2} + \frac{p^2\pi^2}{c^2}\right)}. \tag{7}$$

(6) and (7) give the required values of α and $f(m, n, p)$.

Substitution of these values in the suggested form of solution gives

$$\theta = ht - \frac{64h}{\kappa\pi^3} \sum_{\substack{m=1\\n=1\\p=1}}^{\substack{p=\infty\\n=\infty\\m=\infty}} \frac{\sin\frac{(2m-1)\pi x}{2a} \sin\frac{(2n-1)\pi y}{2b} \sin\frac{(2p-1)\pi z}{2c}}{(2m-1)(2n-1)(2p-1)\left(\frac{(2m-1)^2\pi^2}{4a^2} + \frac{(2n-1)^2\pi^2}{4b^2} + \frac{(2p-1)^2\pi^2}{4c^2}\right)} \times \left[1 - e^{-\kappa\left\{\frac{(2m-1)^2\pi^2}{4a^2} + \frac{(2n-1)^2\pi^2}{4b^2} + \frac{(2p-1)^2\pi^2}{4c^2}\right\} t}\right] \tag{8}$$

the form of the constants having been so changed that m, n and p take all integral values from 1 to ∞.

Case 1a (rectangular rod).—This may be treated separately or deduced from the above by assuming c large as compared with a and b.

$$\theta = ht - \frac{16h}{\kappa\pi^2}\sum_{m,n=1}^{m,n=\infty}\frac{\sin\dfrac{(2m-1)\pi x}{2a}\sin\dfrac{(2n-1)\pi y}{2b}}{(2m-1)(2n-1)\left(\dfrac{(2m-1)^2\pi^2}{4a^2}+\dfrac{(2n-1)^2\pi^2}{4b^2}\right)} \times\left[1-e^{-\kappa\left\{\frac{(2m-1)^2\pi^2}{4a^2}+\frac{(2n-1)^2\pi^2}{4b^2}\right\}t}\right]. \quad (9)$$

Case 1b (thin slab).—This may also be treated separately or deduced from the above by assuming b and c large as compared with a, giving

$$\theta = ht - \frac{4h}{\kappa\pi}\sum_{m=1}^{m=\infty}\frac{\sin\dfrac{(2m-1)\pi x}{2a}}{(2m-1)\dfrac{(2m-1)^2\pi^2}{4a^2}}\left\{1-e^{-\frac{\kappa(2m-1)^2\pi^2}{4a^2}t}\right\}. \quad (10)$$

When t becomes sufficiently large (the exact time will be considered later) the exponential term may be dropped out of any of these equations. When this state is reached differentiation yields

$$\frac{\partial\theta}{\partial t} = h \quad \text{and} \quad \Sigma\frac{\partial^2\theta}{\partial x^2} = h.$$

Every portion is heating uniformly at h degrees per second (steady state).

These simplified differential equations in the case of the slab have as solution

$$\theta = ht - \frac{h}{\kappa}\left(ax - \frac{x^2}{2}\right) \quad (11)$$

$$= ht - \frac{ha^2}{\kappa}\left(\frac{x}{a} - \frac{x^2}{2a^2}\right),$$

as can be immediately verified by differentiation.

Those familiar with Fourier's method of expansion will see the identity of the forms; otherwise this may serve as a proof of the expansion,

$$ax - \frac{x^2}{2} = \frac{16a^2}{\pi^3}\sum_{m=1}^{m=\infty}\frac{1}{(2m-1)^3}\sin\frac{(2m-1)\pi x}{2a}$$

between $x = 0$ and $x = a$.

Case 2 (cylinder).—Let a = radius of cylinder and $2b$ = length of axis. Let the origin of coördinates be at the center. Reasoning similar to the above leads to the equation

$$\theta = ht - \frac{8h}{\pi\kappa}\sum_{m,\,n=1}^{m,\,n=\infty}\frac{1}{(2m-1)R_n\left(\dfrac{R_n^2}{a^2}+\dfrac{(2m-1)^2\pi^2}{4b^2}\right)J_1(R_n)}$$

$$\times J_0\left(\frac{R_nx}{a}\right)\sin\frac{(2m-1)\pi(z+b)}{2b}\left\{1-e^{-\kappa\left[\frac{R_n^2}{a^2}+\left(\frac{(2m-1)^2\pi^2}{4b^2}\right)\right]t}\right\}. \qquad (12)$$

In this the R_n's are the roots of $J_0(x) = 0$ and the values are given in a table in Byerly. Tables of the values of $J_0(x)$ and $J_1(x)$ are also given there, and in Gray and Matthew, Treatise on Bessel's Functions.

For those who wish to verify the formula it may be mentioned that only the following very elementary properties of Bessel's functions are required

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2\times 4^2} - \frac{x^6}{2^2\times 4^2\times 6^2} +, \text{ etc.}$$

$$\frac{dJ_0(x)}{dx} = -J_1(x), \qquad \frac{dJ_1(x)}{dx} = J_0(x) - \frac{1}{x}J_1(x),$$

and

$$\sum_{n=1}^{n=\infty}\frac{2}{R_nJ_1(R_n)}J_0\left(R_n\frac{x}{a}\right) = 1.$$

Case 2a (cylindrical rod).—When b becomes large compared with a the above equation reduces to

$$\theta = ht - \frac{2ha^2}{\kappa}\sum_{n=1}^{n=\infty}\frac{1}{R_n^3J_1(R_n)}J_0\left(\frac{R_nx}{a}\right)\left(1-e^{-\frac{\kappa R_n^2}{a^2}t}\right),$$

which simplifies to

$$\theta = ht\frac{h}{4\kappa}(a^2-x^2) + \frac{2ha^2}{\kappa}\sum_{n=1}^{n=\infty}\frac{1}{R_n^3J_1(R_n)}J_0\left(\frac{R_nx}{a}\right)e^{-\frac{R_n^2}{a^2}t} \qquad (13)$$

The same remarks as regards this simplification apply (*mutatis mutandis*) as in the case 1*b*.

Case 3 (sphere).—Let a be the radius of the sphere and let the origin be at the center. The same method gives

$$\theta = ht - \frac{2ha^2}{\kappa}\sum_{m=1}^{m=\infty}\frac{1}{m^3\pi^3(-1)^{m+1}}\left(1-e^{-\frac{\kappa m^2\pi^2}{a^2}t}\right)\frac{a}{x}\sin\frac{m\pi x}{a}.$$

The only difference in proof from Case (1) is that $x\theta$ takes the place of θ.

Simplification yields

$$\theta = ht - \frac{h}{6\kappa}(a^2-x^2) + \frac{2ha^2}{\kappa}\sum_{m=1}^{m=\infty}\frac{1}{m^3\pi^3(-1)^{m+1}}e^{-\frac{\kappa m^2\pi^2}{a^2}t}\frac{a}{x}\sin\frac{m\pi x}{a}. \qquad (14)$$

The similarity of the equations for cases 1*b*, 2*a* and 3 cannot have escaped the reader's attention. For the sake of comparison we move the origin of coördinates to the center of the slab for case 1*b* and write the equations in the same form:

Inf. Slab. $\theta = ht - \frac{h}{2\kappa}(a^2 - x^2)$

$$+ \frac{2ha^2}{\kappa} \sum_{m=1}^{m=8} \frac{1}{Q_m^3(-1)^{m+1}} e^{-\frac{\kappa Q_m^2 t}{a^2}} \cos\left(Q_m \frac{x}{a}\right). \qquad (15)$$

Inf. Cylinder. $\theta = ht - \frac{h}{4\kappa}(a^2 - x^2)$

$$+ \frac{2ha^2}{\kappa} \sum_{m=1}^{m=\infty} \frac{1}{R_m^3 J_1(R_m)} e^{-\frac{\kappa R_m^2 t}{a^2}} J_0\left(R_m \frac{x}{a}\right). \qquad (16)$$

Sphere. $\theta = ht - \frac{h}{6\kappa}(a^2 - x^2)$

$$+ \frac{2ha^2}{\kappa} \sum_{m=1}^{m=\infty} \frac{1}{S_m^3(-1)^{m+1}} e^{-\frac{\kappa S_m^2 t}{a^2}} \frac{a}{x} \sin\left(S_m \frac{x}{a}\right). \qquad (17)$$

In these formulæ Q_m and S_m are written for the sake of brevity for $[(2m - 1)\pi]/2$, and $m\pi$ respectively.

In conclusion of this section, as special cases, found useful in some practical problems, the heating of a hollow tube and spherical shell will be considered briefly. The complete discussion of these problems is somewhat more difficult than of those already attempted, but for most purposes it is only necessary to consider the solution for the steady state with linear cooling. For this case the equations are, for the tube:

$$\frac{\partial \theta}{\partial t} = h \quad \text{and} \quad \kappa\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x}\frac{\partial \theta}{\partial x}\right) = h$$

and the solution is

$$\theta = ht + \frac{hx^2}{4\kappa} + C_1 \ln x + C_2$$

C_1 and C_2 being constants of integration.

Case (4*a*) Suppose heat transfer to take place only at the outside.

This yields as conditions

$$\theta = ht \text{ at } x = a \quad \text{and} \quad \frac{\partial \theta}{\partial x} = 0 \text{ at } x = a_1,$$

where a_1 and a are the internal and external radii.

The equation becomes

$$\theta = ht + \frac{h(x^2 - a^2)}{4\kappa} + \frac{ha_1^2}{2\kappa} \ln \frac{a}{x}. \qquad (18)$$

Case (4*b*).—Suppose heat transfer takes place equally outside and inside.

The conditions then are

$$\theta = ht \quad \text{at} \quad x = a \quad \text{and} \quad x = a_1.$$

The equation for this case becomes

$$\theta = ht + \frac{hx^2}{4\kappa} + h\frac{\left[a_1^2 \log\frac{x}{a} - a^2 \log\frac{x}{a_1}\right]}{4\kappa \log\frac{a}{a_1}}. \tag{19}$$

For very thin tubes either of these last two equations reduces to the case of a thin slab as given at the beginning of the paper, but for a tube whose internal bore is very small each approximates to the case of a solid cylinder.

Case (5).—Similar reasoning leads, in the case of a spherical shell heated linearly on the outer surface, to the following equation for the steady state:

$$\theta = ht + \frac{h(x^2 - a^2)}{6\kappa} + \frac{ha_1^3}{3\kappa}\left(\frac{1}{x} - \frac{1}{a}\right), \tag{20}$$

where a_1 is the internal radius.

Before making any comments on these equations we give in Tables I., II., III. and IV. the results of a number of numerical calculations from equations (15), (16) and (17).

The character of the temperature distribution in a few cases is illustrated graphically in the curves of Figs. 1 and 2.

Calculated Values[4] *of θ*.—We have taken κ equal to 0.004 cm.^1 per sec. which is an average value for glass. *h* in each case is taken as 0.1 deg. per sec.

TABLE I.

Showing temperature distribution in a slab of glass, 2 cm. thick, heated at a rate of 0.1 *deg. per second.* x/a is the fractional distance from the center, θ is the temperature, t is the time in seconds, and A and B are constants of equation 21. (κ = 0.004.)

x/a.	A.	t = 50.		t = 100.		t = 200.		t = 500.		t = 1,000.	
		B.	θ.	B.	θ.	B.	θ.	B.	θ.	B.	θ.
0.000	12.50	7.87	0.37	4.81	2.31	1.79	9.29	0.093	37.59	0.0008	87.50
0.333	11.11	6.82	0.71	4.16	3.05	1.55	10.44	0.080	38.97	.0007	88.89
0.500	9.38	5.57	1.19	3.40	4.02	1.27	11.89	0.066	40.69	.0005	90.62
0.667	6.95	3.94	1.99	2.40	5.45	0.90	13.95	0.046	43.10	.0004	93.05
0.800	4.50	2.43	2.93	1.49	6.99	0.55	16.05	0.029	45.53	.0002	95.50
1.000	0.00	0.00	5.00	0.00	10.00	0.00	20.00	0.000	50.00	0.0000	100.00

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[1] A convenient table of values of e^{-x} will be found in Becker and Van Ostrand's "Hyperbolic Functions." (Published by the Smithsonian Institution.)

TABLE II.

Similar to Table I. but for a slab 10 cm. thick.

x/a.	A.	$t=500$.		$t=1,000$.		$t=2,000$.		$t=5,000$.		$t=10,000$.	
		B.	θ.	B.	θ.	B.	θ.	B.	θ.	B.	θ.
0	312.5	262.7	0.2	217.0	4.5	146.4	33.9	44.8	232.3	6.22	693.7
.333	277.8	229.3	1.5	188.2	10.4	126.8	49.0	38.8	261.0	5.39	727.6
.500	234.4	188.5	4.1	153.8	19.4	103.5	69.1	31.7	297.3	4.40	770.0
.667	173.6	134.3	10.7	108.9	35.3	73.2	99.6	22.4	348.8	3.11	829.5
.800	112.5	83.4	20.9	67.3	54.8	45.2	132.7	13.8	401.3	1.92	889.4
1.000	0.0	0.0	50.0	0.0	100.0	0.0	200.0	0.0	500.0	0.00	1000.0

TABLE III.

Similar to Table I., but for an infinitely long cylinder of radius 5 cm.

x/a.	A.	$t=500$.		$t=1,000$.		$t=2,000$.		$t=5,000$.	
		B.	θ.	B.	θ.	B.	θ.	B.	θ.
0	156.3	107.1	0.8	68.6	12.3	27.2	70.9	1.70	345.4
.333	138.9	91.6	2.7	58.0	19.1	22.8	83.9	1.44	362.5
.500	117.2	73.3	6.1	45.9	28.7	18.2	101.0	1.14	383.9
.667	86.8	50.2	13.4	31.1	44.3	12.3	125.5	0.77	414.0
.800	56.3	29.8	23.5	18.4	62.1	7.3	151.0	0.46	444.2
1.000	0.0	0.0	50.0	0.0	100.0	0.0	200.0	0.00	500.0

TABLE IV.

Similar to Table I., but for a sphere 5 cm. in radius.

x/a.	A.	$t=100$.		$t=500$.		$t=1,000$.		$t=2,000$.		$t=5,000$.	
		B.	θ.	B.	θ.	B.	θ.	B.	θ.	B.	θ.
0	104.2	94.2	0	56.9	2.7	26.1	21.9	5.38	101.2	0.05	395.8
.333	92.6	82.6	0	47.1	4.5	21.6	29.0	4.45	111.9	0.04	407.4
.500	78.1	68.1	0	36.2	8.1	16.6	38.5	3.43	125.3	0.03	421.9
.667	57.9	48.1	0.2	23.5	15.6	10.8	52.9	2.23	144.3	0.02	442.1
.800	37.5	28.9	1.4	13.3	25.8	6.1	68.6	1.26	163.8	0.01	462.5
1.000	0.0	0.0	10.0	0.0	50.0	0.0	100.0	0.00	200.0	0.00	500.0

The columns headed A give the value of the quadratic term in the equation (15, 16 or 17, according to the solid referred to) and under B the values of the last term are tabulated, *i. e.*,

$$\theta = ht - A + B. \tag{21}$$

x/a denotes the fractional distance from center to surface. θ is the temperature.

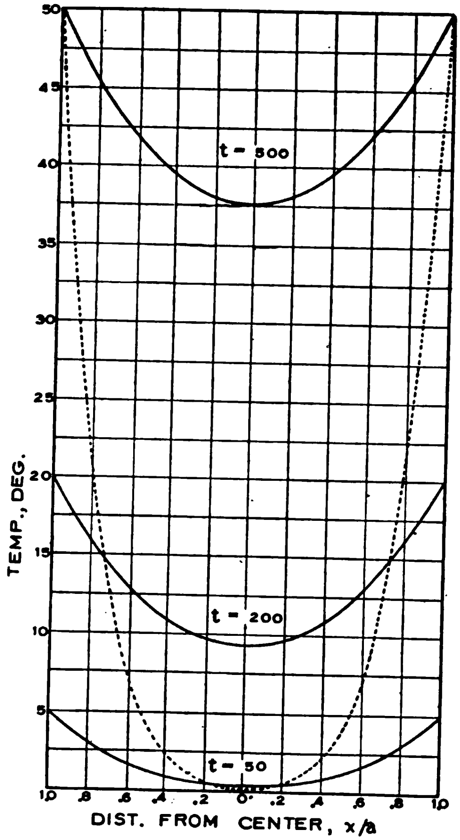

Fig. 1.

Diagram to illustrate distribution of temperature in a slab, the surfaces of which are heated at a uniform rate of 0.1 deg. per sec. The solid lines show the temperature prevailing throughout a slab 2 cm. thick, 50, 200 and 500 seconds respectively after the heating is begun. For the sake of comparison there is included the dotted curve which shows the temperature distribution at the end of 500 sec. in a slab 10 cm. thick which is heated at the same rate. The value of κ, the diffusivity, is taken as 0.004 and the slabs are assumed to be initially at a uniform temperature.

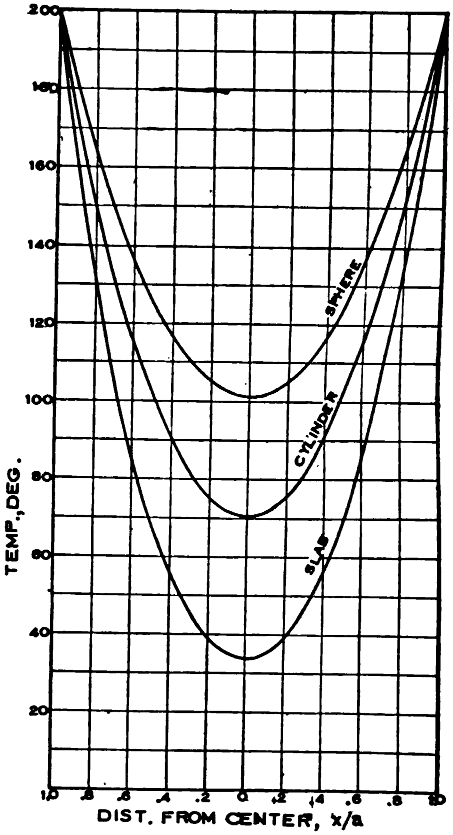

Fig. 2.

Curves showing distribution of temperature after 2000 sec. in the "unidimensional" solids (1) sphere, (2) cylinder of infinite length, and (3) slab. The heating rate is 0.1 deg. per sec., the diffusivity is 0.004, the diameter (or thickness) is 10 cm. and the solids are initially at a uniform temperature throughout.

ADAPTATION OF THESE TABLES TO OTHER NUMERICAL VALUES.

I. *Change of Rate of Heating.*—θ and h are directly proportional, therefore simple multiplication solves this problem. In other words, for a given solid the lag of any point behind the surface temperature is a time lag.

II. *Change of Dimensions.*—If it were not for the exponent in the exponential terms θ would also be directly proportional to a^2. This may be allowed for by changing the time so that by multiplying the values of θ by a_1^2/a^2 we get the values corresponding for the new dimensions at a time equal to $a_1^2/a^2 \cdot t$.

III. *Change of Diffusivity.*—The same remarks apply to κ save that the proportion is inversed. It is therefore necessary to multiply the θ columns by κ/κ_1 to get the values at a time equal to $\kappa/\kappa_1 \cdot t$.

In general[1] if the letters with subscripts apply to the new case

$$\theta_1 = \theta \frac{h_1 a_1^2 \kappa}{h a^2 \kappa_1} \qquad \text{at the time} \qquad t_1 = t \frac{a_1^2 \kappa}{a^2 \kappa_1}.$$

For the three simple cases the temperature differences at the steady state $h/2\kappa \cdot (a^2 - x^2)$, etc., depend on the thickness but the gradients h/κ, $h/2\kappa$, $h/3\kappa$ do not. This evidently means that the temperature distribution in the central inch of say a six-inch slab is the same as in a single one-inch slab, provided h and κ are the same for the two slabs. It follows that in a well-regulated furnace for the case of uniform heating or cooling it is futile to cover slabs of material with sand or other such material in the hope of altering the temperature gradient, as only the lag behind the furnace will be affected and not the actual distribution in the slabs.

At all save the shortest times it is only necessary to use one term of the series in calculating B of equation (21) and hence it is an easy matter to find, for instance, when the value of θ at the center will be within one per cent. of the value at the steady state.

We have only to consider the first term in B and equate it to 0.01 A. This gives in the three simple cases for the values $a = 5$, $\kappa = 0.004$

$$\text{Slab} \qquad \frac{16a^2h}{\pi^3\kappa} e^{-\frac{\kappa\pi^2 t}{4a^2}} = 0.01 \frac{ha^2}{2\kappa},$$

hence $t = 11{,}760$ secs.

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[1] By multiplying both sides of equations (8) to (17) inclusive by κ/ha^2 it is readily seen that the equations may be considered as determining $\kappa\theta/ha^2$ as a function of $\kappa t/a^2$ and x/a, all of which are dimensionless quantities. To illustrate the use of Tables I, II, III, and IV for different values of h, κ, a^2 and t let us take the following example: for a slab, when $x/a = 0.5$, $t = 100$, $\kappa = 0.004$, $h = 0.1$ and $a = 1$, according to Table I, $\theta = 4.02$. Therefore when $x/a = 0.5$ $\kappa_1 = 0.008$, $h_1 = 0.2$, and $a_1 = 5$, it follows that

$$\theta = 4.02 \frac{0.2 \times 25 \times 0.004}{0.1 \times 1 \times 0.008} = 100.5$$

at the time

$$t = 100 \frac{25 \times 0.004}{1 \times 0.008} = 1250.$$

Likewise, Tables I, II, III, and IV can be converted into tables for new values of h, κ and a multiplying t and θ by the factors given in the text. The factor for θ also applies to A and B.

Similarly

Cylinder $t = 5{,}080$ secs.
Sphere $t = 3{,}040$ secs.

Comparison with the tables will show that this fits the facts.

In calculation we found that after t had a value only one hundredth of these values only one term was needed in the calculation of B to an accuracy of about one per cent.

EQUATIONS FOR SUDDEN CHANGE OF SURFACE TEMPERATURE.

The other method of heating considered is by sudden change of the surface from one temperature to another of a solid originally at a uniform temperature. This approximates to a number of physical problems, such as the cooling of the earth, or the cooling of a solid which is plunged into an ice bath.

The equations for such a case can be deduced by the use of exactly the same relations as in the previous discussion. We therefore merely give the equations without comment. The verification is simple by the methods already shown.

In the equations θ is used as before for the temperature at any point. θ_0 is the original uniform temperature and θ_1 is the new temperature of the surface. In all cases the origin is at the center.

(1) Brick shape

$$\frac{\theta-\theta_1}{\theta_0-\theta_1}=\frac{64}{\pi^3}\sum_{m,n,p=1}^{m,n,p=\infty}\frac{\cos\frac{(2m-1)\pi x}{2a}\cos\frac{(2n-1)\pi y}{2b}\cos\frac{(2p-1)\pi z}{2c}}{(2m-1)(2n-1)(2p-1)(-1)^{m+n+p+1}} \times e^{-\kappa\left(\frac{(2m-1)^2\pi^2}{4a^2}+\frac{(2n-1)^2\pi^2}{4b^2}+\frac{(2p-1)^2\pi^2}{4c^2}\right)t}, \tag{22}$$

(1*a*) Rectangular rod.

$$\frac{\theta-\theta_1}{\theta_0-\theta_1}=\frac{16}{\pi^2}\sum_{m,n=1}^{m,n=\infty}\frac{\cos\frac{(2m-1)\pi x}{2a}\cos\frac{(2n-1)\pi y}{2b}}{(2m-1)(2n-1)(-1)^{m+n+1}} \times e^{-\kappa\left(\frac{(2m-1)^2\pi^2}{4a^2}+\frac{(2n-1)^2\pi^2}{4b^2}\right)t}, \tag{23}$$

(1*b*) Slab.

$$\frac{\theta-\theta_1}{\theta_0-\theta_1}=\frac{4}{\pi}\sum_{m=1}^{m=\infty}\frac{\cos\frac{(2m-1)\pi x}{2a}}{(2m-1)(-1)^{m+1}}e^{-\kappa\left(\frac{(2m-1)^2\pi^2}{4a^2}\right)t}, \tag{24}$$

(2) Cylinder.

$$\frac{\theta-\theta_1}{\theta_0-\theta_1}=\frac{8}{\pi}\sum_{m,n=1}^{m,n=\infty}\frac{(-1)^{m+1}}{2m-1}\frac{J_0(R_nx/a)}{R_nJ_1(R_n)}\cos\frac{(2m-1)\pi y}{2a} \times e^{-\kappa\left(\frac{Rn^2}{a^2}+\frac{(2m-1)^2\pi^2}{b^2}\right)t}, \tag{25}$$

(2*a*) Cylindrical rod.

$$\frac{\theta - \theta_1}{\theta_0 - \theta_1} = 2 \sum_{m=1}^{m=\infty} \frac{J_0\left(R_m \frac{x}{a}\right)}{R_m J_1(R_m)} e^{-\kappa \frac{R_m^2}{a^2} t}, \tag{26}$$

(3) Sphere.[1]

$$\frac{\theta - \theta_1}{\theta_0 - \theta_1} = \frac{2}{\pi} \sum_{m=1}^{m=\infty} \frac{\frac{a}{x} \sin \frac{m\pi x}{a}}{m(-1)^{m+1}} e^{-\kappa \frac{m^2\pi^2}{a^2} t}, \tag{27}$$

The last equation is given in Byerly (p. 116, ex. 2).

The next table gives the values of $(\theta - \theta_1)/(\theta_0 - \theta_1)$ $(= F)$ for a sphere for different distances from the center and for different values of $\kappa t/a^2$.

Table V.

Table for calculating the temperature at any time of any point in a sphere originally at a uniform temperature the surface of which is suddenly changed to another temperature.

Values of $F\left(= \frac{\theta - \theta_1}{\theta_0 - \theta_1}\right)$.

x/a.	$\kappa t/a^2 = .000$.	$\kappa t/a^2 = .0040$.	$\kappa t/a^2 = .0160$.	$\kappa t/a^2 = .0360$.	$\kappa t/a^2 = .0640$.	$\kappa t/a^2 = .1000$.	$\kappa t/a^2 = .1960$.	$\kappa t/a^2 = .2560$.	$\kappa t/a^2 = .4000$.	$\kappa t/a^2 = \infty$.
.0000	1.0000	1.0000	1.0000	.9943	.9103	.7071	.2881	.1598	.0386	0.0000
.0500	1.0000	1.0000	1.0000	.9938	.9079	.7046	.2869	.1590	.0385	0.0000
.2500	1.0000	1.0000	.9997	.9790	.8577	.6466	.2596	.1439	.0347	0.0000
.3333	1.0000	1.0000	.9994	.9611	.8133	.6005	.2386	.1321	.0319	0.0000
.5000	1.0000	1.0000	.9896	.8752	.6755	.4745	.1840	.1018	.0246	0.0000
.6667	1.0000	.9997	.9063	.6788	.4727	.3162	.1197	.0661	.0160	0.0000
.7500	1.0000	.9931	.7921	.5312	.3537	.2319	.0869	.0480	.0116	0.0000
.9500	1.0000	.3935	.1791	.1030	.0644	.0411	.0152	.0084	.0020	0.0000
1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

For a sphere of glass where $\kappa = 0.004$ and $a = 10$, the columns give the temperature distribution at 0, 100, 400, 900, 1,600, 2,500, 4,900, 6,400, 10,000 seconds respectively. On the other hand, for a similar sphere where $a^2 = 1{,}000$, the successive times would be 0, 1,000, 4,000,

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[1] The equations for cases (1*b*), (2*a*) and (3) closely resemble the final term in the corresponding equations for linear cooling. This is brought out more clearly by writing them in the shortened notation used above. They become:

(1*b*)
$$F = 2 \sum_{m=1}^{m=\infty} \frac{1}{Q_m(-1)^{m+1}} e^{-\frac{\kappa Q_m^2 t}{a^2}} \cos\left(\frac{Q_m x}{a}\right). \tag{28}$$

(2*a*)
$$F = 2 \sum_{m=1}^{m=\infty} \frac{1}{R_m J_1(R_m)} e^{-\frac{\kappa R_m^2 t}{a^2}} J_0\left(\frac{R_m x}{a}\right). \tag{29}$$

(3)
$$F = 2 \sum_{m=1}^{m=\infty} \frac{1}{S_m(-1)^{m+1}} e^{-\frac{\kappa S_m^2 t}{a^2}} \frac{a}{x} \sin\left(\frac{S_m x}{a}\right). \tag{30}$$

etc. For the earth, if the diffusivity constant be about 0.008, the times would be 0, 6.4×10^9 years, 25.6×10^9 years, etc.

The curves of Fig. 3 were obtained by plotting the fractional temperature difference $(\theta - \theta_1)/(\theta_0 - \theta_1)$ in a sphere, against the distance from the center x/a expressed in fractional parts of the radius for different values of $\kappa t/a^2$. In Fig.[1] 4, on the other hand, $(\theta - \theta_1)/(\theta_0 - \theta_1)$ is plotted against $\kappa t/a^2$ for different values of x/a.

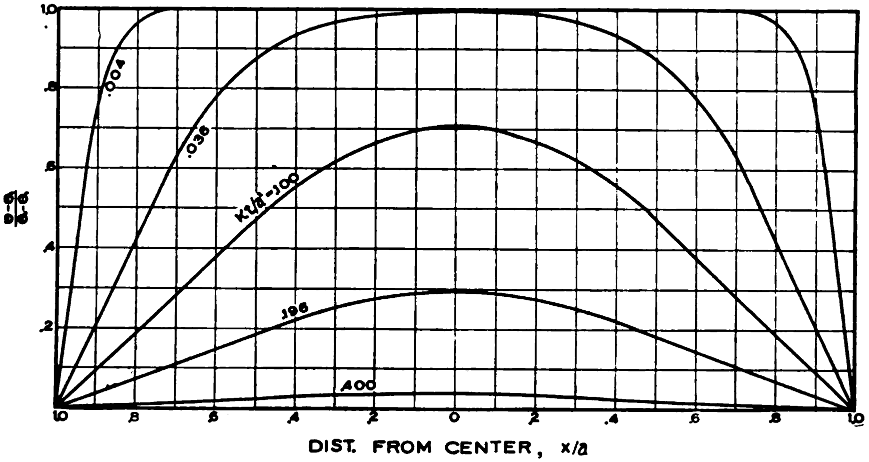

Fig. 3.

Diagram to show variation of temperature θ in a solid sphere after the temperature of the surface has been suddenly changed from θ_0 to θ_1 (temperature of sphere originally uniform throughout). The ordinate is the fractional temperature difference, $(\theta - \theta_1)/(\theta_0 - \theta_1)$, and the abscissa is the distance from the center expressed as fractional parts of the radius. The successive curves are for various values of $\kappa t/a^2$, the diffusivity multiplied by the time and divided by the square of the radius. Example: find the temperature in the center of a sphere of glass (24 cm. in diam.) originally at 100°, after being placed for 1 hour ($=$ 3,600 sec.) in a well-stirred bath at 0°; take $\kappa = 0.004$, hence $\kappa t/a^2 = 0.1$, then from the appropriate curve it is seen that $(\theta - \theta_1)/(\theta_0 - \theta_1) = 0.71$. Hence $\theta = 71°$.

It will be noticed that equations (22)–(27) are considerably less involved than those used for the uniform heating case. In particular, the more involved are products of the less involved, *e. g.*, for Case (2) the equation is simply the product of those for (1*b*) and (2*a*) which are the simplest ones. It is therefore only necessary to calculate for (1*b*) and (2*a*), and the other follows.

Very frequently it is only the temperature at the center that is required, so we have calculated this for a few cases. When $x = 0$ all the cosine and J_0 terms are equal to unity, so the equations reduce to comparatively simple forms containing only the exponentials as variables.

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[1] In plotting $(\theta - \theta_1)/(\theta_0 - \theta_1)$ against t or $\kappa t/a^2$, it is convenient to use "semi-log," paper.

(1*b*) Slab, at center.

$$\frac{\theta - \theta_1}{\theta_0 - \theta_1} = \frac{4}{\pi} \sum_{m=1}^{m=\infty} \frac{(-1)^{m+1}}{2m - 1} e^{-\kappa \frac{(2m-1)^2 \pi^2}{4a^2} t}. \tag{31}$$

(2*a*) Cylindrical rod, at center.

$$\frac{\theta - \theta_1}{\theta_0 - \theta_1} = 2 \sum_{m=1}^{m=\infty} \frac{1}{R_m J_1(R_m)} e^{-\kappa \frac{R_m^2}{a^2} t}. \tag{32}$$

(3) Sphere, at center.

$$\frac{\theta - \theta_1}{\theta_0 - \theta_1} = 2 \sum_{m=1}^{m=\infty} (-1)^{m+1} e^{-\kappa \frac{m^2 \pi^2}{a^2} t}. \tag{33}$$

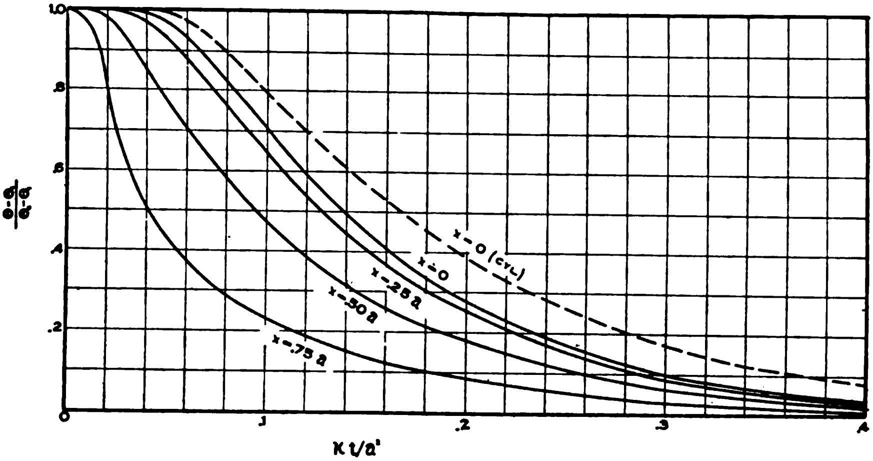

Fig. 4.

For same conditions as Fig. 3 except that $(\theta - \theta_1)/(\theta_0 - \theta_1)$ is plotted against $\kappa t/a^2$ for various values of x/a. These graphs are therefore cooling curves for various points in the sphere. In addition the figure contains a broken-line cooling curve which is a cooling curve for the center of a cylinder whose length is very great compared to its diameter.

Tables of exponentials are easily available, and in Table VI. the first ten values of R_n and $J_1(R_n)$ are given since these may not always be conveniently obtained.

TABLE VI.[1]

Values of R_n (roots of $J_0(x) = 0$) and $J_1(R_n)$.

n.	R_n.	$J_1(R_n)$.	n.	R_n.	$J_1(R_n)$.
1	2.4048	0.51915	6	18.071	−.1877
2	5.5201	− .34026	7	21.212	.1733
3	8.654	.2714	8	24.352	−.1617
4	11.792	− .2325	9	27.493	.1522
5	14.931	.2065	10	30.635	−.1422

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[1] The values in this table are taken from Gray and Matthews, Treatise on Bessel Functions (Macmillan Co.).

In Table VII. are tabulated the values of $(\theta - \theta_1)/(\theta_0 - \theta_1)$ at the center under various conditions and with solids of various shapes.

TABLE VII.

Values of $(\theta - \theta_1)/(\theta_0 - \theta_1)$ *at the center of solids of various shapes for case of sudden change in temperature of surface.* In the heading of column 1, a is the radius, t is the time and κ is the diffusivity constant.

$\kappa t/a^2$.	Slab.	Square Bar.	Cube.	Cylinder of Infinite Length.	Cylinder of Length = Diam.	Sphere.
0	1	1	1	1	1	1
.032	.99983	.9997	.9995	.9990	.9988	.9975
.080	.9752	.9510	.9274	.9175	.8947	.8276
.100	.9493	.9012	.8555	.8484	.8054	.7071
.160	.8458	.7154	.6051	.6268	.5301	.4087
.240	.7022	.4931	.3462	.3991	.2802	.1871
.320	.5779	.3340	.1930	.2515	.1453	.0850
.800	.1768	.0313	.00553	.0157	.00277	.000745
1.600	.0246	.00060	....	.00015	....	....
3.200	.00047	....	....	....	....	....

Approximate Formulæ for Short Times.—When the time interval from the beginning is so small that the heating effect is negligible at the center—at least to the order of magnitude considered—the problem may be considered as that of a heat flow into an infinite solid. For still shorter times when only the surface layers need be considered, the curvature may be disregarded in the case of the cylinder and sphere and the case of the slab alone need be considered. In this case the well-known formula used by Lord Kelvin[1] is very convenient if tables of the so-called probability integral are available. In the notation used in this paper the formula would be

$$\frac{\theta - \theta_1}{\theta_0 - \theta_1} = \frac{2}{\sqrt{\pi}} \int_0^{x/2\sqrt{\kappa t}} e^{-z^2} dz, \tag{34}$$

where z is merely an integration variable.

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Washington, D. C., April, 1919.

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[1] Thomson and Tait's Treatise on Natural Philosophy.

THE SOLUTION OF CIRCUIT PROBLEMS.

Mathematical Methods of Investigation Resulting from the Application of Fourier's Integral.

By Thornton C. Fry.

Synopsis.

Circuit theory; current in any network; expansion of fundamental integral in Bessel's series. This is treated quite fully, and is shown to be applicable to circuits with either lumped or distributed impedances. If the upper limit for the roots of the equation for the impedance function is known, the problem may be reduced to the expansion of a function in a Fourier series. By this method, the principal difficulties of computation are included in those processes which can be mechanically performed. This method is, therefore, a valuable addition to those commonly used in the solution of problems relating to complicated circuits.

Circuit theory; current in any network; expansion of fundamental integral. Summary of treatment (1) using Heaviside-Carson expansion, and (2) using Taylor's series, with a discussion of the limitations of each method.

Integral of $e^{in(t-\lambda)}\,dn/Z(in)$ *from* $-\infty$ *to* $+\infty$; *expansion in Bessel's series.* Treated quite fully, especially in relation to circuit theory.

Telegrapher's equation; solution by expansion of an integral in Bessel's series. This is treated quite fully, with simple examples.

1. Introduction.

The following paper sets forth in a condensed form a number of methods of computation which the writer has found valuable in considering the types of differential equations which arise in connection with electrical circuit theory. Naturally, in dealing with such problems the investigator lays greater stress upon the utility of the methods and results than upon their mathematical elegance; and this paper will reflect that emphasis. However, a number of theoretical problems of real mathematical interest, whose discussion can not be undertaken in this place, suggest themselves in the course of the study. An attempt to consider some of them independently will be found illuminating by those who are mathematically inclined. Perhaps the most attractive of these deal with the application of the modern theories of generalized integrals to the Fourier integral identity. This problem is not yet satisfactorily handled in any of the literature with which the writer is familiar. Others relate to the propriety of differentiating and integrating Stieltjes' integrals under the sign of integration. These problems will be passed with little or no attention, even in those cases where a rigorous investigation of them has been undertaken.

The paper will be found to separate itself into two distinct divisions. That which precedes section 5 is in the main a recapitulation of the work of other writers arranged in such a form as to introduce some of the principal mathematical ideas which are required in the development of the remainder of the paper. Certain portions of Heaviside's electromagnetic theory are very similar in content to sections 3 and 4. His presentation is quite different—Heaviside appears to have had an even greater contempt for analytic functions than for other conventional mathematics—and results in slightly less general formulæ, but otherwise the two are much alike. More recently Mr. J. R. Carson has achieved very similar results by the application of integral equations, using for the most part rigorous mathematical processes and arriving at very general formulæ.[1]

Beginning with section 5, the discussion diverges sharply from the general theory of the subject and is believed to be quite new. So far as the writer is aware, the only investigation which touches similar lines is that of Poincaré in L'Eclair Électrique for 1904.

No attempt has been made in any part of the paper to present extensive applications of the methods developed. Where specific problems are introduced at all, they are of the simplest possible sort and are chosen for their didactic value only. Furthermore, in order to conform with the electrical terminology used throughout the entire discussion, these problems have all been chosen from the domain of electrical circuit theory. It must not be inferred, however, that the methods themselves have no other applications, for there is a wide range of problems in mechanics, heat, elasticity and sound which may be formulated as either ordinary or partial linear differential equations. Many of these equations are analogous to those met in circuit theory and are amenable to like treatment.

It is essential to have a clear conception at the start of what is meant by the expression "circuit theory." In the first place, the term is not limited to a discussion of steady or sustained phenomena only; but includes the consideration of those more or less indefinite periods following abrupt changes during which the circuit is passing from one sustained state to another. In the second place, it is not limited to a discussion of those circuits only in which the impedances may with sufficient accuracy be regarded as concentrated at discrete points, but includes the investigation of the behavior of circuits in which some or all of the impedances are distributed continuously over stretches of line.

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[1] Although Mr. Carson's paper had not appeared in print at the time the above was written, a considerable portion of the contents had been given the author in the form of preliminary memoranda.

In its most general form the circuit problem may be stated somewhat as follows:

At some point of an electrical system an electromotive force is applied. This force, which may vary in any way with the time, causes unique electromotive forces and currents to subsist in all other parts of the system; and these can conceivably be measured as functions of the time. It is required to determine mathematically what the results of such measurements would be, if they were carried out.

Two restrictions upon the breadth of the above statement are generally made in dealing with circuit theory. They are (1) that the set of differential equations satisfied by the electrical system is linear, and (2) that all physical systems are to some extent dissipative. Both of these conditions are supposed to apply to all circuits treated in this paper. The first justifies the principle of superposition of which use is continually made. The second will be found necessary in determining the positions of the poles of certain analytic functions which introduce themselves quite naturally into the discussion.

2. General Theory.

All of the practicable schemes for solving the transient type of circuit problem are made to depend upon a known steady state solution. The same is true of the scheme which is about to be developed. Suppose, then, that an electromotive force of frequency $n/2\pi$ is imposed upon the jth mesh of a network; and that in consequence of this electromotive force a steady state current of amplitude I_k flows in the kth mesh. These are related by the equation

$$Z_{jk}(in)I_k e^{int} = E_j e^{int},$$

which forms the definition of the alternating current impedance $Z_{jk}(in)$. That is

$$Z_{jk}(in) = \frac{E_j}{I_k}.$$

If the current and electromotive force are related by a known differential equation, it is well known that $Z_{jk}(in)$ may be obtained formally by replacing the differential operators $\partial/\partial t$, $\partial^2/\partial t^2$, $\cdots$ by in, $(in)^2$, $\cdots$. Similarly an operator of the type $\int dt$, must be replaced by $1/in$. Since practically all ordinary circuit problems which do not involve distributed constants can be formulated in this fashion, it follows that for all such problems the impedance Z is a single valued, rational algebraic function of n;—that is, it is the quotient of two polynomials. This condition need not be satisfied when distributed constants occur; and indeed is not satisfied in the problem considered in section 7. But whatever the form of Z, the *steady state* equation,

$$I_k e^{int} = \frac{E_j}{Z_{jk}(in)} e^{int}, \tag{1}$$

applies by definition.

Now suppose an electromotive force imposed which is an arbitrary function of the time. With certain restrictions upon the form of this function which will be unimportant for the purposes of this paper it may be expressed by means of the Fourier identity in the form

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dn e^{int} \int_{-\infty}^{\infty} d\lambda E(\lambda) e^{-in\lambda}. \tag{2}$$

This is formally a summation of steady state electromotive forces of the type $A(n)e^{int}$, each frequency $n/2\pi$ having its appropriate amplitude. The number of these component forces is infinite to the order of the linear continuum, for the integral involves every possible real frequency n.

If, however, the principle of superposition applies to a given electrical system so long as the number of component electromotive forces is finite, it is physically improbable that it should fail to apply when they become infinite in number. The assumption that it does apply leads to the identity

$$I(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dn \frac{e^{int}}{Z(in)} \int_{-\infty}^{\infty} d\lambda E(\lambda) e^{-in\lambda}. \tag{3}$$

Of course, where necessary, subscripts may be used upon I, Z and E in the fashion indicated above. Upon inverting the order of integration there results the form of this expression which it will be most convenient to use in the pages following:

$$I(t) = \int_{-\infty}^{\infty} d\lambda E(\lambda) \Omega(\lambda, t), \tag{4}$$

where

$$\Omega(\lambda, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dn \frac{e^{in(t-\lambda)}}{Z(in)}.$$

3. THE Ω-INTEGRAL. FIRST METHOD OF EVALUATION. HEAVISIDE-CARSON EXPANSION.

The problem has now been reduced practically to an evaluation of the function Ω. Several distinct methods of doing this are available. The first to be used in point of time, so far as the writer is aware, is applicable only to those functions whose singularities are not essential, and hence cannot be used in the cable problem; but it is quite simple, and serves as a very convenient introduction to the more complicated ones.

If the function $1/Z(in)$ behaves at infinity in such a way that the integral

$$\int \frac{dn}{Z(in)n}$$

taken about a circle[1] of radius N approaches the finite limit $2\pi il$ as N approaches infinity, then[2]

$$\frac{1}{Z(in)} = l - i\Sigma \frac{1}{(n - n_j)\dfrac{dZ(in_j)}{dn}},$$

the summation being extended over all the roots, n_j, of the equation

$$Z(in) = 0.$$

Substituting in equation (4) we arrive at the result

$$\Omega(\lambda, t) = \frac{l}{2\pi}\int_{-\infty}^{\infty} dn e^{in(t-\lambda)} + \frac{1}{2\pi i}\Sigma \frac{1}{\dfrac{dZ(in_j)}{dn}}\int_{-\infty}^{\infty} dn \frac{e^{in(t-\lambda)}}{n - n_j}.$$

This expression has a meaning if none of the quantities n_j is real. This condition is satisfied, for if any one of these quantities (say n_1) were real, the system would not dissipate disturbances of the frequency $n_1/2\pi$; which is contrary to the hypothesis that all physical systems possess dissipation. What is more, if any n_j belonged to the lower half of the complex plane, it would correspond to an oscillation of constantly increasing amplitude in the absence of any driving force, which is equally absurd. Hence all the roots lie above the real axis.[3]

Now it is possible to show, for a function of this type, that the integral along the real axis is the same as the integral about one of the closed paths shown in Fig. 1, except for the insolated value $\lambda = t$. The result of the integration is therefore zero, if $\lambda > t$; and equal to $2\pi i$ times the residue at n_j if $\lambda < t$. That is

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[1] The assumption is that the path closes, as it will if the singularities are not essential. If it were not to close, a corresponding expansion involving a continued fraction or a divergent series would frequently be possible, according to Stieltjes' theory. This, however, is unimportant from any but a purely mathematical standpoint.

[2] Provided no two roots n_j are equal. If there are repeated roots the expansion still applies, but the formula is slightly more complicated than here stated.

It is to be noticed that the quantity $dZ(in_j)/dn$ has had the constant n_j substituted for the variable n, so that it is no longer a function of this variable.

[3] In those cases where it is desirable to idealize a problem by neglecting dissipation, the path of integration must be regarded as passing slightly below the real axis, and avoiding the real roots of Z.

$$\Omega(\lambda, t) = \frac{l}{2\pi}\int_{-\infty}^{\infty} dn\, e^{in(t-\lambda)} + \Sigma \frac{e^{in_j(t-\lambda)}}{\frac{dZ(in_j)}{dn}}.$$

This is the desired expression for Ω. The corresponding formula for I is obtained by substituting the value of Ω in (4) and identifying the first integral with $E(t)$:

$$I(t) = lE(t) + \Sigma \frac{e^{in_j t}}{\frac{dZ(in_j)}{dn}} \int_{-\infty}^{t} d\lambda E(\lambda) e^{-in_j\lambda}. \tag{5}$$

Formula (5) is a very remarkable formal solution of the circuit problem. By far the most important case of it is that in which the driving force is zero for $t < 0$, and simply periodic thereafter. In this case, if the frequency is $\nu/2\pi$, it may be found by integration and simple transformations that

$$I(t) = lE(t) + \frac{e^{i\nu t}}{Z(i\nu)} + i\Sigma \frac{e^{in_j t}}{(\nu - n_j)\frac{dZ(in_j)}{dn}}. \tag{6}$$

In particular, if $\nu = 0$,

$$I(t) = lE(t) + \frac{1}{Z(0)} - i\Sigma \frac{e^{in_j t}}{n_j \frac{dZ(in_j)}{dn}}. \tag{7}$$

Formula (7) is quite old. It was developed by Heaviside, and has come to be known as Heaviside's expansion. Formula (6) is more modern, and is the work of Mr. J. R. Carson.[1]

Two difficulties present themselves in the use of (5). In the first place it requires a knowledge of the free periods n_j of the system, which more frequently than not are unobtainable. In the second place, in the case of structures of a large number of degrees of freedom, the number of terms becomes so great as to render the labor of actual computation prohibitive. No more space need be devoted to it here, except to make an observation which has a bearing on all of the further work.

It has been tacitly assumed in the above discussion that the isolated value $\lambda = t$ contributed nothing to our solution. This of course is true, provided the value of the n-integral for $\lambda = t$ is finite, since the product of this integral by $d\lambda$ then approaches zero with $d\lambda$. But if the n-integral is not finite a special investigation is always necessary.

As a matter of fact, it is possible to develop by very plausible arguments the following rule:

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[1] PHYS. REV., 1917.

The isolated critical value $\mu = t$ contributes nothing to the value of $I(t)$ unless $Z(\infty) = 1/l$ is a finite quantity. If this condition is satisfied the contribution of the critical value is $lE(t)$. This amount must therefore be added to the result obtained by ignoring the critical value of the integral.

It will be noted that in examining an integral of the type

$$\int_{-\infty}^{\infty} \frac{e^{in(t-\lambda)}}{n - n_j}\, dn,$$

$n - n_j$ takes the place of $Z(n)$ in the general case. Hence for such an integral $Z(\infty) = \infty$, $l = 0$, and the contribution of the critical value is zero.

This investigation must be made in connection with each problem solved; but the rule of thumb given above renders it a matter of a moment's work only. Physically, the result derived will always be zero, unless the circuit permits of the propagation of a discontinuous current. If the circuit does permit discontinuities, they will be cared for by the term introduced by an infinite λ-integrand at $\lambda = t$.

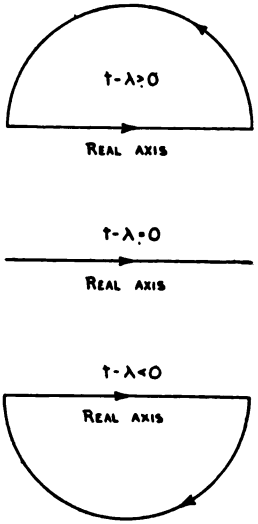

Fig. 1.

4. The Ω-Integral: Second Method of Evaluation. Power Series Expansion.

If the function $Z(in)$ has no singularities at infinity the Ω-integral may be evaluated around one of the paths shown in Fig. 1, whether or not it is algebraic. This fact may be established by applying to the integrals involved an appropriate extension of the Cesaro sum of a divergent series. The only requirement is that there be no singularities of the integrand occurring for finite values of n greater in absolute value than the radius N of the semi-circular path. Since there are no singularities below the real axis, we have $\Omega = 0$, $\lambda > t$, as before; while for $\lambda < t$ the path may be further deformed into a complete circle of radius N, with the origin as its center. Since, however, $Z(in)$ has no singularities of any kind in the portion of the finite plane outside our path of integration, it may be expressed in the uniformly convergent series

$$Z(in) = \sum_{s=0}^{\infty} \frac{a_s}{n^s},$$

which may be integrated term by term, even after multiplication by the factor $e^{in(t-\lambda)}$. This leads us to the formula

$$\begin{aligned}\Omega(\lambda, t) &= \frac{1}{2\pi}\sum_{s=0}^{\infty} a_s \int dn \frac{e^{in(t-\lambda)}}{n^s} \\ &= \frac{a_0}{2\pi}\int_{-\infty}^{\infty} e^{in(t-\lambda)}dn + \sum_{s=0}^{\infty} \frac{a_s i^s}{\underline{|s-1}}(t-\lambda)^{s-1}.\end{aligned} \tag{8}$$

which, substituted in (4) leads to an expansion of $I(t)$ which converges very rapidly for small values of t. The first term is at once recognized as leading to $a_0E(t)$.

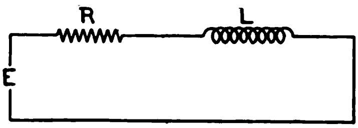

Fig. 2.

For large values of t it is virtually impossible to use this form of expression for purposes of computation; though it has sufficient value otherwise to justify its presentation. It is analogous to expansions derived by other means by Heaviside and by Carson.

As an example of its application we may consider the very simple circuit shown in Fig. 2. The differential equation of this circuit is

$$\left(R + L\frac{\partial}{\partial t}\right)I = E,$$

from which the impedance Z is obtained by replacing $\partial/\partial t$ by in. That is

$$Z(in) = R + Lin.$$

The series for $1/Z$ is therefore

$$\frac{1}{Z} = \sum_{s=1}^{\infty}\frac{(-R)^{s-1}}{(Lin)^s},$$

from which the series (8) becomes

$$\begin{aligned}\Omega(\lambda, t) &= \sum_{s=1}^{\infty}\frac{(-R)^{s-1}}{(L)^s}\frac{(t-\lambda)^{s-1}}{\underline{|s-1}}, \qquad \lambda < t, \\ \Omega(\lambda, t) &= 0, \qquad \lambda > t.\end{aligned}$$

Suppose then that the E.M.F. is of the simple form

$$\begin{aligned}E &= 0, \qquad t < 0, \\ E &= 1, \qquad t > 0.\end{aligned}$$

The equation (4) becomes

$$\begin{aligned}I(t) &= \sum_{s=1}^{\infty}\frac{(-R)^{s-1}}{L^s\underline{|s-1}}\int_0^t (t-\lambda)^{s-1}d\lambda \\ &= -\frac{1}{R}\sum_{s=1}^{\infty}\frac{\left(\frac{-Rt}{L}\right)^s}{\underline{|s}};\end{aligned}$$

a series which is at once recognized as equal to

$$\frac{1 - e^{\frac{-Rt}{L}}}{R}.$$

It may perhaps be interesting for purposes of comparison to apply the Heaviside Expansion to this same problem. For this purpose, Ω is written as

$$\Omega(\lambda, t) = \frac{1}{2\pi Li}\int_{-\infty}^{\infty} dn \frac{e^{in(t-\lambda)}}{n - \frac{Ri}{L}}.$$

Since there is only one pole, for which $n = Ri/L$, this reduces to the single term

$$\Omega(\lambda, t) = \frac{1}{L}e^{-\frac{R}{L}(t-\lambda)}, \qquad t > 0;$$

a result which is identical with the series expansion of Ω in the solution above.

Another interesting example of the use of a power series expansion is

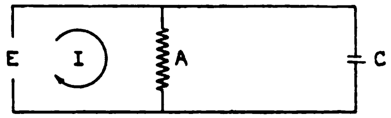

Fig. 3.

furnished by a circuit such as is shown in Fig. 3. Here

$$\frac{1}{Z} = \frac{1}{R} + Cin,$$

which is already expanded in a series in terms of n. But it is to be noted that the series does not satisfy the conditions of the power series solution, since it contains positive powers of n.

Even under these unfavorable conditions however, it is possible to get the correct result by inspection. For upon writing the expression for I the formal value

$$I = \frac{1}{2\pi R}\int_{-\infty}^{\infty} d\lambda E(\lambda)\int_{-\infty}^{\infty} e^{in(t-\lambda)}dn + \frac{Ci}{2\pi}\int_{-\infty}^{\infty} d\lambda E(\lambda)\int_{-\infty}^{\infty} ne^{in(t-\lambda)}dn$$

is obtained. Now the first of these integrals quite evidently represents $1/R\cdot E(t)$. The second is also seen by inspection to be the formal expression for $C\cdot\partial E/\partial t$, obtained by differentiating the identity (2) under the sign of integration. This therefore leads to the result

$$I = \frac{E}{R} + C\frac{\partial E}{\partial t};$$

a result which is quite correct although the method of derivation raises mathematical questions of considerable importance.[1]

It should be noted that the integral of the first term of (8) has been identified with $a_0R(t)$, since it is of the form

$$\frac{a_0}{2\pi}\int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} e^{in(t-\lambda)}E(\lambda)dn,$$

which is the Fourier identity for $E(t)$. This is also in accordance with the rule of thumb given at the close of section 3; which requires a term of exactly this form to appear.

5. The Ω-Integral: Third Method of Evaluation. Bessel Series Expansion.

A third method of evaluation, which in complicated problems is more satisfactory than the other two, depends upon the use of a suitable transformation of the variable of integration. It is the method underlying a treatment of the problem of the cable which will be explained later; although certain slight modifications are necessary in that problem. Consider again the expression

$$\Omega(\lambda, t) = \frac{1}{2\pi}\int \frac{e^{in(t-\lambda)}}{Z(in)}dn,$$

which, if $\lambda < t$, may be evaluated about a circle of sufficiently large radius in the n-plane, and subject it and the path of integration to the transformation $n = \sin\phi$. The result is

$$\Omega(\lambda, t) = \frac{1}{2\pi}\int \frac{e^{i(t-\lambda)\sin\phi}\cos\phi d\phi}{Z(i\sin\phi)}.$$

In order to discuss this integral, it is necessary to know something of the properties of the transformation itself. They are stated here as briefly as possible; they belong to the most elementary part of the theory of analytic functions and are easily derived.

Fig. 4 represents the four quadrants of the n-plane, and a vertical strip of the ϕ-plane. Each is divided into sections which are lettered in such a way as to indicate those regions which correspond under the transformation. It is obvious that the entire n-plane is transformed into any strip of the ϕ-plane of width 2π, extending from the real axis to infinity in one direction only. Which of these strips is chosen is a matter of little moment. In order to be explicit the one marked $ABCD$ will be used.

---

[1] The questions relate to the propriety of differentiating a certain type of Stieltjes integrals under the sign of integration. The writer knows of no literature on this subject, although the problem appears to be capable of successful treatment.

A circle in the η-plane which does not include the points $\eta = \pm 1$ in its interior passes into a closed path in the ϕ-plane; but a circle which does include these points passes into an open path running from some point Φ to another point $\Phi + 2\pi$. It is to be noted that the distances of these points from the real axis are equal; a fact of some consequence, as will be seen in the next paragraph.

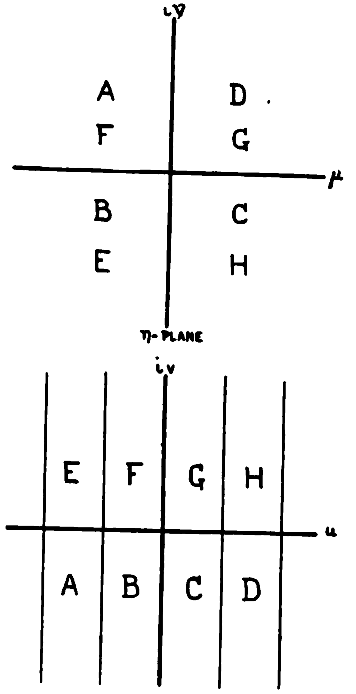

Fig. 4.

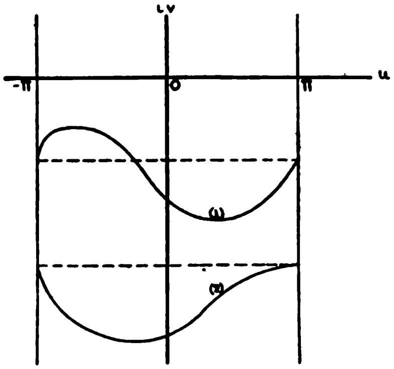

Fig. 5.

The paths (1) and (2) of Fig. 5 are of the type just mentioned. Suppose the integral of some function to be taken along each of them in the same direction. The results of this pair of integrations will differ precisely by the sum of the residues at the singularities included within the closed path formed by them and the verticals between their end points. In particular, if there are no such singularities, the integrals will be identical. Thus, in addition to the possibility of deforming a path of integration continuously, it is allowable to slide its end points along the vertical lines, *so long as they are kept at the same distance from the real axis, and no singularities are encountered by them or the path.*

In particular, then, the path of integration for Ω may be taken as a horizontal line sufficiently far below the real axis that all the poles will lie above it. Let such a path pass from the point $-\pi - i\Phi$ to the point $+\pi - i\Phi$. Then

$$\Omega(\lambda, t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i(t-\lambda)\sin(\phi - i\Phi)}\cos(\phi - i\Phi)d\phi}{Z[i \sin(\phi - i\Phi)]}. \tag{9}$$

By hypothesis, $1/Z[i \sin(\phi + i\Phi)]$ is finite and continuous along this path and may be expanded in a Fourier Series. For brevity it may be denoted by $U(\phi)$, and the coefficients in its Fourier expansion by A_s and B_s, so that

$$\begin{aligned} U(\phi) &= a_0 + \sum_{s=1}^{\infty}[A_s \sin s\phi + B_s \cos s\phi] \\ &= \sum_{s=-\infty}^{\infty} a_s e^{is\phi}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} a_s &= \frac{B_s - iA_s}{2}, \qquad s > 0, \\ a_s &= \frac{B_s + iA_{-s}}{2}, \qquad s < 0. \end{aligned} \tag{11}$$

Substituting (10) in (9), Ω takes the form[1]

$$\begin{aligned} \Omega(\lambda, t) &= \frac{1}{2\pi}\sum_{s=-\infty}^{\infty} a_s \int_{-\pi}^{\pi} e^{i(t-\lambda)\sin(\phi - i\Phi)} \frac{e^{i(s+1)\phi+\Phi} + e^{i(s-1)\phi-\Phi}}{2} d\phi \\ &= -\sum_{s=-\infty}^{\infty} s a_s e^{-s\Phi} \frac{J_s(\lambda - t)}{t - \lambda}. \end{aligned}$$

As a particular example[2] consider the impedance $Z = Lin = Li \sin \phi$.

$$\frac{1}{Z} = \frac{2}{L}\sum_{j=0}^{\infty} e^{-(2j+1)\Phi} e^{-(2j+1)i\phi}.$$

Hence

$$\Omega = \frac{2}{L}\sum_{s=0}^{\infty}(2s + 1)\frac{J_{-2s-1}(\lambda - t)}{\lambda - t}, \qquad \lambda < t.$$

But the expression

$$-2\sum_{s=0}^{\infty}(2s + 1)J_{-2s-1}(\lambda - t)$$

---

[1] Use is here made of the definition of Bessel's functions by means of integrals; and of the addition properties of these functions. These may be found in compact form in Jahnke and Emde, Funktionentafeln mit Formeln und Kurven, p. 165 sqq. Altogether unnecessary restrictions are placed upon the path of integration in the integral definition of $J_p(z)$ as stated in Jahnke and Emde, but this failing is quite outweighed by the advantage of having a well-chosen collection of formulæ compactly arranged.

[2] Note that there is no dissipation. The path of integration is to be taken low enough to miss the pole $n = 0$.

is known to be the expansion of $\lambda - t$. Hence Ω reduces to $1/L$, and

$$I = \frac{1}{L}\int_0^t E(t)dt.$$

This is the solution of the equation

$$L\frac{\partial I}{\partial t} = E(t),$$

as it should be.

A slightly different form of solution may be obtained by making use of the transformation $in = \sin \phi$, instead of the one used above. The result obtained in this case is

$$\Omega = -\sum_{s=-\infty}^{\infty} sa_s' e^{-is\Phi}\frac{J_s[i(t-\lambda)]}{t-\lambda},$$

where the a_s' are the constants derived from the expansion of

$$1/Z[\sin(\phi - i\Phi)]$$

in a Fourier series. In the particular example used above this would result in

$$\Omega = -\frac{2i}{L}\sum_{s=0}^{\infty}(2s+1)\frac{J_{2s+1}[i(t-\lambda)]}{t-\lambda}.$$

This series also converges to the value $1/L$. In fact, in spite of the immense difference in the character of $J_{2s+1}(x)$ and $J_{2s+1}(ix)$, the rapidity of convergence of the series formed from each is about the same.

6. Discussion of the Three Methods of Evaluating the Ω-Integral.

Where it is applicable, and the number of roots of the impedance function is not too great, the Heaviside-Carson method is the simplest for purposes of computation. Unfortunately, it fails in all but a limited number of special cases, owing to the impossibility of determining the roots of the Z-function.

The method of expansion in power series is applicable to nearly all problems which involve only lumped constants, and to some others. It is frequently useful in those discussions in which a knowledge of the character of the first surge of current is sufficient. For other problems the convergence of the series is likely to be too slow to justify its use. It does not involve the determination of the roots of any equations; so that the principal difficulty of the first method is avoided.

The third method, like the second, does not require a knowledge of the roots of the Z-function, although when such a knowledge exists it can sometimes be put to use in a deformation of the path of integration in the ϕ-plane. What is required, however, is a knowledge of an upper limit for the absolute value of the roots of this equation; that is, a knowledge that none of the roots is outside a certain specified circle in the n-plane. This upper limit can generally be determined. The problem is then reduced to the expansion of a function in a Fourier series. Analytically this problem is perhaps as difficult as the determination of the roots of Z. But the existence of harmonic analyzers of the Henrici-Coradi and Michelson types makes this difficulty unimportant. Specifically stated, the third method of evaluation is so designed that the principal difficulties of computation are included in those processes which can be mechanically performed. Furthermore, the use of the binomial expansion and other analytic tools will often make possible the immediate determination of the a's, without the use of the intermediates A and B. This is the case in the examples above given.

When the Ω-function has been evaluated, the problem depends upon a single integration—an operation which may always be performed by the use of a Coradi integraph. If the form of E is complicated, a number of mechanical integrations may be necessary, and the work may take considerable time. But this difficulty is inherent in the problem. It is unreasonable to expect mathematics ever to obtain in a few minutes the complete answer to questions about complicated circuits. The most that can be hoped is that a method of solution can be devised which is sufficiently simple to permit of the answer being calculated by trained computors at an expense commensurate with the importance of the problem. The solution in Bessel's functions satisfies this condition for a large class of problems, and therefore represents an addition to the available methods of computation. Further than this no claim need be made for it.

7. THE PROBLEM OF THE TELEGRAPH EQUATION.

One of the very difficult problems of circuit theory is concerned with the propagation of electrical signals through a telegraph cable. Since this problem can be attacked with considerable success by the methods above, and illustrates the type of modifications necessary in applying these methods to different problems, it will be discussed here in some detail. The actual development of the various formulæ involves the use of algebraic processes which are frequently long, but never difficult. Only so much of this algebra has been included as is needed, in the writer's opinion, to serve as a guide to those who may be interested in filling in the details, and to prevent confusion in the notation.

The differential equations of a pair of conductors containing distributed resistance (R), inductance (L), capacity (C), and leakage (K) are

$$\begin{aligned} -\frac{\partial \bar{E}}{\partial \bar{x}} &= L\frac{\partial \bar{I}}{\partial \bar{t}} + R\bar{I}, \\ -\frac{\partial \bar{I}}{\partial \bar{x}} &= C\frac{\partial \bar{E}}{\partial \bar{t}} + K\bar{E}. \end{aligned} \tag{12}$$

The dashes above the letters are used to indicate that the quantities symbolized by them are measured in a standard system of units. It is much simpler, however, to introduce into the discussion a certain set of units depending upon the properties of the cable, and measure all quantities in this system. If the dash is omitted from the symbols when this is done, the relations which determine the new set of units are:

$$\begin{aligned} E &= \bar{E}, \\ I &= \sqrt{\frac{L}{C}}\bar{I}, \\ X &= R\sqrt{\frac{C}{L}}\bar{X}, \\ t &= \frac{R}{L}\bar{t}. \end{aligned} \tag{13}$$

The differential equations (12) now become

$$\begin{aligned} -\frac{\partial E}{\partial x} &= \frac{\partial I}{\partial t} + I, \\ -\frac{\partial I}{\partial x} &= \frac{\partial E}{\partial t} + kE, \end{aligned} \tag{14}$$

where $k = KL/CR$.

In accordance with section 2, the first step in the discussion of these equations is to obtain their steady state solution.

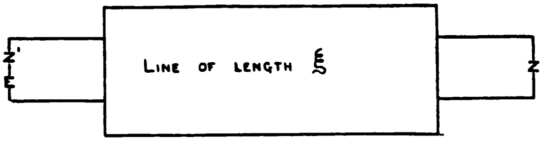

Fig. 6.

Fig. 6 represents diagrammatically a line of length ξ, terminated by an impedance Z'' at one end, and subjected to an electromotive force E which is impressed upon the opposite end through an impedance Z'. If the electromotive force is periodic, of frequency n, it may be represented by

$$E(t) = \frac{\Psi}{2\pi}e^{int}.$$

The corresponding steady state current and electromotive force at a distance x from the sending end of the cable are, respectively,

$$E(x, t) = \frac{f(x)}{2\pi}e^{int}, \qquad I(x, t) = \frac{\phi(x)}{2\pi}e^{int}, \tag{14}$$

where $f(x)$ and $\phi(x)$ are expressible as follows:

$$f(x) = \frac{M_1e^{imx} + M_2e^{-imx}}{\Delta}, \qquad \phi(x) = \frac{in + k}{im}\left[\frac{-M_1e^{imx} + M_2e^{-imx}}{\Delta}\right]; \tag{15}$$

$$im = \sqrt{(in + 1)(in + k)},$$

$$M_1 = -\Psi e^{-im\xi}\left[\frac{in + k}{im}Z'' - 1\right],$$

$$M_2 = -\Psi e^{im\xi}\left[\frac{in + k}{im}Z'' + 1\right],$$

$$\Delta = \begin{vmatrix} -e^{im\xi}\left[\frac{in + k}{im}Z'' + 1\right] & \left[\frac{in + k}{im}Z'' - 1\right]e^{-im\xi} \\ -\frac{in + k}{im}Z' + 1 & \frac{in + k}{im}Z' + 1 \end{vmatrix}.$$

The formal solution for the general problem is built up by using the principle of superposition. Thus, if

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi(n)e^{int}dn, \tag{16}$$

it will be true that

$$E(x, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}f(x, n)e^{int}dn,$$

$$I(x, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi(x, n)e^{int}dn.$$

Of course, in (16), $\Psi(n)$ is determined so as to satisfy the Fourier identity, which requires that it be

$$\Psi(n) = \int_{-\infty}^{\infty}E(\lambda)e^{-in\lambda}d\lambda;$$

and this value must be assigned it in writing the expressions for f and ϕ.

In the following paragraphs it is assumed that $Z' = 0$, and that $x = \xi$,—that is, that the driving force $E(\xi)$ is impressed directly upon the line, and that only the arrival current[1] is desired. Making use of these assumptions it is possible to reduce the expressions for f and ϕ to the form

$$f(\xi, n) = \frac{\Delta(0)}{\Delta(\xi)} \Psi(n),$$

$$\phi(\Phi, n) = \frac{\Delta'(0)}{\Delta(\xi)} \frac{\Psi(n)}{in + 1}.$$

Of course the $\Delta(\xi)$ in these formulæ is simpler than the one previously written because of the assumption $Z' = 0$.

Let us denote by δ_j the quantity

$$\delta_j = \frac{\left[1 + \frac{in + k}{im} Z''\right]^j}{\left[1 - \frac{in + k}{im} Z''\right]^{j+1}}. \tag{16}$$

Then

$$\frac{1}{\Delta(\xi)} = \sum_{j=0}^{\infty} \delta_j e^{(2j+1)im\xi},$$

and therefore

$$f(\xi, n) = -\frac{2im}{in + 1} Z'' \Psi(n) \sum_{j=0}^{\infty} \delta_j e^{(2j+1)im\xi},$$

$$\phi(\xi, n) = -\frac{2im}{in + 1} \Psi(n) \sum_{j=0}^{\infty} \delta_j e^{(2j+1)im\xi}.$$

The physical significance of this expansion will appear later. The apparently arbitrary appearance of the mathematics may be somewhat relieved by anticipating its appearance and stating that the jth term of the series corresponds to that part of the arrival current which has undergone j reflections at the sending end.

Up to this point nothing has been said about the multiple valued nature of m. The form of the expansion of $1/\Delta$ above, however, applies only to that branch of the function for which the signs of the real parts of m and of n are unlike. This convention will be adhered to throughout the work.

Since $\phi(\xi, n)$ is expressed in the form of a series, the same will be true of I, and it will be convenient to denote the separate terms by subscripts corresponding to those used to indicate the various terms of ϕ. Thus,

---

[1] The purpose of these assumptions is purely didactic. Although they result in little if any simplification of the labor necessary to secure a numerical answer to an actual problem, they do permit of considerable simplification of the notation in the explanations following. All the essential ideas of the general discussion are involved in the more special one which is given; so that it is quite as serviceable as a more cumbersome case.

$$I = \sum_{j=0}^{\infty} I_j,$$

where

$$I_j(\xi, t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} dn E(\lambda) e^{in(t-\lambda)+(2j+1)im\xi} \delta_j \frac{2im}{in+1}. \qquad (17)$$

If $Z(in)$ is regular at infinity, as it is in nearly all practical cases, the path of the n-integral can be deformed as explained in section 3,[1] which leads to an Ω-integral of the type

$$\Omega_j(\lambda) = -\frac{1}{2\pi}\int \frac{im}{in+1} \delta_j e^{in(t-\lambda)+(2j+1)im\xi} dn, \qquad (17)$$

which must be evaluated.

8. EVALUATION OF THE Ω-INTEGRAL FOR THE TELEGRAPH EQUATION.

It is easily seen that the Heaviside expansion can not be applied to (17), since the fundamental assumption that all the singularities are poles is violated at the points $n = i$ and $n = ki$. Both the power series expansion and the expansion in Bessel's functions can be carried out, however, provided the exponential $e^{(2j+1)im\xi}$ is properly dealt with.

For the former of these it is necessary to break it up into two factors $e^{-(2j+1)im\xi}$ and $e^{(2j+1)(in+im)\xi}$. The first of these must be included with the term $e^{in(t-\lambda)}$, being equivalent to replacing t in section 4 by $t-(2j+1)\xi$. The second must be included with $im/(in+1)\cdot\delta_j$ in a term analogous to the expression $1/Z(in)$ in section (4). This term is regular at infinity and admits of expansion in a power series. The evaluation of

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[1] Note that when n is very large, $im = -in - \alpha$, and hence

$$e^{in(t-\lambda)+im\xi_j} = e^{-\alpha\xi_j} e^{in(t-\lambda-\xi_j)}.$$

The critical value of λ is therefore $\lambda = t - \xi_j$, or $t-(2j+1)\xi$. For $n = \infty$ the remainder of the integrand becomes

$$-\frac{[1-Z''(\infty)]^j}{[1+Z''(\infty)]^{j+1}},$$

so that a term

$$2e^{-(2j+1)\alpha\xi} \frac{[1-Z''(\infty)]^j}{[1+Z''(\infty)]^{j+1}} E[t-(2j+1)\xi]$$

must be added. This term accounts for the discontinuity at the "head" of the wave, when $Z''(\infty)$ is finite, *i. e.*, when there is no inductance in Z''. The distributed inductance of the line itself, even when it is large, does not prevent the propagation of discontinuities. In fact, under ordinary conditions, increasing the inductance sharpens the head of the wave.

It now appears quite clearly that the current I_j must have been reflected j times from the sending end; for it does not become effective until a sufficient time interval has elapsed to permit the propagation of a disturbance through a distance $(2j+1)\xi$. The only way a disturbance can have been propagated so far, however, is for it to have traveled back and forth from end to end of the line j times after first reaching the receiving end.

Ω in Bessel's functions parallels the theory developed in section 6 to such an extent that detailed explanations of the transformations may frequently be omitted.

As new notation the following symbols will be needed:

$$\xi_j = (2j + 1)\xi,$$

$$\mu = t - \lambda,$$

$$\gamma_j = \sqrt{\mu^2 - \xi_j^2},$$

$$\cosh \phi_j = \frac{\mu}{\gamma_j},$$

$$\sinh \phi_j = \frac{\xi_j}{\gamma_j},$$

$$\alpha = \frac{k + 1}{2},$$

$$\beta = \frac{k - 1}{2}.$$

Most of these differ for different values of j. It is, however, scarcely necessary to be continually adding subscript j's to every symbol used; they will often be omitted when no confusion is possible.

Let the variable in (17) be transformed by the substitution

$$\beta \sin \phi = in + \alpha,$$

which is very similar to the one used in section 6. Under this transformation (17) becomes:

$$\Omega(\lambda) = -\frac{\beta e^{-\alpha\mu}}{\pi} \int_{-i\Phi}^{2\pi - i\Phi} (1 - \sin \phi)\delta e^{\beta\gamma \sin(\phi + i\psi)} d\phi.$$

Of course, δ is now expressed in terms of ϕ instead of n, and Φ is chosen so large that no poles of the integrand lie below the path of integration. This equation is again identical with

$$\Omega(\lambda) = -\frac{\beta e^{-\alpha\mu}}{\pi} \int_{0}^{2\pi} [1 - \sin(\phi - i\Phi)]\delta e^{\beta\gamma \sin(\phi + i\Psi - i\Phi)} d\phi,$$

with an appropriate change in the argument of δ, which leaves it in the form

$$\delta_j = \frac{[\cos(\phi - i\Phi) + iZ - iZ \sin(\phi - i\Phi)]^j}{[\cos(\phi - i\Phi) - iZ + iZ \sin(\phi - i\Phi)]^{j+1}}.$$

This expression may be expanded in a Fourier series, either analytically or mechanically as the situation may warrant;[1] it may be denoted by

$$\begin{aligned}\delta &= a_0 + \sum_{s=1}^{\infty} [A_s \sin s\phi + B_s \cos s\phi] \\ &= \sum_{s=-\infty}^{\infty} a_s e^{is\phi}.\end{aligned} \tag{18}$$

Then

$$\Omega(\lambda) = -\frac{\beta e^{-a\mu}}{\pi} \sum_{s=-\infty}^{\infty} a_s \int_0^{2\pi} [1 - \sin(\phi - i\Phi)] e^{\beta\gamma \sin(\phi + i\Psi - i\Phi)} e^{is\phi} d\phi.$$

This integral is easily evaluated, but the form in which it is obtained in the first instance admits of considerable simplification by algebraic processes. This simplification may be brought about most easily by the introduction of two functions

$$G_s(\xi, \mu) = e^{-\mu} \cosh s\Psi J_s(i\gamma),$$

$$H_s(\xi, \mu) = e^{-\mu} \sinh s\Psi J_s(i\gamma).$$

These functions, being entirely independent of the constants of the line, can be tabulated once for all. It is, however, more convenient to tabulate their μ-derivatives, since these are the functions which explicitly occur in the evaluation of the integral. These derivatives satisfy the formulæ:

$$G_s' = \frac{\partial G_s}{\partial \mu} = \frac{i}{2} G_{s-1} - G_s - \frac{i}{2} G_{s+1},$$

$$H_s' = \frac{\partial H_s}{\partial \mu} = \frac{i}{2} H_{s-1} - H_s - \frac{i}{2} H_{s+1},$$

by means of which they are readily computed, once G and H are known.

Further we introduce the following linear combinations of these terms:

$$\begin{aligned} S_s &= i[G_s'(\beta\xi, \beta\mu) \sinh s\Phi - H_s'(\beta\xi, \beta\mu) \cosh s\Phi], \text{ when } s \text{ is even;} \\ S_s &= i[H_s'(\beta\xi, \beta\mu) \sinh s\Phi - G_s'(\beta\xi, \beta\mu) \cosh s\Phi], \text{ when } s \text{ is odd;} \\ C_s &= i[G_s'(\beta\xi, \beta\mu) \cosh s\Phi - H_s'(\beta\xi, \beta\mu) \sinh s\Phi], \text{ when } s \text{ is even;} \\ C_s &= i[H_s'(\beta\xi, \beta\mu) \cosh s\Phi - G_s'(\beta\xi, \beta\mu) \sinh s\Phi], \text{ when } s \text{ is odd.} \end{aligned} \tag{19}$$

Then, after appropriate algebraic transformations,

$$\Omega(\lambda) = 2\beta e^{-k\mu} [a_0 G_0'(\beta\xi, \beta\mu) + \sum_{s=1}^{\infty} (A_s S_s + B_s C_s)].$$

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[1] There is some opportunity for the exercise of ingenuity in this process, and many short methods will readily suggest themselves. In particular, if the series for in terms of decreasing powers of n is rapidly convergent, it is possible to develop from it a rapidly convergent series in powers of $e^{i\phi+\Phi}$, and integrated each term separately into a G' or an H'. When this is possible the use of the long expressions for S_s and C_s is avoided. The explanation given in the text assumes the necessity of mechanical analysis, and gives the formulæ most adapted for this purpose.

That is, in words: *the value of the* Ω*-integral for the telegraph equation may be built up by the extremely simple process of replacing the functions* sin $s\phi$ *and* cos $s\phi$ *in* (18) *by the functions* S_s *and* C_s *defined by* (19).

9. Example. Receiving End Short Circuited.

The simplest possible example involves a short-circuited receiving end, $Z = 0$. For this case $\delta_j = 1$, and hence

$$\Omega_j(\lambda) = 2\beta e^{-k\mu} G_0'(\beta\xi_j, \beta\mu),$$

$$I(t) = 2\beta \int_{-\infty}^{t-(2j+1)\xi} e^{-k(t-\lambda)} E(\lambda) \sum_{j=0}^{\infty} G_0'[\beta(2j+1)\xi, \beta(t-\lambda)] d\lambda. \quad (20)$$

To this must be added the quantity

$$2\sum_{j=0}^{\infty} e^{-a(2j+1)} E[t-(2j+1)\xi],$$

introduced by the critical value of Ω at $\lambda = [t-(2j+1)\xi]$.

For most existing submarine cables, k is very nearly zero. If, then, k be put equal to zero, and if

$$E = 0, \qquad t < 0,$$
$$E = 1, \qquad t > 0,$$

(20) reduces to

$$I(t) = \sum_{j=0}^{\infty} \Big[2G_0[\beta(2j+1)\xi, t] - 2G_0[\beta(2j+1)\xi, 0] + 2e^{-a(2j+1)\xi} E[t-(2j+1)\xi] \Big].$$

Each term of this summation corresponds to that portion of the arrival current which has suffered reflection j times at the sending end.

More complicated problems involve no greater difficulties than this extremely simple one, excepting those which are involved in the mechanical processes of Harmonic Analysis and Mechanical Quadrature.

10. Infinite Line.

A word may be said as to the form assumed by these expressions if the line is infinite in length, so that there is propagation in one direction only. For this case

$$f = \frac{1}{1 - \dfrac{in+k}{im} Z'} \Psi(in) e^{imx},$$

$$\phi = -\frac{\dfrac{in+k}{im}}{1 - \dfrac{in+k}{im}Z'}\Psi(in)e^{imx}.$$

When these expressions are substituted in the formula for $E(x, t)$ and $I(x, t)$ it is seen that expressions exactly similar to I_0 above result, and the evaluation may be carried out accordingly in exactly the same manner as in the more general case.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE AND TELEGRAPH
COMPANY AND WESTERN ELECTRIC COMPANY, INC.,
NEW YORK CITY, September 23, 1918.

THE L-SERIES IN THE TUNGSTEN X-RAY SPECTRUM.

By Oswald B. Overn.

Synopsis.

Tungsten X-ray Spectrum, L-series; Wave-lengths.—Dershem's results were verified, for the most part within 1/10 per cent., except in the case of 1.777×10^{-8} cm. which was found to have a wave-length of 1.202×10^{-8} cm. Six new lines were discovered, four of which may belong to tungsten: 1.236, 1.213, 1.079 and 0.793×10^{-8} cm. This last line appeared light in contrast to the dark bromine absorption band in which it was located; an explanation of this fact is suggested. Dershem's method was used with a very thin crystal and with very long exposures of the Coolidge tube. Four photographs are reproduced.

Tungsten X-ray Spectrum, L-series; Two Groups of Lines Suggested.—Sixteen lines fall into two groups, each line of one having a wave-length 1.151 times that of of the corresponding line of the other.

Molybdenum X-ray Spectrum, K Series.—Three faint lines were measured which may belong to this series, wave-length 0.711, 0.706 and 0.629×10^{-8} cm.

Bromine X-ray Absorption Band.—Evidence of discontinuity found.

Silver X-ray Absorption Band.—Evidence of discontinuity found.

Thickness of a Soluble Crystal.—Determined by a simple method.

In a recent paper[1] Dershem has given the results of photographs of the tungsten X-ray spectrum obtained by the use of a thin crystal of rock salt wherein he finds this spectrum to contain nineteen wave-lengths in the L-series which he claims to have measured to an accuracy of one tenth per cent. The large number of lines discovered for the first time by Dershem made it seem of importance to verify his work. With this in mind, the present experiments were undertaken with the additional object of finding more lines if possible.[2]

The apparatus was the same as that employed by Dershem. The distance of the plate from the axis of the crystal was kept in the neighborhood of 30 cm. Although slits of various widths were used, the best results were obtained with a slit of the same width as that used by Dershem, namely, .032 cm. Except for a few changes in minor details, the method of measuring the plates was also the same as that used by Dershem. Seed's X-ray plates were used.

The departures from Dershem's method were as follows:

1. The length of exposure was made from two to six times as great

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[1] Physical Review, N.S., Vol. XI., June, 1918, p. 461.

[2] For accurate measurements of the *L* series of tungsten by various investigators the reader is referred to citations in Dershem's article, *loc. cit.*

as the longest exposure used by Dershem. The actual length of exposure ranged from 10 hours to 31 hours per degree of rotation of the crystal, the power input being kept the same as in Dershem's work.

2. The thickness of the paper envelope enclosing the plate was reduced to half.

3. A thinner crystal was used, the crystal used for most of the work having an average thickness of .0077 cm. This was measured as follows: The thickness of the crystal and glass to which it was fastened with wax was measured with a micrometer caliper. The crystal was then dissolved in cold water and the thickness of the glass and sealing wax again measured. The difference was considered the thickness of the crystal.

No difficulty such as that described by Dershem due to the cracking of crystals when ground down to small thicknesses was experienced. Indeed one crystal was ground down to a thickness of less than .003 cm. and was used successfully in making a photograph of the spectrum. This photograph is shown in Fig. 4. The fogging of the plate seems to be materially diminished by the use of a very thin crystal.

EXPERIMENTAL RESULTS.

The wave-lengths corresponding to the lines found on the various plates are given in Table I. Most of the plates were exposed over a limited portion of the spectrum and hence all the lines are not found on any one plate.

Where two plates are listed together they have been exposed on opposite sides of the apparatus and averaged. This, however, does not mean that every line appeared on both plates. The number of plates on which each line was observed is therefore placed in parentheses immediately at the left of each wave-length. The final averages are not weighted but each value is counted once for each plate on which that line appeared from which that value was calculated. Each plate, in turn, represents from five to ten separate observations.

Average values of wave-lengths greater than 1.0×10^{-8} cm. have an estimated accuracy of one tenth per cent. Others are estimated to be accurate within two tenths per cent. Dershem's values are placed in the last column for comparison.

These results not only verify Dershem's work but also show the existence of six more lines not observed by Dershem. If we add to this list the L-series line found by Siegbahn[1] which has a wave-length of about 1.66×10^{-8} cm., the total number of lines exclusive of the K-lines

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[1] Deutsch. Phys. Gesell., Verh. 18, 5, pp. 150–153. 1916.

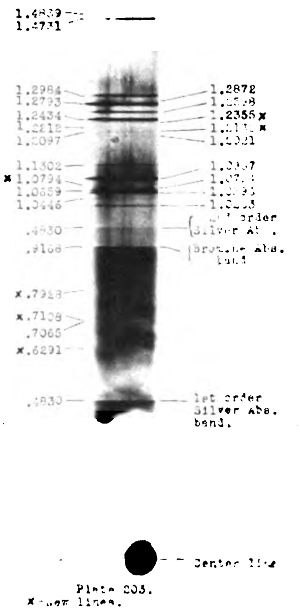

Fig. 1.

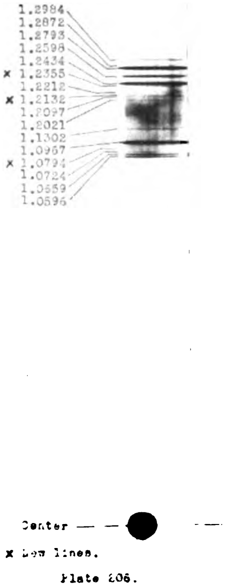

Fig. 2.

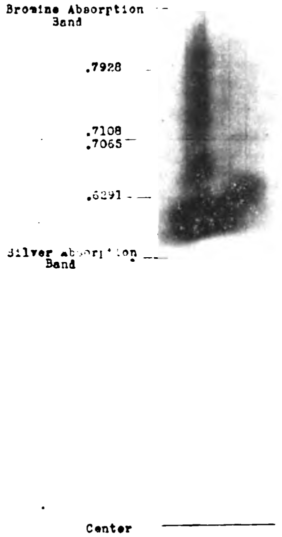

Fig. 3.

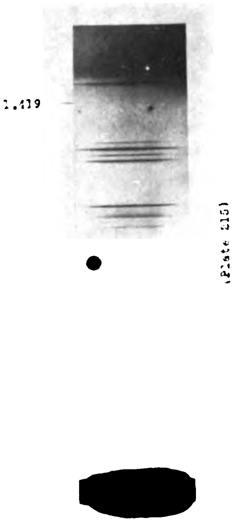

Fig. 4.

OSWALD B. OVERN.

is found to be 25. Whether or not these are all due to tungsten is however open to dispute as will be shown below.

Table I.

Wave-lengths in 10^{-8} *cms. Grating constant for rock salt* = 2.814×10^{-8} *cm.*

Plates 202, 203.	Plates 205, 206.	Plates 210, 213.	Plate 214.	Plate 216.	Average.	Dershem's Values.
(1) 1.484^0				1.483^8	1.483^9	1.482^8
(2) 1.473^1			1.473^5	1.472^6	1.473^1	1.472^2
(1) 1.299^2	(2) 1.298^5			1.296^5	1.298^4	1.297^7
(1) 1.287^6	(2) 1.287^0			1.287^1	1.287^2	1.286^8
(2) 1.279^4	(2) 1.280^8		1.278^3	1.277^0	1.279^3	1.278^4
(2) 1.260^3	(2) 1.260^6		1.259^0		1.259^8	1.258^6
(2) 1.242^5	(2) 1.245^4		1.241^2		1.243^4	1.241^6
(1) 1.235^3	(2) 1.234^8			1.238^0	1.235^5	
(1) 1.221^4	(2) 1.221^6			1.220^3	1.221^2	1.220^3
(1) 1.213^3	(2) 1.213^6			1.212^5	1.213^2	
(1) 1.209^7	(2) 1.210^3			1.208^4	1.209^7	1.209^8
(1) 1.202^0	(2) 1.202^4			1.201^8	1.202^1	1.177^3
(1) 1.130^1	(2) 1.130^4			1.129^9	1.130^2	1.129^2
(2) 1.096^6	(2) 1.097^8		1.094^9	1.096^2	1.096^7	1.095^3
(1) 1.079^2	(2) 1.079^5				1.079^4	
(1) 1.071^9	(2) 1.072^6				1.072^4	1.070^5
(2) 1.065^7	(2) 1.066^8		1.064^6	1.066^0	1.065^9	1.064^8
(2) 1.059^7	(2) 1.060^4		1.058^0	1.059^5	1.059^6	1.058^7
(1) 1.043^0	(1) 1.046^3				1.044^6	1.042^7
(2) 1.025^9	(2) 1.027^0		Plate 211	1.025^7	1.026^3	1.025^3
		(2) $.792^8$			$.792^8$	
(1) $.711^4$		(2) $.709^9$	$.709^9$	$.712^9$	$.710^8$	
(2) $.707^4$		(2) $.705^6$	$.705^5$	$.707^7$	$.706^5$	$.706^8$
		(2) $.629^1$			$.629^1$	
Br. (2) $.916^8$		(2) $.915^7$		$.919^2$	$.916^8$	$.915^9$
Ag. (3) $.483^4$		(1) $.481^9$		$.484^4$	$.483^0$	$.483^3$

The positions of all these lines in the spectrum are shown in Figs. 1, 2, and 3.

It will be noticed that the agreement between the present results and those of Dershem is within one tenth per cent. save in the cases of the wave-lengths, 1.243^4, 1.096^7, 1.202^1, 1.072^4 and 1.044^6. In the case of the first two, this difference can be explained by the fact that these lines, being very intense, are overexposed on my plates with a consequent widening of the image. Since the long wave-length edge of the line is measured, over-exposure will make the results a trifle too large. The discrepancy cannot be explained in this way for the other lines since they did not appear to be overexposed. The line 1.202^1 is listed by Dershem as 1.177^3. Since Dershem's value was found from observations on only one plate and mine from observations on four plates which agree well together (see Table I.) it is probable that Dershem's value for this wave-length is in error far beyond his limit of experimental error.

The wave-length 1.416[8] listed by Dershem has not been included in Table I. because it is the writer's opinion that this is the second order of the wave-length .706[5] found on most of the plates. This second order line has appeared on only one of these plates (see Fig. 4) sufficiently strong to admit of measurement. This plate was exposed in a small apparatus and the measurement is not considered as accurate as the others. The value found is 1.419. If this is a second order line its true wave-length is .709 which does not compare so badly with the wave-length .706[5]. The line .706[5] forms quite a close doublet with the line .710[8] and it would therefore be reasonable to expect both lines to appear in the second order if one does. However, the intensity of the line .710[8] is so small in comparison to that of the line .706[5] that it is very difficult to bring it out even in the first order and hence it could not be expected to appear in the second order. A reference to Fig. 4 will show that the observed line is very faint.

The line of wave-length .792[8] appears light on the plates in contrast to the others which appear dark (see Fig. 3). There may be two explanations for this:

1. It may be the edge of an absorption band due to some substance through which the rays pass before reaching the plate. The rays pass through glass, air and paper. The edge of the absorption band would be at the short wave-length end of the spectrum of some element contained in these substances but the K-series wave-lengths of these substances are far too large for this region. The "J radiations" for carbon, oxygen and aluminium, evidence for which has been found by Barkla and White,[1] are far too short for this region. Therefore this assumption is not very probable.

2. It may be the convergence wave-length of the L-series or some other series in the tungsten spectrum. The end of a series would appear as a dark band with a lighter region at the short wave-length end. This sudden change from dark to light is probably what has been observed here. As seen from the photographs, this line is in the region of the bromine absorption band and the general blackening of the plate makes it hard to observe.

The line of wave-length 1.2132 makes a close doublet with the line 1.209[7] already found by Dershem (see Figs. 1 and 2). The close proximity of these two lines which are clearly resolved shows that the resolving power of the apparatus with the crystal used was at least 336 for that wave-length.

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[1] Phil. Mag., 34, p. 270, Oct., 1917.

An examination of the bromine and silver absorption bands reveals several sudden but faint changes in the intensity of the blackening of the plate. This may indicate that this region is rich in spectrum lines. An attempt has been made to measure the wave-lengths of some of these but the work has not progressed far enough to make the results of any value.

Of the lines which stand out prominently in this region, the wave-lengths $.710^{8}$ and $.706^{5}$ are within two per cent. of the wave-lengths of the K-series alpha lines of molybdenum found by other observers.[1] The wave-length $.629^{1}$ is within two per cent. of the value given by Blake and Duane[2] for the edge of the absorption band of molybdenum and hence may correspond to the K-series beta line of molybdenum. It is therefore probable that these three faint lines are the K-series lines of molybdenum produced by the molybdenum used in the construction of the Coolidge tube.

Two Groups in L-series.

A scrutiny of the wave-lengths thus far discovered revealed the fact that they may be divided into two groups which are exactly similar. Since the α line of Moseley is the principal line of one group and the β line holds the same relative position in the other group, they have been called the "α group" and "β group" respectively. Their arrangement is shown in Table II.

Table II.

α Group.		β Group.	
λ.	λ/λ_α.	λ.	λ/λ_β.
1.4839	.993	1.2872	.994
1.4731 = λ_α	1.000	1.2793 = λ_β	1.000
1.2984	1.134	1.1302	1.132
1.2598	1.169	1.0967	1.167
1.2434	1.185	1.0794[3]	1.185
1.2355[3]	1.192	1.0724	1.193
		1.0659	1.200
1.2212	1.206	1.0596	1.207
1.2132[3]	1.214		
1.2097	1.217		
1.2021	1.225	1.0446	1.225
		1.0263	1.246

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[1] Kaye, "X-Rays," second edition, Longmans, p. 226.
Also Malmer, Phil. Mag., 28, 1914, p. 787.

[2] Physical Review, X., Dec., 1917, p. 700.

[3] Lines found for the first time in this work.

Table II. is unique and gives evidence to show that there are at least two distinct series of lines in the L-series. The similarity of the two groups is shown by the second and fourth columns. There are two lines missing from each group to make it correspond to the other group which gives a possible suggestion as to where to look for new lines.

Any line in the β group may be found from the corresponding line of the α group by dividing its wave-length by 1.151.

A further discussion of these line groups will be left for a later paper.

In conclusion the writer wishes to thank the Staff of the Physics Department of the State University of Iowa for the encouragement and inspiration received from them and especially Professor G. W. Stewart who suggested the problem. He is also indebted to Dr. Elmer Dershem for his initiation into the technique of the experimental work.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA,
July, 1918.

THE SPEED OF SOUND PULSES IN PIPES.

By Arthur L. Foley.

Synopsis.

Speed of sound pulses in short tubes placed near a spark source; measured by the photographic method. By a modification of this method, an instantaneous photograph was obtained of a sound pulse, part of which had come through a tube, while another part had come through free air. In this way the effect of short tubes of various sizes was determined when their near ends were from one to five centimeters from the spark. Two photographs are reproduced which show that in some cases the pulse may travel faster through a short tube than in free air, and faster through the smaller of two tubes than through the larger. The speed through a tube was found to depend chiefly on the intensity of the pulse as it entered the tube, and this was true even when the motion of the air as a body was prevented by a thin collodion membrane stretched across one end of the tube. The mean speed was less the longer the tube.

Speed of sound in tubes; historical summary with a discussion of the Kirchoff formula and a table of the results obtained by various methods for tubes of various sizes and materials, and for various pitches.

THE speed of sound in pipes has been experimentally determined directly by measuring the time required for a sound (1) to pass from one to the other end of a pipe; (2) to return to the source after reflection from the closed distant end; (3) to pass through a U tube and return to the source. Of the several indirect methods one may name the calculation of sound speed from the length and frequency of an organ pipe, or the change in length necessary to restore the pitch of a pipe sounding a harmonic of known frequency. The so-called resonance tube method is practically the same as the organ pipe method except that the air column is thrown into vibration by means of a tuning fork or some other source of known frequency. The Kundt's tube has been used with the familiar dust figures and it has been modified to permit the location of the nodal and anti-nodal points by using branch tubes to the ear, to a manometric flame, or to a capsule covered with a thin membrane carrying a tiny mirror. A further modification consists in using a double Kundt's tube, a tube on each end of a vibrating rod which is rubbed at its middle point. Of the several interference methods Quincke's double tube method is the simplest. A sound passing through a tube divides at a Y and is reunited at a second Y after the two parts have traveled through branch tubes of unequal length, adjusted to give either constructive or destructive interference as desired.

In 1862–63 Regnault,[1] in Paris, made an elaborate series of experiments on the speed of sound in newly laid water pipes. As sources of sound he used a pistol, explosions and musical instruments. Both ends of the pipes were closed and the sound was produced at one end. The wave passed back and forth through the pipe many times, its time of arrival at the end being recorded on a chronograph drum by a stylus operated electrically when the sound wave impinged on a thin membrane and closed an electric circuit. From these experiments Regnault concluded that the speed of sound is greater in large pipes than in small ones; that the speed tends to a lower limit as the distance from the source increases and therefore as the intensity of the sound decreases; and that the limiting speed is the same for all sources. Rink objected to Regnault's conclusions on the ground that the first few coursings of the sound through the pipes was affected by the motion of the air in a body, caused by the explosion. Omitting the first two passages Rink[2] concluded from Regnault's experiments that neither the distance from the source nor the charge of powder used had any effect on the speed of the wave, in other words, that the speed of sound is independent of its intensity. On this point, however, most subsequent experimenters agree with the conclusion of Regnault. They agree also that the speed of sound in a pipe is a function of its diameter. They differ from Regnault by finding that the speed is a function of pitch, being greater for notes of high pitch.

In Table I. the writer has tabulated a few of the results obtained by various experimenters. It will be observed that the speed was found to depend on the material of the walls of the tube, on the diameter of the tube, and on the pitch of the sound. Most of the work referred to in the table was done for the purpose of arriving at an equation connecting these variables. The problem was attacked theoretically by both Helmholtz[3] and Kirchoff,[4] who arrived at equations reducible to the form

$$u = v\left(1 - \frac{c}{2r\sqrt{2\pi n}}\right),$$

where u is the speed of sound of frequency n in a pipe of radius r, v being the speed in free air. According to Helmholtz c is the viscosity of the gas while Kirchoff makes it depend on heat conduction between the gas and the wall of the tube. Kayser[5] found that the equation holds

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[1] V. Regnault, Mem. de l'Acad. Paris, 37, I., 3, 1868; Comptes Rendus, 66, 209, 1868; Phil. Mag., S. 4, V. 35, p. 161, 1868.

[2] H. J. Rink, Pogg. Ann., V. 149, p. 533, 1873.

[3] H. v. Helmholtz, Wissench., Abhandl., V. 1, p. 338, 1882.

[4] G. Kirchoff, Pogg. Ann., V. 134, p. 77, 1868.

[5] H. Kayser, Wied. Ann., V. 2, p. 218, 1877.

TABLE I.

Speed of Sound Waves in Pipes.

Observer.	Method Used.	Source of Sound.	Kind of Pipe.	Frequency.	Diameter of Pipe, cm.	Speed, cm./sec.
Wertheim,[1] 1844	Organ pipe		Brass		1.0	329.12
			"		2.0	330.11
			Glass		2.0	330.23
			Brass		4.0	332.10
Le Roux,[2] 1867	Modified Regnault method	Tap on membrane over end of U tube			7.0	330.66
Kundt,[3] 1868	Double Kundt's tube, dust figures.	Longitudinal vibrations of a glass tube.	Glass	1850	0.35	305.42
			"	"	0.65	323.00
			"	"	1.30	329.47
			"	"	5.50	332.80
			"	5550	0.65	328.14
			"	"	1.30	330.87
			"	"	5.50	332.80
Regnault,[4] 1868	See text above	Pistol, charge of .4 gm. powder	Iron		10.8	330.99
			"		10.8	326.66
			"		33.0	331.64
			"		33.0	328.96
			"		110	334.16
			"		110	330.52
Seebeck,[5] 1870	Resonance tube, sliding piston lateral ear tube to locate nodes.	Tuning fork	Glass	320	0.34	317.26
			"	320	0.90	328.02
			"	320	1.75	329.24
			"	512	0.34	322.98
			"	512	0.90	328.44
			"	512	1.75	330.92
Blaikley,[6] 1879	Closed resonance tube, length adjusted to pitch of open tube.	Sound produced by blowing across end of tube itself.	Brass	506.8	1.10	323.73
			"	506.8	2.65	328.28
			"	506.8	5.28	329.48
Blaikley,[7] 1883	Special form of organ pipe.	Organ pipe blown with constant air pressure.	Brass	256	1.17	324.56
			"	256	1.95	326.90
			"	256	3.25	328.78
			"	256	5.41	329.72
			"	256	8.82	330.13

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[1] Pogg. Ann., Bd. 77, s. 427, 1844.

[2] LeRoux, Ann. Chim. Phys., (4), 12, 345, 1867.

[3] A. Kundt, Pogg. Ann., Bd. 135, s. 333 u. 527, 1868; Phil. Mag., V. 36, p. 4, 1868.

[4] V. Regnault, *loc. cit.*

[5] A. Seebeck, Pogg. Ann., Vol. 139, p. 104, 1870; Ann. Chim. Phys., s. 4, Vol. 15, p. 487 1870; Abstract, Phil. Mag., s. 4, Vol. 40, p. 231, 1870.

[6] D. J. Blaikley, Phil. Mag., V. 7, p. 343, 1879.

[7] D. J. Blaikley, Phil. Mag., V. 16, p. 447, 1883.

TABLE I (*Continued*)

Observer.	Method Used.	Source of Sound.	Kind of Pipe.	Frequency.	Diameter of Pipe, cm.	Speed, cm./sec.
Law,[1] 1894	Resonance tube, branch tube to ear.	Tuning fork.	Glass	256	0.93	320.60
			"	512	0.93	323.60
			"	1023	0.93	325.29
			"	256	1.71	325.24
			"	512	1.71	326.70
			"	1023	1.71	327.80
			"	256	2.80	327.29
			"	512	2.80	328.33
			"	1023	2.80	328.68
Stevens,[2] 1902	Quincke's Interference method.	Tuning fork.	Porcelain	600	4.00	331.29
			"	508	4.00	331.54
			Iron	508	4.00	330.95
Müller,[3] 1903	Resonance tube. Dust figures.	Tuning fork electrically driven.	Glass	903	0.37	317.2
				903	0.68	322.9
				903	1.55	327.3
				2482	0.37	323.0
				2482	0.68	325.4
				2482	1.55	330.2
Schulze,[4] 1904	Quincke's Interference tubes.		Glass	384	0.101	258
			"	384	0.151	282
			"	512	0.101	265
			"	512	0.101	290
			Brass	384	0.099	189
			"	384	0.148	230
			"	512	0.099	208
			"	512	0.148	253
			Rubber	512	0.150	195

if c is placed equal to 0.0235 instead of the theoretical value 0.00588. Schneebeli[5] and Seebeck[6] found the equation true only so far as it connects speed and pipe diameter, and that the decrease in speed is inversely proportional to the square root of the cube of the frequency. According to Müller[7] the Helmholtz equation has no general validity. According to Schulze[8] the "constant" c was found to range between 0.0075 and 0.025, depending on the diameter and nature of the tube.

[1] J. W. Low, Ann. der Phys., V. 52, p. 641, 1894.

[2] E. H. Stevens, Ann. der Phys., V. 7, p. 285, 1902.

[3] J. Müller, Ann. der Phys., V. 11, p. 331, 1903.

[4] F. A. Schulze, Ann. der Phys., V. 13, p. 1060, 1904.

[5] H. Schneebeli, Pogg. Ann., V. 136; p. 296, 1869.

[6] *Loc. cit.*

[7] *Loc. cit.*

[8] *Loc. cit.*

Sturm[1] found that the Kirchoff formula was not valid for different tubes and frequencies while the results obtained by Wertheim[2] and by Blaikley[3] support the theoretical equation. Thus it will be seen that there is no consensus of opinion on the question of the speed of sound waves in tubes and the theoretical equations connecting speed, diameter and pitch have not been verified. It will be noted, further, that the speed of sound waves in tubes of small diameter has been found in every instance by the Kundt's tube or some resonance tube method. The conditions obtaining for a stationary wave are so different from those in the case of a traveling wave that the speed calculated for the former is not necessarily the true speed of the latter. The direct methods of Regnault and others have been applied to large tubes only, and they are open to the objection that the wave recording devices are subject to a time lag which depends on the intensity and type of the wave, both of which vary with the distance from the source. To avoid these various sources of error it occurred to the writer to photograph simultaneously two portions of the same wave, one part traveling in free air, the other passing through a tube, and then compare their speeds directly.

The general plan of the experiment is shown in Fig. 1, in which the light spark, the sound producing spark and the sound wave are shown in section while the tubes T_1, T_2, T_3, T_4, through which the sound passed, and the dry plate P, are shown in perspective. B, B' are brass rods, bent as shown, and tipped with platinum points to form a sound spark gap about 2.5 cm. long. To prevent light from this spark from falling on the photographic plate a hard rubber disk or button was placed on each terminal of the gap. A spark passing between the platinum points generates a cylindrical wave with hemispherical ends, shown in cross section at WW'. Suppose, while the wave is in this position, that a spark occurs at the light gap. The light rays above x and below z are practically unaffected. The entire plate is therefore about equally illuminated, except for the region lying between where the rays x and z strike the plate. If the wave is a condensation the ray Y is bent toward the center. Remembering that the wave is cylindrical we should have on the plate a dark ring shadow DR outside and a light ring LR inside, both concentric with D, the shadow of the rubber disk. If the wave were a rarefaction the dark ring would be on the inside. This fact enables us to know whether or not in a given case, there is any change of phase on reflection, etc. EE' are terminals of an electric machine capable of yielding sparks 20 cm. long. Sparks from these terminals pass across the gaps *GG′* and through the sound gap and light gap, which are in series. The spark in the latter is made to occur a little later than the one producing the sound wave, by a capacity *K*. The reader is referred to an earlier article[1] for fuller details of the method. I shall describe here only what is special for this investigation.

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[1] J. Sturm, Ann. d. Phys., V. 14, p. 822, 1904.

[2] *Loc. cit.*

[3] *Loc. cit.*

Concentric with the sound gap is a half ring cut from a piece of brass tubing 3 cm. in diameter and 5 cm. long. The half ring is supported by a rod *R* and itself supports radially four brass tubes T_1, T_2, T_3 and T_4 respectively 0.25 cm., 0.48 cm., 0.8 cm., and 1.15 cm. in internal diameter, each 2.5 cm. long. The shadow of the half ring and the four tubes is shown on the plate. The supporting rods *B*, *B′* and *R* were arranged in a line with the light gap so as to show but one shadow on the plate. With this arrangement one half the sound wave travels in free air. The other half is cut off except for the portions which enter the tubes.

Fig. 2 of Plate I. shows the result, when the sound wave was what I shall call intense—produced when six "three quart" leyden jars *CC′* were used on each electrode in addition to the jars with which the electric machine was equipped, and the capacity *K* consisted of eighteen jars. A weaker wave was obtained by removing five jars from each terminal of the machine and by reducing the capacity *K* from eighteen jars to six jars. The difference in the sound intensities in the two cases was very noticeable to the ear. The difference in the light intensities was quite marked. With the heavier discharge a ten-minute tank development of the dry plate gave a much denser negative than a thirty-minute development in the case of the lighter discharge.

To show at a glance just what has happened I have drawn on Fig. 2 a broken line circle with the sound gap as center. To avoid confusion the circle is drawn just outside the free wave. It will be noted that the waves through the tubes lie well outside the circle, showing that their speed is *greater than the speed of the wave in free air*, and that the speeds of the waves through the four tubes are apparently the same, although the tube diameters are in the approximate ratios of 1, 2, 3 and 5.

Fig. 3 was obtained by replacing the four short tubes by two longer ones of 0.25 and 1.15 cm. internal diameter, respectively, each 10 cm. long, and adjusted radially as in Fig. 2. The ends of the two tubes included but half the length of the sound gap, so that the other end of the cylindrical wave traveled outward in free air. It will be noted that here again we have a greater speed through the tubes than in free air, and what is more, that the speed through the smaller tube is the greater.

On the original negative from which Plate I. is a reduced print, the

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[1] PHYSICAL REVIEW, V. 35, p. 373, 1912; Proc. Indiana Acad. Sci., p. 305, 1915.

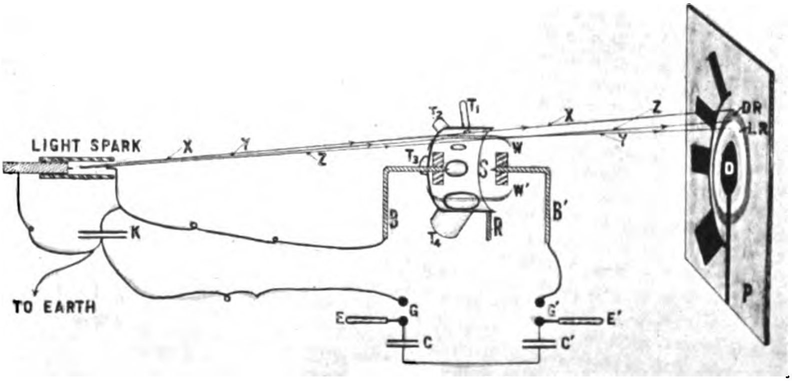

FIG. 1.

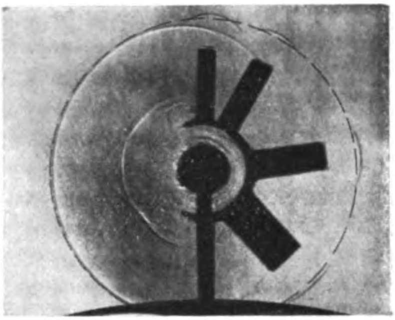

FIG. 2.

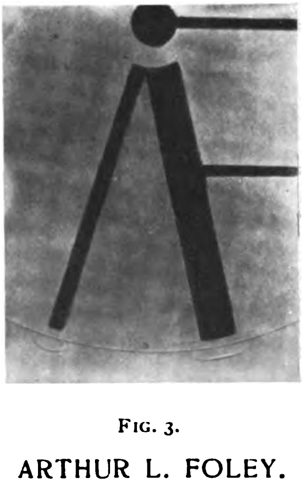

FIG. 3.

ARTHUR L. FOLEY.

tube shadow is 4.56 cm. long and the tube wave 0.48 cm. in advance of the free wave. Assuming the entire gain in space traversed occurred while the wave was inside the tube, an assumption which I think is not altogether true, the relative increase in speed is the ratio of 0.48 to 4.56, or 10.5 per cent. In the case of Fig. 3 the shadow length is 16.5 cm. and the gain of the wave in the smaller tube .89 cm., corresponding to a speed increase of 5.4 per cent. Comparing Figs. 2 and 3, it would appear that more than half the gain was made in the first fourth of the tube's length, and that if the tube were long enough the speed might drop to the values obtained by other experimenters, or even below—for their results are averages through tubes of considerable length.

The writer makes the above calculations merely as an illustration of what occurred in the case of Figs. 2 and 3, and not because he attaches any significance whatever to the values given. As a matter of fact they have no significance. The writer has photographs which show sound waves through pipes all the way from twenty per cent. behind to twenty per cent. ahead of the portions of the wave that traveled in free air. The speed is chiefly a question of the *intensity* of the wave as it *enters* the pipe. In the space near a sound source the intensity of a wave decreases very rapidly with increase in distance. Inside a pipe a wave can not spread out and so its intensity, if the pipe is smooth and rigid-walled, varies slowly. If, then, the end of a short pipe is near a sound source, such as an electric spark, the average intensity of the portion of the wave inside the pipe is greater than the average intensity of the wave outside, and the former, therefore, moves the faster. The writer tested this conclusion in several different ways, a few of which will be given.

1. An intense spark gave a greater difference between the speed in tubes and free air than a weak spark, particularly in the case of long tubes.

2. Pictures of waves through tubes varying from 2.5 cm. to 20 cm. in length showed that the tube speed was greater at first than the free air speed, but that it gradually decreased and eventually became less than the free air speed. An attempt to photograph waves at different points *inside* the tubes failed. Neither glass nor transparent quartz tubes would transmit enough light from the illuminating spark to affect a dry plate.

3. When two tubes of the same diameter and length were placed radially to the sound gap and with their ends near and at the same distance from the gap, each of the tube waves showed the same gain over the free air wave. When one of the tubes was moved radially toward the spark so that the intensity of the wave entering it was greater

than the intensity of the wave entering the other tube, the wave through the former gained on the wave through the latter. When both tubes were moved away from the gap the gain in each was correspondingly decreased. At a distance of two to five centimeters, depending on the intensity of the spark and to a less extent upon the size of the tube, the tube speed became less than the free air speed.

4. Two tubes, each 20 cm. long, were soldered together side by side and one sawed in two, crosswise, near the middle, making a cut about 2 mm. wide. When a wave passed the cut a part of its energy was lost and the wave emerged from the far end of the divided tube behind the wave through the continuous tube.

5. When a thin membrane of collodion, as used in making the lenses described in a former paper, was stretched across the end of a tube the wave speed through the tube was diminished. The decrease was greater when the membrane was on the end of the tube next the sound spark, than it was when placed on the far end. This result is readily explained when one considers the twofold effect of a membrane closing the end of a tube. In the first place, it either entirely or partially prevents the air in the tube moving bodily, depending respectively on whether the membrane is on the end of the tube next to or farthest from the sound spark. In the second place, if the membrane is placed next the spark so much of the wave energy is reflected at its surface that the intensity of the wave through the tube is greatly diminished. Inasmuch as the writer obtained photographs when the membrane was next the spark gap showing wave speeds greater for intense than for weak sparks, it is clear that Rink and others are wrong in their contention that sound speeds above normal are necessarily to be attributed to a motion of the air in a body.

While the speeds determined in this investigation are relative only, the experiment establishes the following conclusions:

1. The speed of a sound pulse in a short pipe may be greater than the same pulse in free air, and it may be greater in a small pipe than in a large one.

2. The speed of a sound pulse in a pipe depends on the intensity of the pulse as it enters the pipe rather than upon the intensity of the sound source.

3. The speed of a spark wave through a pipe may be increased by the motion of the air in a body, but strong sparks give an increased wave speed near the spark, even when such motion is prevented.

4. The speed of sound in tubes can not be represented by the Helmholtz equation or by any other equation which is independent of intensity.

5. Accurate determination of the speed of a traveling wave can not

be obtained by measuring wave lengths and frequencies of standing waves.

In a paper to be published soon the writer will describe an experiment in which he has measured the absolute speed of sound by a photographic method.

Waterman Institute for Research,
Indiana University,
March, 1919.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NINETY-SEVENTH MEETING.

THE ninety-seventh meeting of the American Physical Society was held at the Bureau of Standards, in Washington, D. C., on Friday and Saturday, April 25 and 26, 1919. In accordance with a resolution of the council, the meeting was given over entirely to special papers and exhibits of apparatus, illustrating the application of physical principles to the solution of problems arising from war conditions. The program and arrangements were entirely in the hands of a special committee consisting of the following: G. K. Burgess, chairman, F. A. Wolff, in charge of exhibits, G. O. Squier, J. S. Ames, E. B. Rosa, and A. W. Duff.

This meeting, which was attended by about three hundred members and visitors, was perhaps the most interesting and important which the Society has ever held. It is doubtful whether such an amount of important scientific work has ever before been presented at any scientific meeting in this country. There were four sessions for the reading of papers, the presiding officers being President Ames and past-President Bumstead.

The program consisted of fifty-two papers presented upon invitation of the committee, and of one hundred and eighty-eight additional titles which were submitted as a record of work accomplished, but which for various reasons could not be presented in the form of papers. The larger part of the work described has been carried on under the direction of various government departments and bureaus, and many of the papers were of a semi-confidential nature and were "released for verbal presentation only," it being understood that there would be no further publication by the Physical Society except by special permission. A few of the papers of general scientific character, by members of the Society, will be published in the usual manner.

The exhibits were displayed in fifteen or more rooms in the new Industrial Building of the Bureau of Standards. The arrangements were most satisfactory in every respect. There were fifty-six exhibitors, and several thousand exhibits were listed under about five hundred and fifty titles. The exhibition was open on Friday and Saturday, including Friday evening, April 25 and 26; it was also open on Monday and Tuesday, April 28 and 29, on account of the meeting of the National Academy of Sciences. It is estimated that about two thousand persons visited the exhibition.

At a meeting of the council held on April 26, 1919, the following elections took place: *elected to regular membership*, Dinsmore Alter; *transferred from associate to regular membership*, Millard B. Hodgson and Theodore T. Smith; *elected to associate membership*, Harmon S. Boyd, William G. Brombacher, Robert Burt, V. Bush, J. A. Dent, Harold P. Donle, A. L. Fitch, P. H. Graham, W. R. Gregg, W. G. Housekeeper, M. J. Kelly, Emran J. Kolkmeyer, John E. Morecroft, Raymond Morgan, Mary C. Nail, H. S. Read, Harry D. Smyth, Toshio Takamine, W. D. Tillyer, A. D. Udden.

The program of papers presented and read is as follows:

Experimental Work on Airplane Photography, U. S. Air Service. H. E. IVES.

Photography of Trajectories. A. W. DUFF and L. P. SIEG.

Psychology and Physics Applied to the Flyer. K. DUNLAP.

Determination of Position of Shell Burst by Photography at Night with the Stars. F. R. MOULTON.

Forces on Projectiles, Due to Air Streams of High Velocities. G. F. HULL and L. G. BRIGGS.

Progress in the Design of Artillery Projectiles. O. VEBLEN.

Method of Testing Drop-bombs. A. W. EWELL.

The Pressure Waves Due to the Discharge of Large Guns. D. C. MILLER.

An Interesting Observation of Light Reflection on Water and its Applications. F. C. BROWN.

Tree Telephony and Telegraphy. G. O. SQUIER.

Quantity Production of Vacuum Tubes for Military Use. N. H. SLAUGHTER.

Determination of Geographic Position of Airplanes by Astronomical Observations. J. P. AULT.

Radio Direction Measurements with Particular Reference to the Effects of Daylight and the Use of Undamped Waves and Spark Transmitters. C. KINSLEY and A. SOBEY.

High Frequency Currents on Wires. J. O. MAUBORGNE.

Inclinometers for Airplanes. D. L. WEBSTER.

General Investigation of Altitude Instruments. M. D. HERSEY, A. H. MEARS and H. B. HENRICKSON.

General Investigation of Air Speed Indicators. F. L. HUNT.

General Investigation of Tachometers. G. E. WASHBURN.

Accuracy Obtained in Sound Ranging in the A. E. F. A. TROWBRIDGE.

Airplane Navigation. H. N. RUSSELL.

Location of Aircraft by Sound. G. W. STEWART.

Electrical Transmission of Information in Airplane Detection by Binaular Method. J. P. MAXFIELD.

Production of Helium. H. N. DAVIS.

Electrostatic Dangers to Balloons. W. F. G. SWANN and G. S. FULCHER.

The Principles of Camouflage. M. LUCKIESH.

Transatlantic Flight from the Meteorologist's Point of View. W. R. GREGG and C. F. BROOKS.

Effect of Winds and Other Atmospheric Conditions on Airplanes. C. F. BROOKS.

The Meteorological Work of the American Army. R. A. MILLIKAN.

The Use of the Oscillograph for the Study of Ballistic Problems. H. L. CURTIS and R. C. DUNCAN.

Use of Coil Aerial as Direction Finder and Transmitting Device. F. A. KOLSTER and L. E. WHITTEMORE.

Improvements in Precision Measurements at Radio Frequencies. J. H. DELLINGER.

Report on the Applicability of Ultra-violet Radiation to Signaling. I. G. PRIEST and K. S. GIBSON.

Detection of Invisible Objects by Heat Radiation. S. O. HOFFMAN.

Infra-red Signaling. W. W. COBLENTZ.

The Use of Interference Methods in the Calibration of End Standards. C. G. PETERS.

High Precision Reference Blocks. W. E. HOKE.

High Power Searchlamps and Their Characteristics. E. KARRER.

Application of Physical Laboratory Methods to the Study of Aircraft Power Plant Problems. H. C. DICKENSON and STAFF.

Study of Flame Propagation in Cylinders of Aircraft Engines. D. MACKENZIE, R. K. HONAMAN, F. B. SILSBEE and L. G. SAWYER.

Characteristics of Vapor-pressure Thermometers. C. W. WAIDNER, E. F. MUELLER and R. M. WILHELM.

Rapid Methods of Binocular Testing. W. O. Lytle.

Determination of Refractive Indices by the Tank Immersion Method. W. H. TAYLOR.

Non-Rectilinear Propagation of Long Electromagnetic Waves. A. H. TAYLOR.

A Method and Instrument for the Measurement of the Visibility of Objects. L. A. JONES.

Submarine Detection Problems. M. MASON.

Characteristics of a High Speed Signal Lamp. A. G. WORTHING.

A Precision Study of Quenching. N. B. PILLING.

Evolution and Absorption of Gases in Glasses. D. ULREY.

High-velocity Jets of Gas, Applicable to Rockets. R. H. GODDARD.

Modulation for Radio Signaling. R. A. HEISING.

Phase Relations in Vacuum Oscillators. M. K. AKERS.

High Frequency Bridges. C. R. ENGLUND.

The following papers were presented by title only:

Bureau of Aircraft Production: Military Aeronautics. Compasses for Airplanes, R. C. WILLIAMSON and J. A. THOMSON. Ultra-violet Signals and Trench Signalling, J. T. TATE. Leak-Proof Tanks for Airplanes, G. S. FULCHER. Infra-red Signalling (Dyscrasite), T. CASE. An Internally Damped Pendulum, R. W. WILSON. A Gyroscopic Inclinometer, A. H. COMPTON. An Air-speed Meter Independent of Density, C. F. KETTERING. A Gyroscopically Damped Pendulum, A. W. Duff and W. A. Hyde. Double-drift Methods for Ground-speed, A. W. Duff. Bubble Sextant for Airplane, R. W. Wilson. Gyroscope with Double Magnetic Erector, A. N. Lucian. A Gyroscopically Stabilized Camera, W. A. Hyde. A Liquid Oxygen Container, C. A. Kraus. A Gyroscopic Integrator for Ground-Speed; Electrical Method for Ground-speed, C. Barus. Photographic Method for Air-speed Determination, L. P. Sieg. Drift Sight for Cross-wind Flying, D. L. Webster. Color Filters for Detecting Camouflage, F. A. Saunders. Signalling Lamp Devices, R. W. Wood.

Naval Radio Laboratory. Short Wave Reception and Transmission on Ground Wires; Long Wave Reception and the Elimination of Strays on Ground Wires; Variation in Direction of Propagation of Long Electro Magnetic Waves; A. H. Taylor. Application of Subterranean Radio Reception to Remote Control Stations, A. H. Taylor and A. Crossley.

Ordnance U. S. Army. Certain Results on Air Resistance as Affecting the Flight of Drop Bombs, F. C. Brown. Optical Glass Developments, F. E. Wright. Window and Mirror Position Finders and Accuracy of Results, A. L. Loomis. The Use of the Acoustic and Magnetic Effects of a Projectile in Flight in Determining the Complete Path of the Projectile and its Velocity. and Retardation at Any Point, G. F. Hull. The Development of the Aberdeen Chronograph, P. E. Klopsteg. Physical-Chemical Developments in Explosives, A. W. Hixon.

Weather Bureau. Short Report of the Climatology of France and Belgium, E. C. Day. Meteorology in Aëronautics, W. R. Blair. Mean Values of Free Air Barometric and Vapor Pressures, Temperatures and Densities over the United States, The Turning of Winds with Altitude, W. R. Gregg. Physics of the Air; Holes in the Air; Summer and Winter Temperatures, Pressures, Humidities and Densities at Various Elevations in the Region of Northern France, W. J. Humphreys.

Bureau of Standards. An Electrical Control System for Aircraft Guns which Shoot Between the Blades of the Propeller, F. Wenner and W. R. Wright. A Study of a Hydraulic Machine Gun Synchronizing Gear. H. L. Curtis and R. Morgan. A Study of Naval Gun Firing Circuits, H. L. Curtis. A Determination of the Time Elements in the Firing of Primers, H. L. Curtis, Margaret Rogers. Methods of Controlling the Time of Primer Fire, H. L. Curtis, R. C. Duncan and H. H. Moore. Study of the Dispersion of Fire of Heavy Naval Guns, H. L. Curtis, R. C. Duncan, H. H. Moore and E. S. Purrington. A Time pressure Gage for Measuring Gun Pressure, H. L. Curtis, F. E. Kester and R. C. Duncan. Location of Metallic Bodies by Electrical Inductance Methods, H. L. Curtis and M. C. Batsel. A Study of the Cotton-Weiss Sound Ranging System, H. L. Curtis and L. W. McKeehan. Instruments for the Detection of Mining Operations, H. L. Curtis, E. B. Stephenson, E. C. Crane, W. G. Dow. The Motion of a Rotating Projectile, H. C. Richards. Electrical Conduction in Porcelain, F. B. Silsbee and R. K. Honoman. Notes on the Physics of the High-tension

Magnets, F. B. SILSBEE. The Use of Subsidiary Spark Gaps in Ignition Systems, W. S. GORTON. Development of Induction Signalling for Airplane Landing, F. A. KOLSTER. Development of Apparatus for Submarine Radio Communication, J. A. WILLOUGHBY and P. D. LOWELL. Electric Wave Transmission Formulas for Antenna and Coil Aerials, J. H. DELLINGER. Properties of the Insulating Materials of the Phenol Type, J. H. DELLINGER and J. L. PRESTON. Inductance, Capacity and Resistance of Coils at Radio Frequencies, L. E. WHITTEMORE and G. BREIT. Applications of the Cathode-ray Oscillograph in Radio Work, L. M. HULL, L. E. WHITTEMORE. Electrical Performance of Dry Batteries, G. W. VINAL and L. R. KOLLER. Factors Determining Service- and Shelf-life of Dry Cells, H. D. HOLLER. Studies of Radium Luminous Materials, N. E. DORSEY, P. T. WEEKS, T. B. BROWN and ELIZABETH YUNG-KWAI. Study of X-Ray Protective Materials, N. E. DORSEY, W. S. GORTON, P. T. WEEKS and NINA M. ALDERTON. An Expeditious Graphical Method Applicable to Radium Measurements, N. E. DORSEY. Precision Micrometers, J. A. ANDERSON and C. A. BRIGGS. Arrangement of Projection Lantern for Measuring Screw Threads, C. A. BRIGGS and H. L. VAN KEUREN. Elastic Indentation of Steel Balls, C. A. BRIGGS, H. G. HEIL, W. C. CHAPIN. Length Changes of Iron and Steel on Passing Through the Critical Regions between 700 and 950° C., W. SOUDER and P. HIDNERT. Thermal Expansivity of a Few Insulating Materials, W. SOUDER, P. HIDNERT. Physical Properties of Methane at Low Temperatures, N. S. OSBORNE, C. W. KANOLT, H. F. STIMSON, T. S. SLIGH and C. S. CRAGOE. A Device for Improving the Operation of Thermoregulators, T. S. SLIGH. The Influence of Temperature on the Velocity of Inversion of Sucrose, R. F. JACKSON and CLARA L. GILLIS. A New Cadmium Lamp for Continuous Operation, F. BATES. The Magnetic and Natural Rotation of Quartz at High Temperatures, F. BATES and F. B. PHELPS. A New Point on the Thermometric Scale; the Transformation Temperature of Quartz, F. BATES and F. P. PHELPS. The Magnetic Rotation of the Iron-oxygen System at High Temperatures, F. BATES and F. P. PHELPS. Optical and Photographic Methods for the Detection of Invisible Writing, I. G. PRIEST and E. P. T. TYNDALL. The Color Grading of Red Signal Flares, I. G. PRIEST. The Spectral Transmission of Various Selective Ray Filters Used to Detect Camouflage or Improve Visibility, K. S. GIBSON, E. P. T. TYNDALL and H. J. MCNICHOLAS. Radiant Power Life Tests of Quartz Mercury Vapor Lamps, W. W. COBLENTZ, M. B. LONG and H. KAHLER. Portable Volume Standard for Gases, M. H. STILLMAN. Photographic Method of Detecting Changes in a Group of Objects with Especial Reference to Camouflage, M. H. STILLMAN. Optical Properties of Balloon Fabrics, W. W. COBLENTZ, M. B. LONG and H. KAHLER. Photoelectric Sensitivity of Molybdenite and Various Other Substances, W. W. COBLENTZ and H. KAHLER. Amplification of Bolometer Current for Signaling Purposes, W. W. COBLENTZ. Protection of Moving Picture Film from Heat of Lamp, W. W. COBLENTZ. Reflecting Power of Stellite and Lacquered Silver Mirror, W. W. COBLENTZ and H. KAHLER. Instruments and Methods of Radiometry, III.—Selective Radiometers, W. W. Coblentz. Effect of Ultra Violet Light upon Lacquers for Protecting Tarnishable Metallic Surfaces, W. W. Coblentz and H. Kahler. Thermal-radiophonic Signaling Devices, W. W. Coblentz. Note on the Distribution of Energy in the Visible Spectrum of a Cylindrical Acetylene Flame, W. W. Coblentz. Comments on Spectral Radiation Formulæ, W. W. Coblentz. Note on the Coefficient of Total Radiation of a Uniformly Heated Enclosure, W. W. Coblentz. The Size of Smoke Particles, P. V. Wells and R. H. Gerke. Measurements on the Index of Refraction of the Saturated Vapor of Ammonia and of Ethyl Chloride in the Temperature Interval, −50° to +50° C., C. G. Peters and C. S. Cragoe. General Study of Aëronautic Instruments, M. D. Hersay, F. L. Hunt. On the Theory of Irreversible Time Effects, M. D. Hersay. Ink Substitutes for Recording Instruments, F. L. Hunt. Vibration Stands for Instrument Testing, F. L. Hunt, J. B. Peterson, G. E. Washburn. A Study of Vibrometers, C. Nusbaum and H. O. Stearns. Experiments on Diaphragms, H. N. Eaton and J. R. Freeman, Jr. A Theorem Relating to the Effect of Temperature on Bodies of Constant Shape, M. D. Hersay. Errors of Barometric Altitude Determination, M. D. Hersay. Computation of Standard Altitude-pressure Tables, W. G. Brombacher. Testing of Altitude Instruments, A. H. Mears, H. B. Henrickson, W. G. Brombacher. Mercurial Altimeters for Laboratory Standards, G. E. Washburn and J. B. Peterson. Theory of the Aneroid Barometer, M. D. Hersay. A Method for Converting Low Altitude Into Altitude Barographs, M. D. Hersay. Designs Relating to Improvements in Aneroid Barometers, J. B. Peterson. 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Dayton C. Miller,
Secretary.

A New Chronograph for Measurement of Projectile Velocities and Short Time Intervals.[1]

By Paul E. Klopsteg and Alfred L. Loomis.

The Aberdeen chronograph was developed for measurements of projectile velocities at the various proving grounds established during the war, particularly at the Aberdeen Proving Ground. Its principal features are: (1) A constant speed direct current motor with automatic governor, holding the speed constant to within .1 per cent. of 25 revolutions per second; (2) a shallow cylindrical aluminum drum, mounted on the vertical shaft of the motor, the *inside* circumference of which is 500 mm. The record strip is spun into the drum while the latter is rotating, and is held there very smoothly and firmly due to the rotation. Its linear speed is 12,500 mm. per second. (3) A record strip of blue paper, coated white on one side with paraffin, on which a spark produces a bright blue spot, making the record marks easily visible. (4) A simple fall apparatus for checking the constancy and measuring the speed of the record strip. (5) A "make-circuit" arrangement by which the projectile causes a spark to record the instant at which its tip perforates a special contact target. The target consists of two sheets of lead foil or tin, separated by a layer of paraffined paper. When the metal sheets are brought in contact, a condenser, charged through high resistances from the 110- or 220-volt direct-current power line, is discharged through the low resistance primary of an induction coil, resulting in a spark from the secondary. A condenser across the line to the contact target greatly increases the intensity of the spark. (6) A direct reading velocity scale which enables the velocity to be obtained, without computation, from the record strip. (7) Portability of the instrument and freedom from error due to shock from the gun. This permits carrying the instruments into the field and using them within fifty feet of the gun under test.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

The combination of the above features with proper organization of the work permitted testing to be done three times as rapidly and with considerably more accurate results than was possible with the former types of instrument; it also made possible other kinds of measurements, such as speeds of shell fragments and of trench mortar bombs, which could not be measured with the regulation chronograph.

The work on the development of instruments for projectile velocity measurement was begun at the Bureau of Standards and the Sandy Hook Proving Ground, and was continued at the Aberdeen Proving Ground. The laboratory facilities of the Leeds & Northrup Company were offered and accepted for part of the development work. This company also contributed certain valuable features of design, and manufactured the instruments which were supplied the government.

Several slow speed instruments, for a linear travel of the record strip of 100 mm. per second, were also built, for measuring times of flight of machine gun bullets.

Evolution and Absorption of Gases by Glass.[1]

By D. Ulrey.

The absorption of various gases by glasses and their subsequent evolution with vacuum-heat treatment have been studied particularly with respect to the production and maintenance of high vacua.

The following conclusions are drawn:

1. The rates of evolution of the different evolution products are, in general, different functions of the time at any temperature, and, for a given baking period, are different functions of the temperature.

2. There is a large variation in the quantity of gases evolved per square centimeter from different samples of the same kind of glass.

3. Annealing in air, at atmospheric pressure in an electric furnace always lessens the quantity of gases evolved on subsequent vacuum heat treatment, although the glass may be under normal atmospheric conditions for an interval of several months between the two processes.

4. At a given temperature and pressure, glass is in equilibrium with a definite amount of H_2O. Water vapor evolved at high temperatures is reabsorbed after the temperature is reduced.

5. Glass from which practically all absorbed gases have been removed by

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

melting in vacuo subsequently reabsorbs gases from the atmosphere at room temperature.

6. At temperatures up to the softening point, diffusion of gases of the atmosphere through glass does not take place.

Airplane Inclinometers.[1]

By David L. Webster.

AIRPLANE inclinometers may be divided into two main classes, those for the inclination of the longitudinal axis of the machine and those for the transverse axis. Of the longitudinal ones little need be said. For aërodynamic experiments they are of course valuable, but in ordinary piloting any reliance on such an instrument as an index of how to fly would be a positive source of danger.

The transverse inclinometers may be subdivided into two classes; first, indicators of deviation from the correct bank during a turn, and second indicators of the absolute vertical during a turn. Closely related to the latter in their use are the turn indicators, designed to show when the airplane is turning, without regard to correctness of bank.

In an open machine the best correct bank indicator is merely the wind on the pilot's face, as the direction of this wind is very sensitive to the slightest errors in banking and the pilot cannot fail to notice it. For closed machines a spirit level of moderate radius and about a 20-degree arc is the most accurate substitute for this wind that has yet been devised, but no pilot should depend on such an instrument for his safety.

An indicator of absolute vertical is of course most important in bombing. Another use for it is in cloud flying, where the erratic behavior of the compass on northerly courses makes a need for something to show when the airplane is turning. Assuming correct banking, any instrument that will indicate the absolute vertical is a good indicator of turns. For this the essential requirement is a pendulum of such a long period that it can not be much influenced by centrifugal effects during the time required for a turn. But it must not have such a long period as to refuse to respond to the rotation of the earth. Theoretically such an instrument might be made by hanging any object of large moment of inertia on a pivot very near its center of gravity, but practically the effects of pivot friction eliminate this possibility. Practically the pendulum must involve a gyroscope. The French Toupie Garnier, a small wind-driven gyro on a single point pivot, is fairly satisfactory in this way, but it has the tendency common to all gyro pendulums, to get into circular oscillations that persist for a long time after a turn and interfere seriously with the use of the instrument. Such oscillations can not be damped with a dash pot attached to the case of the instrument without introducing strong disturbing forces when the airplane banks. The problem has, however, been solved in several ingenious ways. In the Duff-Hyde short period gyro pendulum it is done by dash pots on a free pendulum. In the long period Gray bomb sight stabilizer it is done by weights shifting under the influence of the centrifugal force of a turn. In the Titterington long period gyro pendulum it is done by a blast of air pumped by the gyro itself and directed by small vanes hung on an ordinary pendulum. This problem can therefore be considered as in a fairly satisfactory state, although these instruments are of recent development and may still be considerably improved.

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In cloud flying, however, an absolute inclinometer is not, so far as the author's piloting experience indicates, an ideal turn indicator. For although the pilot can easily know when the machine is banked wrong, the air in clouds is liable to be so "bumpy" that sudden rolling and yawing can not be prevented even with such knowledge. This makes a need for an instrument to indicate the rate of turn of the airplane whether it is correctly banked or not. One good instrument of this sort is an English device consisting of a tube running along the wings and having openings at each end designed to take up the static pressures of the air without errors due to its motion. When the airplane turns, the centrifugal effect on the air in this tube tends to drive it along the tube, and this tendency can be utilized by means of a very sensitive pressure gauge, to tell the direction and speed of the turn. An attempt has been made, with some success, to increase the pressure available by the use of Venturi tubes on the ends of the wings. But even with this help the pressure is small and the errors large, and the instrument is not easy to install accurately. However, it is a workable instrument, and can be made reliable enough if carefully installed.

Another valuable turn indicator, suggested by A. H. Compton and improved by C. E. Mendenhall, is a gyroscope on a horizontal axis transverse to the airplane, with the top of the rim moving forward. This gyro is free to swing on an axis longitudinal to the airplane, and is held by a light spring. When the airplane turns, the precession forced by that motion makes the axis of the gyroscope tip in the direction opposite to that in which the plane must bank. Thus, although a momentary deflection may be due to a sudden roll of the airplane, any sustained deflection necessarily means a turn. It might be supposed that the momentary deflections due to quick rolling might be confusing to the pilot, who would think they meant a quick turn. But as a matter of fact in cloud flying if the airplane rolls to one side, she will soon begin to turn toward that side, because she will sideslip first and then the action of the side wind on the tail, as well as an instinctive response to side wind by the pilot, will both tend to produce the turning motion. Hence this instrument may almost be said to predict turns as well as indicate them. This instrument is easy to install and can be made very reliable and sensitive to the slightest turning motion, and it enables the pilot to fly northerly courses in clouds with the required straightness even with a quick and erratic compass.

As to which type of turn indicator is the better, it is difficult to make a definite statement. The author has done a considerable amount of piloting with each type and prefers the gyroscopic one, but as another pilot who tried them preferred the other, it may be largely a matter of personal taste.

Massachusetts Institute of Technology,
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The Detection of Invisible Objects by Heat Radiation.[1]

By Samuel O. Hoffman,
Master Signal Electrician, Science and Research Division, U. S. A.

Trench warfare on the western front was peculiarly a thing of night activities. At night, No Man's Land was alive with working parties and patrols, but any movement in daylight was promptly followed by hostile fire.

If there had been a means of giving at night something of the visibility of daylight, which rendered extensive movements unsafe, it is apparent that methods of trench warfare would have been entirely changed.

Early in 1918 work was started on the problem of detecting men and other objects at a higher temperature than their background, in the dark, by their thermal radiation.

It is to be noted that for detecting invisible objects by thermal radiation, only a receiving instrument is required; that is, it is not necessary to project a beam of radiation on to the body, the body furnishes its own radiation and without special precautions it cannot prevent its radiation escaping and its presence becoming known. Furthermore an object sighted on has no means of ascertaining that it is under observation as there is nothing proceeding from the receiving instrument that could be used for locating it.

Experimental Results.—The apparatus consisted of a Hilger thermopile mounted in the focus of a 14-inch (36 cm.) parabolic mirror (silvered on the surface), together with a d'Arsonval galvanometer.

Men were detected with ease at 600 feet (180 m.). A man lying in a depression in the ground at a distance of 400 feet (120 m.) was detected unfailingly as soon as he showed the upper part of his face above ground. These results were repeated on different nights and with different backgrounds during April, 1918. The mirror was a Navy Searchlight parabola, focus 7 1/2 inch (19 cm.), the Hilger pile had a sensitive surface 0.1 × 1 cm., resistance 3 ohms. The galvanometer sensitivity was 5 mm. per microvolt at 1 m. scale distance; period 6 seconds, critical damping resistance 60 ohms. Naturally it was rather sluggish in action.

Previously tests had been made with a 4-inch (10-cm.) concave mirror, relative aperture $f/2$, same thermopile and galvanometer. Extremely large deflections (8 cm.) were obtained for a chimney at 300 feet (90 m.). Men were detected at 50 feet (15 m.), and it was readily possible to distinguish between the painted and unpainted sides of a piece of sheet iron on fairly cloudless nights, but not at all possible on a heavily overcast night.

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Secret signaling was accomplished by simply covering and uncovering the face. At moderate distances a man could not cross the range of the instrument without giving a marked indication. This was the same whether he crawled slowly or ran rapidly past. This suggested an arrangement for preventing raiders from creeping into our lines undetected; simply ranging the instrument parallel to our front line and about 30 yards in advance thereof.

In general, on directing the instrument in any direction a large permanent deflection is obtained, due to the background. This is compensated for by a slide wire potentiometer, keeping the spot of light at the center of the scale.

The Apparatus.—Work was at once started on standardizing and constructing the equipment. Full use was made of the excellent papers by Coblentz on bismuth-silver thermopiles (Bull. Bur. Standards, 9, p. 15, 1911). No bismuth wire being obtainable, it was made by squirting without any great difficulty. A little trouble was experienced in getting high conductivity bismuth. The final product was about equal to the imported in thermo-E.M.F., (69 to 79 microvolts per degree C.) and superior in conductivity, the resistance of 0.07 mm. wire being 273 ohms per meter. The piles were constructed under a binocular microscope using special tools and jigs and the work was so standardized that 10 couple piles could be made in about three hours. These were so uniform that after a time sensitivity tests were discontinued. The following design was arrived at as the result of constructing some 40 different types.

Bismuth wire, 0.07 mm. diam. × 3 mm. long,
Silver wire, 0.025 mm. diam. × 5 mm. long,

Receiving surfaces, 0.018 mm. silver foil, 1/2 mm. × 1 mm., blackened with lampblack paint and smoked over camphor. About 1.5 mm. of 0.05 mm. silver wire was inserted between the end of the bismuth wire and the copper terminal. This, together with leaving a little slack, reduced the breakage during construction to practically nil. No piles of the final design have been broken in use.

These piles reached their maximum deflection in a little less than a second and as the weight of the lampblack paint is one-fourth that of the silver foil, it is believed that this is near the limit of rapidity of action for this type of construction. The resistance of a 10-couple pile is about 6 1/2 ohms. The cold junctions are directly attached to copper plugs, the circuit throughout being strictly copper to avoid stray thermo-E.M.F.'s. The heavy enclosing case is air-tight, the radiation entering through a varnished rocksalt window. No trouble from drift was experienced when everything was air-tight.

The galvanometers used were specially constructed Leeds & Northrup, high sensitivity d'Arsonval instruments of two types:

1. Sensitivity 15 mm. per microvolt, period 5 seconds, critically damped

on 5 ohms. Coil consists of 7 turns of 0.003 inch Belden enameled copper wire. Suspensions, strip rolled from silver wire 0.0175 mm.

2. Sensitivity 5 mm. per microvolt, period 1 1/2 seconds, critically damped on 4 ohms, coil 6 turns same wire as above, suspensions, strip from 0.0375 mm. silver wire.

Both had 1/4-inch (6 mm.) plane mirrors, coil was 1 cm. × 2.5 cm., length of upper suspension 10 cm. Multiple reflection was used to give a meter optical lever with the scale 30 cm. distant.

The mirrors used were of two sizes, 24-inch (60-cm.) and 12-inch (30-cm.) cut down from standard navy searchlight mirrors of respectively 36-inch (90-cm.) and 18-inch (45-cm.), silvered on the surface. The silver tarnished readily, but this did not seem to affect the sensitivity and was of advantage in that it lowered the visibility of the mirror considerably.

This apparatus was shipped overseas in August, 1918.

Ranging on Aircraft by Thermal Radiation.—Tests on a model having shown that there would probably be no trouble in picking up aircraft on clear nights, a special instrument was constructed, provided with a 24-inch (60-cm.) mirror and having slow screw movements in azimuth and elevation. Listening horns were provided for rough location by sound, but were not used. "Cross-hair" thermopiles were made, the separate elements of which (vertical and horizontal) are separate piles, electrically insulated from each other and connected to separate galvanometers. Each line of receivers consists of 32 couples with receivers 1 mm. square. The center point is shared in common by both piles.

Night flying tests were made in January, 1919, at Langley Field, Virginia. The flying was done at an altitude of 3,500 feet (1,100 m.) on a course about a mile long and about 500 yards from the receiving instrument. Time was six P.M. and quite dark. The weather was very damp and hazy. The slant distance of the plane varied from 4,000 feet to well over a mile. The plane used was a Curtiss JN 4 H, wing spread about 44 feet (14 m.); the motor, rated at 150 H.P. was throttled down to 1,200 R.P.M. giving an air speed of 55 miles an hour, at which speed the motor developed somewhat less than 50 H.P. This was done to allow for the relative nearness of the plane; a heavy bombing plane would develop over ten times this power with a consequent increase in the radiated energy.

No trouble was experienced in picking up the plane or in keeping the dark image on the pile. The deflections were very large, averaging about 10 cm. with a maximum of 25 cm. They were not proportional to the distance, apparently depending on the aspect of the plane and not falling off greatly at the maximum distance. The movement of the spot started sharply, but died down gradually, due to the effect of the hot exhaust gases. So long as clouds did not drift across the field, the galvanometer was admirably steady, but the slightest whiff of cloud gave a warm indication, as large as a plane would give. This rendered sweeping the sky for possible planes somewhat uncertain. The impulse given by the clouds however, was of a different nature from that given by a plane. The characteristic abrupt jump at the start was absent.

The ease of picking up the planes showed that the elaborate thermopile was unnecessary; a much smaller one would have done as well.

In conclusion I wish to express my sincere thanks to Dean George B. Pegram, of Columbia, in whose laboratory this work was done, and who has given much helpful advice, to Major Joseph Sears, Post Commander at Columbia, to Lt. Col. R. A. Millikan, to Lieut. Victor Fleming for assistance in the early field trials, and particularly to the following soldiers for their loyal and efficient work as mechanics and laboratory assistants; Sergeant 1st Class, George Richards, Air Service, and Sergeants Harold MacCracken and George Gotti, Signal Corps.

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LOCATION OF AIRCRAFT BY SOUND.[1]

BY G. W. STEWART.

THE solution of the problem of the location of aircraft by sound involves at least three accomplishments. These are: first, the production of a sound locator which will operate satisfactorily under practical conditions; second, the application of a sufficiently rapid method of correcting for the velocity of the airplane and for the influence of meteorological conditions; third, the development of recording methods whereby the information is available not only of the past position of the aircraft, but also its present and future computed positions. This abstract will refer only to the locator itself and even more narrowly, to the scientific aspects thereof.

The chief requirements for the locator are sufficient accuracy and range, satisfactory freedom from the interference of noises other than the one whose source is to be located, and a faithfulness of reproduction of the quality of sound. These requirements have been met in this country as herein described.

Accuracy has been secured by the application of the fact, known for a number of years, that a difference of phase, intensity constant, produced at the two ears produces an apparent displacement of the source of sound from the median plane.

Range has been secured by a combination of resonance and concentration with equipment that gives appreciable resonance for any frequency within the range desired. This device is a long conical horn.

Freedom from "side" noises has been secured by receivers having large openings, thus introducing interfering phase differences for wavefronts not parallel to the opening.

Faithfulness of reproduction to a satisfactory degree has been obtained by using as receivers conical horns of the order of 15 feet in length which give such a satisfactory sound spectrum distribution of sensitivity.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

The results achieved in the field may be briefly summarized as follows:

1. With receivers 12 feet apart, the accuracy of locating the wave front is correct to within a few tenths of a degree; there is no practical limit to the frequency with which such an apparatus will operate, tones as high as 15,000 d.v. having been tried; the sharpness of the location is improved by the complexity of the sound. An additional fact of importance which must be considered under accuracy is that the reaction time causes a lag of from 0.5 to 0.8 seconds, with good observers.

2. With the horns 18 feet in length and diameter of opening 4.5 feet the range under fair night conditions for airplane elevation of 6,000 feet was approximately three times that of the unaided ear. Experiments indicated, however, that the amplification of the apparatus was 100, thus showing great diffusion in the atmosphere.

3. Similar airplanes could be separately located by such a device if more than 5° apart, and if engine noises were sufficiently marked a separate location could be secured with only 1° actual separation.

4. The device gives a greater faithfulness of reproduction than is necessary for present methods of defense against bombing aircraft, but this characteristic can no doubt be utilized to a greater degree, involving an improvement in defense.

Physical Laboratory,
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Progress in Design of Artillery Projectiles.[1]

By O. Veblen.

The paper is a report on experimental work carried out at the Aberdeen Proving Ground, which led to improvements in artillery projectiles. Among the questions investigated were the effects of false ogives, of boat-tailed or stream lined bases, and of devices for modifying the yaw. Considerable increases in range were obtained by these methods, but the most important gain was obtained by a modification of the rotating band by the removal of some excess copper. This resulted, in the case of the 6-inch gun, in an increase in range of about 2 1/2 miles, and in a diminution of dispersion by a factor of 8. Similar results were obtained in several other guns. It is intended to publish an account of these studies on rotating bands in an article by Major Veblen and Lieutenant Alger in The Journal of the United States Artillery.

These investigations were of an empirical nature, and were carried out, for the most part, without the guidance of a mathematical theory. The absence of a mathematical theory is particularly to be underlined in the case of the rotating band experiments, because semi-official popular accounts of this work have been published in which it is stated that the results were obtained by pure mathematics. The work at the proving ground was done in close collaboration by several men, among whom may be mentioned Lieutenants P. L. Alger, F. E. Fish and H. Galajikian, and Captain J. W. Alexander. Although the coöperation among all concerned was so close as to make it impossible to trace ideas to their sources, the largest share of the credit must be assigned to Lieutenant Alger, particularly in the rotating band investigations. The results on the 6-inch gun were obtained independently in France by Captain R. H. Kent, after studying the Aberdeen results for the 8-inch howitzer.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

NOTE ON THE DISTRIBUTION OF ENERGY BY THE VISIBLE SPECTRUM OF A CYLINDRICAL ACETYLENE FLAME.[1]

BY W. W. COBLENTZ.

UNDER the above title Hyde, Forsythe and Cady[2] have discussed data on the spectral energy distribution of an acetylene flame published by the writer,[3] which data they say will not agree with that of a black body at any temperature to better than 7 to 8 per cent. The statement is misleading, for no attempt is made to discuss the data in terms of the accuracy attainable in making the radiometric observations, in different parts of the spectrum. As shown in a previous paper,[4] in the yellow to red part of the spectrum (0.6 μ to 0.7 μ), where the intensities are high, an accuracy of 1.0 to 0.5 per cent. is attainable. On the other hand, in the blue and violet (0.4 μ), where the deflections are small an accuracy of only 5 per cent. to 10 per cent. is attainable.

Computations made in this laboratory some years ago, indicated that in the spectral region of 0.45 μ to 0.7 μ the energy distribution of the acetylene flame used was close to that of a black body at about 2100° C. (2375° K). Using the constant $C_2 = 14{,}350$, and the temperature 2360° K. recommended by Hyde, the computed spectral energy curve fits the observations close to 1 per cent., throughout the spectrum from 0.50 μ to 0.74 μ, which is remarkable considering the difficulties of observation. The observations at 0.45 μ to 0.4 μ are high by 5 to 10 per cent. This may be owing, in part, to instrumental errors, and it may be owing to a high emissivity, as indicated in a previous paper[5] and as shown in the spectrophotometric measurements on a cylindrical flame, by Nichols and Merritt.[6]

The independent check by Hyde and his collaborators confirms the direct radiometric observations on the spectral energy distribution of the acetylene flame, in region of 0.50 μ to 0.75 μ; and shows that the difference in our visibility curves, in this part of the spectrum, cannot be explained on the basis of an erroneous energy evaluation.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Hyde, Forsythe & Cady, PHYS. REV., 13, p. 157, 1919.

[3] Bull. Bur. Standards, 13, p. 355, 1916.

[4] Bull. Bur. Stds., 7, p. 252, 1911.

[5] Bull. Bur. Stds., 7, p. 275, 1911.

[6] Nichols and Merritt, PHYS. REV., 30, p. 328, 1910.

Bureau of Standards,
Washington, D. C.,
March 17, 1919.

PYROMETRIC PRACTICE. I., THE TEMPERATURE SCALE; II., THERMOELECTRIC PROMETRY.[1]

By Paul D. Foote, T. R. Harrison and C. O. Fairchild.

THE following lists the topics which are treated at length from a practical standpoint. The paper will appear as a series of articles in one of the trade journals. Temperature Scale.—Thermocouple; Metals used for thermocouples; Constancy of calibration; Reproducibility of couples; Mounted thermocouples. Indicating instruments; Galvanometer method; Resistance of indicating instruments; Use of portable test set; Galvanometer with variable series resistance. Potentiometer method.—Semi-potentiometric method; Northrup pyrovolter; Brown Heatmeter: Temperatures of the cold junctions of thermocouples; Compensating leads; Potentionetric compensation; Compensation by shunt; Wheatstone bridge compensation. Methods employed in correcting for variations in the temperature-E.M.F. relation of different couples of the same type; Adjustment by series resistance; Adjustment by shunt resistance; Adjustment by shunt and series resistance. Thermocouple installations; Common return; Wiring diagrams of multiple installations: Commutating switches; Distributing box; Temperature of buried cold junction; Depth of immersion of couples; Couples purposely insufficiently immersed. Protecting tubes for thermocouples. Tables.

A STUDY OF THE VELOCITY OF FLAME PROPAGATION IN THE CYLINDERS OF AIRCRAFT ENGINES.[1]

By D. MacKenzie and R. K. Honaman.

A METHOD has been devised for determining the average velocity with which the igniting flame is propagated over various paths in a one-cylinder Liberty engine. The cylinder is fitted with three spark plugs, only one of which is used to supply ignition. The other two are connected to condensers charged to a potential just below the breakdown potential of gaps in the compressed mixture. When the spreading flame reaches these gaps, ionization ensues and the condensers are discharged. The discharged currents, as well as the current which produces ignition in the first spark plug, cause deflections of an oscillograph vibrator. The time interval between each of these events is thereby determined and from it the average velocity of the flame over a known path. Velocities of 5 to 25 meters per second have been observed. Further work must be done in order to ascertain the effect of different variables of engine operation upon the velocity of flame propagation.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

Washington, D. C.,
June 16, 1919.

Inductance, Capacity and Resistance of Coils at Radio Frequencies.[1]

By L. E. Whittemore and G. Breit.

The inductance coil used in a radio circuit may usually be considered as a pure inductance shunted by a capacity representing the distributed capacity of the coil. The pure inductance can most conveniently be determined from the values of the apparent inductance as obtained from the capacity which is required to give resonance at certain frequencies. The apparent inductance is very large at wave-lengths which are near the natural wave-length of the coil, decreasing as the wave-length is increased until a very long wave-length is reached at which the inductance increases again slightly due to the absence of skin effect. The pure inductance and distributed capacity of the coil may be obtained from the data used for plotting the apparent inductance curve by the use of the following formulas:

$$C_0 = \frac{(\Sigma C_1)(\Sigma\lambda_1^4) - (\Sigma\lambda_1^2)(\Sigma C_1\lambda_1^2)}{-N\Sigma\lambda_1^4 + (\Sigma\lambda_1^2)^2},$$

$$L_0 = \frac{0.2814[N\Sigma\lambda_1^4 - (\Sigma\lambda_1^2)^2]}{-(\Sigma C_1)\,(\Sigma\lambda_1^2) + N\Sigma C_1\lambda_1^2}.$$

C_0 and C_1 being expressed in micromicrofarads, λ_0 in microhenries, and λ in meters. N is the total number of observations from which C_0 and L_0 are calculated. C_1 and λ_1 are corresponding values of the capacity in series with the coil and the resonant wave-length. The assumptions made in the derivation of this least square formula are that the error in the capacity measurement is constant and that the percentage of accuracy in the wave-length measurements is constant.

For measuring the capacity of a coil it is convenient to use a generating circuit in which the current contains harmonics of the fundamental frequency. The coil whose capacity is to be measured is connected to the terminals of a condenser and the circuit is tuned to the fundamental frequency, the capacity of the condenser being C_1. Let the circuit then be tuned to the second harmonic by the use of the capacity C_2. The capacity of the coil C_0 must be such that $C_0 + C_1 = 4\ (C_0 + C_2)$. While it would seem that the accuracy obtainable in this measurement would be greater if higher harmonics were used, it has been found that usually on account of the reaction of the measuring circuit upon the generating circuit it is better to use the second harmonic as described above.

If a curve is plotted giving the capacity which it is necessary to use with the coil to obtain resonance at various wave-lengths against the square of the wave-length, this curve is a straight line which cuts the axis of capacity on the negative side of the origin. The point at which this curve cuts the axis gives the value of the capacity of the coil.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

If the coil whose capacity is to be obtained is immersed in a liquid of dielectric constant K, the capacity of the coil C_0 is

$$C_0 = \frac{C_1 - C_2}{K - 1}.$$

Where C_1 and C_2 are the capacities required to tune the coil to resonance with a certain frequency before and after the coil is immersed in the liquid. Care must be taken in this measurement to avoid the presence of solid dielectrics in the field of the coil. It is also necessary to know accurately the value of the dielectric constant K.

The following may be given as examples of measurements which have been made on coils suitable for use in radio circuits:

A solenoidal coil having 569 turns, 11.1 cm. radius, length 70 cm., had a measured pure inductance of 1.949×10^4 microhenries and a distributed capacity of 25 micromicrofarads. The direct current inductance computed from a current sheet formula is 1.972×10^4 microhenries. The fact that the pure low frequency inductance is smaller than the computed inductance is possibly accounted for by the skin effect or by eddy currents.

A coil suitable for use as a direction finder and consisting of ten turns, spaced 1 cm. apart, wound in a single layer on a frame 124 cm. square had a capacity of 50 micromicrofarads.

A coil 12.2 cm. long and 31 cm. in diameter having 76 turns in a single layer was found to have a capacity of 14.7 micromicrofarads.

By immersing in oil and water, the capacity between the turns of a certain small coil was found to be 0.6 micromicrofarad.

The capacity of a coil to ground is important in many cases. If the two condensers, C_1 and C_2, are connected in series with a coil and the point of connection between the condensers is grounded, then, at resonance, the capacities of these condensers are related by the expression:

$$\alpha C_1 + \beta C_2 + \lambda C_1 C_2 + \delta = 0,$$

where α, β, λ and δ are constants of the coil and its position.

For a symmetrical coil the capacity to ground is given by the expression:

$$C_g = 4\left[\frac{C_1C_2}{C_1 + C_2}(\text{max.}) - \frac{C_1C_2}{C_1 + C_2}(\text{min.})\right].$$

The maximum is obtained when $C_1 = C_2$. The minimum value of C_1C_2/C_1+C_2 is obtained when either C_1 or C_2 is short-circuited.

It is interesting to note that a coil oscillate freely at several different wave-lengths. For a number of double layer coils which were tested, it was found that these wave-lengths had the relation

$$1, \frac{1}{K}, \frac{1}{K\sqrt{K}},$$

where K is a constant for the coil in question, and for a given coil had a value of 4.47.

The resistance of the coil is most conveniently measured by the resistance-variation method described in Bureau of Standards Circular 74. The resistance depends very much upon the capacity of the coil. The greatest accuracy is obtainable in the use of the resistance-variation method when the inserted resistance is 0.6 of the resistance of the circuit. That is, when by its insertion the galvanometer deflection is reduced to about 0.4 of the maximum deflection. The following empirical expression was found to represent the curve between the resistance of the coil and the wave-length

$$R = A + \frac{B}{\frac{\lambda^2}{\lambda_0^2} - 1},$$

where R is the resistance at wave-length λ and where A, B and λ_0 are constants of the coil.

WASHINGTON, D. C.

THE SIZE OF SMOKE PARTICLES.[1]

BY P. V. WELLS AND R. H. GERKE.

IT is one of the familiar properties of ultramicroscopic particles that some portion of any given type are electrically charged. Moreover, it is now accepted that the charges carried by any such particles are always multiples of the elementary unit, the electron. When the particles are not ionized by some powerful agent, the charged particles form but a fraction of the whole number and most of the charged particles possess but a single electron. Compared with the number carrying one electron, in statistical work the number doubly charged is negligible. The observed motion of the particles in a given electrostatic field is thus a measure of their size.

A method is presented for measuring the size of ultramicroscopic particles by their oscillation amplitudes in an electric field which is reversed by a rotating commutator. This eliminates the uncertainties in previous methods due to the effects of Brownian motion and to passing out of focus. Moreover, it renders possible the precise determination of the Stokes' law size of a single particle.

The general properties of gaseous dispersoids and the application of this method to their statistical study are briefly discussed.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

AN EXPEDITIOUS GRAPHICAL METHOD APPLICABLE TO RADIUM MEASUREMENTS.[1]

By N. E. Dorsey.

IN the determination of the radium content of sealed specimens that have not reached equilibrium it is very desirable to ascertain promptly the concordance of each observation with those that have preceded it. For several years the following graphical method has been used at the Bureau of Standards with great satisfaction.

The excess of intensity of the gamma radiation from the specimen at the time t_2 above that at the time t_1 is given by the equation

$$R_2 - R_1 = 1.0089\epsilon^{-\lambda t_1}R_m\{1 - \epsilon^{-\lambda(t_2-t_1)}\},$$

where R_2 and R_1 are the intensities of the radiation at the times t_2 and t_1 respectively, R_m is its equilibrium value and λ is the decay constant of radium emanation. It is assumed that at the time t_1 the specimen had been sealed for at least 5.5 hours. $1.0089\,\epsilon^{-\lambda t_1}R_m$ is the amount by which the gamma radiation at the time t_1 falls short of the equilibrium value. The equation states that the growth in the radiation during the interval t counted from an arbitrary origin is proportional to $(1 - \epsilon^{-\lambda t})$; the factor of proportionality being the amount the radiation still had to grow at the arbitrary origin of time.

Hence if the observed values of R be plotted on any convenient scale against the corresponding values of $(1 - \epsilon^{-\lambda t})$, t being measured from an arbitrary origin the values of R will lie on a straight line and the equilibrium value will be the intersection of this line with the coördinate corresponding to t equal to infinity. For convenience in plotting, a time scale consisting of values of $(1 - \epsilon^{-\lambda t})$ corresponding to suitably selected values of t may be laid off once for all along the edge of a card and marked with the corresponding times.

While this graphical method enables the equilibrium value to be determined in most cases with satisfactory precision it is the custom of the bureau finally to compute this value by a least square reduction of the observations.

Bureau of Standards,
Washington, D. C.,
April 18, 1919.

PHOTOGRAPHIC METHOD OF DETECTING CHANGES IN A COMPLICATED GROUP OF OBJECTS WITH ESPECIAL REFERENCE TO CAMOUFLAGE DETECTION.[1]

By M. H. Stillman.

THE method consists of making two negatives of approximately equal density of the group of objects, one before the change, the other after the change; printing a positive from tone of the negatives and then superimposing this positive upon the other negative.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

When these photographs are properly made, those parts of the picture which have not changed in the interval between exposures will show up as a field of practically uniform density while the changes are clearly shown up by a considerable departure from this uniform density. When it is impracticable to make the scale of the original negatives the same, as in aircraft work, results can still be obtained by enlarging or reducing, by matching the negative with the projection of the positive, or vice versa.

This method has long been in use in astronomy for the detection of variable stars. It was independently devised by a member of the Bureau of Standards staff and developed for camouflage detection, engineering, and other purposes in coöperation with Captain H. E. Ives of the United States Air Service.

PORTABLE VOLUME STANDARD FOR GASES.[1]

BY M. H. STILLMAN.

THIS instrument, which is described in Technologic Paper No. 114 of the Bureau of Standards, was developed by this bureau particularly for use in testing gas-meter provers. It is, however, adapted to much wider uses. Among the incidental uses to which it has been put are the accurate measurement of volumes of gas in connection with the poison gas investigations of the army, and the direct calibration of laboratory gas meters.

The instrument consists essentially of three telescoping vertical cylinders, one of which is arranged to move relative to the other two, this motion being accurately limited by stops.

This instrument is now being manufactured by one or more firms.

NOTE ON THE COEFFICIENT OF TOTAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE.[1]

BY W. W. COBLENTZ.

UNDER the above title the writer published[2] a value of the so-called Stefan-Boltzmann constant of radiation from a uniformly heated enclosure or so-called black body. The value is $\sigma = 5.72 \times 10^{-12} + 0.012$ watt cm.$^{-2}$ deg^{-4}. It is based upon about 600 measurements, made with 10 receivers, which are summarized in Table 6 of a previous publication.[3] The data obtained with receivers no. 8 and 9 were not included because the apparatus was defective. The data obtained with these 10 receivers were corrected for radiation lost by reflection, which loss amounts to 1.2 per cent. for receivers covered with lampblack (soot) and 1.7 per cent. for receivers covered with platinum-black. It does not include a set of measurements made on an unblackened radiator. The reflection from a receiver covered with platinum-black, then smoked, is 1.2 per cent. These corrections were determined by direct measurements upon some of the receivers and by comparison of the surfaces of the other receivers with samples of lamp-black whose reflection losses had been determined in a previous investigation.[1]

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Proc. Nat. Acad. Sci., 3, p. 504, 1917.

[3] Coblentz and Emerson, Bull. Bur. Stds., 12, p. 549, 1916.

Experiments were made on atmospheric absorption and it was shown that if any correction for atmospheric absorption is to be made to these data, it can hardly be greater than 0.1 per cent.

Recently a new determination[2] of this radiation constant was brought to my attention, and in view of the fact that this paper contains inaccurate statements concerning my own work a few comments are permissible. For example, the statement is made that the only novelty in the apparatus employed by Coblentz and Emerson (*loc. cit.*) was a thermopile with a continuous receiving surface; which is of secondary importance. As a matter of fact, the crucial part of the apparatus was a receiver with potential terminals attached thereto, at a sufficient distance from the ends to avoid the question of heat conduction to the electrodes. These potential wires, which were from 0.003 mm. to 0.02 mm. in diameter, accurately defined the length of the central part of the receiver which was utilized in the measurements. By exposing the whole length of the receiver to radiation, conduction losses, did not enter the problem. The writer is not aware of anyone having used a similar apparatus which compares with this receiver in nicety of construction, and reproducibility of results under given conditions.

The receiver used by Kahanowicz was placed at the center of a spherical mirror, with an opening in one side to admit radiation. In this manner the correction for reflection was eliminated. The shutter was close to the receiver. If its temperature was different from that of the water-cooled diaphragm, which was before the radiator, errors in the radiation measurements would occur. As mentioned in previous papers, the shutter should be placed between the water cooled diaphragm and the radiator, to avoid a change in surroundings facing the receiver when the shutter is raised for making the radiation measurements. The temperature range was from 260° C. to 530° C. The distance from the radiator to the receiver was 35 to 55 cm. A series of 28 measurements gave an average value of $\sigma = 5.61 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$. Of this number 11 gave a value of $\sigma = 5.7$. Out of a series of 4 measurements made in December, 1916, with the distance $d = 56$ cm., three gave a value of $\sigma = 5.7$.

No corrections were made for atmospheric absorption, which for the temperatures used is not negligible.

In a previous paper[3] it was shown that on removing the moisture (vapor pressure of 10 to 12 mm.) from a column of air 52 cm. in length, the radiation constant was increased from $\sigma = 5.41$ to 5.55 or about 2.6 per cent. For the the spectral region transmitted by rock salt, to $\lambda = 15\,\mu$, the absorption is about 1 per cent.[4] Other measurements mentioned in these papers indicate an absorption of 2 to 3 per cent. of the radiations emitted at 1000° C. for the average humidity of Washington.

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[1] Bull. Bur. Stds., 9, p. 283, 1913.

[2] Kahanowicz, Nuovo Cimento, (6) 13, p. 142, 1917, Naples.

[3] Bull. Bur. Stds. 12, p. 576, 1916. See Table 3: Series CLXXX. to CLXXXII.

[4] Proc. Nat. Acad., *loc. cit.*

Dr. H. H. Kimball, of the U. S. Weather Bureau, very kindly sent me comparative data, showing that the vapor pressures at Naples are considerably higher than at Washington. From these data it would appear that the correction for atmospheric absorption must be at least 1 per cent. For the low temperatures at which the radiator was operated, a fair estimate of the correction to the radiation data obtained by Kahanowicz is 1.5 to 2 per cent., or a value of $\sigma = 5.69$ to 5.72×10^{-12} watt cm.$^{-2}$ deg.$^{-4}$. In other words the Naples value of the coefficient of total radiation is comparable with other recent determinations which indicate a value of $\sigma = 5.7 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$.

Bureau of Standards,
Washington, D. C.,
March 8, 1919.

The Electrical Fume Mask.[1]

By W. W. Strong.

It has been well known that fine fumes are harder to absorb than gases under many practical conditions. An example of this difficulty is that of the absorption of suphuric acid mist by an alkaline solution such as that of hydroxide of ammonia. The usual process of absorption is to pass fine bubbles of the mist through the ammonia. Fine mist particles in these bubbles do not come into contact with the ammonia and thus pass though the solution unaffected.

A problem of this nature arose in connection with the poisonous matters used in the war. The charcoal gas mask was adapted for the absorption of gases and for the absorption of fumes it was necessary to add an extra filter that increased the difficulty of breathing.

It was suggested by the chemical warfare service that possibly the process of electrical precipitation might be adapted in some way to the problem at hand and a number of suggestions such as the development of a small electrostatic machine or a magneto or an induction coil to operate a precipitator were made.

The writer took up the problem and as it was closely related to the problem of developing the maximum corona pressure, previous knowledge as to the proper type of electrodes for the development of such a corona pressure made the early development of suitable electrodes possible. In the arrangement used a wire electrode is made to provide the corona while the passive electrode serves for the collection of the fume. In service the amount of poisonous fume present is so small that the choking of the electrodes with precipitated fume hardly appears possible.

Early in May of last year an electrical fume mask was developed that answered the fume precipitation problem as far as the proper type of electrodes was concerned, but was only a beginning as regards the limits that were set for the weight and the life of the apparatus. The apparatus consists of a small induction coil, a special type of active and passive electrodes and a battery for the operation of the induction coil. The mask proper would contain the ordinary parts of the gas mask that control the incoming and outgoing air currents or the electrical parts could be added to those of the ordinary gas mask. By the use of silver and alkali any trouble with ozone or oxides of mitrogen could be avoided.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

The weight of the whole fume task has been reduced to about two pounds and when the armistice was signed the life development of the mask was several hours, a time that was entirely too short and which constitutes the remaining problem for solution.

It is planned to have the working parts of this apparatus to exhibition at the meeting. For a short account of electrical precipitation and a brief bibliography the reader is referred to the Trans. of the American Electrochemical Society, Vol. 31, pp. 514–430, 1917.

MECHANICSBURG, PA.

SOME NUCLEAR CHARGES CALCULATED FOR L-RADIATION.[1]

BY FERNANDO SANFORD.

IT has previously been shown that on the assumption that x-rays are emitted by electrons which are revolving in elliptical or circular orbits about positive sub-atoms the central positive charges may be calculated, in the case of K-radiation from the two equations $Q = 2.882 \cdot 10^{-12}/\sqrt{\lambda}$ and $Q = 2e(N - b)$. From the measurements of Blake and Duane the value of b is 3.6.

Element.	*N*.	λ, Obs.	λ, Calc.	Dif.
Sn	50	2.889	3.040	+.151
Pr	59	1.933	1.920	−.013
Nd	60	1.775	1.830	+.055
Eu	63	1.590	1.582	−.008
Gd	64	1.558	1.548	−.010
Tb	65	1.470	1.480	+.010
Dy	66	1.418	1.414	−.004
Ho	67	1.365	1.370	+.005
Er	68	1.316	1.316	
Yb	70	1.223	1.223	
Lu	71	1.183	1.184	

In Friman's measurement of the wave-lengths of the L-radiation band for a considerable number of elements[2] he has indicated what he regards as corresponding lines in the different spectra. The shortest of these lines for any considerable number of elements is marked γ^3. This line is indicated in the spectra of eleven elements. The central charges for these lines may be calculated from the equation $Q=e(N-15.4)=2.882\cdot 10^{-12}/\sqrt{\lambda}$. In the preceding table the wave-lengths calculated from this equation are compared with Friman's measured wave-lengths.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Phil. Mag., 32, 497, Nov., 1916.

PHONOTELEMETER; A DEVICE FOR LOCATING THE SOURCE OF A SOUND WAVE FROM A SET OF SOUND-RANGING DATA.[1]

BY H. M. DADOURIAN.

ONE of the most interesting applications of physics to modern warfare is the locating of enemy guns by the method of sound-ranging. This consists in observing the time of arrival of the report of the gun at three or more observation posts, and from these computing the position of the gun.

Set T_1, T_2, T_3, etc., be the instants at which the report reaches the observation posts, O_1, O_2, O_3, etc. Further, let P denote the position of the gun, and O_1 the post nearest to the gun. Then if distances are laid off from O_1, O_2, O_3, etc., on the lines of O_1P, O_2P, O_3P, etc., equal to respectively $v\ (T_1 - T_1)$, $v(T_2 - T_1)$, $v(T_3 - T_1)$, etc., these points will lie on a circle of which P is the center, and PO_1 the radius. This fact is made use of in the following simple mechanism for locating the source of a sound wave from a set of sound-ranging data. To each of the points O_1, O_2, O_3, etc., on a military coördinate map of the sector which is being served, a perfectly flexible and inextensible string is attached. These strings go through the small hole of a bead and terminate at a moveable point A. The length of the string attached to any one of the posts O_i is so adjusted that it is greater than the shortest string by a length equal to $v(T_i - T_1)$. Then if the point A is pulled zaway from the posts in general only three of the strings will be taut. But if the bead is moved while the point A is pulled away from the posts a position will be reached at which all the strings are equally taut. This, then, will be the position of the gun on the map.

If only two of the strings are put through the bead at a time, the bead describes the hyperbola which contains the position of the gun. By making use of all the combinations in pairs, $n(n - 1)/2$ hyperbolas can be drawn from n strings. Theoretically all these hyperbolas pass through the point which represents the position of the gun, but in practise they form in the neighborhood of this point a sheaf, the middle point of the narrowest part of which represents the position of the gun. The phonotelemeter is so constructed as to lend itself to corrections for wind, temperature and altitude.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

Some Experiments Bearing on the Nature of γ-Rays.[1]

By Alois F. Kovarik.

EXPERIMENTS were performed to find out whether or not the observed effects of γ-rays take place simultaneously in all directions at the same distance from the source. This was done by using the method of autoregistration of γ-ray effects described before this society last December. It was found that they do not occur simultaneously, although the average number over a certain length of time is the same for positions at equal distance from the source. The experiment was conducted as follows: Two independent point discharge chambers were made of thin aluminium for one set of experiments, of thick brass for another, and of thin paper for a third. One of these chambers as connected with the self-registering device and the other with a string electrometer; the one produced audible clicks and the other visible motion of the fiber of the electrometer. Radium enclosed so as to emit γ-rays only into the space around it was placed midway between the two chambers at various distances in different experiments. Each apparatus registered the same number of "effects" per minute, *i.e.*, if these "effects" are due to individual pulses or trains of pulses, the same number of such γ-ray pulses gave up their energy to each counting chamber in the same time, but these effects did not occur simultaneously. One chamber was kept fixed in position and the other was moved around a circle at whose center was the radium source; the two were also placed side by side, one above the other, and one behind the other and in no case were the effects simultaneous. On an ether-string or a corpuscular hypothesis one may readily find an explanation of the observations by assuming a definite quantum and a definite direction for the individual γ-ray pulse or train of pulses and by assuming that some atom in line with this traveling energy is in a condition to receive this quantum of energy and emit an electron with the proper speed to correspond to this energy. On the wave theory there seem to be greater difficulties. First of all, since the ejected β-particles have a definite amount of energy and since these experiments show that atoms in the chambers do not eject the β-particles simultaneously, and since the density of the energy in the wave front becomes smaller the farther the wave front is from the source, the atoms must absorb the energy and store it until a definite quantum is reached. Furthermore, some element of the atom must have initially unequal quantities of energy. While these are possible suppositions they seem more difficult to conceive.

Experiments were also tried to find the effect of materials of which the chamber is made both on the natural effect and on that produced by the γ-rays. A chamber was made of different thin materials. Part of the chamber was common to all, namely, the point with its ebonite holder and a short piece of brass tube, over which equal sized tubes of paper, aluminium, brass, lead, tin and platinum were placed. The chambers thus contained the same volume of air, had a portion of material common, and did not absorb the γ-rays appreciably in any case. The natural effects were markedly different, being for paper only a few per cent. of the effect for platinum, and the order of increasing "natural count" being paper, aluminium, brass, lead, tin, platinum. The γ-ray effect was also smaller for the paper chamber than for the platinum, but relatively of the same order of magnitude. These results seem to indicate that while the material of the chamber is of importance it is of smaller importance than air in the chamber; and that the natural effect (natural ionization in ordinary ionization experiments) is due mainly to rays emitted by the materials, whether this be caused by outside penetrating radiation acting on the material or by rays starting initially in the substance itself. In view of the following experiment it seems that it may be due to the latter cause. Counts were made using an ordinary brass chamber. Then a carbonized paper was inserted, fitting closely into the brass chamber. Records were made both for the natural and the γ-ray effects under the two conditions. The results showed a smaller number, although not very much smaller, when the paper coating was used indicating a stoppage, to some extent, of radiation from the brass.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

ELECTRIC WAVE TRANSMISSION FORMULAS FOR ANTENNA AND COIL AËRIALS.[1]

BY J. H. DELLINGER.

IN a radio transmitting or receiving set, either the condenser or the inductance coil is made of large dimensions. It is then called the aërial, and effects the transfer of power between the radio circuits and the ether. The coil aërial has the inherent advantage of serving as a direction finder and interference preventer, but is less effective quantitatively as a transmitting or receiving device than the condenser aërial, commonly called the antenna. The practical question, how far can communication be maintained by a coil in comparison with an antenna, is answered by the following formulas, derived from electromagnetic theory. A flat-top antenna is considered, and a rectangular coil. Let h = height of antenna or coil, l = horizontal length, and N = number of turns of wire of coil aërial, I = current, λ = wave-length, d = distance apart of transmitting and receiving aërials, R = resistance of receiving aërial circuit. The subscripts s and r refer to sending and receiving, respectively. All lengths are in meters.

Antenna to antenna:

$$I_r = \frac{188.\, h_s h_r I_s}{R\lambda d}.$$

Antenna to coil:

$$I_r = \frac{1184.\, h_s h_r l_r N_r I_s}{R\lambda^2 d}.$$

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

Coil to antenna:

$$I_r = \frac{1184. h_s l_s h_r N_s I_s}{R\lambda^2 d}.$$

Coil to coil:

$$I_r = \frac{7450. h_s l_s h_r N_s N_r I_s}{R\lambda^3 d}.$$

For distances greater than about 100 km. multiply any of these formulas by the factor $\epsilon^{-0.0000047(d/\sqrt{\lambda})}$ based on empirical results of Austin.

These formulas have been verified by experiments made at the Bureau of Standards and elsewhere. In some experiments the received current will be found to be greater than the formulas indicate, because the coil aërial acts both as a coil and as an antenna by virtue of its capacity to ground. This double action of the coil structure is likewise indicated by values obtained for radiation resistance and by the observed directional properties.

The use of the coil aërial is particularly advantageous, as may be seen from the formulas, for short waves. In most cases, the usefulness of the coil increases as its dimensions are made to approach the order of magnitude of the wave-length. An advantage of the coil not apparent from the formulas is that its resistance can usually be made lower than that of the corresponding antenna.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

IMPROVEMENTS IN PRECISION MEASUREMENTS AT RADIO FREQUENCIES.[1]

BY J. H. DELLINGER.

THE methods of measurement at radio frequencies are unique, in that no mechanical system can follow the alternations of current themselves. The methods are essentially indirect and hence specialized. They include methods which depend upon (*a*) balancing of reactance, (*b*) thermal effect, and (*c*) rectifying effect. In general the methods are very simple in principle, but very difficult in practice.

The radio work which the Bureau of Standards was called on to do during the war involved many kinds of measurements. As a result of the work done here and elsewhere on radio measurements, rapid progress has been made in this fairly new science. High-frequency phenomena are now susceptible of accurate calculation. On account of the general use of continuous rather than damped waves the theory of the subject is a direct application of low frequency theory. The principles of radio measurements are coextensive with the principles of radio engineering and operation to an extent not true in other fields. For example, the wavemeter is a complete though miniature transmitting and receiving station, consequently progress in measurements contributes to progress in the general subject of radio.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

I can give here a mere table of contents of the work which has been done in this subject. The principles of the subject are given in the publication "Radio Instruments and Measurements," issued by the Bureau of Standards in March, 1918. The work described here refers chiefly to work done at the Bureau of Standards since that time.

In the general field of the development of apparatus, depending upon reactance and the balancing of reactances, a special study has been made of the inductance and capacity of inductance coils used in radio. It has been found that not only must the distributed capacity of the coil be considered, but also the capacity of the coil to the surroundings. The resistance of coils has also been studied, and empirical expressions for the variation of resistance with frequency determined. New types of wavemeters have been designed. These included direct-reading wave-meters of simple type and a long-wave wavemeter employing special and very compact type of coil.

Included under the general head of measurements depending upon the thermal effect of the current, studies have been made of the methods of measuring those factors which give an expression for the power loss in a radio circuit. These quatities are power factor, resistance, phase difference, decrement, and sharpness of resonance. Direct-reading methods have been worked out and scales constructed by which a direct-reading phasemeter or decremeter may be readily made out of any ordinary wavemeter. Convenient measuring sets have been developed. Studies have been made of the precision of resistance measurements by least-square methods. A particularly large amount of work has been done on the power factor of insulating materials, and a general study has been made of the high-frequency properties of such materials. In connection with these measurements work has been done on the shielding of radio measuring circuits. The principal source of error in radio measurements arises from the presence of electrostatically induced E.M.F.'s. These methods produce effects which cannot be handled by calculation and have heretofore been a source of great difficulty and error. They can now be eliminated by a suitably grounded and designed metallic shield which may entirely surround the measuring circuit or may be merely placed between the measuring circuit and the generating circuit. Certain measurements also have been greatly improved by the use of a screened cage which entirely surrounds the measuring apparatus and observer.

A type of measurement which is not included under the classes mentioned above depends on the use of a system which does actually follow the alternations themselves. As previously stated this cannot be a mechanical system, but is a beam of cathode rays. Reference is made to the cathode ray oscillograph which is utilized to plot Lissajou's figures on a fluorescent srceen—the figures depend on phenomena taking place in radio circuits. This means the phase and high-frequency relations in radio circuits can be accurately and visibly studied. Progress has been made in the development of rectifying devices and methods of measurement. The measurement of intensity of signals in a telephone receiver has been developed into an accurate method by use of the principles of comparison of sound intensities. The crystal detector has been thoroughly investigated, and means devised for improving its constancy. A large amount of work has been done on the vacuum tube as a generator of oscillations, a detector, and an amplifier. The great improvements in radio measurements and engineering which have resulted are well known. In the laboratory the use of the vacuum tube gives the experimenter an entirely new power over radio phenomena, and the combination of this with the cathode-ray oscillograph opens up fields of measurement which have boundless possibilities.

Bureau of Standards,
Radio Laboratory,
Washington, D. C.

The Use of the Oscillograph for the Study of Ballistic Problems.[1]

By H. L. Curtis and R. C. Duncan.

At the request of the Bureau of Ordnance of the Navy Department the Bureau of Standards undertook investigations of the possible causes of dispersion of the Navy's 14-inch guns under actual service conditions. After a careful analysis of the problem, it was decided to design apparatus for the following: (1) To determine the firing time intervals. By determining the time of the occurrence of various events which indicate the completion of certain steps in the progress of the firing. (2) To determine the motion of the gun during the firing of single gun salvos and also of turret salvos. This involves the study of the recoil of the gun and also its lateral motion in both vertical and horizontal directions, usually called jump and whip, respectively.

The nature of the work demanded that the apparatus be self-recording and that the recording apparatus be placed some distance from the guns. The regular General Electric oscillograph was chosen as the most suitable instrument for the recording of the measurements. However, it was necessary to modify it to meet the special conditions of the investigation.

A timing system was developed which causes flashes of light to fall across the film at regular intervals, thus ruling the film in time units. These units may be made either 100th or 1,000th of a second as desired.

Electric circuits were so arranged that one oscillograph element recorded the make of the current through the firing circuit, the break of the primer bridge, the explosion of the primer, the beginning of the recoil of the gun and the ejection of the shell. From this record, the various firing intervals were determined.

In the measurement of the motion of the gun a point by point method was used throughout. A system of contacts was arranged with a resistance element connected between each of them. A battery was connected in series with the oscillograph element, a sliding contactor and this series of contacts. The motion of the gun caused the sliding contactor to move along the series of contacts thus changing the current step by step in the oscillograph circuit. The apparatus was calibrated in advance so that the amount of motion required to produce a certain definite step in the current value was known. As the timing lines were ruled on the same film, this record furnished sufficient data to plot a distance-time curve of the motion of the gun.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

In the actual work aboard ship a room four decks below the main deck was used as the laboratory. In most cases three oscillographs were used simultaneously and in some cases four. A 14-wire cable connected the oscillograph elements to the apparatus on the guns while another cable connected the operating circuits of the oscillographs to the firing key. The closing of the firing key electrically operated the oscillograph shutters and also operated a relay which fired the guns. This relay was so timed that the firing circuits were closed a few 1,000ths of a second after the shutters were opened.

Data has been taken during the experimental firing of two battleships. The results have been such that it is planned to carry on investigations during other experimental firings.

A time-pressure gage is also being developed for studying the variation of pressure inside the powder chamber of the gun. The pressure is measured by the motion of a piston working against a strong spring. The motion of the piston is recorded on an oscillograph film by means of a step by step contact system, similar to the one used in studying the motion of the gun. Sufficient data is obtained from the oscillograph record to plot a curve showing the variation of pressure with time.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 3, 1919.

THE SPECTRAL COMPOSITION AND COLOR OF CERTAIN HIGH INTENSITY SEARCHLIGHT ARCS.[1]

BY IRWIN G. PRIEST, W. F. MEGGERS, H. J. MCNICHOLAS, K. S. GIBSON, AND E. P. T. TYNDALL, IN COÖPERATION WITH THE SEARCHLIGHT INVESTIGATION SECTION, CORPS OF ENGINEERS, U. S. A.

THE development of high intensity arcs, by the use of specially prepared carbons, has resulted in the production of a very blue light from such sources. In a general investigation of searchlights, the effectiveness and suitability of illumination for various purposes, it appeared desirable to have at least approximate quantitative data as to the quality of the light (spectral distribution, color).

There are considerable difficulties in the spectrophotometry of such a source. It was therefore decided to obtain some preliminary data by the short cut color matching method of rotatory dispersion.[2] By this method one may determine quickly a spectral distribution which gives the same color as the source being studied. The spectral distribution for equivalent color of a regular army Sperry 45-volt, 75-amp. searchlamp obtained in this way was found to be very close to that of the noon sun at Washington,[1] *i.e.*, approximately that of a "black body" at 6000° K.

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[1] Abstract of a paper presented by title at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Arons, Ann. der Phys., *39*, 545, 1912. Priest, PHYS. REV., 2, 10, 208, 1917.

Preliminary spectrophotometric measurements with the Koenig-Martens spectrophotometer were made in April, 1918; but inasmuch as we had no description or specification of the carbons other than the maker's name, it was decided to make further measurements with carbons subjected to spectroscopic analysis for the sake of greater definiteness of description.

The relative spectral distribution of radiant energy throughout the visible spectrum was measured for the following arcs: Sperry searchlight arc, Columbia carbons, 75 amperes; Sperry Searchlight arc, Speer carbons, 75 amperes; Ordinary arc, Electra carbons, 10 amperes. These relative values of radiant power were obtained visually by a spectrophotometric comparison to a standard Mazda C 500-watt lamp (B.S. Lamp no. 1717), the spectral distribution of which had been accurately determined radiometrically by W. W. Coblentz of this bureau. This comparison was made by a substitution method, the light from the arc and the standard lamp in turn being measured in terms of that of a second 500-watt Mazda C lamp. The apparatus and method may be briefly described as follows:

Light from the arc (or the standard lamp) on one side and the comparison lamp on the other, entered the two halves of an illumination box. These two compartments were separated from each other by a brass partition. The interior of each compartment was coated white with magnesium oxide, and blocks of magnesium carbonate were placed with their surfaces at 45° to the direction of the incident light from the two sources.

Light from these two illuminated carbonate surfaces entered the double slit of a König-Martens polarization spectrophotometer, through small openings in the illumination box. The whole interior of the box was of course illuminated by diffuse reflection from the magnesium carbonate blocks, and two beams of light from the sides of the box next to the spectrophotometer passed through openings in the opposite side and were compared by means of a Martens polarization photometer.

A red glass filter was placed over the eyepiece of this photometer to eliminate the color difference between the arc and the comparison lamp, and a ground glass diffusing screen used to make the photometric field still more uniform and of the proper brightness. A rotating sector was used to reduce the illumination from the arc to the proper value. In some of the runs a 5-millimeter diaphragm was placed over the arc to cut off all light except that coming from the center of the positive carbon.

Five people were necessary in making the observations on the arc, one to watch and control the arc, recording the readings of a voltmeter, a second to regulate the current through the arc, recording the readings of an ammeter, a third to observe the illumination through the Martens photometer, a fourth to take readings on the spectrophotometer, and a fifth to record these readings and keep the proper voltages on the lamps by means of a potentiometer. The procedure for taking a series of observations under these conditions was as follows:

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[1] Abbot's data. See Priest, Phys. Rev., 2, 11, 502, 1918.

The arc was allowed to run a few minutes in order that conditions might become as steady as possible. While this was being done, the comparison lamp was kept at its proper voltage, and the variations of the illumination from the arc followed on the Martens photometer. Finally the photometer was set for equality of brightness between the two sources at what seemed to be the best mean value of the illumination from the arc. This setting was then unchanged during the run. Now for any given wave-length, the variations in the illumination from the arc were followed on the spectrophotometer, the attempt being made to keep the two halves of its comparison field always equal in brightness. Whenever conditions were right as shown by the Martens photometer, a sharp signal was given and the reading of the spectrophotometer taken at that instant.

At each wave-length from four to ten readings were taken on the spectrophotometer, the number depending on the agreement obtained among the readings. Measurements were made from 430 or 440 to 710 millimicrons, usually at every 20 millimicrons.

After this series of measurements was completed, the arc was replaced by the standard lamp and another series of measurements was made at the same wave-lengths as previously on the arc. Only two observers were now necessary, one to make the measurements on the spectrophotometer, the other to record the data and keep the voltages constant with the potentiometer.

In this manner all the data were obtained, either a complete run or check points being taken on the standard lamp between any two of the runs on the arc.

These measurements thus gave the spectral distribution of radiant power of the arc relative to that of the standard lamp; and knowing the values for the latter, the true relative values of the spectral distribution in the arc were obtained.

In the following table are given some of the values thus obtained, all being reduced to 100 at 590 millimicrons. These are the final averages of all the runs, both diaphragmed and undiaphragmed. As noted below, there is little difference in the two cases.

Perhaps the most striking characteristic of the spectral distribution of radiant power of the 75-ampere arcs is the preponderance of the blue. This was to be expected from the bluish color of the arc when run at that current The sudden increase of radiant power below 450 millimicrons is too large to be due to errors of measurement, in spite of the difficulty of obtaining accurate values in this region. Spectrograms of these arcs taken at 10 amperes show that this increase in the blue is due to a selective emission band analogous to the carbon bands in the plain arc at shorter wave-lengths.

Wave-length, Millimicrons.	Relative Radiant Power.		
	Columbia Carbons 75 Amp. Ave. of 4 Runs.	Speer Carbons 75 Amp. Ave. of 3 Runs.	Electra Carbons 10 Amp. Ave. of 2 Runs.
710	98.6	103.9	146.1
670	93.4	98.1	125.5
630	98.6	101.7	110.3
590	100.0	100.0	100.0
550	115.0	116.6	83.4
510	149.7	141.4	67.2
490	154.6	155.6	61.8
470	169.6	155.2	55.1
450	182.1	151.7	47.8
440	262.6	229.2	47.7
430		372.8	

Spectroscopic analysis revealed the fact that the Columbia and Speer carbons were cored with a mixture of rare earth elements among which cerium, thorium, lanthanum, and yttrium were represented. The emission spectra of these rare earths are very rich in lines, especially in the blue and violet spectral regions, so that a combination of them gives practically a continuous spectrum. Spectroscopic analysis of the incandescent vapors in the center of the arc showed that the rare earths in the Columbia and Speer carbons carried most of the current and suppressed the spectrum of carbon, while the electra carbons, which contained no rare earths, gave a very discontinuous spectrum in which the carbon bands of short wave-lengths were most prominent.

There seems to be no consistent difference between the data for the Columbia carbons and for the Speer carbons. In both cases the values obtained when the diaphragm was used, are lower in the blue than those found with the arc undiaphragmed. In the red, however, the diaphragmed values are higher than the undiaphragmed values in one case (Columbia carbons) and lower in the other (Speer carbons).

The plain carbon arc at 10 amperes shows little selective emission over this range. It is to be regretted that time was lacking to make measurements on this arc at various currents up to 75 amperes.

This work would have been extended, but was discontinued on account of demobilization.

A complete report illustrated with diagrams of apparatus, spectral distribution graphs and photographs of emission spectra was issued to the Corps of Engineers, U. S. A., and others, Jan. 23, 1919 (Bull. No. 4, " B. S. Bulletins on Visibility and Camouflage "). It is expected that the complete report will also be published later in the military reports of the Bureau of Standards.

Bureau of Standards,
Washington, D. C.,
April 24, 1919.

OPTICAL AND PHOTOGRAPHIC METHODS FOR THE DETECTION OF INVISIBLE WRITING.[1]

BY IRWIN G. PRIEST, AND E. P. T. TYNDALL.

OPTICAL and photographic methods for the examination of suspected papers and documents, developed by the authors and used by the Office of Naval Intelligence and Military Intelligence during the war, included the following: (1) Examination by intense illumination at grazing incidence, (2) examination by regular reflection, (3) examination by fluorescence, (4) ultra-violet photography.

Special apparatus was designed and constructed at the Bureau of Standards to facilitate rapid and convenient routine examination by these methods. This is to be exhibited at the Physical Society meeting, April 25–26. Four confidential technical reports on this subject were made to the Office of Naval Intelligence, February to May, 1918, and a later report (August) by one of the present authors (Tyndall) in conjunction with Dr. N. E. Dorsey on the use of the electroscope and X-ray photography in detecting secret writing. A more detailed account of this work will appear in the forthcoming military reports of the Bureau of Standards.

NATIONAL BUREAU OF STANDARDS,
April 20, 1919.

REPORT ON THE APPLICABILITY OF ULTRA-VIOLET RAYS TO SIGNALING.[2]

BY IRWIN G. PRIEST AND K. S. GIBSON.

SEVERAL months before the entry of the United States into the war, the Bureau of Standards was requested to advise the Navy as to the selection and specification of a lamp to be used as a stern light, so designed that it would be visible to other ships of the same squadron at 1,000 yards (914 meters) astern while not visible at greater distances and not easily picked up by enemy ships.

This request suggested to us the possibility of screening the source with a filter transparent to ultra-violet, while opaque to the visible, and observing it with a fluorescent detector. Experiments made at the bureau, February to May, 1917, showed in general the feasibility of this idea; and a letter to this effect was delivered to Admiral R. S. Griffin, June 1, 1917. A detailed technical report was made to the Director of the Bureau of Standards March 5, 1918, and transmitted by him to Admiral Griffin, March 6, 1918. Most of the following information is taken from this report. For a more detailed illustrated account reference must be made to the forthcoming military reports of the Bureau of Standards.

The experiments were made under the following conditions:

1. *Sources.*—A right angled carbon arc with current up to 25 amperes and a

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[1] Abstract of a paper presented by title at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

vertical iron arc with current up to 15 amperes were used. (A mercury arc would perhaps be more favorable than either of these, but was not used because of the *a priori* conclusion that such a lamp would not be rugged enough for actual service at sea.) No condensing, collimating or directing mirrors or lenses were used in these trials, although they were recommended for use in service.

2. *Screens.*—Corning glass G-55-A-62[1] and similar glasses made by the Corning Glass Works at our request; aqueous solution of nickel sulphate; aqueous solution of p-nitrosodimethylaniline. The wave-lengths actually used were in a narrow band centered at about 340–350 millimicrons.

3. *Range.*—Experiments were made in an open range of about 654 meters. Rotating sectors were employed to give effect of greater distance. To facilitate the experiments the sending and receiving station were in direct telephonic communication.

4. *Receiver.*—The receiver consisted of a short-focus quartz lens forming an image on the fluorescent screen, which was observed by a low power positive ocular from the opposite side. The observer's eyes were completely shaded by a close-fitting shade, and a thin piece of "purple-ultra" glass (Corning G–55–A–62) was inserted between the objective and the screen. This cuts out extraneous illumination, showing the fluorescent image on a dark background *and is essential to sensitiveness.*

5. *External Conditions.*—All tests were made at night, but under various conditions of light from sky, moon, etc.

Our conclusions may be stated as follows:

1. *General Conclusions.*—Working on the above range and with the apparatus mentioned we have observed clearly visible fluorescent images when the source was nearly or quite invisible to the unaided eye, and the fluorescent image still remained visible when a 4 per cent. rotating sectored disk was interposed before the source. However, the following qualifications of this general conclusion must be noted.

It is very difficult to reach any definite conclusions as to the absolute invisibility of such a source to the unaided eye. The circumstances of the visibility or quasi-visibility of the source with filters used in our experiments were as follows:

Even with our best ultra-violet filters the source was always visible when near it. At the receiving station, when the source was nearly invisible, it could never be focused as a blue or violet point source by the unaided eye although the same source with a red filter could easily be seen as a red point source. However, when carefully looked for, an indefinite blue-gray haze, covering a large area of sky, could be seen. This phenomenon is apparently due to fluorescence of the eye media rather than to true visibility of the light transmitted by the ultra-violet filter. The personal equation in the visibility of this blue haze is considerable; it may be invisible to some and quite visible to others. This visibility is a function of the observer's age. The blue haze was much more visible to Mr. Filgate, age twenty, than to Mr. Priest, age thirty-one. Owing to the senile yellowing of the crystalline lens, it is still less visible to older observers. Its appearance in the sky is of such an elusive nature that its presence can only be determined by the method of right and wrong answers. The darkness or lack of darkness of the night, of course, affect this visibility to the unaided eye while not affecting the detector as we use it. The advantage of this detector over the unaided eye is thus greatest on moonlight nights or with other lights in the neighborhood of the sending station.

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[1] The suitability of this glass for the purpose was known to us from tests of its visual and ultra-violet spectral transmission which we made in another connection in 1196.

The source may, of course, be found and sharply focused by "night glasses" when it is invisible to the unaided eye. However, the fluorescent image may be seen in the detector when the source can not be seen using this same optical system simply as a telescope (fluorescent screen removed).

2. *Special Conclusions.*—(*a*) An absolutely dark (or invisible in perfectly dark surroundings) ultra-violet source for practical signaling through the earth's atmosphere, is not possible because the source is disclosed by the fluorescence of the eye itself, although the appearance is very vague and ill-defined.

(b) As position and identification lights designed to be inconspicuous (if not quite invisible) and capable of having their visibility increased to one provided with the fluorescent detector, the ultra-violet screened sources may be recommended as worthy of trial in practice.

(*c*) The method could be used very efficiently at night and the source be made entirely invisible to the unaided eye (probably also to ordinary telescopic examination) if the sending station were "camouflaged" by bright yellowish lights (carbon filament lamps or arcs screened with "no-glare" glass) placed close to it. The glare from such sources would obliterate the faint blue haze. It seems probable that this procedure would be quite practicable in certain military and naval circumstances.

Regular lighthouses along our coast might send ultra-violet signals which could be picked up by our own ships and never be suspected by any one not equipped with the fluorescent detector.

(*d*) Although we have made no trials by daylight, it would appear that conditions might be more favorable than night for obtaining the desired

(*e*) The best practical signaling apparatus is apparently the iron ar the Corning glasses and nickel sulphate. The p-nitrosodimethylanil not necessary. Silver films have been tried as ultra-violet screens w practical success. The uranyl sulphate was the most sensitive detect by us.

NATIONAL BUREAU OF STANDARDS,
April 20, 1919.

A New Formula for the Spectral Distribution of Energy from a Complete Radiator. II., Concerning the Value of the Constant D_2.[1]

By Irwin G. Priest.

IN two previous papers,[2] the author has proposed the following formula for the radiant energy from a black body as a function of temperature and wave-length.

$$E_\lambda = D_1 T^5 e^{-D_2[A^{-1/3}-(\lambda T)^{-1/3}]^2}.$$

In these papers tentative values of the constants A and D_2 were indicated as follows, on the basis of Coblentz' spectral distribution data:

A = 2,920 to 2,940 micron degrees,

D_2 = 4,481 (micron degrees)$^{2/3}$.

Tables of computed spectral distribution were given for $A = 2{,}940$ and $D_2 = 4{,}481$.

Dr. E. P. Hyde has called the author's attention to the fact that Dr. Forsythe in his laboratory has found that the proposed equation with $A = 2{,}940$ and $D_2 = 4{,}481$ does not lead to the correct experimental value for the isochromatic intensity ratio:

$$\left[\frac{E_{(\text{Palladium point, } T=1828)}}{E_{(\text{Gold point, } T=1336)}}\right] \text{ for } \lambda = 0.665\,\mu.$$

The experimental value for this ratio is given by Dr. Hyde as 77.5. Wien's (or Planck's) equation for $C_2 = 14{,}350$; "gold point" 1,336 K; "palladium point" 1,828 K; gives this ratio as 77.2. The proposed equation for the same temperatures with $A = 2940$ and $D_2 = 4481$ as pointed out by Dr. Forsythe and verified here, gives 70.5.

This circumstance has led the author to a more careful consideration of the determination of D_2, resulting in the following conclusions:

1. The data considered in the previous papers is best satisfied by $A = 2{,}940$ micron degrees in the proposed equation.

2. The constant A being determined by the isothermal distribution data as 2,940, the value of D_2 required to satisfy the above mentioned isochromatic ratio for the gold and palladium points at $\lambda = 0.665$ microns is 4,640 [micron degrees]$^{2/3}$.

3. *This value of D_2 satisfies the isothermal spectral distribution data just as well as the previous value*, 4481. See accompanying table.

4. The gist of the matter is simply this:

The isochromatic ratio data give a much more sensitive criterion for the determination of D_2 than the isothermal distribution data for *relative* energy. In the previous papers no attention was given to isochromatic data and the value of D_2 was obtained solely to satisfy the isothermal distribution data.

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[1] Abstract of a paper presented by title at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Optical Society of America, Baltimore, December 27, 1918; American Physical Society, New York, March 1, 1919. Phys. Rev., 2, 13, 314.

It so happened that the value chosen did not accurately satisfy the isochromatic data; but both sets of data may be satisfied by a single value, viz: $D_2 = 4{,}640$.

	I. λT	II. $e^{-4640[2940^{-\frac{1}{2}}-(\lambda T)^{-\frac{1}{2}}]^2}$	III. Exp. Data for Rel. Energy Arbitrary Scale. Coblentz B. S. Sci. Pap. 284 Fig. 2 $T=1596.5K$.	IV. Residuals, II-III.
	942.	0.0082	0.0147	−.0065
	1102.	.0339	.0344	−.0005
	1213.	.0700	.0570	+.0130
	1325.	.1238	.1080	+.0158
	1485.	.2288	.2288	−.0000
	1676.	.3844	.3879	−.0035
	1788.	.4810	.4861	−.0051
	1900.	.5752	.5705	+.0047
	2044.	.6873	.6874	−.0001
	2155.	.7649	.7827	−.0178
	2427.	.9067	.9123	−.0057
	2714.	.9841	.9761	+.0080
$\lambda_m \longrightarrow$	2986.	.9997	.9800	+.0197
	3257.	.9743	.9457	+.0286
	3512.	.9268	.9231	+.0037
	3624.	.9021	.8946	+.0075
	3848.	.8462	.8484	−.0022
	4071.	.7866	.7571	+.0295
	5843.	.3871	.3899	−.0028
	6003.	.3624	.3673	−.0049
	6434.	.3018	.3074	−.0056
	7089.	.2318	.2396	−.0078
	7408.	.2039	.2141	−.0102
	7695.	.1816	.1935	−.0119
	7983.	.1627	.1758	−.0131
	8238.	.1475	.1571	−.0096
	8493.	.1341	.1444	−.0103
	8765.	.1214	.1296	−.0082
	9930.	.0805	.0845	−.0040

Mean residuals			
	$\lambda < \lambda_m$	Arith.	+.00673
		Alg.	+.00019
	$\lambda > \lambda_m$	Arith.	+.01056
		Alg.	−.00009
	All	Arith.	+.00898
		Alg.	+.00002

The form of the equation remains unchanged. Only the value assigned to the constant D_2 has been revised. These values of A and D_2 must of course, still be considered tentative owing to the limited data considered.

Errata in previous paper (PHYS. REV., 2, 13, 314–317).

Line 6, page 317, for "are" read "an."

Line 10, page 317, for "D^2" read "D_2."

Please note also in equations 1 and 2 that $-1/3$ always occurs as an exponent, which is not clearly shown in printed equations. The form of the equation is correctly given in lower left corner of Fig. 1, page 315.

BUREAU OF STANDARDS,
April 16, 1919.

DEVELOPMENT OF LOOP AËRIAL FOR SUBMARINE RADIO COMMUNICATION.[1]

BY J. A. WILLOUGHBY AND P. D. LOWELL.

IN the fall of 1917 a series of experiments was made using a closed insulated loop with the object of developing an apparatus for the detection of enemy submarines. In the course of this work it was discovered that radio signals could be received by means of the loop, whether the loop was in air or was submerged in fresh water. As the value of such a device for the reception or transmission of radio signals under water with or between submerged submarines was immediately apparent, an extensive series of experiments under actual working conditions was made in coöperation with the Navy at the submarine base, New London, Conn., during the spring and summer of 1918.

The loop as finally perfected consisted of two insulated wires grounded at the extreme ends of hull, carried over suitable supports to the bridge and thence through radio lead-ins to the receiving and transmitting apparatus.

The experiments showed that communication at sea can be carried on under all conditions more efficiently with such a loop than with the ordinary antenna now in general use. As the loop is of simple construction and requires no masts, it does not interfere with submergence.

When the submarine is submerged, any North American or European station can be received as distinctly as when it is on the surface; the maximum depth of submergence at which signals can be received is determined by the wave-length. To receive short wave-lengths it is necessary that the top of the loop be near the surface of the water, whereas a wave-length of 10,000 meters can be received when the top of the loop is submerged 21 feet. Still longer wave-lengths can presumably be received at still greater depths.

Signals can be transmitted from the loop, using the standard 952-meter wave-length, at a distance of ten or twelve miles when the submarine is completely submerged, the maximum distance being obtained when the top of the loop is practically at the surface, and the distance decreases to two or three miles when the top of loop is eight or nine feet below the surface.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

Submergence of the submarine during reception or transmission of a message does not alter the wave-length. If the maximum depth for the wave-length in question is exceeded, communication, of course, is interrupted.

When the submarine is running awash or on the surface, signals can be transmitted (with a 1-K.W. spark set), a distance of at least a hundred miles even under very stormy conditions. Grounds or short circuits caused by heavy seas, which render the ordinary antenna useless in stormy weather, do not affect the closed loop.

The closed loop can also be used as a direction finder, either on the surface or submerged, the maximum strength of signal being obtained when the submarine is pointing toward the transmitting station and the minimum signal when at right angles.

The foregoing results were so satisfactory that the Navy adopted the device, and in the summer of 1918, started equipping its submarines with this type of loop.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

AN INVESTIGATION OF THE PHYSICAL PROPERTIES OF COTTON AIRPLANE FABRICS.[1]

BY R. W. DUNCAN.

THIS experimental study of cotton airplane fabrics was a part of a larger investigation which was undertaken at the Bureau of Standards in order to obtain, if possible, a satisfactory substitute for linen as a covering for airplane wings. This undertaking became necessary since the supply of linen was not sufficient to meet the demands of the countries at war.

Owing to the fact that in airplane service the fabric is "doped," that is, treated with a solution of cellulose acetate, or nitrate, the chief interest lay in the characteristics of the doped fabric. Strips, cut from fabric proposed for airplane use, were tested by a regular series of loading. The results obtained from these doped samples were represented graphically in a load-stretch curve. This curve is characteristic of the particular fabric, since its shape is determined not only by the total per cent. elongation for a given load, but also by the manner in which this total is made up of its two parts, viz., the returnable stretch, and the permanent set. These characteristic differences which are very important in determining the behavior on an airplane wing are due to the quality and make-up of the yarn, the kind of weave, the characteristics of the dope, etc., that is, to differences in material, yarn number, twist, ply, crimp, lateral contraction, etc. Therefore these curves, when drawn for a series of fabrics, offer a means of comparing fabrics as to their characteristic properties and also of studying in detail the effect which each of these factors has in determining these properties.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

It may be pointed out here that Hooke's law does not hold for fabrics. What little elasticity there is may be attributed chiefly to the dope, while the permanent set is due not so much to the yarns themselves as to the crimp which is introduced in the process of weaving. In no case, yet observed, was it possible to duplicate results on any particular sample, for after undergoing a simple series of tests, or any previous strain whatsoever, the fabric had been so changed that if put through another similar test, the curve obtained was quite different. For purposes of comparison, therefore, it is necessary to use a "fresh" sample in each case. Other variations, which must always be taken into account, are those due to the non-uniformity of samples cut from the same piece of cloth, differences in concentration and application of the dope, and the "creep" or gradual increase in length with time while under stress. All measurements must be taken in a room where temperature and moisture content are under control.

One very important matter which has received little attention in work with fabrics is elastic hysteresis. The shape of the hysteresis loop and the "fatigue" breakdown of the fabric under rapidly repeated stresses. A machine for studying the fatigue properties in this way was constructed and was used in testing lacing cords of cotton and linen.

Tests for fabric characteristics were made upon a rectangular area of the fabric which was stretched upon a frame and doped. The deflection (by air pressure) of the middle point of this rectangle was observed and pressure-deflection and hysteresis curves were drawn. These curves are characteristic of the behavior of the fabric under conditions quite similar to those upon the plane itself. Much valuable information with regard to the performance of a given fabric upon an airplane wing may be obtained from the load-stretch, the pressure-deflection, and the hysteresis curves, together with a knowledge of the fatigue properties of the fabric.

Experiments were undertaken to measure the initial tension (before deflection) produced by the dope upon drying and to determine the effect of this tension upon subsequent tensions and deflections due to a given wind pressure. For high values of the initial tension (drumhead tightness) the doped fabric is quite elastic, and the deflection produced by a given wind pressure is determined almost entirely by the initial tension and is approximately independent of the material of the sample. Thereotical curves drawn to represent the deflections for a series of wind pressures for any fabric whatever checked closely with the experimental results taken upon number of quite different fabrics. Therefore, since the deflection may be taken as an approximate measure of the tightness, a tautness indicator was constructed and was tried out in testing the effect of weathering upon tightness. This indicator was afterwards made in a somewhat improved form to be used for standardizing production in airplane factories.

I wish to express my appreciation of the hearty coöperation of Mr. E. D. Walen, who had charge of all the work in the Textile Section of the bureau.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
January, 1919.

THE USE OF SUBSIDIARY SPARK GAPS IN IGNITION SYSTEMS.[1]

BY W. S. GORTON.

ADDITIONAL or subsidiary gaps have frequently been used in jump spark ignition systems in order to cause the resumption of sparking in fouled spark plugs. The series gap, to which the greater part of this report is devoted, is a subsidiary gap in the connection between the high tension terminal of the plug and that of the magneto or coil. A brief account is given of the use of this gap up to the present time and also of the statements concerning it which have gained some currency, most of which are shown to be erroneous. The simple theory of the action of the series gap is discussed, and a detailed account given of the effect upon the sparking ability of the plug produced by changes in the values of the electrical resistance of the fouling and of the capacities in parallel with the plug and with the magneto or coil. Experiments confirm the main features of the simple theory. The points of difference between the conditions as postulated in the simple theory and the conditions actually existing are enumerated and their bearing upon the design and operation of the series gap discussed. The ensuing section is devoted to a discussion of the design of series gaps.

It is concluded that the series gap may be used as a remedy for a considerable part of the trouble due to the fouling of plugs which is met with in practice, since it has been found possible by the use of a series gap on an average ignition system to spark a plug having a fouling resistance of only 4,000 ohms.

NOTES ON THE PHYSICS OF THE HIGH TENSION MAGNETO.[1]

BY F. B. SILSBEE.

IN connection with the study of various problems relating to ignition apparatus used on aviation engines, it became necessary to develop a fairly complete and definite theory on the phenomenon occurring in the operation of the high tension magneto. Literature on this subject is very meager and the information in regard to it has been confined to a few manufacturers, in particular, the German Bosch Company.

The cycle of operations can be divided into six periods as follows: (1) The building up of the current in the primary winding; (2) the rise of voltage during

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

the short interval between the opening of the primary breader contacts and the breakdown of the spark gap; (3) the initiation of the spark and the initial discharge of the primary and secondary capacities into the spark gap; (4) the continuation of the spark and the discharge of the magnetic energy of the coil into the spark gap; (5) the brief period during which the spark is extinguished and the condensers further discharged; (6) the period of rest prior to the beginning of the next cycle.

During each of these periods the conditions of the electrical circuits are definite and the theory of the phenomenon can be worked out to a greater or less extent. Between each period and the succeeding one there is an abrupt change in the circuit conditions and the differential equations proper to one period do not apply to the others.

Methods of testing are needed for measuring the various constants of the circuit which determine the performance of the system. Such methods have been developed for measuring the ratio of turns and magnetic coupling between primary and secondary windings, the maximum voltage developed at break, and the heat energy in the spark.

The oscillograph has proved a very powerful tool in analyzing the operation of the device, but the ordinary forms are not of sufficiently high period to indicate the variation of the current during periods 2–3–5. Methods are now being developed which it is hoped will indicate the magnitude of the eddy currents in the iron core and the secondary capacity of the winding. The results of these investigations have been published at intervals in the series of Aeronautic Power Plants Reports issued by the Bureau of Standards, and in the reports of the National Advisory Committee for Aeronautics.

Bureau of Standards,
Washington, D. C.,
April 9, 1919.

Electrical Conduction in Porcelain.[1]

By F. B. Silsbee and R. K. Honaman.

THIS report describes in some detail the preliminary experiments which were made at the Bureau of Standards on the conductivity of spark plug insulators in order to develop a satisfactory comparative method for testing various materials. The measurements were made at temperatures between 200 and 900° C., and with both alternating and direct current at voltages as 2,000 volts.

The results obtained confirmed the experiments of earlier observers at lower temperatures, in indicating a very rapid decrease of resistance with increase of temperature in porcelain, mica, fused silica, and similar materials. This decrease is, however, a gradual one and there is no definite temperature at which the material suddenly changes its properties. The results of conductivity measurements can be most conveniently expressed by stating the temperature (T_e) at which the material has the arbitrarily selected resistivity of one megohm per centimeter cube.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

The measurement with direct current showed the presence of disturbing polarization effects which make the apparent resistance of the specimen vary with the magnitude and time of application of the measuring voltage. This effect can be eliminated by the use of alternating current in making the measurements.

There is a wide field for further investigation of this subject, as the mechanism of conduction in this class of materials is very complex.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 9, 1919.

THE MOTION OF A ROTATING PROJECTILE.[1]

BY HORACE C. RICHARDS.

I. *The effect of rotation on the path of a projectile.*

DURING the flight of a rotating projectile its motion is continuously modified by the action of the air upon it. This may be conveniently considered as made up of three parts: (1) The principal action is the resistance in the axial plane (the plane through the tangent to the path and the axis). This produces three effects: it diminishes the speed, it curves the path in the axial plane toward the line of the axis, and it produces precession of the axis around the line of flight. (2) The second action is the frictional force due to to the rotation. The principal effect of this is to decrease the rate of spin. If however the axis of the projectile does not coincide with the line of flight, the force is greater on the advancing side, so that there will be a resultant force tending to curve the path at right angles to the axial plane and a couple tending to decrease the tilt of the axis. (3) Finally there is the shift of pressure due to the rotating air. This will produce a force and couple which will in general oppose and at low velocities overbalance those due to the friction. At high velocities it is probable that this effect is of much less relative importance.

The naval 14-inch armor piercing shell was taken as a type and estimates made as to the magnitudes of the various effects. If data of sufficient accuracy were available the motion of the axis could be followed by a step-by-step method. More experimental data on center of pressure, air friction, etc., at high velocities is therefore very desirable.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

II. *Effects due to disturbances at or near the muzzle.*

In addition to the disturbances which continue during the whole flight of the projectile, a number of causes acting when the projectile is at or near the muzzle modify the initial motion and so affect the subsequent flight. The most important effect is an unsymmetrical periodic displacement or wabble of the axis. The amount of this effect was estimated in turn for the following causes: (1) Angular displacement of principal axis of inertia, (2) lateral displacement of center of gravity, (3) vibrations of the gun, (4) jump and whip of the gun, (5) blast from a neighboring gun, (5) irregular escape of powder gases. It was concluded that in the case of the 14-inch shell the effect was probably small in each case.

III. *The Decrease of Spin.*

A review was made of the scanty evidence available concerning the loss of spin of a projectile. The only data published refer to shells of from 3 to 4 1/2 inches caliber with velocities of 1,600 ft./sec. or less. While theory suggests that the logarithmic decrement of the spin varies inversely as the caliber, it is doubtful how far these results may be extrapolated for shells of different size or for higher velocities.

Bureau of Standards,
April 23, 1919.

Properties of Insulating Materials of the Phenol Type.[1]

By J. H. Dellinger and J. L. Preston.

A GENERAL investigation of the electrical and mechanical properties of bakelite dilecto, bakelite micarta, formica, and similar materials, has been in progress at the Bureau of Standards for the last year and a half. The principal object in view was to secure data which would assist in the selection of suitable insulating materials for use in radio apparatus. These materials are made by the impregnation of layers of paper with a particular kind of phenol varnish under fairly high pressure and temperature. Samples of the materials manufactured by the various principal companies were included in the investigation.

Measurements were made upon the electrical, mechanical, and thermal properties of the actual sheets as regularly supplied for commercial use. The data obtained are thus directly applicable, not being vitiated by having to manufacture samples especially for test purposes. For the measurements at radio frequencies, condensers were made up, using an actual slab of the material such as is used in the construction of apparatus. The principal high-frequency property measured is the power factor. Dielectric constant is simultaneously obtained. Measurements of the effects of high voltage at radio frequencies were also made. The other properties measured included volume resistivity, surface resistivity at three humidities, density, moisture absorption after twenty-four hours' immersion, thermal expansivity, tensile strength, transverse strength, permanent warping, impact strength and hardness.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

Special methods and apparatus for rapid measurements were devised in connection with a number of parts of this investigation. A number of interesting properties were observed. These include the effects of varying percentage of varnish in the completed product. An electrical characteristic of much interest is the flashover voltage. It is impossible to assign a particular value of flashover voltage to a material, because it varies with a great many properties, and it depends furthermore upon the shape and size of sample and electrodes and many other functions. Nevertheless, it is found that the phase difference is a very good criterion of the flashover voltage. Partially on this account, the most of the work has been concentrated upon the phase difference measurements. The manufacturing companies have coöperated in this investigation, and have, in some cases, utilized the data obtained to make improvements in their product. Some of the same kinds of measurements have been made on moulded materials using the same basic product, and the future work upon this investigation will include a large number of moulded materials.

Second Series *September, 1919* *Vol. XIV., No. 3*

THE

PHYSICAL REVIEW.

FLUORESCENCE AND ABSORPTION OF THE URANYL ACETATES.

By E. L. Nichols, H. L. Howes, and Frances G. Wick.

Synopsis.

This paper deals with the fluorescence spectra and absorption spectra of the two known forms of uranyl acetate and of thirteen double uranyl acetates as they appear when excited at the temperature of liquid air. The essential identity of the spectra of the double acetates of lithium, potassium, calcium, manganese and strontium, both as regards the location of the principal bands and the structure of the fluorescence groups is established and the deviations from this type occurring in the spectra of the uranyl acetates containing barium, ammonium, rubidium, sodium, magnesium, zinc, silver and lead are considered.

The approximate identity of frequency intervals for all series and for all salts is likewise established.

THE uranyl acetates afford a broader field for investigation than the chlorides or nitrates, the spectra of which have been considered in previous papers.[1]

In addition to two forms of the single acetate $UO_2(C_2H_4O_2)_2$ we have studied the double salts of all the alkali metals except cæsium and the double salts of calcium, barium, strontium, magnesium, zinc, lead, silver and manganese.

In the fluorescence spectra of the acetates, as in the case of all uranyl salts thus far studied the broader bands observed at room temperature are resolved into groups when the substance is excited at the temperature of liquid air and the constitution of these groups, which repeat themselves at regular intervals from the red to the region in the blue where absorption begins to replace fluorescence is very similar in the acetates to that of the groups in the spectra of the compounds already discussed.

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[1] Nichols and Merritt, Phys. Rev. (2), VI., 358, 1915; Nichols and Howes, Phys. Rev. (2), VIII., 364, 1916; Nichols and Merritt, Phys. Rev. (2), IX., 113, 1917; Howes and Wilber, Phys. Rev. (2), X., 348, 1917; Nichols and Howes, Phys. Rev. (2), XI., 285, 1918.

THE SINGLE ACETATE.

Two distinct varieties of this salt were available for observation. The finely powdered anhydrous form $UO_2(C_2H_3O_2)_2$ and the crystalline form $UO_2(C_2H_3O_2)_2.2H_2O$.

The spectra of the two are very similar in appearance; each being characterized by two strong well-defined series forming a set of doublets. They are easily distinguished however by the widely different location

TABLE I.

Uranyl Acetate (anhydrous) at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$.)					
	3.	4.	5.	6.	7.	Average Interval.
A		1717.5	1802.6	1888.5	1975.4	85.97
B		1724.3	1810.5	1896.6	1983.6	86.40
C_1		1739.4	1824.6	1912.5	1997.4	86.00
C	1656.4	1742.6	1828.5	1914.4	2000.6	86.00
D	1663.5	1750.5	1836.4	1922.4	2008.6	86.27
E	1673.6	1758.5	1843.6	1929.4	2015.6	85.5
F	1680.6	1765.7	1851.5	1937.5	2024.3	85.91
G			1868.5	1954.4		85.90
H			1880.5	1965.5		85.00
General average						85.96

TABLE II.

Uranyl Acetate (Crystalline; $2H^2O$).

Series.	Groups and Frequencies ($1/\mu \times 10^3$).						
	3.	4.	5.	6.	7.	8.	Average Interval.
A_2						2015.3	
A_1				1846.4			
A					1935.7		
B			1770.5	1857.0	1942.3	2028.5	86.00
C			1776.1	1863.3	1949.2	2035.5	86.45
C′				1866.7	1952.5	2039.2	86.25
D				1872.1	1958.0	2044.1	86.00
E_1		1706.6	1791.2	1876.5	1962.3		85.23
E	1623.9	1709.8	1793.6	1878.9	1964.6		85.18
F_1				1885.0	1970.4	2056.3	85.65
F	1630.5	1716.8	1801.9	1886.7	1973.5	2058.8	85.66
F′				1890.7	1976.7	2062.7	86.00
F″			1808.8				
G			1816.9	1901.9	1989.1	2073.2	85.42
H			1827.0	1911.7	1998.2		85.60
I			1837.6				
General average							85.72

of the doublets. In the spectrum of the anhydrous variety these occur near the group centers of the alkaline double salts whereas in the crystalline form they fall nearly midway between these groups.

The strong series of the crystalline salt, which we have denoted as E and F, frequently appear in great reduced intensity in the spectra of the double salts, due doubtless to the presence of traces of the single acetate. The strong doublets C, D, of the anhydrous acetate, if they ever appear in the spectra of the double salts would be more difficult to detect as they would overlap bands in the groups of the latter.

Frequencies of the fluorescence bands in the spectra of these two forms of uranyl acetate are given in Table I. and II. and the average intervals for the various series are likewise recorded.

Studies of a Single Group.

Since the acetates, like the chlorides and nitrates, discussed in the previous papers already cited, have spectra consisting of similar recurring groups, it is convenient and sufficient, in the study of the structure of the ensemble of the fluorescence to consider a single group. For this purpose group seven, which is in the brightest part of the spectrum and is free from the complications due to the overlapping of fluorescence and absorption in the reversing region is most favorable. In Fig. 1 the spectral region of this group is plotted for the anhydrous and crystalline forms of the single acetate to depict the remarkable displacements brought about by the presence of water of crystallization and the consequent modification of crystal structure. The effect is very similar both as regards the direction of the shift of the groups and the amount of shift, to that already

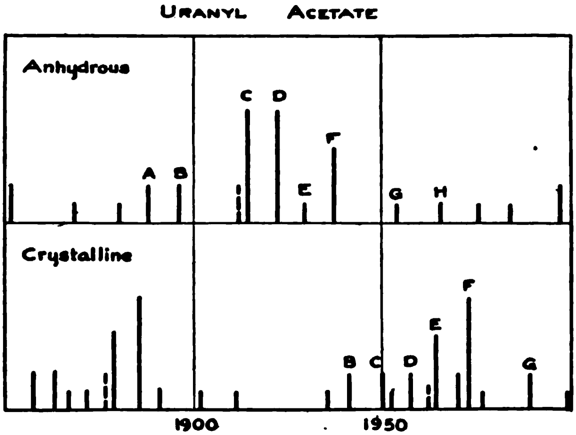

Fig. 1.

described in the case of the nitrates. (Compare the diagram on page 355, volume X. of the PHYSICAL REVIEW.)

In both instances it is not the transfer of the groups towards the blue without change in their structure that occurs but something much less obvious. In fact, it is not possible to identify any of the bands in the spectra of the hydrated form with those belonging to the anhydrous salt.

To produce this change in the spectrum it is only necessary to add a drop of water to a small portion of the anhydrous powder and to compare the fluorescence of the dry and moistened substance when excited in the usual way at − 185°.

FREQUENCY INTERVALS OF THE SINGLE ACETATES.

The frequency intervals of the two forms of the single acetates are given in Table III. from which it will be seen that the two forms of the acetate appear to have the same interval. The difference between the weighted averages is much less than the uncertainties in the determination of the intervals of the dim bands of the weaker series.

TABLE III.

Intervals of Series in the Fluorescence Spectra of the Single Acetates at − 185°.

Anhydrous.		Crystalline ($2H_2O$).	
Series.	Interval.	Series.	Interval.
A	85.97	B	86.00
B	86.40	C	86.45
C_1	86.00	C′	86.25
C	86.00	D	86.00
D	86.27	E_1	85.23
E	85.50	E	85.18
F	85.91	F_1	85.65
G	85.90	F	85.66
H	85.00	F′	86.00
		G	85.42
		H	85.60
Weighted average 85.96		Weighted average 85.72	

THE DOUBLE ACETATES.

The fluorescence spectra of these salts have as a rule lower frequency intervals than the two forms of single acetate. The average interval as may be seen from Table IV. is below 85 as compared with 85.7 for $UO_2(C_2H_3O_2)_22H_2O$ and 85.9 for the anhydrous single acetate.

The group structure is in general less symmetrical than that of the double chlorides or the double nitrates and precise comparisons are therefore more difficult.

Corresponding groups in the majority of cases however occupy very nearly the same position in the spectrum and the system of designating the various bands employed in the discussion of the chlorides and nitrates has been used.

If we neglect some of the weaker outlying bands, the group structure of several of the double acetates is found to consist of four nearly equidistant bands the wave-length of which is almost if not quite independent of the metal which enters into the composition of the double salt. The substances which most nearly conform to this type are the double acetates containing lithium, potassium, calcium, manganese and strontium (see Fig. 2).

In the spectrum of the barium double acetate the groups are shifted bodily towards the red about 5 frequency units. In the spectra of the ammonium and rubidium salts band *D* is doubled.

The double acetates of sodium, magnesium, zinc, silver and lead (Fig. 3a) are made up of groups which while they overlap are by no means identical either as to the location or arrangement of their bands.

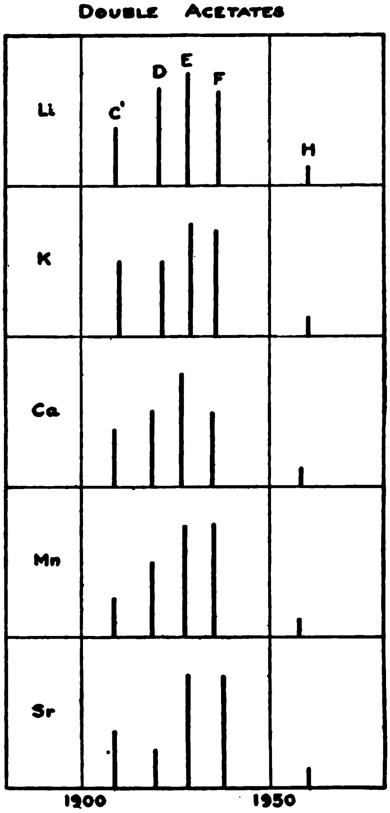

Fig. 2.

The spectra of these five salts agree however in this. They contain in each group five bands which correspond so closely with the bands B, C, D, E and F of the double acetates depicted in Fig. 2, that by a bodily shift of the group as a whole they may be made to conform to the uniform arrangement so far as those bands are concerned, quite as well as do the groups in Fig. 2. This may be seen from Fig. 3*b*, in which the dotted vertical lines indicate the positions of the bands in the uniform type while the group in each case has been shifted so as to register approximately.

The distinction between the spectra under discussion and those previously considered, which were described as having group spectra conforming to an essentially uniform type both as to location of bands and group-structure, is two fold: (*a*) There is a shift of the groups as a whole. (*b*) There are additional series varying in intensity, some of which are among the strongest in the spectra and which are characteristic of the individual salt.

It should be reiterated in this connection that neither the bands B, C, D, E and F which, although sometimes uniformly shifted, are common to the spectra of the double acetates, nor the additional bands, are found in the spectra of the single acetates. The spectrum of neither the anhydrous acetate nor the crystalline form can be made to conform to the uniform type by a general shift.

Double Acetates

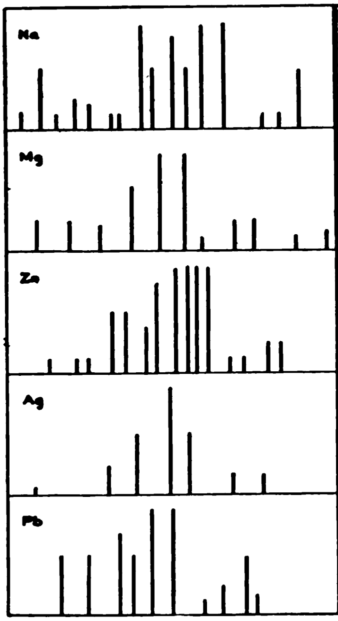

Fig. 3a.

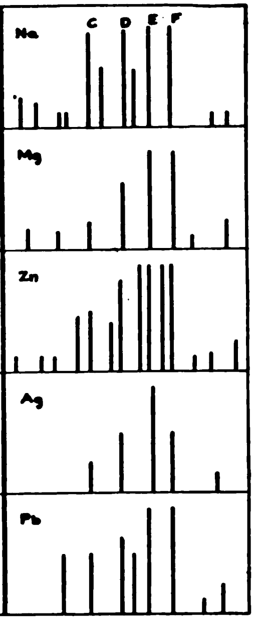

Fig. 3b.

A Possible Relation to the Metallic Spark Spectra.

It appears from the foregoing that any metal capable of forming a double uranyl acetate modifies the constitution of the fluorescence spectrum both as to the composition of the groups and their location. Certain metals such as lithium, potassium, calcium, manganese and strontium produce one and the same modification irrespective of the metal which is present. Other metals shift the group slightly (*e.g.*, barium) or vary slightly the relative distances between neighboring bands without otherwise changing the structure of the group. The presence of still other metals such as sodium, magnesium, zinc, silver and lead results in a considerable general shift and the introduction of new series into the spectrum characteristic of the particular metal in question and existing only in the double salt of which it forms a part. Some of these groups are much more complex than the uniform type depicted in Fig. 2. The others are accompanied by strong bands or minor groups of bands lying outside the usual boundaries.

One might imagine, to account for this type of spectrum, that in addition to the metal in combination as a part of the double salt, there are in solution certain other radiators. If these are uncombined particles of the metal existing in a condition akin to the gaseous state, one might expect a type of radiation, under excitation, similar to that discovered by Wood[1] in sodium vapor; *i.e.*, series of constant frequency made up of bands instead of lines because of damping. One member of each such series should coincide or nearly coincide, with some line in the arc or spark spectrum of the metal.

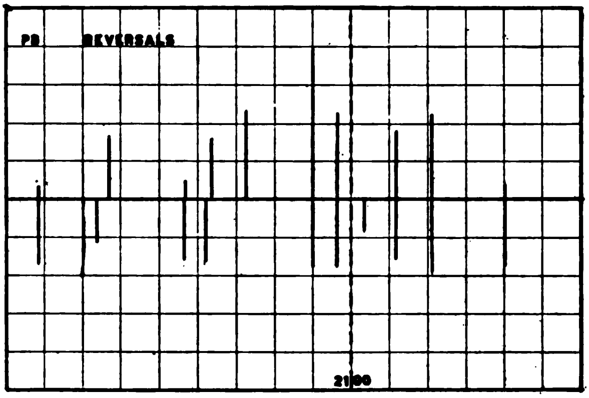

Fig. 4.

Now there are in fact various coincidences or approximations thereto close enough to bring lines of the emission spectrum well within the brighter portion of one of the fluorescence bands in question. In the spectrum of silver uranyl acetate, for example, there is a strong series which does not coincide with any series in the fluorescence spectra of the other acetates thus far observed. One member of this series coincides with the brightest visible line in the spark spectrum of silver (Haschek .54655 μ; frequency number 1829.6). Our reading of the corresponding band, made before we had any suspicion of the possible relation here suggested was 1829.8.

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[1] R. W. Wood, Phys. Rev. (2), XI., p. 76.

The rather bright line (.51838) and the neighboring doublet (.51729–.51675) in the spark spectrum of magnesium correspond similarly to bands 1928.9 and 1934.4 of the fluorescence spectrum.

In the spark spectrum of lead, of the nine lines listed by Haschek which lie in the fluorescence region, seven are within one frequency unit of our readings of the corresponding bands. Four of these are in practically perfect coincidence, the departures from the crests of the bands being only one or two tenths of a unit.

Table IV.

Lithium Uranyl Acetate at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$).						
	3.	4.	5.	6.	7.	8.	Average Interval.
A	1638.0						
C	1657.4	1742.1	1824.1	1909.0	1993.7	2080.0	84.52
D		1751.0	1834.4	1919.8	2005.1	2090.5	84.89
E	1677.0	1760.5	1844.0	1928.9	2013.1		84.03
F		1769.0	1852.0	1935.8	2021.1		84.05
F′						2104.8	
G			1864.7	1950.2			85.50
H	1707.0	1789.0	1874.1	1960.2	2045.6		84.65
General average							84.50

Table V.

Sodium Uranyl Acetate at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$).							
	2.	3.	4.	5.	6.	7.	8.	General Average.
B_1		1643.5	1729.5	1815.0			2074.1	86.12
B		1648.0	1735.0	1820.2	1905.0	1989.2		86.43
B′					1907.5			
C		1659.0	1744.9	1828.8	1915.3	2001.0	2085.1	85.22
C′		1664.8	1749.0	1834.1	1919.3	2004.9	2089.2	84.88
D		1672.8	1756.5	1840.9	1926.3	2011.3	2097.0	84.84
D′			1758.5	1844.3	1929.9	2014.2	2100.0	85.37
E	1597.0	1681.1	1766.1	1850.6	1985.6	2020.8		84.76
F		1688.0	1772.5	1857.8	1942.5	2028.0	2112.5	84.90
F′			1777.8	1862.0		2033.6		85.27
G_1		1695.5	1781.4	1866.0				85.25
G		1702.0	1785.9	1871.0	1955.0	2040.9	2124.5	84.50
G′			1789.0	1875.5	1961.0	2044.5		85.17
H		1710.5	1796.9	1882.0	1967.5	2051.5		85.20
I	1637.5	1722.5	1808.6	1898.0	1980.8	2064.3		85.36
I′				1897.8				
General average								85.22

Of the twenty-five spark lines of zinc, within the fluorescence region fifteen are certainly not related to fluorescence in the manner here under consideration, four in somewhat doubtful coincidence, and six are in close approximation. Of these last, five are consecutive lines of the spark spectrum all of which are in group 7 of our fluorescence system.

The search for possible coincidences in the case of sodium led to the discovery of a striking arrangement which seems to be peculiar to that element. The doublets and triplets of the spark spectrum while they do not form constant frequency series are so located that they could be excited to radiation of the type described by Wood, with a common

Table VI.

Magnesium Uranyl Acetate at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$).							
	2.	3.	4.	5.	6.	7.	8.	Average Interval.
A		1635.4	1721.2	1806.1	1890.8	1975.1		84.93
B		1647.0	1731.8	1815.9	1900.9	1985.8	2071.4	84.88
C				1826.5	1912.0	1997.1	2082.6	85.37
C′							2087.1	
D		1666.1	1751.8	1836.0	1920.9	2006.8		85.18
E		1674.8	1759.2	1844.0	1928.9	2013.8	2099.0	84.84
F			1763.2	1849.0	1934.4	2019.6	2103.6	85.10
G		1687.6	1773.1	1858.0	1943.8	2028.7	2112.6	85.00
H		1695.7	1781.6	1866.5	1950.1	2037.4	2124.5	85.76
I	1627.0	1715.0	1799.8					86.40
General average........								85.19

Table VII.

Ammonium Uranyl Acetate at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$.)						
	3.	4.	5.	6.	7.	8.	Average Interval.
A	1640.2						
B		1727.5	1811.5	1897.0	1981.0		84.50
C	1654.5	1739.3	1823.5	1907.5	1992.0	2076.5	84.40
D		1751.5	1834.0	1918.5	2002.5	2089.0	84.38
D″				1923.0	2007.0	2093.0	85.00
E	1675.5	1760.3	1843.5	1927.0	2012.0		84.13
F	1682.0	1766.5	1850.5	1934.0	2019.0	2102.5	84.10
G			1859.2	1944.8	2029.0		84.90
H		1781.5	1865.5	1950.5			84.50
I	1704.5	1789.2	1873.5	1958.9	2042.0		84.38
General average........							84.40

interval equal to the fluorescence interval of the acetates; *i.e.*, about 85, the result would be a well defined group spectrum of the type of the fluorescence spectrum of the uranyl salts.

The individual coincidences with the bands of the sodium uranyl acetate are however, only two. The evidence of any significant relation based upon these coincidences is obviously far from conclusive. They are mentioned here solely in the view of possible developments in the further study of the connection between fluorescence and temperature radiation.

TABLE VIII.

Potassium Uranyl Acetate at – 185°.

Series.	Groups and Frequencies.						
	3.	4.	5.	6.	7.	8.	Average Interval.
A	1641.2						
B			1814.2	1900.3	1983.9		84.83
C	1656.1	1742.8	1823.4	1909.3	1993.8	2078.9	84.56
D		1754.0	1837.0	1922.3	2005.7	2091.5	84.63
E	1673.5	1761.5	1845.0	1929.6	2013.3		84.95
F	1684.7	1769.1	1852.2	1937.0	2021.0	2105.9	84.24
G				1946.3	2031.3	2116.8	85.25
G′		1781.3	1845.0	1952.7	2036.7		85.13
H	1707.8	1791.3	1876.0	1960.4	2044.7		84.26
General average							84.57

TABLE IX.

Calcium Uranyl Acetate at – 185°.

Series.	Groups and Frequencies (1/μ × 10³.)						
	3.	4.	5.	6.	7.	8.	Average Interval.
C		1742.2	1823.8	1908.4	1993.2	2076.8	83.87
C′	1662.0						
D	1670.3	1753.5	1834.5	1918.6	2004.0	2087.7	83.91
E	1675.6	1759.9	1842.6	1926.8	2012.1		84.12
F	1683.8	1767.1	1850.8	1934.6	2019.0	2103.0	83.85
G′						2114.6	
G″		1781.9	1865.3	1949.3	2033.3		83.80
H	1703.9	1788.9	1873.0	1958.1	2042.9		84.66
I					2051.0		
J	1717.9						
K_1				1976.7			
General average							83.88

FLUORESCENCE SERIES IN THE SPECTRA OF THE DOUBLE ACETATES.

In Tables IV. to XVII. the frequencies of the fluorescence bands in the spectra of the double salts are arranged by series together with the average intervals. It will be seen by comparison with Table III. that the average interval for the double acetates is more than one frequency unit less than for the single acetates; also that the intervals for the various double salts differ from the general average for all (Table XVII.) by an amount no greater than the differences between the intervals of the various series present in the spectrum of a given salt.

In brief, whatever real differences of interval may exist are too small to be determined with certainty, from our data.

TABLE X.

Manganese Uranyl Acetate at − 185°.

Series.	Groups and Frequencies $(1/\mu \times 10^3)$.					
	4.	5.	6.	7.	8.	Average Interval.
C		1821.0	1908.4	1992.8	2078.1	85.70
D	1748.9	1833.5	1918.4	2002.6	2089.4	85.12
E	1757.0	1842.3	1927.9	2011.7		84.90
F		1849.1	1935.0	2019.4	2104.8	85.23
G_1				2025.9		
H		1873.0	1957.9	2042.9		84.95
General average						85.19

TABLE XI.

Zinc Uranyl Acetate at − 185°.

Series.	Groups and Frequencies $(1/\mu \times 10^3)$.							
	2.	3.	4.	5.	6.	7.	8.	Average Interval.
A		1640.0	1724.0	1809.0	1895.7	1980.3	2064.8	84.96
B			1730.1	1814.1				84.00
C		1654.3			1905.0	1989.7	2075.1	84.16
C′			1739.5	1824.0	1909.1	1995.8	2079.6	85.03
D				1831.6	1916.1	2000.9	2085.5	84.48
D′				1835.0	1919.9	2004.9	2089.3	84.77
E	1590.3		1754.0	1841.6	1925.6	2009.1	2094.0	83.95
E′		1675.8	1759.0	1845.0	1929.9	2014.0	2101.9	85.22
F		1682.9	1766.6	1848.0	1934.0	2017.2		83.80
F′				1851.9	1938.0	2021.9		85.00
G_1							2107.0	
G		1689.0	1773.1	1858.0	1942.0	2026.3		84.33
H		1697.0	1780.6	1865.0	1949.1	2033.4		84.20
I		1703.7	1788.0	1872.0	1956.9	2041.4		84.45
J				1875.9	1961.3	2045.9		85.00
General average								84.51

TABLE XII.

Rubidium Uranyl Acetate at – 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$).						
	3.	4.	5.	6.	7.	8.	Average Interval.
A			1800.6				
A′		1720.9					
B	1641.2		1810.0				84.40
C		1738.2	1821.8	1908.0	1991.2	2077.7	84.88
C′		1742.5	1826.2	1912.4			84.45
D		1749.2	1832.3	1917.9	2002.0	2087.7	84.62
D′		1753.3	1836.5	1922.0	2006.4	2092.1	84.70
E	1673.1	1759.0	1842.3	1927.5	2011.7		84.65
F	1679.3	1766.2	1850.0	1934.2	2019.0	2104.8	85.10
G		1773.4	1859.8	1944.0	2029.6	2114.2	85.20
H		1780.0	1865.7	1949.7	2034.2		84.73
H′	1701.0						
I		1788.3	1873.4	1957.3	2043.7		85.13
Generage average							84.86

TABLE XIII.

Strontium Uranyl Acetate at –185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$).							
	2.	3.	4.	5.	6.	7.	8.	Average Interval.
C			1738.6	1823.9	1908.6	1993.2	2078.2	84.90
D			1749.9	1834.0	1919.0	2004.0	2088.1	85.05
E		1674.0	1758.5	1842.6	1928.0	2012.5		84.63
F		1680.5	1765.8	1849.4	1934.5	2020.4	2105.0	84.90
G			1780.8	1864.5	1949.0	2034.0		84.40
H_1		1703.0						
H			1789.1	1874.9	1959.1	2045.9		85.60
I	1636.1	1720.7		1886.8	1969.4			83.32
General average								84.74

TABLE XIV.

Silver Uranyl Acetate at – 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^3$).						
	3.	4.	5.	6.	7.	8.	Average Interval.
A		1722.5					
C		1735.0	1818.4	1904.9	1989.2	2073.0	84.50
D		1745.1	1829.7	1913.2	2002.2	2085.1	85.00
E		1754.7	1839.4	1923.9	2008.6		84.63
E′	1672.5						
F	1677.5	1761.1	1846.0	1931.0	2016.1	2101.5	84.80
G			1859.0	1945.1	2028.6		84.80
G′		1777.7					
H	1701.2	1785.7	1871.0	1955.7	2040.0		84.70
General average							84.74

Table XV.

Barium Uranyl Acetate at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^4$).						
	3.	4.	5.	6.	7.	8.	Average Interval.
C	1651.5	1734.9	1818.8	1904.8	1990.0	2075.5	84.80
D		1744.3	1827.8	1913.5	2000.0	2085.5	85.30
E	1668.6	1754.4	1837.9	1922.7	2008.8		85.05
F	1677.0	1763.0	1846.7	1930.5	2016.5	2101.3	84.86
G	1688.3	1773.7	1856.3	1941.7	2027.2	2113.7	85.08
H	1698.4	1785.1	1869.5	1954.7	2040.4		85.50
General average							85.08

Table XVI.

Lead Uranyl Acetate at − 185°.

Series.	Groups and Frequencies ($1/\mu \times 10^4$.)							
	2.	3.	4.	5.	6.	7.	8.	Average Interval.
B		1643.1	1727.7	1812.1	1897.8	1983.5	2067.4	84.86
C_1				1818.2	1904.0	1989.0	2073.0	84.93
C		1653.4	1738.7	1821.9	1907.6	1992.4	2077.4	84.75
C′			1741.8	1827.5	1911.9	1997.0		85.07
D		1662.5	1747.7	1831.7	1916.2	2001.8	2087.5	85.00
E_1					1922.7	2006.8		84.10
E		1669.1	1754.3	1838.7	1925.0	2009.6	2095.7	85.32
E′				1844.0	1928.3		2098.2	84.30
F				1848.3	1935.0	2021.1	2105.9	84.40
G		1684.1	1770.1	1855.1	1942.1	2025.8	2111.0	85.38
H		1692.4	1778.9	1862.4	1949.1	2034.0		85.40
H′		1697.5	1782.3	1868.5	1953.4			85.30
I	1621.6	1606.8	1788.7	1875.1	1960.8	2046.7		85.02
L			1796.3					
General average								85.12

Table XVII.

Summary of Average Intervals of the Double Acetates.

Substance.	Interval.	Substance.	Interval.
$LiUO_2(C_2H_3O_2).3H_2O$	84.50	$Zn(UO_2)_2(C_2H_3O_2)_6.7H_2O$	84.51
$NaUO_2(C_2H_3O_2)_3$	85.22	$RbUO_2(C_2H_3O_2)_3$	84.86
$Mg(UO_2)_2(C_2H_3O_2)_6.7H_2O$	85.19	$Sr(UO_2)_2(C_2H_3O_2)_6.6H_2O$	84.74
$NH_4UO_2(C_2H_3O_2)_3$	84.40	$AgUO_2(C_2H_3O_2)_3$	84.74
$KUO_2(C_2H_3O_2)_3$	84.57	$Ba(UO_2)_2(C_2H_3O_2)_6.6H_2O$	85.08
$Ca(OU_2)_2(C_2H_3O_2)_6$	83.88	$Pb(UO_2)_2(C_2H_3O_2)_6$	85.12
$Mn(UO_2)_2(C_2H_3O_2)_6.6H_2O$	85.19	General average	84.76

ABSORPTION SPECTRA OF THE ACETATES.

The fluorescence and absorption of the acetates are related to each other in a manner entirely similar to that already established in the case of the other uranyl compounds.

The absorption bands occur in series of constant interval and this interval is much shorter than that of the fluorescence series. Fluorescence and absorption overlap in the reversing region with numerous coincidences and an interlocking of the fluorescence and absorption intervals. Reversals both exact and of the well-known displaced type are more frequent, perhaps, than in any family of uranyl salts as yet studied. A notable group occurs in the spectrum of lead uranyl acetate (see Fig. 4). Fluorescence bands are represented by vertical lines above, absorption by lines below the base line.

The absorption spectra fall into two fairly well-defined classes:

(1) (Double acetates of Li, NH_4, Na, K, Ca, Zn, Rb, Sr, Ag, Ba).

In this class the system of bands having a series interval of 70 +, terminates at about 2180, where is located the head of the strongest series.

Band E of the fluorescence series which is usually missing in group 8 is supplanted by a strong absorption band (1) located 85 ± frequency units from the terminating absorption band mentioned above, which is designated as e in accordance with the convention adopted in previous papers.

An excellent instance of this type of transition from fluorescence to absorption is afforded by the spectrum of barium uranyl acetate (Fig. 5).

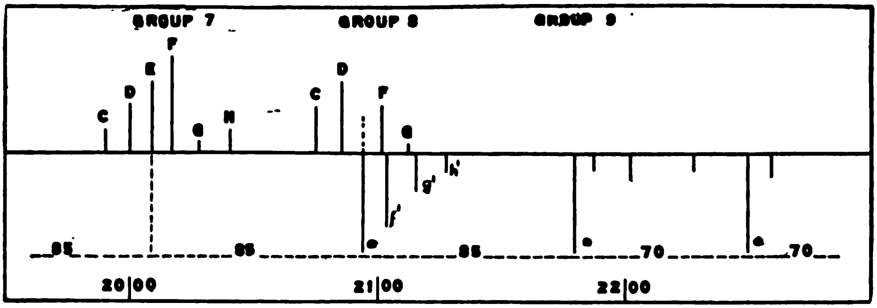

Fig. 5.

Here an exact reversal of band E (group 8) occurs, and the very strong absorption band which takes its place is 85 frequency units from the first member of the strong e series, which extends for several groups toward the ultra violet with the usual absorption interval of 70 units.

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[1] This band E_8 may appear either as fluorescence or as absorption according to the conditions of illumination, etc. It is commonly seen as fluorescence in the spectrum of the zinc uranyl acetate and as absorption in the spectra of the other salts of this class.

Displaced reversals of F, G, and H also occur, an indication of the probably complex structure of these bands. The corresponding bands are likewise 85 units from the first members of the f′, g′ and h′ series of the absorption spectrum.

There is almost as notable a resemblance between the absorption spectra of this class as between their fluorescent spectra. The resolution is however, not so good, and all the members of the various series are not so easily located. Almost without exception, the bands which can be observed are definitely related in the manner just described, to the fluorescence series.

(2) The single acetate $UO_2(C_2H_4O_2)$ and $UO_2(C_2H_3U_2) + 2H_2O$ (double acetates of Mg, Mn, Pb).

Here the absorption system (interval 70 ±) distinctly overlaps the fluorescence system extending into the region of groups 8 and 7 beyond, without change of interval.

The various series of absorption bands located in our visual and photographic studies of the actetates are contained in tables XVIII. to XXXI. inclusive. Frequencies and average intervals are given for each salt; the series being designated as usual by small letters which indicate their relation to fluorescence series denoted by the corresponding capital letter. The three examples of bands or series not thus related to visible fluorescence are indicated by means of the Greek letters, γ.

From the list of general, weighted averages of the intervals for the various salts (Table XXXIII.) it appears that the frequency interval of the single acetates is larger than the general average, corresponding in this respect with the larger interval of their fluorescence spectra as has

Table XVIII.

$UO_2(C_2H_3O_2)_2$ Anhydrous.

Series.	Frequencies of Absorption Bands at −185°. (Groups 7 to 15.)					Intervals.
b	2275.3	2343.4	2411.4	2556.5		70.75
c_1	2208.5	2282.5	2352.3	2424.5	2495.5	71.00
d_2	2075.6					
d_1	2077.7	2149.4	2219.4	2359.4		70.43
d	2010.5R	2364.8	2436.5	2505.5		70.71
e	2235.5	2303.5	2374.5	2446.6	2515.5	70.00
f	2025.0	2097.5	2169.5	2243.5		71.30
γ	2321.5	2392.5				
g_1	2104.4					
g	2397.5					
h_1	2258.5					
h	2049.4	2121.5	2192.5	2264.3		71.63
General weighted average						71.04

Table XIX.

$UO_2(C_2H_3O_2)_2\cdot 2H_2O$.

Series.	Frequencies of Absorption Bands at −185°. (Groups 8 to 15.)							Intervals.
a	2242.5							
a'	2082.5R	2164.5	2213.4	2383.8	2456.8	2527.0	2598.5	72.29
b_1	2174.4	2245.5						71.55
c	2182.4	2253.5	2550.5	2621.5				73.18
c'	2258.5	2334.5	2406.6					73.70
d	2045.5R							
e	2123.9	2195.6	2269.4					72.75
f	2058.7	2129.5	2202.5					71.90
f'	2062.5	2132.5	2205.1	2278.4	2350.5			72.00
f''	2426.9							
g_1	2357.5	2501.7	2571.5					71.33
g	2074.6	2147.5	2220.0	2292.6	2363.5			72.39
h	2081.6							
h'	2159.2	2230.4						71.2
General weighted average								72.35

Table XX.

$Li(UO_2)C_2H_3O_2)_3\cdot 3H_2O$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 7 to 15.)					Intervals.
c	2234.5	2375.0	2446.5	2514.8	2584.8	70.06
c'	2085.0					
e	2097.8R	2185.0	2256.6	2325.0	2396.2	70.40
e'	2100.0					
f'	2026.5R	2108.8				
f''	2110.8					
g_1	2113.0					
g	2117.8	2201.5	2623.8			70.39
h	2124.0					
h'	2129.0					
General weighted average						70.27

Table XXI.

$NH_4(UO_2)(C_2H_3O_2)_3$.

Series.	Frequency of Absorption Bands at 185°. (Groups 7 to 15).						Intervals.
b_1	2145.6						
γ	1996.5R	2082.4R	2163.4	2299.2	2511.6		69.62
c	2232.4	2374.0	2446.3				71.73
e	2096.2	2183.2	2253.8	2322.8	2394.2		70.12
f'	2107.8						
g	2114.8						
h	2119.8	2207.0	2278.8	2348.5	2422.8	2487.5	
i'	2047.8R	2130.0					
General weighted average							70.19

TABLE XXII.

$Na(UO_2)(C_2H_3O_2)_3$.

Series.	Frequencies of Absorption Bands at −185°. (Groups 8 to 15.)							Intervals.
b	2229.2							
b'	2375.3							
c	2239.6	2311.1	2382.1	2454.0	2524.0	2591.8		69.8
d_1	2093.4							
d	2395.8							
e_1	2328.3							
e	2102.6R	2190.1	2259.4	2332.1	2403.8	2472.8	2542.4	70.46
e'	2265.0	2336.4	2409.1	2475.9				70.3
f	2111.0R	2478.9						
f'	2117.7R	2277.4	2343.0	2415.3				
g	2126.8R	2211.4	2285.7	2350.7	2423.7	2569.1		71.54
h	2137.7R	2217.8	2291.0	2358.5	2430.1	2499.4		70.15
i	2366.3							
General weighted average								70.46

TABLE XXIII.

$Mg(UO_2)_2C_2H_3O_2)_6\cdot 7H_2O$.

Series.	Frequencies of Absorption Bands at −185°. (Groups 8 to 15.)			Intervals.
c_1	2161.0			
c	2168.3	2239.2		70.90
c'	2172.8	2315.0		71.10
d'	2092.8			
e	2182.8			
f	2102.2			
g_1	2108.4			
g	2266.0	2336.2		70.20
g'	2115.3R	2199.3	2269.0	69.70
General weighted average				70.60

TABLE XXIV.

$K(UO_2)(C_2H_3O_2)_3$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 7 to 11).					Intervals.
c	2166.0	2234.0				68.00
c'	2084.0R					
γ	2000.0R					
d	2176.1					
e_1	2011.0R					
e	2017.0	2100.0	2186.0	2256.3	2326.5	70.25
f_1	2025.0R	2109.3				
g	2116.0R	2203.0				
g'	2124.0R	2211.6	2292.2	2349.0		68.70
h	2131.8R					
h'	2061.8R					
General weighted average						69.18

Table XXV.

$Ca(UO_2)_2(C_2H_3O_2)_6 \cdot 8H_2O$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 7 to 13).					Intervals.
γ	2023.1	2092.5	2166.8			71.85
d	2173.0	2245.2	2312.7	2384.9		70.63
e	2180.1	2249.2	2317.5	2390.1		70.00
f	2104.8R					
f_1	2185.0	2254.3	2325.6	2395.8		70.27
g_1	2198.8	2340.3	2409.6			70.27
g	2032.1R	2115.1	2203.6	2274.3	2415.5	70.63
h_1	2123.6R	2278.4	2349.1			70.70
h	2126.3R					
i	2436.6					
General weighted average..						70.54

Table XXVI.

$Mn(UO_2)_2(C_2H_3O_2)_6 \cdot 6H_2O$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 7 to 11).					Intervals.
c	2425.0					
e	2011.5R	2094.0	2178.4	2236.2	2307.5	71.62
e'	2311.3					
g	2110.5	2181.2	2251.5			70.5
General weighted average..						71.25

Table XXVII.

$Zn(UO_2)_2(C_2H_3O_2)_6 \cdot 7H_2O$.

Series.	Frequencies of Absorption Bands at −185°. (Groups 8 to 12.)					Intervals.
c	2372.5	2442.6				70.10
e	2096.4R	2181.5	2251.7	2322.9	2392.5	70.10
f'	2188.2					
g	2263.0					
h_1	2178.6	2343.6	2415.0			71.40
h'	2205.6	2277.9	2350.7			72.5
i	2129.9					
General weighted average..						70.77

been previously noted. The determination of intervals is, however, somewhat less accurate than in the case of the fluorescence bands and as in that instance no differences, between various series, or various salts, can be considered as positively established.

Table XXVIII.

$Rb(UO_2)(C_2H_3O_2)_3$.

Series.	Frequencies of Absorption Bands at −185°. (Groups 7 to 12.)					Intervals.
a	2059.3R					
b_1	2070.8R					
c	2158.9	2292.2	(2299.4)	2373.6	2440.8	70.27
c′	2081.6R					
d′	2093.8R	2246.7	2317.0			70.35
d″	2392.3					
e	2186.7	2259.4	2326.1	2399.8		71.03
g_1	2027.2R					
g′	2115.1					
i_1	2041.7R					
i	2129.0R	2209.5	2279.5	2350.2	2423.1	71.20
General weighted average						70.78

Table XXIX.

$Sr(UO_2)_2(C_2H_3O_2)_6 \cdot 6H_2O$.

Series.	Frequencies of Absorption Bands at −185°. (Groups 8 to 12.)					Intervals.
c_1	2371.2					
c	2375.6					
e_1	2390.0					
e	2094.5R	2181.4	2251.9	2322.5		70.55
e′	2185.7	2254.1				68.40
e″	2399.9					
h	2273.8	2412.7				69.45
h′	2206.0	2347.8				70.90
i	2209.0	2279.1	2350.0	2420.0	2490.0	70.25
General weighted average						70.11

Table XXX.

$Ag(UO_2)(C_2H_3O_2)_3$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 8 to 11).				Interval.
c	2231.4	2369.7			69.15
e	2180.0	2251.3	2321.2	2390.1	70.03
f′	2107.0R	2191.4			
g	2116.0R				
h	2125.0R				
i	2133.0	2204.1	2276.3	2344.7	70.39
General weighted average					70.01

Table XXXI.

$Ba(UO_2)_2(C_2H_3O_2)\cdot 6H_2O$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 7 to 13).					Interval.
b	2294.1					
c	2227.2	2370.8				71.81
e	2094.7R	2180.1	2250.2	2322.5	2393.5	71.13
f'	2105.3R	2187.2	2260.0			72.80
g'	2115.1R					
γ	2204.1	2275.8	2419.0	2486.9		70.70
General weighted average						71.36

Table XXXII.

$Pb(UO_2)_2(C_2H_3O_2)_4$.

Series.	Frequencies of Absorption Bands at 185°. (Groups 8 to 15).							Intervals.
b	2065.0							
c₁	2072.0	2142.5	2355.0	2426.5	2495.0			70.50
γ	2365.0	2437.5	2506.0					71.50
e	2094.5	2236.5	2309.8	2380.0	2450.0	2521.0	2594.5	71.43
e'	2098.0	2168.5	2240.0	2313.5				71.83
e''	2101.5	2389.0						71.87
f	2105.5	2176.5	2248.4	2321.5	2392.5			71.75
g	2110.5							
h	2119.5							
h'	2125.0	2194.5	2264.0	2332.5				71.00
k	2201.5	2272.0	2341.0	2413.5	2482.0	2552.5		71.00
General weighted average								71.21

Table XXXIII.

General Weighted Averages of Intervals of Absorption Series in Spectra of the Acetates at − 185°.

Salt.	Interval.	Salt.	Interval.
		Double acetates of	
Uranyl acetate (anh)	71.04	Manganese	71.25
" " ($2H_2O$)	72.35	Zinc	70.77
Double acetates of		Rubidium	70.78
Lithium	70.27	Strontium	70.11
Ammonium	70.19	Silver	70.01
Sodium	70.46	Barium	71.36
Magnesium	70.60	Lead	71.21
Potassium	69.18		
Calcium	70.54		
Average interval for all acetates			70.68

Summary.

1. The spectra of the uranyl acetates consist of the usual equidistant fluorescence bands.

2. When excitation occurs at the temperature of liquid air, these bands are resolved into groups the homologous members of which form series of constant frequency interval.

3. There are two single acetates, a finely powdered, anhydrous variety, and a crystalline form with two molecules of water of crystallization, whose spectra differ widely, particularly as to the groups of fluorescence bands.

4. Of the double acetates, those containing lithium, potassium, calcium, manganese and strontium have spectra which may be regarded as essentially identical both as regards the location of the principal bands and structure of the fluorescence groups. The only distinctions between their spectra are in the sharpness of resolution and relative brightness of the various components.

5. The spectrum of barium uranyl acetate differs from the above, in that the groups are shifted, as a whole, about five frequency units towards the red.

6. In the spectra of the double acetates of ammonium and rubidium, band D in each group is doubled, but there is no shift of the groups.

7. The presence of sodium, magnesium, zinc, silver and lead modifies the group structure by the addition of bands characteristic of the metal and causes slight relative displacements of the group system as a whole. Otherwise the spectra resemble those mentioned under (4).

8. The frequency interval for the fluorescence series of the double acetates is probably the same for all series and for all salts, the departures of the general averages for the various salts being less than one frequency unit from the average for all; *i.e.*, 84.76.

The same is probably true of the absorption series, the general average for which is 70.68.

9. The frequency intervals, both in the fluorescence and absorption spectra, are larger by more than one frequency unit for the single acetates than for the double acetates.

Physical Laboratory of Cornell University,
June, 1919.

THE QUENCHING BEHAVIOR OF LIQUIDS.

BY NORMAN B. PILLING.

SYNOPSIS.

An experimental study of the mechanism of heat transfer from a hot body plunged into a liquid was made with the aid of a string galvanometer used as an autographic recording pyrometer. From cooling curves obtained it was found that cooling takes place at the most in three distinct steps, corresponding respectively to (1) the formation and persistence of an enveloping vapor sheath, the thickness of which is, within limits, proportional to the initial liquid temperature and through which heat is transmitted mainly by thermal conduction; (2) its disappearance followed by very rapid cooling in which active vaporization of the liquid, aided by local convection, plays a dominant part; (3) the final cooling to temperatures below the boiling point limited by thermal conduction through the liquid. During each of these stages the cooling follows a simple exponential relation involving the nature and temperature of the respective cooling agents.

WHEN a small metallic body is quenched from a temperature above a red heat in a liquid, it follows certain laws of cooling which are surprising in their simplicity when consideration is given to the many factors of possible importance affecting the rapid extraction of its heat. An essential feature of such cooling is that it is a discontinuous process taking place in a series of well-defined steps differing from each other in the mechanism of heat abstraction and that in any one of these steps heat is being extracted from the hot body in accordance with one predominant law. The temperature of the liquid bath has a profound effect on the relative arrangement of these steps, an increase in its temperature, in general, bringing about a lowering of the temperature in which the several cooling stages are operative.

The experimental procedure involved the accurate registration of the time-temperature cooling curve of a small metallic cylinder when quenched from a certain fixed temperature in a bath maintained at various temperatures. The cylinder was 6 mm. in diameter, 50 mm. long and had an axial hole to its center through which a small platinum-platinrhodium thermocouple was introduced with the bare hot junction in good thermal and electrical contact with the base of the hole. The cylinder was screwed on to a combined protective tube for the thermocouple and supporting tube for the cylinder by which it was held in the heating furnace (electric resistance) and rapidly removed by an appro-

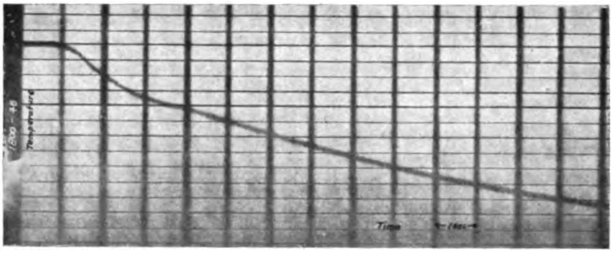

FIG. 1*a*. Water temperature 0° C.

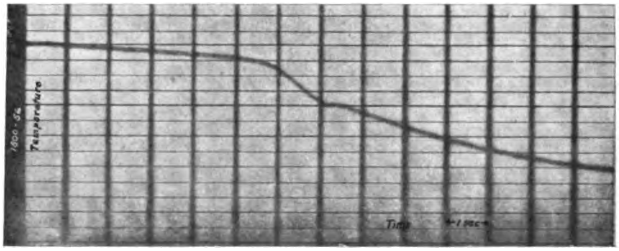

FIG. 1*b*. Water temperature 58° C.

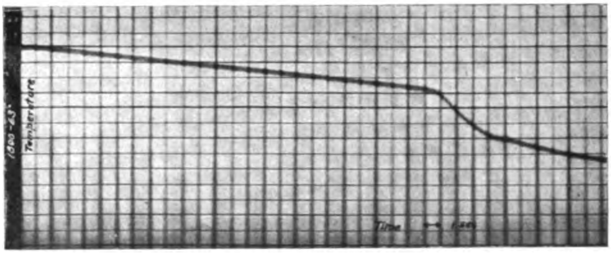

FIG. 1*c*. Water temperature 99° C.

NORMAN B. PILLING.

priate device synchronizing with the registration apparatus. The transfer from the heated furnace to a stationary position in the quenching bath took approximately one half second. The initial temperature of the thermocouple at the center of the cylinder was measured with a L. & N. potentiometer indicator; during quenching it was recorded by the deflection of the image of a silver wire Einthoven galvanometer projected on to a turning cylinder bearing an oscillograph film. The cylinder used as a heat reservoir was made of nickel + 5 per cent. silicon which was found by the usual thermal analysis to be free from transformation points, insuring constancy in the available supply of heat. Silicon not only suppresses the nickel transformation but confers the additional desirable property of resistance to oxidation at high temperatures, thus eliminating uncertain surface conditions due to the formation of a scaly oxide.

Several autographic cooling curves taken from an initial temperature of 830° C. in water at 0°, 58° and 99° C. are given in Fig. 1.[1]

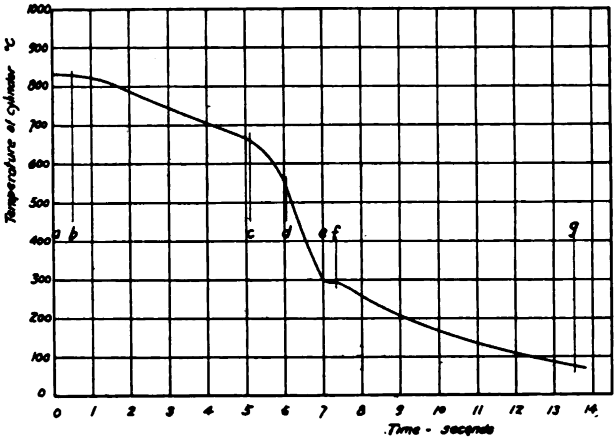

Fig. 2.

Water, 58° C.

The mechanism of heat abstraction during quenching is shown by an analysis of one of these curves in which there is a full development of all of the cooling steps. In Fig. 2 the cooling curve of the standard cylinder quenched from 830° C. in water at 58° C. has been replotted to a uniform temperature scale. The record starts at *a* with the cylinder leaving the furnace; at *b* it enters the quenching bath and at this instant the water in immediate contact with the surface of the cylinder is quickly raised to the boiling point and vaporized, forming an envelope of steam around the cylinder. The cooler layers of water strive to condense this steam while the heat flow from the hot cylinder tends to superheat it. The survival of this steam jacket depends upon the relative rates at which heat is delivered through the jacket to the boundary surface between steam and water and is removed from it by the adjacent cooler layers of water. In this case

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[1] The horizontal tracings on the records are reference lines and do not have a direct relation to the temperature scale.

$$\frac{dH}{dt}\,(\text{water}) < \frac{dH}{dt}\,(\text{vapor}),$$

and the vapor jacket with its relatively poor thermal conductivity persists. The branch of the curve between *b* and *c* represents the sojourn of the cylinder in this atmosphere of steam. At *c* the two rates become equal, reverse in magnitude and at *d* the vapor jacket disappears. There is now liquid contact between the heated surface and the water, and as the temperature of the metal surface is far above the boiling point, vigorous boiling ensues. From *d* to *e* a steady heat loss by this means is maintained and the liquid shows a great capacity for absorbing heat rapidly. The quick cooling terminates at *e*, when 0.3 second is consumed in cooling the concentric layer of water at the boiling temperature before the cooling of the cylinder is resumed at *f*, and from *f* to *g* the heat absorption, relatively slow, is limited by thermal conduction through the water.

To summarize, the rather complicated mechanism of heat transfer during quenching may be divided into three stages, as follows:

Mechanism.	Character.	Designation.
Cooling in vapor	Slow	A
" by vaporization	Rapid	B
" in liquid	Slow	C

For convenience in reference we may call these three modes of cooling A, B and C respectively.

If a series of cooling curves are taken, with the water temperature varying from 0° to 100° C., and separated into the several stages some interesting relations are developed. Each of the three branches may be rectified when plotted to an appropriate logarithmic function. On the A and B branches the limiting temperature of cooling is the boiling point of the cooling liquid and the function is log $(\theta - 100)$; on the C branches the limiting temperature is the bulk temperature of the liquid and the function is log $(\theta - \theta_L)$.

The general equation of cooling for each stage is then

$$\log(\theta - \theta_0) = - Ct + \text{const},$$

or

$$\theta = Ae^{-Ct} + \theta_0,$$

and the quantity C defining the character of cooling for each curve is of course the slope of the logarithmic straight line. This quantity C may be split into several factors of which those pertaining to the dimensions and material of the cylinder, all constant, may be lumped as c

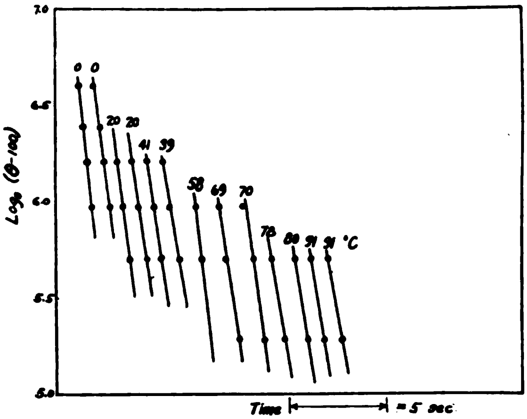

Fig. 3.

B stage.

and another quantity k be reserved for the property or properties of the agent active in cooling.

The family of straight lines thus derived from the B stage is plotted in Fig. 3 from which it is apparent that not only has rectification been achieved but that parallelism prevails. This indicates that k_B is not only constant for each curve but is independent of the temperature of the cooling bath.

Similarly, for the C branches rectification and parallelism exist (Fig. 4) leading to the same conclusion that k_c is constant and independent of the water temperature.

In the case of the A branches (Fig. 5) there is rectification, but the family of lines is divergent; k_A, while constant for any one cooling, increases with diminishing temperature of the water, corresponding to a decrease in the effective thickness of the vapor sheath around the cylinder characterizing this cooling stage. This thickness seems to be directly proportionate to the bulk temperature of the water, and plotting to a new function, θ_L [log $(\theta - 100)$], brings about the parallel condition (Fig. 6). For this stage then, the cooling law is

$$\theta = Ae^{-\frac{k_A}{\theta_L}ct} + \theta_0.$$

From these relations it follows that the first action of a liquid in which a highly heated body is plunged is the formation of a continuous envelope of vapor forming a thermally insulating layer between the heated surface and the liquid, the effective thickness of which is, within limits, proportional to the initial temperature of the liquid. Transfer of heat across this vapor layer is accomplished by thermal conduction in great excess over the effect of radiation and convection. This jacket persists with constant thickness until its disappearance, determined by a quasi-equilibrium in the rates of heat transmission between vapor and liquid. Its disappearance initiates a period of rapid heat abstraction in which the cooling process is a continuous cycle of vaporization and condensation in which local convection currents of large magnitude operate to increase

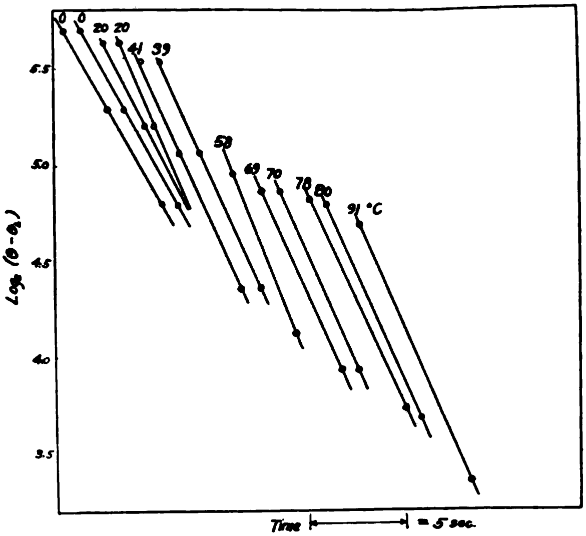

Fig. 4.

C stage.

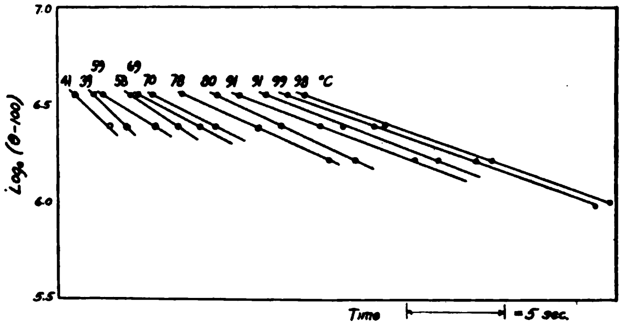

Fig. 5.

A stage.

the effective surface of heat transfer enormously. The rapid transfer of heat in excess of the maximum rate at which it can be conducted away from the zone of action leads to the formation of a concentric layer of water at the boiling point which finally includes the entire zone of agitation tending toward the re-establishment of the vapor sheath and the quick cooling ceases at a temperature considerably above the boiling point. In the last stage the rate of heat transfer is limited by the thermal conductivity of the water.

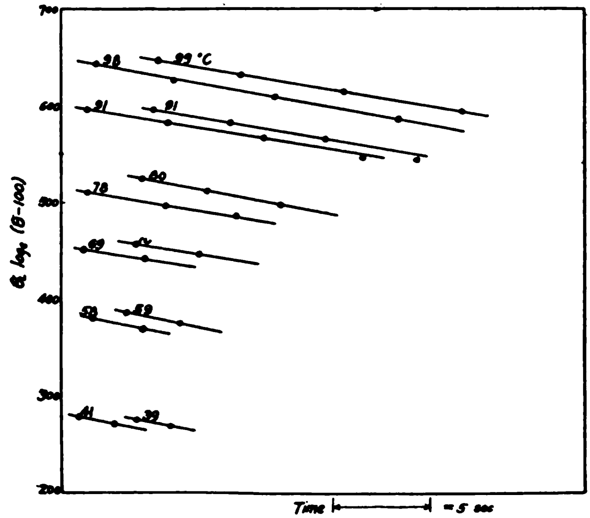

Fig. 6.

A stage.

The operation of these laws, given in detail for water, has been confirmed by the behavior of a number of aqueous solutions. With oils, the lack of a definite boiling point, resulting in a more or less continuous vaporization to temperatures well below the nominal boiling point, prevents a clear separation of the two latter cooling stages.

The discontinuous quenching behavior of a liquid has its corollary in the interpretation of cooling curves of quenched alloys. The tacit assumption seems to have been made by certain investigators of the properties of rapidly cooled alloys[1] that an irregularity on a cooling curve had its inception of necessity in a heat effect in the cooling alloy, and the possibility of its origin in the thermal action of the liquid does not appear heretofore to have been emphasized.

Westinghouse Research Laboratory,
April 21, 1919.

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[1] Benedicks, Jour. Iron & Steel Inst., Vol. II., 1908, p. 153. Portevin and Garvin, Comptes Rendus, Vol. 164, p. 885, 1917. Dejean, Comptes Rendus, Vol. 165, p. 182, 1917. Law, Jour. Iron & Steel Inst., Vol. 97, p. 333, 1918.

THE BUNSEN ASPIRATING PUMP AND THE BERNOUILLI PRINCIPLE.

BY WILL C. BAKER.

SYNOPSIS.

The usually given application of the Bernoulli theorem to the Bunsen aspirating pump is shown to be fallacious. Experiments are described that show it to be an impact pump.

MOST text-books in dealing with Bernouilli's theorem of the pressure-velocity relations in stream-line motion, cite the Bunsen aspirating pump as an example. It is usually stated that, in accordance with the principle in question, the pressure at the smallest section of the nozzle must be very much less than that of the atmosphere, owing to the great velocity of the water at that point; that air is accordingly drawn through the side tube provided there; and that it is carried away by the rush of water in the enlarged channel.

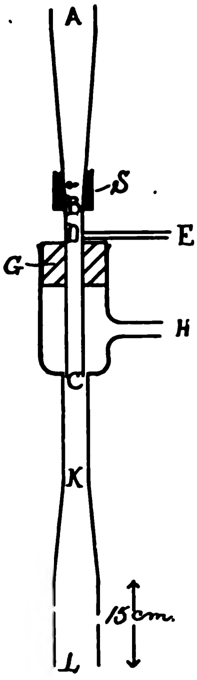

Fig. 1.

Doubting the application of the principle to this pump, it was thought well to construct one in which the constriction should be a cylinder long enough for the attachment of a side tube through which pressure measurements might be made; and to compare the actual water pressures at the narrowest part of the flow with those obtaining in the pump itself. It will be at once evident that the average velocity must be the same at all sections of the cylinder, and, if Bernouilli's conditions hold, that the pressures must therefore be the same at all sections. Either viscous or eddy friction, if appreciable, would cause a pressure gradient even in this region of uniform average velocity.

The experimental pump was as indicated in Fig. 1. A glass nozzle, *AB*, tapering from about 1 cm. diam. to 0.364 cm. in about 4 cm. of its length, led the water, through a smooth joint to a brass tube, *BC*, 4.2 cm. long and 0.364 cm. internal diameter. The joint was made by melting sealing wax, *S*, into a cup soldered to the top of the brass tube.

while the brass tube and glass nozzle were held centered by means of a metal rod that just fitted both. Nine millimeters below the point *B*, a hole .5 mm. in diameter was bored and a side tube, *DE*, was soldered on over it; the inside of the main tube being broached out smooth after the soldering. The brass tube *BC*, projected through a rubber stopper *G* into the vacuum chamber. The throat of the pump *CK* was 4.38 mm. in diameter and 2 cm. long, and was blown onto a tube *KL*, 8 mm. in diameter and 15 cm. long. Mercury manometers (open-end) were attached to the vacuum chamber at *H* and to the side tube at *E*. The mean velocity of flow through *BC* for any condition was determined from its area of cross section, and from the volume of water discharged per second. The latter quantity came from the measurement of the water caught below *L* in a known time interval.

As the pump was connected to the city water supply, the velocity was not quite constant and the pressures shown by the manometers varied from time to time for any given setting of the tap that controlled the flow of water into the pump. Thus while the pressures could be read at any given instant, the best one could do with the velocity in the cylinder was to take its mean value over a period of the order of a minute. This accounts for the fact that some of the experimental points do not fall as closely on the line as might be wished and for the fact that when a point falls below the *Q* line on the graph, the corresponding point is equally below the *R* line. (See Fig. 2).

Table I. gives a set of readings of the manometers attached to *H* and to *E* and the corresponding velocities of the water in the cylinder *BC*. All these pressure-differences are in cm. of mercury, and are all *less than that of the atmosphere.*

TABLE I.

Velocity in *BC*. Cm. per sec.	Pressure at *H*. Cm. of Mercury less than Atmospheric.	Pressure at *E*. Cm. of Mercury less than Atmospheric.
405	4.4	3.2
770	12.7	8.7
1002	20.5	14.2
1120	24.1	17.0
1280	32.5	23.2
1515	45.1	32.9
1680	55.4	40.7
1780	62.9	46.0
1902	74.2	45.6
2230	74.5	38.0

The glass part of the apparatus (*H* to *L*) was now removed so that the water discharged into the open air at *C*, and a set of readings of the manometer at *E* was taken for various rates of discharge. These are set forth in Table II.

TABLE II.

Velocity in BC. Cm. per sec.	Pressure at E. Cm. of Mercury greater than Atmospheric.
598	2.8
1270	7.6
1820	16.3
2000	20.1

The graph in Fig. 2 contains these data plotted as dots (and a second set as crosses). The abscissæ are *squares of the velocity* in units of 10^5, and the ordinates are the actual pressures in cm. of mercury. The dotted line at 76.1 gives the atmospheric pressure at the time of the experiment. Curve P contains the values given in Table II.; curve Q those from Table I. for the manometer at E; and curve R the pressures from Table I. for the manometer at H. It will be noted that when the pressure at C is atmospheric (curve P) the pressure at D *rises* as the *square of the speed* of the water. This of itself shows that the condition for Bernouilli's principle does not obtain, and that the increased pressure in the small tube is mainly due to eddy current friction. If Bernouilli's conditions existed, the pressure at D would in all cases be the same as that of C, *i.e.*, atmospheric, for the mean velocities of the water at both points is the same. The second important result is shown by the curve Q where, for any given velocity, the pressures at D are always higher than those at H (curve R) *by the same difference as that shown for the velocity in question by curve* P, and this holds even after R ceases to fall when Q bends up parallel to P. In other words, the difference of the pressures at the points D and C is dependent only on the velocity, *i.e.*, only on the eddy current loss between the two sections. When the pump begins to reduce the pressure at the point C, the pressure at D falls by the *same* amount, as the plot shows. Thus the pressure at D is the pressure at C *plus* that required to overcome the eddy friction between the two points.

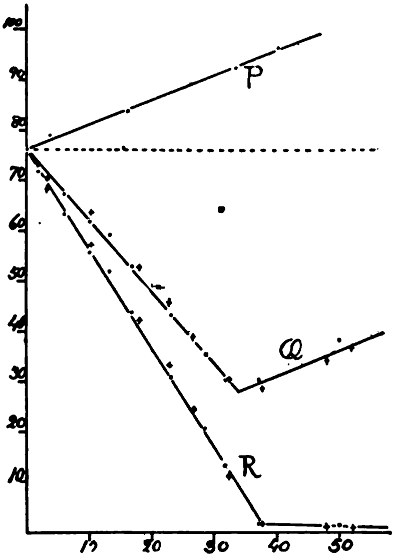

Fig. 2.

The whole effective work of the pump is therefore done after the water has left C, and the ordinary application of Bernouilli's principle to this instrument is fallacious. It is an impact pump, the action of which is made plain by the following observation. If the pump be held vertically and the water turned on carefully it is possible to get a jet that travels down the axis of the tube without touching the sides. In this case there can be no pumping, as there is a free air passage between the jet and the glass walls right up to the vacuum chamber. If now the finger be introduced for an instant into the jet at L, the splash at once runs up the tube LK; the water forms a barrier completely filling the cross section and this is driven down by the impact of the oncoming jet. The splash at impact continually forms fresh barriers that in turn are driven down against the rapidly growing pressure-gradient in KL. The action is that of a series of continually formed "pistons" of this sort that carry the enclosed air down with them. The function of the baffle plate in so many pumps is obviously to set up this action.

Consider now the effective region from the throat K to the open end at L. The motion is turbulent in the extreme, and the usual Bernouilli principle cannot be applied. Nevertheless, if we consider an approximately steady state in which the pressure-difference between L and K are constant, we may apply the conservation of energy after Bernouilli's fashion *if we take account* of that passing into heat through eddy friction, and carried out *as heat* at L. Taking the origin at K and measuring h positively along the axis KL we may write the average velocity at any section (of radius r) in terms of the velocity (V) and radius (R) at the point L,

$$v = V\frac{R^2}{r^2}$$

for now the pump is carrying away no air (by the hypothesis of no change in the pressures at H and L) and the tube runs full of water. The force required to overcome eddy friction for a length dh along the tube near this section, is given by

$$C\ 2\pi r\ dh\rho v^2,$$

where ρ is the density of the water, and C is a constant. Thus the rate at which energy is passing into heat here is

$$C\ 2\pi r\ dh\rho V^3R^6/r^6,$$

and the total heat so formed between L and K is

$$C\ 2\pi\ V^3R^6\rho\int_K^L\frac{dh}{r^5};$$

but as r is a function of h, the quantity under the integral is purely geometric and is constant for a given pump. We may then write this more simply as $C'V^2$, where C' is the product of the constant terms in the longer expression. Adding this term for the energy carried out of the region at L, expressing both mean velocities in terms of that at L, and proceeding in the usual way we find that

$$\frac{V^2R^4}{2r^4}+\frac{P_K}{\rho}=\frac{V^2}{2}+gh+\frac{P_L}{\rho}+C'V^2.$$

This at once becomes

$$V^2\left(\frac{R^4}{2r^4}-\frac{1}{2}-C'\right)=\frac{P_L-P_K}{\rho}+gh,$$

giving the linear relation between V^2 and the difference of pressure at L and K, that the graph shows.

It might be mentioned that the laboratory has some metal pumps in which the side tube has been very carefully inserted just at the point of greatest constriction. These have evidently been constructed by some one who relied on the text-book statement in regard to Bernouillis' principle. The pumps however flare out in a cone from this point and give no opportunity for the action described above. Needless to say they are very poor pumps but are fine examples of a mistaken application of theory.

It may be of interest to record that in an early pump the water passed suddenly from a tube of internal bore 8 mm. to the brass tube BC. With this arrangement the pressures at E were greater than those at H *only* until a velocity of 1,500 cm. per sec. was reached. At this point the manometer at E shot up to nearly its maximum height while that

TABLE III.

Velocity in BC. Cm. per Sec.	Pressure at H. Cm. of Mercury Less than Atmospheric.	Pressure at E. Cm. of Mercury Less than Atmospheric.
420	4.8	3.1
725	10.0	7.2
1080	21.9	16.6
1300	30.5	23.3
1360	35.1	26.4
1400	37.6	28.1
1450	40.0	29.8
1540	42.0	72.4
1630	50.8	73.6
1710	61.0	73.8
1760	67.5	74.0
1880	72.7	74.1
1860	73.3	73.6

at *H* followed more slowly. Indeed that at *H* did not rise quite so high as the other even at the highest speed then attainable. The facts will be most easily seen from Table III. The effect is attributed to the suppression of the *vena contracta* as it completely disappeared when a tapering nozzle was substituted for the sudden change of section.

Physical Laboratory,
Queen's University,
Kingston, Ont.,
May 12, 1919.

SOFT X-RAYS.

By H. M. Dadourian.

Synopsis.

Method.—Soft X-rays were produced by the impact of electrons from a hot-lime cathode against a platinum anode. Potentials of 20 to 1,000 volts were used to give the electrons the desired velocity of impact. The properties of the rays were studied by observing the velocity and number of electrons produced on a brass plate by a narrow beam of soft X-rays.

Experimental Results.—(1) Soft X-rays were obtained with as low potentials as 20 volts. (2) Radioelectric curves were obtained under different experimental conditions. These curves have the general shape of photoelectric curves but differ from them in several important particulars. (3) The velocity distribution of radioelectrons was studied and it was found that only a negligible fraction of the electrons have velocities comparable with the velocity of impact of the cathode electrons. (4) Evidence was obtained of the presence of characteristic L-radiations of copper and zinc of approximate wave-lengths of 15×10^{-8} and 13×10^{-8} respectively and of softer characteristic rays of platinum of approximate wave-lengths of 21×10^{-8} and 31×10^{-8}. (5) It is pointed out that the method used in this investigation, which is called the radioelectric method, lends itself to the study of characteristic radiations of elements of low atomic numbers.

Introduction.

The term *soft X-rays* is used in this paper to denote roughly that region of general radiation which falls between the shortest known ultra-violet rays and the longest waves of characteristic X-radiation studied to date, that is, the region between the wave-lengths 600×10^{-8}, observed by Theodore Lyman, and 12.3×10^{-8}, the characteristic L-radiation of zinc. According to the well-known quantum equation, $Ve = h\nu$, this region represents, approximately, X-rays produced by cathode rays moving with velocities corresponding to a fall of potential varying from 20 volts to 1,000 volts.

A number of new terms are introduced in this paper which are conducive to conciseness and lucidity in descriptions of phenomena related to the production of electrons by X-rays. These phenomena are similar to photoelectric effect; therefore in introducing the new terms the author was guided by the terminology used in photoelectricity. Electrons produced by X-rays are called *radioelectrons*, while the phenomenon of the production of radioelectrons is called *radioelectric effect*. With these definitions the meanings of the terms *radioelectricity*, *radioelectric current*, *radioelectric curve*, and *radioelectric chamber* become self-evident. In order to differentiate clearly between radioelectrons and the electrons forming a beam of cathode rays which give rise to X-rays, the latter are called *cathode electrons*. Further, the potential through which the cathode electrons fall before they give rise to X-rays is called *cathode potential*, and the retarding potential necessary to bring an electron to full stop *halting potential*.

Relatively little has been published on the important region of radiation represented by soft X-rays; consequently a rapid review of the work done by other investigators is feasible.

Wehnelt and Trenkle[1] showed in 1904 that electrons from a hot-lime cathode give rise to X-radiation when they impinge against a metal target with a velocity corresponding to a cathode potential of 400 volts. Dember[2] found in 1911 that photoelectrons, moving with a velocity corresponding to a potential drop of 250 volts, produce X-rays when stopped by a metal obstacle. Later[3] he obtained evidence for the production of X-rays by photoelectrons moving with velocities corresponding to 18.7 volts. Dember also studied the velocities of radioelectrons produced by soft X-rays by applying retarding electrostatic fields to the radioelectrons and found that as the cathode potential was increased from 250 volts to 9,000 volts the halting potentials of the radioelectrons increased from 0.7 volts to 3.6 volts. Whiddington[4] showed in 1913 that the impact of electrons from a hot-lime cathode against air molecules gives rise to soft X-rays. He used potentials of 128 to 218 volts to accelerate the cathode electrons and found that the corresponding halting potentials ranged from 100 to 128 volts. In 1914 Sir J. J. Thomson[5] produced soft X-rays by slow moving positive rays and by electrons from a hot-lime cathode moving with as low velocities as that corresponding to a cathode potential of 10 volts. Miss Laird[6] studied soft X-rays produced by cathode electrons moving with velocities corresponding to potentials of 220 to 1,300 volts and showed that the rays are polarized. She was unable, however, to obtain evidence for the production of X-rays with cathode potentials of less than 200 volts and arrived at the conclusion, "various experiments make the results of previous observations of a Röntgen radiation produced by cathode rays of less than 200 volt-velocity appear doubtful." The four last named observers showed that soft X-rays are very absorbable but are capable of passing through extremely thin films of glass, celluloid, etc.

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[1] A. Wehnelt and W. Trenkle, Sitz. Phys.-Med. Soc. Erlangen, 37, 1905.

[2] H. Dember, Verh. Deutch. Phys. Ges., 13, p. 601, 1911.

[3] H. Dember, Verh. Deutch. Phys. Ges., 15, p. 560, 1913.

[4] R. Whiddington, Camb. Phil. Soc., 17, p. 144, 1913.

[5] J. J. Thomson, Phil. Mag., 28, p. 620, 1914.

[6] Elizabeth R. Laird, Ann. der Phys., 46, p. 605, 1915.

The present investigation was undertaken: (1) to confirm by an electrical method the results obtained by Sir J. J. Thomson who used a photographic method, (2) to determine the relation between the cathode potential of the electrons producing soft X-rays and the halting potential of the radioelectrons which the X-rays produce on a metal surface, and (3) to study the relation between the cathode potential and the saturation current of radioelectrons. A preliminary statement of some of the results obtained in this investigation was given in a paper[1] read before the American Physical Society at its eighty-eighth meeting.

APPARATUS.

The final form of the apparatus adopted and the scheme of electrical connections used are represented by the sketch of Fig. 1. The main part of the apparatus consisted of a brass tube 20 cm. long and 6 cm. in diameter, divided into three compartments by the two brass discs *D* and *D'*. The left-hand compartment formed the X-ray tube of the apparatus, the right-hand compartment the radioelectric chamber, and the middle compartment the electrostatic trap used to prevent the passage of electrons from the X-ray tube to the radioelectric chamber.

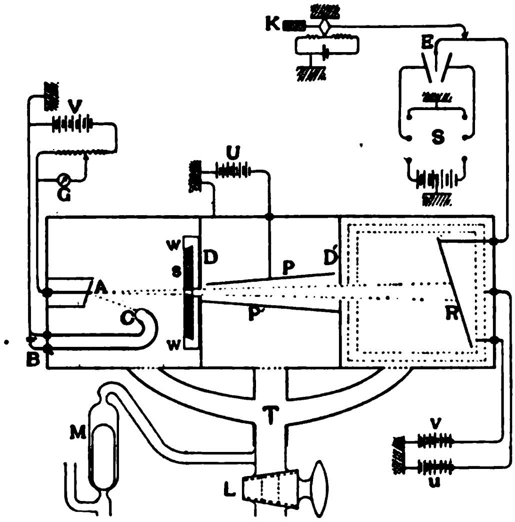

Fig. 1.

The X-ray tube is provided with a platinum anode *A* and a hot-lime cathode *C*. The latter consists of a thin strip of platinum foil with a small patch of barium oxide deposited on it by burning away a tiny speck of Bank of England sealing wax. The heating current for the hot-lime cathode is supplied by the battery *B*, and the cathode potential, which gave the cathode electrons the desired velocity, by the battery *V*. The cathode current is measured with the galvanometer *G*.

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[1] H. M. Dadourian, PHYS. REV., 9, p. 503, 1917.

The partitions separating the compartments are provided with central windows to permit the passage of a narrow beam of X-rays from the anode, *A*, into the radioelectric chamber. To the partition *D* is attached a magnetic shutter of which two other views are shown in Fig. 2. The shutter proper, *s*, consists of a rectangular plate of soft iron with bevelled sides which fit in the bevelled ways of the piece of aluminum *WW*, and is free to slide back and forth between the pegs *p* and *p'*. The shutter is provided with two circular holes *o* and *o'*. When the shutter is in contact with the peg *p* the hole *o* is in line with the windows in *D* and *D'*. On the other hand, when the shutter is in contact with *p'*, the hole *o'* is in line with the windows. The hole *o'* is covered with a thin film of celoidin, while *o* is open. The shutter can be moved, by means of a magnet, from one extreme position to the other, or made to take a midway position, thus entirely closing the window in the partition. When the cylinder *c* is in the position indicated in Fig. 2 the shutter can be moved from one extreme position to the other but it cannot be made to take a midway position by means of a magnet. If the cylinder is pulled against *WW*, however, the shutter can be moved only from its extreme position on the right to its midway position. The upper half of the cylinder is made of soft iron and the lower half of brass so that it may be moved by means of a magnet without its "freezing" on the shutter.

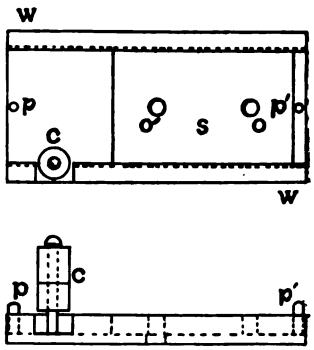

Fig. 2.

The radioelectric chamber is provided with a brass plate, *R*, and a cylindrical cage of bronze wire-gauze. (In some of the experiments the chamber was provided with two cages as indicated in Fig. 1.) A wire-gauze could not be obtained which had sufficiently small meshes and a considerable part of the area of which was not taken up by the wire itself. Therefore commercial wire-gauze of 1 mm. meshes was used in making the cages after the wire gauze was thinned down by dipping it in nitric acid until the wire of the gauze did not take up more than 25 per cent. of the total area.

The three-forked brass tube, *T*, connects each of the three compartments of the main apparatus directly to a charcoal-liquid-air bulb through a glass stop-cock with a bore one cm. in diameter. A ground glass valve *M* is placed between the liquid-air-charcoal bulb and the pumping system in order to reduce the distillation of mercury vapor into the charcoal bulb after the pump is stopped.

The measuring instrument was a gold-leaf electroscope, *E*, which could be given a sensitiveness of 5,000 divisions per volt with a fair degree of stability. A description of the electroscope has already been given by H. A. Bumstead[1] and need not be repeated here. The conditions under which this unusual sensitiveness was obtained, however, might be stated to advantage. The outside of the electroscope was covered with a thick coating by dipping it several times in molten paraffine. This made temperature changes within the electroscope slow and thus eliminated convection currents of air which disturb the gold leaf. The electroscope was provided with a milled head for raising and lowering the gold leaf in order to change its sensitiveness. It was found, however, that raising and lowering the gold leaf is invariably accompanied by slight rotational displacements of the gold leaf about its axis which often cause greater changes in the sensitiveness than the vertical displacement. Therefore after a fairly favorable position of the gold leaf had been obtained changes in sensitiveness were made by altering the potentials applied to the plates of the electroscope. The gold leaf was kept at the position of equilibrium by tilting the electroscope by means of a screw of fine pitch. The electroscope and the microscope were mounted upon a solid metal platform and a screw attachment was devised by means of which the microscope could be moved laterally without disturbing the electroscope in the slightest degree.

EXPERIMENTAL RESULTS.

I.

In the course of the experiments described in the following pages evidence was obtained for the production of X-rays by electrons moving with velocities corresponding to a drop of potential of from 1,000 volts to 20 volts. There was nothing to indicate that 20 volts is the lower limit of potential with which an electron has to be made to impinge against an atom in order to give rise to X-rays. The fact that the effect observed did not disappear suddenly as the cathode potential was reduced, but its magnitude decreased gradually until it became equal to the magnitude of the errors of observation, indicates that 20 volts is not the lower limit in question if there is indeed a limiting potential.

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[1] H. A. Bumstead, Phil. Mag., 22, p. 910, 1911.

It has been suggested that the discrepancy between Miss Laird's[1] negative results and the positive results of investigators who have been able to observe radioelectric effects with cathode potentials lower than 200 volts may be explained by assuming that the effects observed by the latter at these potentials might have been due to a diffusion of electrons from the X-ray compartment to the radioelectric chamber. In order to settle this point by testing the effectiveness of the electrostatic trap the following experiments were carried out.

The shutter *s* was pulled to its middle position, thus closing the window in partition *D*, then the X-ray tube was operated at a number of different potentials. No effect was observed on the electroscope, showing that no electrical or electromagnetic communication could take place between the extreme compartments except through the window. The X-ray tube was then operated at a given potential and alternate readings of the deflection of the electroscope were taken with the shutter first at one extreme position and then at the other. During these experiments the walls and the wire cage of the radioelectric chamber were connected to earth, while the potential applied to the plate *P* was increased from zero to any desired value. The results obtained with three different cathode potentials are shown in Fig. 3, where the abscissas represent the potentials applied to the plate *P* and the ordinates the rate of deflection of the gold leaf due to positive charging of the radiator *R*. The continuous curves correspond to readings taken with the window open, while the discontinuous curves correspond to readings taken with the window closed by means of a thin film of celoidin. The cathode potential was 80 volts for

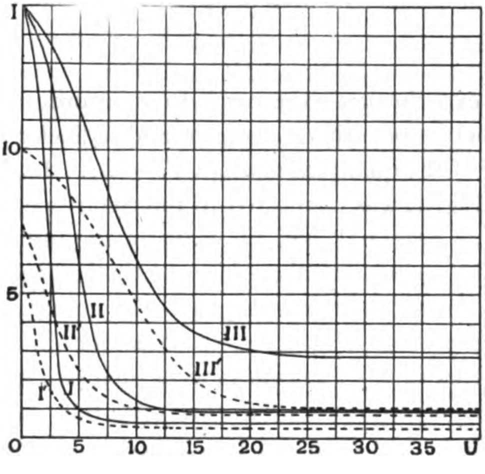

Fig. 3.

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[1] *Loc. cit.*

curves I and I', 190 volts for curves II and II', and 380 volts for curves III and III'. The continuous curves were plotted on different scales so as to make their initial points coincide, while each of the discontinuous curves was plotted on the scale of the corresponding continuous curve. It will be observed that every one of the curves becomes perfectly horizontal before the potential of the electrostatic trap attains one tenth of the value of the corresponding cathode potential. The electrostatic trap is therefore perfectly effective in preventing the passage of electrons from the X-ray compartment to the radioelectric chamber.

The celloidin film was prepared in the following manner. Celloidin was dissolved in a solution of equal parts of ether and absolute alcohol. A minute drop of the resulting solution was dropped on the surface of quiescent water by means of a glass pin point. The drop of solution spread over the surface forming a thin film. The shutter s was then removed from its ways, dipped under the film and lifted from the water so that the film covered one of the two holes in the shutter. The shutter was then set aside to dry. If the film was too thin it broke during the stretching process due to drying. After several attempts an unbroken film was obtained which was not much thicker than those which did break. The thickness of this film was estimated to be less than 10^{-6} cm. The film was examined with a microscope and was found to have no holes.

II.

A number of experiments were carried out in order to find a relation between the velocities of cathode electrons and radioelectrons and thereby test the validity of the quantum relation $Ve = h\nu$. In these experiments the radiator R, Fig. 1, was connected to the electroscope, the wire-gauze cage to a variable potential, and the walls of the radioelectric chamber to earth. This experimental disposition of the radioelectric chamber is indicated in the sketch inserted in Fig. 4. The current of electrons from the radiator to the cage was then measured for different accelerating and retarding potentials. The results were then plotted, with the radioelectric currents as ordinates and the potentials as abscissas. The radioelectric curves thus obtained for cathode potentials varying from 60 to 1,000 volts were all of the same general shape as that of curve I, Fig. 4, which corresponds to a cathode potential of 390 volts. It will be observed that curve I is similar to the typical photoelectric curve in its general form but differs from it in several particulars, namely, the radioelectric curve (*a*) attains its positive-saturation value at a higher accelerating potential, (*b*) intersects the potential-axis at a greater potential, (*c*) dips well below the potential-axis and (*d*) reaches a negative-saturation value. These characteristics of the radioelectric curve led the author to the conclusion that a part of the X-radiation incident upon the radiator was reflected and scattered by the latter and produced electrons on the wire-gauze cage and the walls of the radioelectric chamber. In order to test this conclusion certain experiments were carried out the results of which are represented by the curves of Figs. 4, 5 and 7.

Curve *II*, Fig. 4, was obtained under the same experimental conditions as curve *I* with this exception—that in the experiment represented by curve *II* the walls of the radioelectric chamber were insulated and connected to the cage. Since at zero potential experimental conditions were identical for both curves they were plotted so as to make their points of intersection with the current-axis coincident. The result is interesting. Curve *II* has a greater negative saturation, rises to positive saturation more rapidly, and has the same value for positive saturation as curve *I*. Let us see how these facts fit in with the assumption that a fraction of the X-rays are scattered by the radiator. On this assumption the negative-saturation current of curve *I* represents the electron-current from the cage to the radiator, electrons produced on the walls of the chamber being driven back by the field between the walls and the cage. The negative-saturation current of curve *II*, on the other hand, represents the electron-current from the cage plus the current of electrons from the walls of the chamber which pass through the meshes of the cage. Taking into account the area occupied by the material of the cage we find that (in the experiment represented by curve *II*) about 30 per cent. of the X-radiation was scattered by the radiator *R* and that about one third of the electrons produced on the walls of the chamber passed through the cage. The relatively rapid rise of curve *II* to positive saturation indicates that for accelerating potentials of under 20 volts

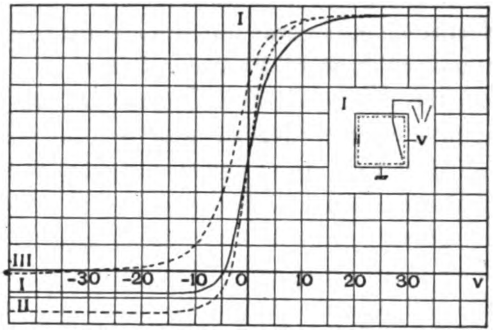

Fig. 4.

electrons produced on the walls of the chamber have a greater chance to be captured by the wire-gauze under the experimental conditions of curve *II*. In other words these electrons have a better chance to reach the radiator if they are accelerated before passing through the wire-gauze. When the accelerating field is high enough, however, all the electrons produced on the walls and the cage are prevented from reaching the radiator under the experimental conditions of both curves; therefore they have the same value for positive-saturation.

The negative saturation current may also be accounted for by assuming that the radioelectric chamber was not sufficiently free of gas molecules and that these molecules were ionized. But if the negative current were due to ions the condition of the surface of the cage and of the walls of the chamber would have no effect on the saturation current; on the other hand if it were due to electrons produced on the cage and the walls by scattered radiation the condition of these surfaces would have considerable effect. This fact was made use of to decide between the two hypotheses in an experiment in which the walls of the chamber and the cage were blackened with smoke from an asceteline lamp, the radiator *R* being kept polished. The result of this experiment, indicated by curve *III*, is decidedly in favor of the hypothesis that the negative current was due to radioelectrons produced by scattered radiation.

The curves of Fig. 5 were obtained with a cathode potential of 60 volts and with an experimental disposition of the radioelectric chamber indicated in the inserted sketch. In the experiments corresponding to both curves the outer cage was connected to a positive potential of 200 volts. In the experiment represented by curve *II* the walls and the cages were blackened, while in the case of curve *I* this was not done. It will be observed that curve *II* does not cross the potential-axis but approaches it asymptotically, indicating that the number of radio-

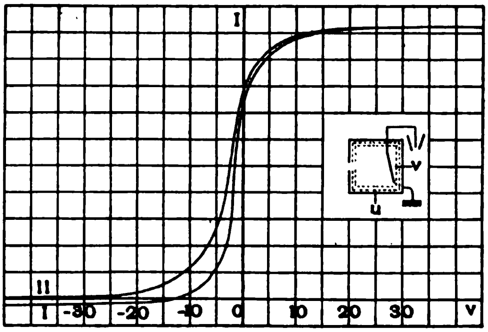

Fig. 5.

electrons which have velocities comparable with the velocity of the cathode electrons producing the X-rays is vanishingly small. This is in agreement with results obtained by Duane, and Hunt[1] who have shown for hard X-rays that energy due to the maximum frequency of hard X-rays given by the quantum equation $Ve = h\nu$ is infinitesimal for a given cathode potential V. This result is to be expected if transformation of energy from radiant energy to energy of motion of electrons and vice versa is not a perfectly reversible process. When the cathode electrons impinge against the anode only a small fraction give up their entire energy to an X-ray pulse in their first collision with an atom. Therefore the primary pulses produced by the original cathode electrons contain many wave-lengths. A fraction of this radiant energy is absorbed by the anode and the inner walls of the X-ray tube, thereby giving rise to radioelectrons and these in their turn give rise to softer X-rays. As the result of this process of progressive exchange of energy between radiation and radioelectrons the beam of X-rays which enters the radioelectric chamber becomes anything but monochromatic. The process of degradation of energy is continued in the X-ray chamber and results in a pseudo-maxwellian distribution of velocity among the radioelectrons, with a maximum velocity determined by the quantum equation. A rough idea of the relative number of electrons having a given velocity may be obtained from Fig. 6, where the abscissas denote accelerating and retarding potentials and the ordinates the corresponding values of the slope of curve *II*, Fig. 5. In view of foregoing considerations it is easy to see why Whiddington[2] was able to obtain halting potentials comparable with the cathode potentials he used, while the other investigators

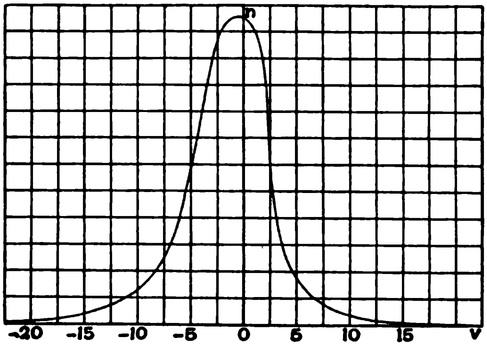

Fig. 6.

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[1] Phys. Rev., 6, p. 166, 1915.

[2] *Loc. cit.*

[3] Irving Langmuir, Phys. Rev., 2, p. 450, 1913.

of soft X-rays did not. In Whiddington's experiment X-rays were produced by the impact of cathode electrons with molecules of air at low pressure. Therefore the rays were more nearly monochromatic than the rays produced by the impact of cathode electrons against metallic targets.

It should be noted that a considerable accelerating potential is necessary to obtain positive saturation even when the electron current from the walls of the radioelectric chamber and the cage are eliminated, as was the case in the experiment represented by curve *II*, Fig. 5. This lag in saturation can not be accounted for by contact difference of potential, especially in view of the fact that both the radiator and the wire-gauze cage were both brass. Two possible factors suggest themselves which may have contributed to the lag. One of these is the negative gradient due to space-charge effect caused by the radioelectric current, and the other is the infiltration of the accelerating field into the depressions on the surface of the radiator. The part played by the first of these factors must have been relatively small because the radioelectric currents involved in these experiments were not large. Therefore the lag must have been due in a large measure to the second factor. Experiment as well as theory shows that electrons produced by a beam of X-rays may have initial velocities having any direction. There can be no doubt that there are electrons among those liberated from atoms on the surface of the radiator which are captured by neighboring atoms and which would leave the radiator if a slight impulse could be given to them in the proper direction. This is especially true when the surface of the radiator is not perfectly smooth and consequently some of the surface atoms are in depressions relative to their neighboring atoms. The accelerating field infiltrates into the depressions on the surface of the radiator and pulls away from the radiator electrons whose velocities make small angles with the plane of the radiator and which without this impulse would have been just barely captured by neighboring atoms which project out on the surface. All the electrons which, in the absence of an accelerating field, would have been captured by a projecting atom after crossing a depression on the surface are pulled away from the radiator when the accelerating potential equals the saturation potential.

On the infiltration hypothesis the value of the saturation potential will depend upon the distance across which the accelerating potential acts and upon the condition of the surface of the radiator. It is possible therefore to give it a further test by obtaining radioelectric curves with different distances between the radiator and the gauze, and with radiators of the same material but with different surface conditions.

The curves of Fig. 7 represent the results of experiments in which the experimental disposition of the radioelectric chamber was further altered as indicated by the inserted sketches. In the experiment represented by curve *I* a positive potential of 200 volts was applied to the outer cage.

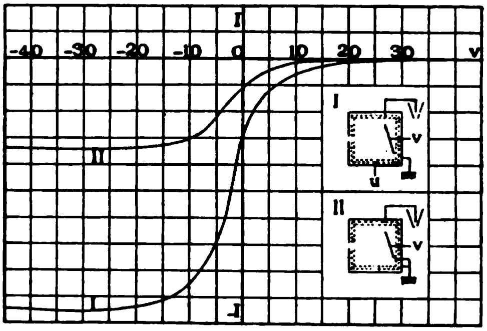

Fig. 7.

The cathode potential was 200 volts for both curves. The general character of these curves is in agreement with the results of the experiments described in the foregoing pages.

III.

The relation between the cathode potential and the radioelectric current was investigated and the results indicated by curves *I* and *II* of Fig. 8 were obtained. In these curves the abscissas denote the cathode potential in the X-ray compartment and the ordinates the positive saturation current of electrons from the radiator to the wire-gauze cage. In the experiment represented by curve *I* the window in partition *D*, Fig. 1, was open, while in the case of curve *II* it was closed with a thin celloidin film. The readings for these curves were taken alternately, that is, a reading for a given cathode potential was taken for each curve before altering the potential for another pair of readings.

Both curve *I* and curve *II* show the presence of characteristic X-radiations. In curve *I* there are three well-defined peaks at 400, 580 and 800 volts. The first two of these peaks are shifted to the right in curve *II* indicating that the rays responsible for these peaks have suffered partial absorption in passing through the celloidin film. They must therefore be due to characteristic radiations from the platinum anode in the X-ray compartment. The third peak is, in all likelihood, due to L-radiation of copper, which forms with zinc the two elements in the brass radiator. The potential at which this peak appears is near enough the computed value of the potential at which L-radiation of copper is to be expected. Using

Kossel's[1] formula $L_\alpha = K_\beta - K_\alpha$, the quantum relation, and the known values of K_α and K_β for copper this potential is found to be 778 volts. The corresponding potential for zinc is 930 volts; therefore the sudden rise of both curves in the neighborhood of 950 volts may be due to L-radiation of zinc.

It may be stated here parenthetically that the radioelectric method described in this paper lends itself very readily to the detection of K- and L-radiations of elements of low atomic numbers, and the lower series of characteristic radiations of elements of high atomic numbers. In the usual spectrometer method the beam of X-rays has to pass through absorbing matter before entering the ionization chamber where the effect to be observed is produced. In the radioelectric method there need be no absorbing material in the path of the beam before entering the radioelectric chamber where the observed effect is produced. The author hopes to use this method in studying characteristic radiations of low frequency using a modified apparatus especially adapted for the purpose.

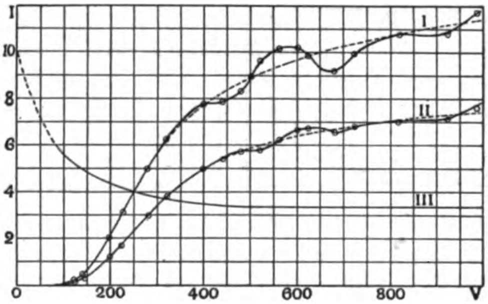

Fig. 8.

Curve *III*, Fig. 8, shows the absorption of soft X-rays by the celloidin film. The ordinates of this curve represent a coefficient of absorption defined by

$$\alpha = \frac{I_1 - I_2}{I_1}$$

where I_1 and I_2 denote, respectively, the ordinates of curves *I* and *II*. It will be observed that the coefficient of absorption is greater at lower potentials.

I wish to thank Sir J. J. Thomson for suggesting this investigation and for placing the facilities of the Cavendish Laboratory at my disposal during the preliminary stages of the investigation. The experiments described in this paper were carried on in the Sloane Physical Laboratory while I was still a member of the Physics Department of Yale University.

New York City, April, 1919.

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[1] Kossel, Deutsch. Phys. Gesell. Verh., 16, p. 953, 1919.

THE SIZE AND SHAPE OF THE ELECTRON.

By Arthur H. Compton.

Synopsis.

In this paper it is pointed out that Sir J. J. Thomson's explanation of the fluorescent absorption of X-rays on the basis of incident radiation consisting of short double pulses, taken in connection with Moseley's law relating atomic number and the wave-length of characteristic X-radiation, leads to Owen's experimental law for the fluorescent absorption of moderately soft X-rays, namely that $\tau/\nu = KN^4\lambda^3$, τ/ν being the atomic fluorescent absorption coefficient, N the atomic number, λ the wave-length of the incident radiation, and K a constant. Thus, though Thomson's explanation cannot be considered complete, it affords a provisional theoretical basis for Owen's law which has heretofore been lacking.

It is then shown that the law of absorption which holds for moderate frequencies must be modified in order to apply at very high frequencies in view of the fact, shown in a previous paper, that the wave-length becomes comparable with the radius of the electron. For absorbing elements in which excess scattering of the primary beam is inappreciable, the following law of absorption is derived: $\mu/\nu = K\varphi N^4\lambda^3 + \sigma/\nu$ where μ/ν is the atomic total absorption coefficient, σ/ν is the atomic scattering coefficient, and φ and σ are functions of the ratio of the wave-length to the radius of the electron which are evaluated in the paper.

It is found that this formula agrees satisfactorily with the experimental values for the absorption of high frequency radiation in aluminium, the only substance for which the requisite data are available to make an adequate test, if the electron is taken to be a ring of radius $(1.85 \pm .05) \times 10^{-10}$ cm. This result is in good agreement with the value 2×10^{-10} cm. previously estimated by the writer on the basis of measurements of the scattering of high frequency radiation.

II. The Absorption of High Frequency Radiation.

In the first paper of this series[1] it was pointed out that the experimental observations on the scattering of high frequency radiation by matter could be explained only on the hypothesis that the radius of the electron is comparable with the wave-length of hard γ-rays. The phenomena of scattering were found to be quantitatively accounted for, within the probable errors of observation, if the electron was considered to be a flexible ring of electricity with a radius of 2×10^{-10} cm. The present discussion will deal chiefly with the modifications to be expected in the law of absorption of high frequency radiation if the electron is considered to have appreciable dimensions. Before studying these modifications, however, it will be instructive to consider the law of absorption for wave-lengths which are long compared with the radius of the electron.

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[1] A. H. Compton, Phys. Rev., 14, 20 (1919).

The absorption of high frequency radiation is due to two independent processes. The more important of these is usually the energy absorbed in exciting corpuscular and fluorescent radiation. There is always, however, a certain amount of energy removed from the primary beam because of scattering by the electrons in the absorbing screen. Thus the total absorption coefficient may be written as

$$\mu = \tau + \sigma,$$

where τ represents the fluorescent absorption, and σ is the absorption due to scattering. In order to compare directly the absorption by different elements, it is customary to consider the "atomic absorption coefficient," μ/ν, where ν is the number of atoms per unit volume of the absorbing screen and μ is the linear absorption coefficient. We may then write

$$\frac{\mu}{\nu} = \frac{\tau}{\nu} + \frac{\sigma}{\nu}.$$

Owen has shown[1] that the atomic fluorescent absorption coefficient follows the experimental law,

$$\frac{\tau}{\nu} = KN^4\lambda^3, \tag{1}$$

where N is the atomic number of the absorber, λ is the wave-length of the incident rays, and K is a constant over certain ranges but changes abruptly when λ passes the critical wave-length required to excite a characteristic radiation in the absorber. The quantity σ/ν has been shown by Barkla and Dunlop[2] to be calculable for the lighter elements and for moderately hard X-rays according to Thomson's formula,

$$\frac{\sigma_0}{\nu} = \frac{8\pi}{3}\,\frac{e^4N}{m^2C^4}, \tag{2}$$

where e is the charge and m the mass of an electron and C the velocity of light. The total atomic absorption coefficient, for the elements of low atomic weight and for X-rays of moderately short wave-length, obeys therefore the experimental law,

$$\frac{\mu}{\nu} = KN^4\lambda^3 + \sigma_0/\nu. \tag{3}$$

The absorption which is expressed by relation (1) is generally supposed to be due chiefly to a transformation of the incident energy into fluorescent radiation of longer wave-length. As in the similar case of ordinary light, no satisfactory explanation of this fluorescent absorption has been proposed. Sir J. J. Thomson has suggested a partial explanation, however, on the basis of an incident X-ray pulse of special wave-form, which leads to the experimental law for the variation of the fluorescent absorption with the wave-length of the incident rays and with the atomic number of the absorber. He shows[1] that if the pulse consists of an electric intensity X through a distance d followed by an intensity $-X$ for a distance d, after which the field due to the pulse vanishes, and if the distance $2d$ is short compared with the wave-length of the radiation excited by the absorbing electron, the atomic fluorescent absorption coefficient is

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[1] E. A. Owen, Proc. Roy. Soc., 94, 522, 1918.

[2] C. G. Barkla and J. G. Dunlop, Phil. Mag., 31, 222, 1916.

$$\frac{\tau}{\nu} = 4\pi^3 \frac{e^2}{mC^2} d^3 \sum_k \frac{n_k}{\lambda_k^2}. \tag{4}$$

In this expression $2d$ may be taken as the wave-length of the incident radiation, and n_k is the number of electrons per atom of the type k, the wave-length of whose free radiation is λ_k.

Moseley has shown that the square root of the frequency of either the K or the L characteristic X-radiation is nearly proportional to the atomic number of the radiator. The same relation holds for Siegbahn's M radiation. We may therefore write $1/\lambda_k^2 = c_k N^4$, where N is the atomic number of the absorbing element, and the constant c_k has different values for the different types of characteristic radiation, but it is the same for all elements. The factor $\Sigma n_k/\lambda_k^2$ may therefore be written as $N^4 \Sigma c_k n_k$. Since the number of the electrons of any type k is probably the same for all the elements which have electrons of this type, and since those terms in the summation which are due to the most rigidly bound electrons are the most important, the factor $\Sigma c_k n_k$ may be considered to be practically constant for all elements except possibly the very light ones, which may not possess electrons of the same high frequency types as do the heavier elements. Writing also for the thickness $2d$ of the incident X-ray pulse its equivalent λ, Thomson's expression for the fluorescent absorption coefficient becomes:

$$\frac{\tau}{\nu} = K_1 N^4 \lambda^3, \tag{5}$$

where

$$K_1 = \frac{2}{\pi^3} \frac{e^2}{mC^2} \Sigma c_k n_k.$$

As thus stated, Thomson's result is identical in form with the experimental law expressed by equation (1).

The incompleteness of this solution of the problem is evident, however, from the following considerations. In the first place, the solution by hypothesis does not apply if the natural period of any of the electrons in the absorbing material is comparable with or shorter than the period of the incident radiation. That is, the result is derived strictly for only the case where the wave-length of the incident radiation is considerably shorter than the wave-length of the K radiation characteristic of the absorbing material. In the second place, the value of the coefficient K_1 is too great by a factor of about 10.[1] In the third place, the basic hypothesis of incident X-rays consisting of very short pulses is inconsistent[2] with the fact that crystal analysis shows no X-radiation of frequency higher than that given by the relation $h\nu = eV$.

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[1] Sir J. J. Thomson, Conduction of Electricity through Gases, 2d ed., p. 325.

The exact similarity of form of the two expressions, and the fact that even the coefficient K_1 is not far from the proper order of magnitude, are nevertheless coincidences too remarkable not to have some physical significance. Thus, for example, we could hardly expect such an agreement if this absorption were not really due principally to fluorescent transformations. In any case this explanation affords a provisional theoretical basis for Owen's Law which has heretofore been lacking.

Expression (2) represents the scattering coefficient if the electrons in the atom are far enough apart to scatter independently, and if the wave-length is great enough for the electrons to act sensibly as point charges. The first of these conditions has been shown by the experiments of Barkla and Dunlop to be satisfied for X-rays of ordinary hardness in the case of absorbing elements of lower atomic weight than copper. For the heavier elements they find the scattering to be greater than thus calculated. This is probably due chiefly to a grouping of the electrons so close together in the heavy atoms that the waves scattered by the different electrons are nearly in the same phase. The increased scattering may also be partly due to the fact that there are in the heavier atoms certain electrons whose natural frequency is near that of ordinary X-rays. The second condition, that the electron must be small compared with the wave-length of the incident beam of X-rays, has been shown in the first paper of this series to hold for moderately hard radiation but not to hold if the wave-length of the incident rays is shorter than about 0.3 A.U.

In order to obtain an expression for the total absorption which will be valid at shorter wave-lengths, two modifications must be made in formula (3). In the first place, as just suggested, the value of the atomic scattering cannot be considered constant, but must decrease when the wave-length becomes comparable with the diameter of the electron. If the electron is assumed to have the form of a flexible ring of electricity, it has been shown in the first paper of this series that the scattering will be expressed by the formula

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[1] If, however, as suggested by Thomson's recent theory of atomic structure (Phil. Mag. 37, 419, 1919), not all the atoms possess electrons of a given type K, it is possible that K_1 may be equal to K.

[2] Cf. D. L. Webster, PHYS. REV., 7, 609, 1916.

$$(6)\quad \frac{\sigma}{\nu} = \frac{\sigma_0}{\nu}\left\{1 - 29.61\left(\frac{a}{\lambda}\right)^2 + 524.2\left(\frac{a}{\lambda}\right)^4 - 5{,}398\left(\frac{a}{\lambda}\right)^6 + \cdots\right\},$$

where σ_0/ν is the atomic scattering coefficient as expressed by equation (2), a is the radius of the electronic ring, and λ is the wave-length of the incident radiation. The values of the coefficients of the powers of a/λ have been given in detail in the previous paper.[1]

The second modification concerns the coefficient of λ^2 in the term representing the fluorescent absorption. In the derivation of Thomson's expression (4) for the fluorescent absorption, he shows that the energy absorbed from the incident ray by an electron is proportional to the square of the acceleration to which the electron is subject. It would seem that this relation must hold in whatever manner the fluorescent absorption is calculated. The acceleration of a comparatively large electron will be less than that of a small electron of the same mass, however, when both are traversed by X-rays of the same intensity, since in the former case the phase of the incident ray will not be the same at all parts of the electron. The fluorescent absorption due to a large electron will therefore be less than the value $KN^4\lambda^2$ by the factor,

$$\varphi = \left\{\frac{\text{acceleration of large electron}}{\text{acceleration of small electron}}\right\}^2.$$

If the large electron is assumed to have the form of a ring, its accelera-tion when traversed by an X-ray may be calculated in the following manner. Let us consider the motion of the electron represented by the heavy ring in Fig. 1, when traversed by an electromagnetic wave propa-

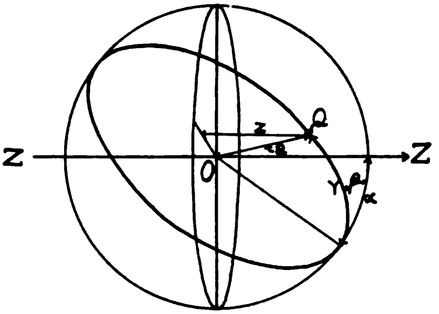

Fig. 1.

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[1] A. H. Compton, *loc. cit.*, p. 42.

gated in the direction $-ZOZ$. The electric intensity at a given instant at the point $Q(a, \alpha, \beta, \gamma)$ on the ring is

$$X = A \cos 2\pi \left(\frac{\delta - Z}{\lambda} \right),$$

where A is the amplitude of the incident electric intensity, $2\pi\delta/\lambda$ is the phase angle at the point O and z is the Z coördinate of the point Q. But

$$z = a\,(\cos \alpha \cos \gamma + \sin \alpha \cos \beta \sin \gamma),$$

where a is the radius of the electron. The acceleration of the electron at this instant is therefore

$$\frac{Aa\eta}{m} \int_0^{2\pi} \cos \frac{2\pi}{\lambda} \{\delta - a\,(\cos \alpha \cos \gamma + \sin \alpha \cos \beta \sin \gamma)\} d\gamma,$$

where η is the charge per unit length along the circumference of the electron, and m is the electron's mass.

This integral is obviously a maximum when δ is zero. The amplitude of the acceleration is therefore,

$$f = \frac{Aa\eta}{m} \int_0^{2\pi} \cos \left\{ \frac{2\pi a}{\lambda} (\cos \alpha \cos \gamma + \sin \alpha \cos \beta \sin \gamma) \right\} d\gamma.$$

This quantity may be written

$$f = C_1 \int_0^{2\pi} \cos\,(k \cos \gamma + l \sin \gamma) d\gamma,$$

where

$$C_1 = Aa\eta/m,$$

$$k = 2\pi \frac{a}{\lambda} \cos \alpha,$$

$$l = 2\pi \frac{a}{\lambda} \sin \alpha \cos \beta,$$

or

$$f = C_1 \int_0^{2\pi} \cos\,\{M \sin\,(\gamma + \Delta)\} d\gamma,$$

where $M = \sqrt{k^2 + l^2}$ and Δ is the appropriate phase angle. Since the integration extends from 0 to 2π, the value of Δ is immaterial, and may therefore be put equal to zero. The integral then becomes:

$$f = C_1 \int_0^{2\pi} \cos\,(M \sin \gamma) d\gamma$$

$$= 2\pi C_1 J_0(M),$$

where

$$J_0 M = 1 - \frac{M^2}{2^2} + \frac{M^4}{2^2 4^2} - \frac{M^6}{2^2 4^2 6^2} + \cdots,$$

i.e., Bessel's J function of the zero order. Substituting

$$M = \frac{2\pi a}{\lambda}\sqrt{1 - \sin^2 \alpha \cos^2 \beta}$$

and

$$2\pi C_1 = \frac{A}{m} \cdot 2\pi a \eta = Ae/m,$$

where e is the total charge on the electron, we obtain

$$f = \frac{Ae}{m} J_0\left(\frac{2\pi a}{\lambda}\sqrt{1 - \sin^2 \alpha \cos^2 \beta}\right).$$

The ratio of the acceleration of the ring electron to that of a sensibly point charge electron is therefore

$$J_0\left(\frac{2\pi a}{\lambda}\sqrt{1 - \sin^2 \alpha \cos^2 \beta}\right).$$

The required value of φ is obviously the mean square of this quantity averaged over all angles α and β at which the ring electron may be oriented, *i. e.*,

$$\varphi = \frac{1}{\pi^2}\int_0^\pi \int_0^\pi J_0^2\left(\frac{2\pi a}{\lambda}\sqrt{1 - \sin^2 \alpha \cos^2 \beta}\right) d\alpha d\beta.$$

This is the same integral as that which enters into the calculation of the scattering by a ring electron, the solution being

$$\varphi = 1 - \alpha\left(\frac{2\pi a}{\lambda}\right)^2 + \beta\left(\frac{2\pi a}{\lambda}\right)^4 - \gamma\left(\frac{2\pi a}{\lambda}\right)^6 + \cdots,$$

where the constants α, β, $\gamma \cdots$ have the values given on page (42) of the first paper of this series. For purposes of calculation this function may be more conveniently expressed as

$$\varphi = 1 - n\frac{a^2}{\lambda^2} + o\frac{a^4}{\lambda^4} - p\frac{a^6}{\lambda^6} + \cdots$$

where

$n = (2\pi)^2\alpha = 14.8044,$
$o = (2\pi)^4\beta = 93.6041,$
$p = \cdots = 306.6932,$
$q = 608.7241,$
$r = 807.8924,$
$s = 766.5255,$
$t = 545.4744,$
$u = 301.8218,$
$v = 133.5450,$
$w = 48.3256,$
$x = 14.5661,$
$y = 3.7134,$
$z = .8111,$
$a = .1535,$
$b = .0254,$
$c = .0037,$
$d = .0005,$
$e = .0001.$

The values of φ, and of the ratio σ/σ_0 of the scattering by a ring electron to that of an electron of negligible size, are shown for different values of λ/a by curves *I* and *II* respectively of Fig. 2.[1]

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[1] In a preliminary paper on the absorption of high frequency radiation (Phys. Rev., 13, 296, 1919), the quantity $\sqrt{\sigma}$ was used in place of the quantity φ. This was justified by the fact that for values of λ/a greater than 8, with which that paper was concerned, φ is almost exactly equal to $\sqrt{\sigma/\sigma_0}$.

The total absorption of high frequency radiation by the lighter elements may therefore be expressed by the formula,

$$\frac{\mu}{\nu} = K\varphi N^4\lambda^3 + \frac{\sigma}{\nu}, \tag{8}$$

where σ/ν and φ have the values assigned by equations (6) and (7), and depend upon the radius of the electron. If this radius is taken as .02 A.U., as estimated from determinations of the scattering, K is the only arbitrary constant involved in this expression; though it must be admitted that the exponents of N and λ have no conclusive theoretical support.

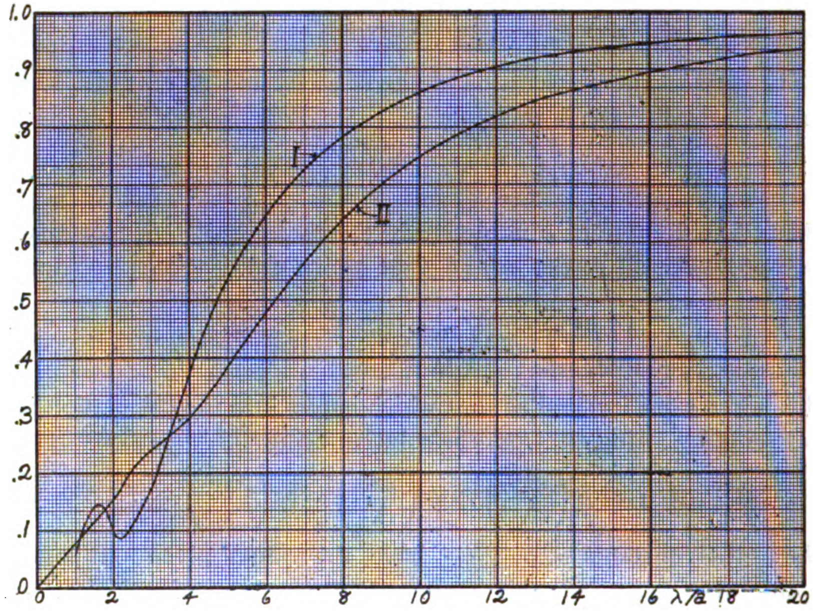

Fig. 2.

The only reliable experimental data which are of value in testing expression (8) are those which refer to the mass absorption coefficients in aluminium.[1] In Fig. 3 are plotted the cube roots of the mass absorption coefficients for this metal against the wave-length. The data include all the published measurements for strictly homogeneous rays of known wave-length which I have been able to find. The dotted line represents

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[1] It is unfortunate that the measurements by Barkla and White (Phil. Mag., 34, 270, 1917) were not made with strictly homogeneous rays of known wave-length. They determine the effective wave-length of their rays by the absorption in copper, using the values given for this material given by Hull and Rice. Since, however, the variation of the absorption coefficient with wave-length of the light materials, aluminium, paper, paraffin, used by Barkla and White is much smaller than the variation in the case of copper, the effective wave-length of a heterogeneous beam will not be the same in the different materials. Moreover, the effect of inflection in the experimental absorption curves renders the true absorption for any definite wave-length uncertain. It would be of great interest to have the measure-

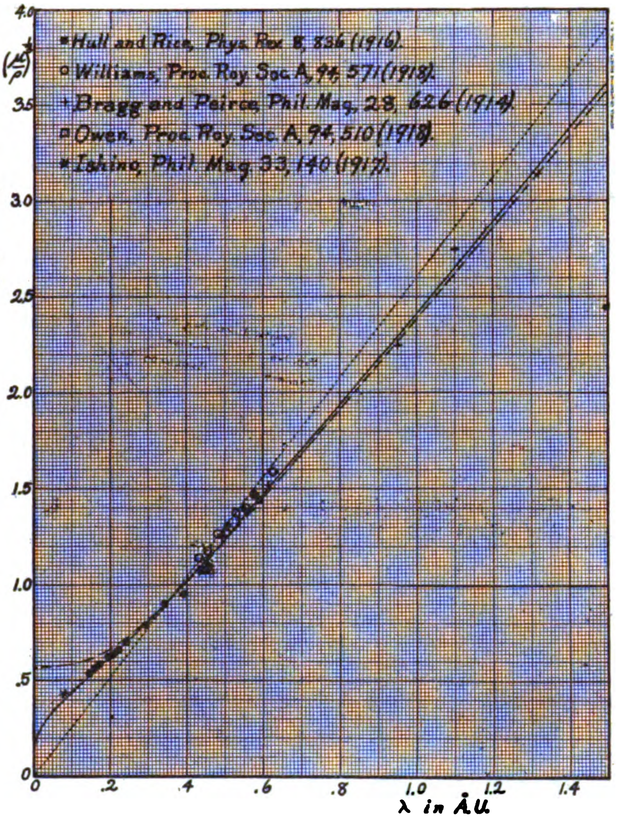

Fig. 3.

the simple cube law as expressed by equation (1), which neglects the absorption due to scattering. The broken line represents Owen's formula (3), which takes into account the scattering but is based on the hypothesis of a relatively small electron. The solid line represents equation (8), calculating the absorption on the basis of a ring electron of radius .02 A.U. as estimated from determinations of the scattering.[1] These curves are obtained from the values of μ/ν as calculated from expressions 1, 3 and 8 respectively, by multiplying by the factor ν/ρ, taking ν for alumin-

ments on these absorbers of low atomic weight repeated with strictly homogeneous radiation, since the predominance of the scattering over the fluorescent absorption by the light elements makes the measurements made on them of maximum value in determining the radius of the electron.

[1] Ibid.

ium to be 6.06 × 10^{22} and ρ to be 2.71. For the longer wave-lengths the three formulas differ but little, and the experimental variations are of little significance. It is at the shorter wave-lengths that the effects of the different hypotheses become evident. Here it is apparent how failure to take into account the scattering in equation (1) and the effect of the electron's size in equation (3) makes the dotted and the broken lines depart seriously from the experimental values, whereas the solid line shows a very fair agreement.

The mass absorption coefficients of aluminium for wave-lengths less than .35 A.U. are plotted in Fig. 4 in larger scale, in order to show the

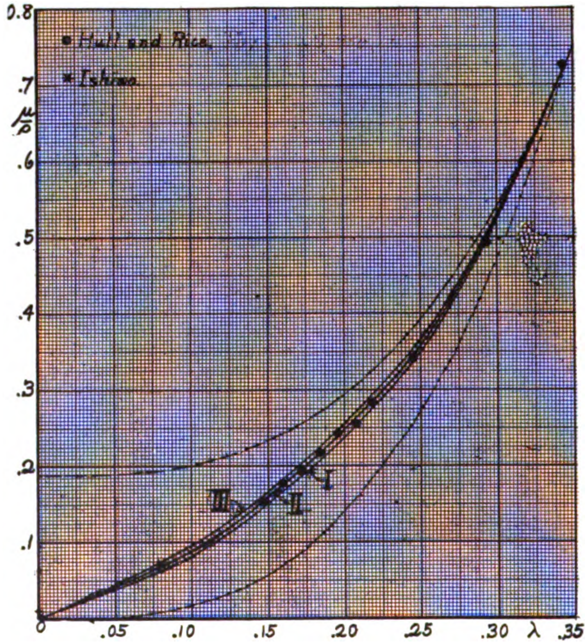

Fig. 4.

effect on the form of the theoretical curve of changing the value assumed for the radius of the electron. Here, as in Fig. 3, the dotted and the broken curves represent formulas (1) and (3) respectively, and both are obviously inaccurate for these very hard rays. It is of particular interest to note that the fact that the total absorption falls below the value of σ_0/ρ, required for the scattering coefficient alone on the basis of a small electron shows the hypothesis of a sensibly point charge electron to be untenable. Of the three solid lines, curve *I* is calculated on the basis of an electron of 2.0 × 10^{10} cm. radius, curve *II* for a radius of 1.85 × 10^{10} cm., and curve *III* for a radius of 1.7 × 10^{10} cm. Unless there is some consistent error in the experimental figures, we can, on the basis of the agreement of these curves, take the radius of the electron to be $(1.85 \pm .05) \times 10^{10}$ cm.[1]

[1] Attention has been called repeatedly (cf. Nature, 100, 510, 1918; H. S. Allen, Proc. Phys. Soc., 30, 143, 1918; 31, 53, 1919) to the fact that my original estimate of the radius of the electron (Jour. Wash. Ac. Sci., 8, 1, 1918) as 2.3×10^{-10} cm. was based on the value 9×10^{-10} cm. for the wave-length of the hard rays from radium C as determined by Rutherford and Andrade (Phil. Mag., 28, 263, 1914) instead of a value one third to one tenth as large more recently estimated by Rutherford (Phil. Mag., 34, 153, 1917) in light of measurements on the absorption of hard X-rays. In these latter measurements the X-radiation was obtained from a Coolidge tube operated by an induction coil, and the rays were filtered through considerable thicknesses of iron and lead. The wave-length of the end radiation was estimated according to the relation $\lambda = hC/eV$, where h is Planck's constant, C is the velocity of light, e the electronic charge, and V is the voltage across the X-ray tube. The value of V used in Rutherford's calculations was the *maximum* voltage across the tube, which obviously is considerably greater than the effective voltage. Furthermore, the filtering method of obtaining the radiation which has the shortest wave-length is uncertain in its results, as is apparent from the false indications obtained by this method by Rutherford, Barnes and Richardson (Phil. Mag., 30, 339, 1915). Both of these sources of error lead to a calculated value of the effective wave-length which is smaller than the true value.

That these errors are actually present in Rutherford's work is suggested by the following considerations. (1) For $\lambda = .086$ A.U. (maximum voltage = 144,000) he was unable to measure the intensity of the beam accurately after it had passed through 5 mm. of lead, whereas Rutherford and Andrade (Phil. Mag., 28, 266, 1914) find that homogeneous radiation shorter than 1.16 A.U. penetrates 6 mm. of lead without great loss.

(2) The absorption coefficient measured by Rutherford "is intermediate between μ and $\mu + \sigma$ (where μ is the true [fluorescent] absorption coefficient and σ the scattering coefficient), and probably closer to the former. The value of μ as given by Hull and Miss Rice corresponds to $\mu + \sigma$ in the above notation." That is, using the notation of the present paper, the data of Hull and Rice refer to the total absorption μ, while the figures given by Rutherford refer more nearly to the fluorescent absorption τ. Nevertheless, Rutherford points out that his measurements on both aluminium and lead agree with those of Hull and Rice for the wave-lengths which overlap. These wave-lengths are, however, in the neighborhood of .135 A.U., where, whether the electron is taken to be very small or of the size here estimated, by far the greater part of the total absorption in the case of aluminium is due to scattering. Rutherford's values for the absorption coefficient should therefore have been much smaller for aluminium than the values of Hull and Rice.

(3) That the minimum wave-length produced by the tube is not separated out by the filtering process is apparent from the following data on the absorption coefficient in lead for various thicknesses of the lead screen, as taken from Rutherford's paper (p. 154):

Max Voltage.	Range of Thickness in Lead, mm.	Absorption Coefficient.	Max Voltage.	Range of Thickness in Lead, mm.	Absorption Coefficient.
170,000	3.1–3.7	18	196,000	4.3–5.5	13
	3.7–4.3	17		5.5–6.4	12
	4.3–5.5	16		7.8–9.2	10
				8.8–10.0	8.5

If the absorption coefficients for 196,000 volts, for example, are plotted against the reciprocal of the mean thickness of the absorption screen, the curve shown in Fig. 5 is obtained. There is apparently no tendency for the absorption coefficient to approach a constant finite value for large thicknesses of the absorption screen. In fact these figures would rather lead to the con-

If a similar calculation is made in the cases of copper and lead, the absorption coefficients of which for very short wave-lengths have been determined by Hull and Rice, the agreement for long wave-lengths is satisfactory, but for shorter wave-lengths the calculated values are somewhat too low. The difference is greater in the case of lead than in the case of copper. This discrepancy is doubtless due at least in part to the fact, pointed out by Barkla and Dunlop,[1] that a considerable excess scattering occurs in these metals when traversed by X-rays, while

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clusion that the absorption coefficient of the rays, after passing through a very thick lead screen, approaches zero. It is certain, at least, that the true absorption coefficient corresponding to the shortest wave-length produced at 196,000 volts is much less than the value 8.5 used by Rutherford.

It appears from these considerations that the wave-lengths assigned by Rutherford to correspond with the different observed absorption coefficients are considerably too short.

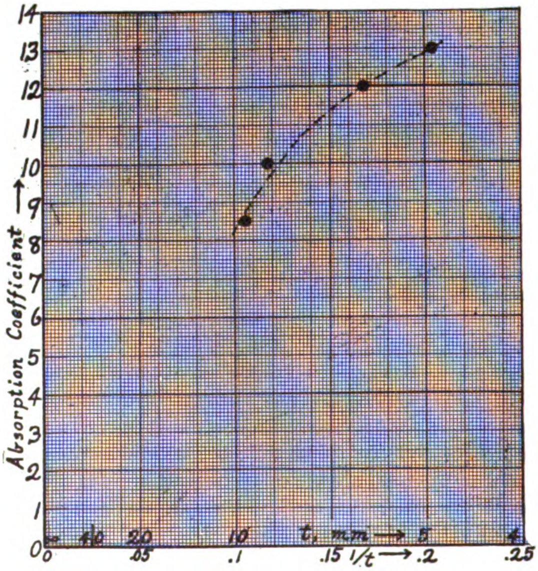

Fig. 5.

For this reason I have not considered his estimate of the wave-length of hard γ-rays, based on these absorption measurements, as reliable as the direct determinations of Rutherford and Andrade.

Note added September 3, 1919: After reading over these comments, Professor Rutherford still believes in the accuracy of his later measurements rather than in the earlier determinations by himself and Andrade. He writes me, "When I recall the faintness of the radium *C* lines and the difficulty of fixing them. . . . I am inclined to think that a mistake could easily arise. . . . I am inclined to give a good deal more weight to the Coolidge Tube experiments than you do." As Rutherford suggests, the question is one which must finally be answered by more refined measurements of the absorption of very hard homogenous rays of known wave-length.

[1] Barkla and Dunlop, *loc. cit.*

it does not occur to an appreciable extent in the case of aluminium for these wave-lengths. It is therefore impossible to test on the basis of measurements on lead and copper the validity of the assumptions underlying formula (8), which applies strictly to only those absorbing materials in which excess scattering does not occur.

Conclusions.—The excellent agreement of the absorption as calculated by equation (8) with the experimental values, in the only case (aluminium) in which the requisite data are available to make an adequate test, constitutes a strong support of the fundamental assumption of an electron of a size comparable with the wave-length of short X-rays. This agreement is the more significant since it is impossible to account for the low values of the absorption observed for very short X-rays if the electron is assumed to be sensibly a point charge of electricity.

These results must also be considered as a partial confirmation of the formula (8) proposed from theoretical considerations for the absorption coefficients of elements which do not show appreciable excess scattering.

Assuming the validity of this formula and the accuracy of the measurements of Hull and Rice on the absorption of hard X-rays in aluminium, and considering the electron to have the form of a flexible ring of electricity, the radius of the electron is calculated to be $(1.85 \pm .05) \times 10^{10}$ cm.

Research Laboratory,
Westinghouse Lamp Company,
May 24, 1919.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE PASADENA MEETING.

THE ninety-eighth meeting of the American Physical Society was held in the Physics lecture room of Throop College, Thursday, June 19, in connection with the meeting of the Pacific Division of the American Association for the Advancement of Science. On Friday morning a joint meeting with the Astronomical Society of the Pacific was held. Dr. J. A. Anderson presided at both meetings. There was an attendance of about forty. Members of the society were entertained at luncheon by Throop College on both days. On Friday they were invited to a garden party by Dr. and Mrs. Hale.

The National Research Council's laboratories at Throop College and various industrial laboratories in the vicinity were open to visitors. On Saturday morning many members of the society inspected the physical laboratory of the Mount Wilson Observatory in Pasadena, and later went by stage to the observatory, where they were entertained at luncheon and given an opportunity to inspect the equipment. Those who remained until evening made observations with the 60- and 100-inch telescopes.

On adjourning a vote of thanks was extended to the authorities of Throop College and Mount Wilson Observatory for their courtesies.

The following papers were read:

The Stability of a Rotating Parachute or Falling Body Rotating about a Vertical Axis of Symmetry. H. BATEMAN.

The Most Probable Value of the Planck Constant *h*. RAYMOND T. BIRGE.

The Stark Effect for Metals in the Ultra-Violet. J. A. ANDERSON.

Absorption Effects with the Electric Furnace as Related to Temperature Classification. A. S. KING.

The Electric Furnace Spectra of Metals in the Infra-Red. A. S. KING.

The Spectra of Krypton and Xenon in the Infra-Red. PAUL W. MERRILL.

A Study of the Comparative Intensities of Spectrum Lines in Different Regions of the Arc as Compared with their Behavior in Other Sources. PAUL W. MERRILL.

The Vapor Pressures of Hydrogen Chloride and Hydrogen Bromide above Their Aqueous Solutions. S. J. BATES and H. D. KIRSCHMAN.

Recent Observations of Tube-Arc Spectra, Especially in the Infra-Red. ARTHUR S. KING and PAUL W. MERRILL.

The Reflective Power of Metals and Dielectrics in the Ultra-Violet. E. P. Lewis and A. C. Hardy.

Determination of the Normal Temperature by Means of the Equation of the Seasonal Temperature Variation and the Thermograph Record. F. L. West, N. E. Edlefsen and S. P. Ewing.

Snow Crystal Studies. John C. Shedd.

A Study of the Reversible Pendulum. John C. Shedd and Paul Kirkpatrick.

The Movement of Moisture in Soil by Capillarity. Willard Gardner.

Physical Constants Pertaining to the Ocean. George F. McEwen.

Sparking Voltage of Point-to-Sphere Discharge by Means of a High-Voltage Direct Current Generator. E. R. Wolcott.

Formula for the Wave-Length of M-Radiation. Fernando Sanford.

The Effect of Chemical Treatment on the Rate of Percolation of Water through Soils. A. E. Vinson and C. N. Catlin.

E. P. Lewis,
Local Secretary.

THE SPECTRAL TRANSMISSION OF FILTERS USED TO DETECT CAMOUFLAGE OR IMPROVE VISIBILITY.[1]

By K. S. Gibson, E. P. T. Tyndall and H. J. McNicholas.

During the war, the spectral transmissions of a large number and variety of filters were measured by the Bureau of Standards. Among the many uses to which these filters were put, were the detection of camouflage or the improvement of visibility. By the latter is meant increasing the ability of the eye or the photographic plate to detect the details of distant objects.

Examples of the different types of filters used for these purposes are given in the accompanying figures. In each figure is given also the relative visibility curve of the average human eye, that is, the relative sensibility of the eye to radiant energy of different wave-lengths.

In Fig. 1 are shown a number of types of filters used to improve visibility. This is done by the absorption of the shorter wave-lengths, blue haze or brilliant glare being thus partially or completely eliminated. Goggles whose transmissions are represented by curves *1* to *4* were much used by aviators, and those represented by curves *2* to *8* in photographic work, especially from airplanes. Numbers 1 to 8 are different shades of Corning glass G 38 (Noviol), G 36, G 34, and G 24. This same type of curve, in many of the goggles examined, was produced by dyed gelatine or celluloid between glass.

In Fig. *2* are given curves for certain types of aviator goggles known as "Brock" goggles, the color being produced by means of dyed gelatine between glass. No. 23 and No. 73 are similar to those of Fig. 1 and are doubtless used for the same purpose. The others, except No. 79, are dichromatic and used for the detection of camouflage. In most of the dichromatic screens successfully used for this purpose, the two colors transmitted are approximately equal in luminosity. Other Brock goggles examined were No. 93, No. 14, and No. 25. Filters represented by curves *9* (gelatine dyed with methyl violet) and *15*, and others similar have been of considerable value. In addition to the dichromatic screens, monochromatic filters are often of value, especially red (curve *8*), violet or ultra-violet.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

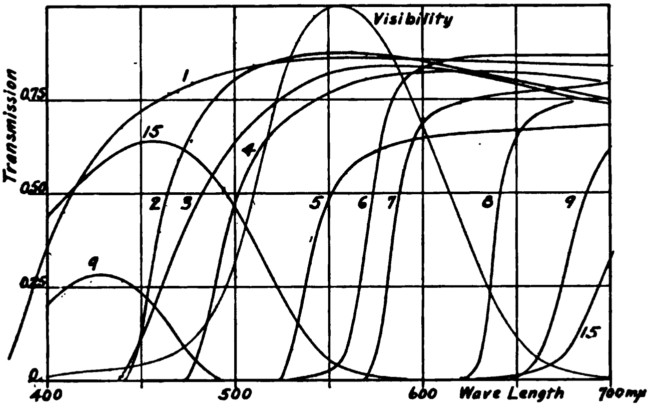

Fig. 1.

Spectral transmission of filters used to detect camouflage or improve visibility.

Other dichromatic filters of very low transmission are shown in Fig. 3. These are valuable for producing chromatic contrast between greens which match in daylight but have different reflecting powers for red.

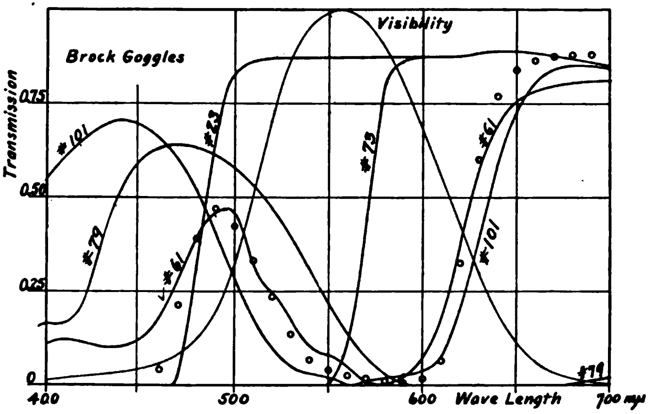

Fig. 2.

Spectral transmission of filters used to detect camouflage or improve visibility.

Considerable time and effort was spent at the bureau during 1918 in making filters to detect camouflage, both in reproducing filters by request when the composition was unknown and only the spectral transmission determined, and in producing new ones. In Figs. 2 and 3 are shown, by circles, optical reproductions of Brock No. 61 and another filter of unknown composition (curve *14*). The other filters shown in Fig. 3 were also made at the bureau as well as many others not shown. Capt. W. E. Mathewson, U. S. A. Ordnance, who was stationed at the bureau, and Irwin G. Priest, coöperated in the preparation of these dyed filters. The compositions of these various filters are as follows:

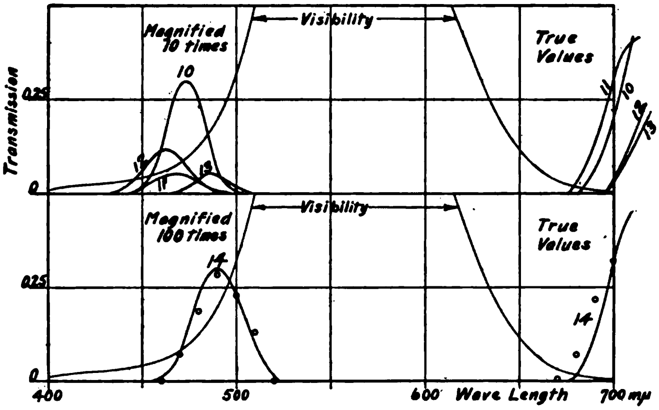

Fig. 3.

Spectral transmission of filters used to detect camouflage or improve visibility.

Filter	Composition
Filter made to duplicate Brock No. 61.	Separate gelatine solutions of iris violet and picric acid flowed on glass and cemented together with Canada balsam.
Made to order to duplicate the transmission of filter submitted (Curve 14, Fig. 3).	Three separate films of gelatine on celluloid dyed respectively with iris violet, tartrazine, malachite green.
Curve 10. Made to show chromatic contrast in viewing greens.	
Curve 11. Made to show chromatic contrast in viewing greens.	Film dyed with guinea green B + film dyed with naphthalene red.
Curve. 12.	Glass, cobalt blue D and Noviol C.
Curve 13.	Glass, cobalt blue C and 2 samples Noviol C.

Bureau of Standards,
Washington, D. C.,
April 21, 1919.

A METHOD FOR THE COLOR GRADING OF RED FLARES.[1]

BY IRWIN G. PRIEST.

THE development of a method for the color grading of red flares was requested by the Colored Lights Section, American University Experiment Station. The flares submitted were classed as follows: (1) carbonate flares, burning with a yellowish red flame; (2) nitrate flares, burning with a comparatively purer red flame; (3) a standard railway red fusee. It is understood that the red flame is due to strontium.

Fundamentally considered, the ideal method of expressing the color of these flares would be by means of an energy-wave-length function showing the relative energy at each point in the visible spectrum. On account of the extremely erratic burning and consequent enormous instantaneous variations of intensity, such determinations are quite impracticable. For the same reason a monochromatic or trichromatic analysis would be difficult.

Determinations by means of the Arons Rotatory Dispersion Quartz Plate Chromoscope[2] have been found feasible and apparently satisfactory. As actually used the arrangement of apparatus is as follows:

1. *The Comparison Light Source* is a vacuum tungsten lamp. This illuminates a magnesium carbonate block which reflects the light to the eye through the train of nicol prisms, quartz plates, and the Lummer-Brodhun cube constituting one arm of the chromoscope. This lamp is operated at a constant voltage such that the light in the photometric field is color-matched with a standard acetylene flame[3] when the quartz plates are removed from the path.

2. *The Train of Nicols and Quartz Plates* in order from the source to the eye is: Nicol No. 1, nicol no. 2, quartz plate no. 1, nicol no. 3, quartz plate no. 2, nicol no. 4. Each quartz plate is 2 mm. thick. Rotation of nicol no. 1 with respect to no. 2 serves to vary brightness. Rotations of nicols nos. 2 and 4 with respect to no. 3 when the quartz plates are in place serve to vary spectral distribution. These rotations are measured from zero for the extinction position with quartz removed.

3. *The Light from the Flare* is reflected by another magnesium carbonate block to the eye through a pair of nicols and the Lummer-Brodhun cube. The photometric field is thus illuminated, in one part by the light from the flare and in the other part by light the spectral distribution of which can be varied by rotation of the nicols nos. 2 and 4.

For the color variation in the flares submitted, it was found that nicol no. 2 could be kept set constantly at 80° while the color variations could be followed by rotation of nicol no. 4. By using both hands the observer can control both total intensity (nicol no. 1) and spectral distribution (nicol no. 4).

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Ann. der Phys., 39, 545, 1912.

[3] Coblentz and Emerson, B. S. Sci. Paper 279, 1916.

For the flares submitted the following rotations of nicol no. 4 gave color-matches:

Carbonate flares	70°
Nitrate flares	54°
Railway fusees	57°

From the above data, the relative spectral energy distribution of acetylene[1] and the rotatory dispersion of quartz, *the relative spectral energy distribution of the light found to color-match the light from the flare can be computed.*[2] These computations have been made and the results are shown in the accompanying figure. To facilitate comparison, the ordinates are all arbitrarily made 100 for wave-length 720 millimicrons. These computations would ordinarily be very tedious, but are greatly facilitated by extensive tables which have previously been prepared and are on file at the bureau.

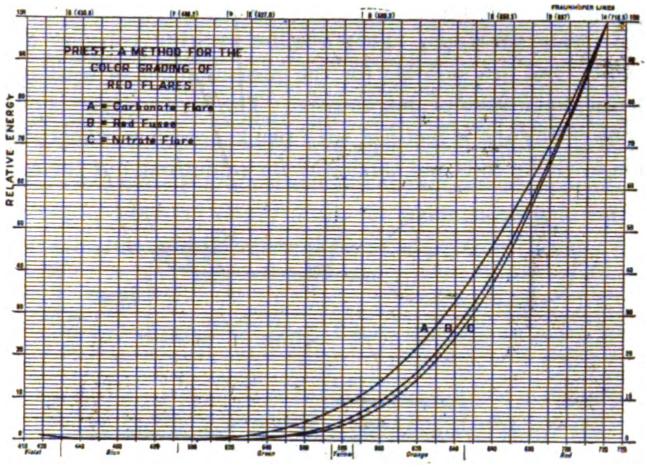

Fig. 1.

The method thus determines not the actual spectral energy distribution of the flare but *a spectral energy distribution which color-matches it.* This apparently meets all present actual needs for color-grading these flares.

The apparatus could also be adapted to determine the candlepower of the flares.[3]

The report which is abstracted above was made to the American University Experiment Station, July 6, 1918.

National Bureau of Standards,
April 22, 1919.

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[1] Coblentz and Emerson, B. S. Bull. 13, 363.

[2] See also Priest, Phys. Rev., (2), 10, 208, August, 1917.

[3] Priest, Phys. Rev. (2), 6, 64; 9, 341.

Applications of the Cathode-Ray Tube in Radio Work.[1]

By L. E. Whittemore and L. M. Hull.

THE cathode-ray oscillograph furnishes the only means of mapping wave forms and of determining directly the phase relations of currents and voltages in radio frequency circuits. It is particularly useful in studying generating or receiving circuits in which continuous oscillations are maintained.

Two general types of cathode-ray tube have been used in such investigations at the Bureau of Standards, those having the usual plane cathode of metal and those having a cathode consisting of a heated filament from which the electron emission is spontaneous. Tubes of the former type are constructed in thc usual way, with aluminum electrodes, a screen coated with calcium tungstate, and aluminum plates for electrostatic deflections of the cathode beam sealed inside the tube. They require exciting voltages of from 8,000 to 20,000 volts, and are suitable for the measurement of oscillating currents of the order of magnitude of one ampere. The high voltage is obtained from a 60-cycle power line and is rectified with two kenotron rectifiers with a suitable combination of choke coils and capacities for smoothing out the rectified wave.

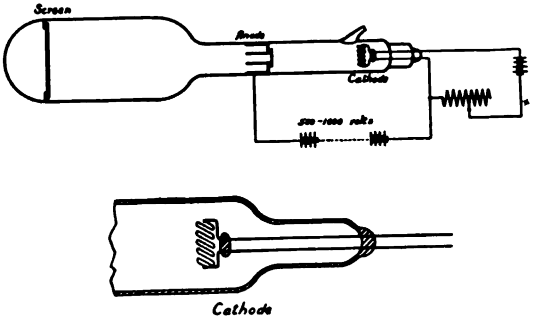

Fig. 1.

A successful hot cathode tube has been constructed using a platinum filament coated with a mixture of calcium and barium oxides. A diagram of this tube and the exciting circuit is shown in Fig. 1. The diaphragm is constructed of metal and is made the anode. For the particular type of diaphragm construction shown in the figure we are indebted to the General Electric Company. A concentrating coil of 500 ampere turns is used with the tube and an extremely fine, sharp spot is produced on the screen. Since relatively low exciting voltages are used, the cathode stream is quite sensitive to electrostatic or electromagnetic deflecting fields, and the tube is suitable for measuring the oscillating electron currents obtained in three-electrode electron tubes, which are of the order of magnitude of 0.05 ampere.

Oscillograms are made with these tubes as a part of the regular tests performed upon radio transmitters submitted to the Bureau of Standards. The quenching action of different spark gaps is determined directly by passing the gap currents through suitable deflecting coils placed around the tube below the anode, a time axis perpendicular to the current deflection thus produced being provided by connecting the electrostatic deflecting plates across the capacity in the closed oscillatory circuit.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

The cathode-ray oscillograph is now being used in an investigation of the harmonics produced in antenna circuits by electron tube generators. Dynamic characteristic curves are obtained for a given generator by impressing the oscillating grid voltage upon the deflecting plates and allowing the electron currents to flow through the deflecting coils. The wave forms of the antenna currents are determined by providing a sinusoidal time axis from a separate generator tuned to a lower frequency of which the frequency of the current oscillation is a suitable multiple.

The combination of an electron tube generator, which is capable of furnishing currents at any frequency up to that corresponding to a wave-length of a few centimeters, with a cathode-ray tube having a slow-moving electron stream, whose deflections are closely proportional to the instantaneous values of these oscillating currents, offers an opportunity for much profitable research, with possibilities of valuable contributions to modern electron theory.

Comments on Spectral Radiation Formulæ.[1]

By W. W. Coblentz.

At the New York meeting of this society (March 1, 1919) Mr. I. G. Priest presented an empirical radiation formula,[2] and data purporting to show that this formula fits the experimental observations at least as well as, if not better than, does the Planck spectral energy equation from which it was obtained.

There are two sets of experimental data available for determining the merits of a proposed spectral radiation formula; (1) the data published by the writer,[3] which pertain to the spectrum extending from 0.9 μ to 1.5 μ and from 3.6 μ to 5.5 μ, and especially (2) the data published by Rubens and Kurlbaum[4] which relate to the spectrum at 24.0 μ, 31.6 μ and 51.2 μ. In fact, any formula which is not in agreement with the data at these long wave-lengths is usually given but little consideration.

Mr. Priest makes the mistake of computing his data in terms of the maximum emission, which point, for the temperatures attainable, involves the greatest errors owing to spectral impurity and the greatest atmospheric absorption.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

[2] Priest, Phys. Rev., 13, p. 314, 1919; Jour. Opt. Soc. Amer., 2. p. 18, 1918.

[3] Coblentz, Bulletin Bureau of Standards, 13, p. 459, 1916; see Figs. 1 and 2.

[4] Rubens and Kurlbaum, Sitzber Akad. Wiss. Berlin, XLI., p. 929; 1900.

And he makes the further mistake of using the single, highest point, $E_m = 463$ of Fig. 2 (*loc. cit.*) instead of the more probable value, $E_m = 450$, which is in common with the two series of measurements. Now it so happens that the value $E_m = 463$ is high because at this point the temperature of the radiator was slightly above the average for the series. The rest of the observations are in close agreement. They do not differ systematically throughout the two curves which were obtained at an average temperature difference of only 0.°5 C. (4 microvolts by the thermocouple). The published difference in temperature, of 1°, is the result of a supposed change in the thermocouple calibration. As a result, the "experimental data," given in col. VI. of his table, are erroneous. For example, at $\lambda T = 1900$, $E_\lambda = 0.605$ instead of 0.581.

Instead of the excellent agreement claimed by Priest, using the most probable radiation constants $C_2 = 14350$ and $A = 2890$ $(= \lambda_m T)$ the computed curve falls below the observed curve by 3 to 4 per cent. at 3.6 μ by 8 per cent. at 4.5 μ and 10 to 11 per cent. at 5 to 6.2 μ. (See his tables for $\lambda T = 6{,}000$ to 9,900, column (II.)–(III.).) Upon my insistence that the formula must fit the data closer than 5 to 10 per cent., he adopted the value of $A = 2940$, which is the old Lummer Pringshein value, long since abandoned as erroneous because of an erroneous temperature scale, and wave-length calibration curve.

Using the arbitrary constant, $A = 2940$, brings a closer agreement, say 1 to 2 per cent. at 4 to 5 μ, but then there are greater deviations at 0.9 to 1.5 μ. This is merely a compromise. In these two spectral regions (0.9 to 1.5 μ and 3.6 to 5.5 μ) which comprise the most reliable data obtainable at present, computations made by using the Planck equation, fit the smooth curve drawn through the observations to ± 0.3 per cent. to ± 0.7 per cent. Readers in doubt about the matter can see a least square reduction of these data by consulting a paper by Mr. H. M. Roeser[1]; and can make their own computations using the writer's experimental data, there published.[2]

Rubens and Kurlbaum (*loc. cit.*), using radiations of wave-lengths 24.0 μ, 31.6 μ (residual rays from fluorite) and 51.2 μ (rock salt) and a temperature range of − 188° C. to + 1500° C. found that computations based upon Planck's equation fitted the observed curves so closely that they resorted to tabulation in order to show the small deviations. For the impure residual rays from fluorite, the maximum deviation of the computed from the observed data was 3 to 5 per cent., for the whole temperature range; while in the temperature range of 750° to 1500° C. the deviation was from 0 to 1.5 per cent. For the residual rays, obtained by reflection from rock salt ($\lambda = 51.2\,\mu$) the maximum deviation of the computations from the observations, in the temperature range from − 80° C. to + 1500° C. is 1.7 per cent. and the predominating deviation is ± 0.9 per cent.—a truly remarkable agreement when one considers the difficulties involved in the experiments.

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[1] Roeser, Bul. Bur. Stds., 14, p. 237, 1917; see Fig. 1, p. 241. These same data are plotted in Fig. 1; B. S. Bul., 13, p. 474.

[2] (*Added by author subsequent to meeting.*) In a paper presented at this meeting Mr. Priest has modified his formula in order to fit certain experimental requirements in the visible spectrum. Although the writer's "experimental data" have been adjusted to fit the formula, there is a difference of 5 to 8 per cent. between the computed and observed data, at 4.6 μ to 5 μ ($\lambda T = 7{,}408$ to 7,983) and a still greater discrepancy at 50 μ. The futility of attempting to make things fit in one part of the spectrum at the expense of another part should be apparent.

On the other hand, Priest's tabulated data show that, for $\lambda = 51.2\,\mu$ and for a temperature range of − 80° C. to + 1680° C. ($\lambda T = 10{,}000$ to $100{,}000$), using $A = 2{,}940$, the difference between the data computed by his empirical formula and those computed by Planck's formula (which is in agreement with the experimental data) is − 10 per cent. at 118° C., − 3 p.c. at 507° C., + 7 p.c. at 900° C., + 33 p.c. at 1290° C., and + 53 p.c. at 1680° C. The agreement is somewhat closer when using $A = 2{,}890$. This empirical formula offers no important advantages in simplifying experimental work, and in computing experimental data. The advantages claimed for the use of frequencies instead of wave-lengths are open to question.

The radiation constants are now known to such an accuracy that no new investigations should be undertaken unless the laboratories concerned are prepared to carry on the work for years, in order to attain a still higher accuracy. It will, therefore, be left primarily to the national laboratories. Some of these laboratories have their plans already formulated and since much preliminary computation has already been completed, using the Planck equation, which is known to fit the observational data for a spectral range of 0.6 to 50 μ, with an accuracy of 1 to 3 per cent., which is the accuracy of the experimental data, experimenters will be loath to turn aside to test an empirical formula which, in the same parts of the spectrum, shows a disagreement of 10 to 50 per cent. with the observations.

The Planck equation is based on theoretical foundations, and after almost two decades of discussion it remains unchanged. It has been indicated that numerous other formulæ can be set up which will represent the experimental data. But it seems futile to set up a formula, without a theoretical foundation, especially when the accuracy attainable is as limited as in using the empirical formula just discussed.

Bureau of Standards,
Washington, D. C.,
March 4, 1919.

The Stability of a Rotating Parachute or Falling Body Rotating About a Vertical Axis of Symmetry.[1]

By H. Bateman.

It is found that when the falling body or parachute is rotating about a vertical axis the stability may be studied most conveniently by using a simple transformation of the equations of motion in which the coordinates used are complex quantities. The stability is thus seen to depend on the nature of the roots of a cubic equation with complex coefficients.

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[1] Abstract of a paper presented at the Pasadena meeting of the American Physical Society, June 19, 1919.

In the case of no rotation a study is made of the effect of varying some of the resistance coefficients while the others are kept constant. The effect of rotation on stability is discussed and particular attention paid to conditions in which the effect of rotation is entirely favorable.

THE STARK EFFECT FOR METALS IN THE ULTRA VIOLET

BY J. A. ANDERSON.

USING a large quartz spectrograph the spectra of Ag, Al, An, Cd, Co, Cu, Fe, Mg, Mo, and Ni have been photographed.

Some work has been done using a small 26,000-volt transformer with synchronous rectifier instead of the direct-current generators.

Direct photographic comparison of the spectrum given by the tube with that of the arc and spark has been made for iron with results so interesting that similar comparisons will be made for the other elements studied.

ABSORPTION EFFECTS WITH THE ELECTRIC FURNACE AS RELATED TO TEMPERATURE CLASSIFICATION.

BY ARTHUR S. KING.

A SERIES of experiments have been made in which a graphite plug was placed in the tube of a resistance furnace, producing absorption spectra of metallic vapors which the tube contained. By this means a study has been made of the absorptive power of spectrum lines at various temperatures, the dependence of the phenomenon on wave-length, and a comparison with the emission spectrum of the furnace at corresponding temperatures. A close connection appears between the absorptive power of a line and the temperature class in which it had been placed. A comparison with solar and stellar lines is in some respects simplified.

THE ELECTRIC FURNACE SPECTRA OF METALS IN THE INFRA-RED.

BY ARTHUR S. KING.

THE electric furnace spectra of Fe, Ni, Cr, Ti, Ba, Sr and Ca have been photographed by the use of plates dyed with dicyanin, an extension into the infra-red beyond λ 9000 being obtained. A considerable number of lines for each element have appeared at different furnace temperatures. Lines of special interest are those which maintain their intensity at the lower temperatures and those which are relatively stronger in the furnace than in the arc. For Ca, Ba and Sr, the temperature grouping of lines has proved useful in selecting lines which fit into series having first members in this region.

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[1] Abstract of a paper presented at the Pasadena meeting of the American Physical Society, June 19, 1919.

RECENT OBSERVATIONS OF TUBE-ARC SPECTRA, ESPECIALLY IN THE INFRA-RED.[1]

BY ARTHUR S. KING AND PAUL W. MERRILL.

THE presence of spark lines in the tube-arc spectrum, as previously observed by one of us has been found to extend through a long range of spectrum including the ultra-violet and the infra-red. The carbon spark lines are strong and much sharper than in the spark. Hence their wave-lengths can probably be more accurately determined from the tube-arc than from any other known source. Several new carbon spark lines have been identified. An interesting feature in the infra-red is the presence of numerous air lines, including the well-known oxygen triplet at λ 7770.

THE SPECTRA OF KRYPTON AND XENON IN THE INFRA-RED.[1]

BY PAUL W. MERRILL.

THIS paper describes photographic measurements of the spectrum lines of krypton and xenon, principally in the red and infra-red, made at the National Bureau of Standards. There are numerous strong lines in this region in both spectra, most of which had not been previously observed.

Attention is called to a probable analogy between the spectra of the rare gases neon, argon, krypton and xenon which this investigation has brought to light.

A STUDY OF THE RELATIVE INTENSITIES OF SPECTRUM LINES IN DIFFERENT REGIONS OF THE ARC AS COMPARED WITH THEIR BEHAVIOR IN OTHER SOURCES.[1]

BY PAUL W. MERRILL.

THIS paper deals with data concerning the strengthening of spectrum lines of iron, cobalt and nickel at the positive pole of the electric arc as compared with an adjacent region. It was found, in confirmation of results of other observers that "enhanced" or "spark" lines appear at the positive pole of the arc. Among the other lines the amount of strengthening at the pole was found to bear an interesting relation to Dr. King's classification based on the temperatures required to produce the lines in the electric furnace.

THE VAPOR PRESSURES OF HYDROGEN CHLORIDE AND HYDROGEN BROMIDE ABOVE THEIR AQUEOUS SOLUTION.[1]

BY S. J. BATES AND H. D. KIRSCHMAN.

BY employing a method which avoided the errors due to the effect of spray and those due to the uncertainty in the volume of gas passed thru the apparatus, the vapor pressures of hydrogen chloride and of hydrogen bromide above their aqueous solutions was determined to two or three thousands of a millimeter over the concentration range from 4 to 10 mols per 1,000 grams of water. By combining the values for hydrogen chloride with the electromotive force data of Ellis, the free energy of hydrogen chloride is computed to be—22, 690 calories.

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[1] Abstract of a paper presented at the Pasadina meeting of the American Physical Society, June 19, 1919.

PHYSICAL CONSTANTS PERTAINING TO THE OCEAN.[1]

BY GEORGE FRANCIS MCEWEN.

PHYSICAL constants are fundamental data of the science of physics. They are essential for quantitative description of the behavior of substances. Refinement in physical measurements continually yields more accurate values. It also reveals the variable nature of quantities formerly regarded as constant, and shows the need of new constants. Under controlled conditions, approach seems to be had to constants, dependent only upon the nature of substances. Actually, however, all are more or less influenced by external conditions.

Quantitative expression of laws describing those cosmical or terrestrial phenomena involving correspondingly enormous magnitudes, require physical constants whose evaluation under controlled conditions is impossible. Coefficients of viscosity, conductivity, and diffusion applicable to oceanic conditions are of a much higher order of magnitude than values obtained from experiments necessarily limited to much smaller dimensions.

Moreover, ocean conditions found in a given locality or season are not sufficiently typical to be representative of other regions or seasons.

The high variability of these coefficients with respect to conditions in the sea, demands extensive and continuous "field" observations.[2]

Complicated as these facts are, encouraging results have already come from quantitative studies, not only in oceanography, but also in other geophysical investigations.

THE REFLECTIVE POWER OF METALS AND DIELECTRICS IN THE ULTRA-VIOLET.[1]

BY E. P. LEWIS AND A. C. HARDY.

WHILE testing the photoelectric properties of methyl violet deposited as a mirror on a glass plate from an alcoholic solution it was found that 2 cm. of quartz cuts out about 90 per cent. of the effective radiation. Since Schumann found that this thickness cuts out practically all radiation below λ 1860, it seemed possible, by using a test electrode of this material with a sensitive electroscope, to determine the reflecting power of substances somewhat beyond the limit reached by Hulburt, about λ 1850. The source was a mercury arc in quartz, which appears to transmit radiation to about λ 1800 or slightly beyond. In some experiments the light was incident at an angle of 13 degrees, in others at 45 degrees. The methyl violet showed little evidence of fatigue, but the surface was renewed from time to time. In one series of experiments a polished iron electrode was used, in another one of zinc. As indicated by the greater reflecting power of the metals and the smaller reflecting power of other substances, the effective radiation with iron and zinc was of longer wave-length than in the case of the methyl violet. The silicon mirror was the plane polished surface of a large crystal, probably less perfect than the surfaces used by Hulburt. The speculum surface was that of an old mirror, repolished. At the limit observed by Hulburt, about λ 1870, the reflecting power of silicon was found to be about 60, but the curve falls so rapidly in this region that his results are not inconsistent with those given below, provided the effective radiation was of slightly shorter wave-length.

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[1] Abstract of a paper presented at the Pasadena meeting of the Physical Society, June 19, 1919.

[2] For a detailed discussion of physical constants pertaining to the ocean see (McEwen, G. F., 1919, Ocean temperatures, their relation to solar radiation and oceanic circulation: Quantitative comparisons of certain empirical results with those deduced by principles and methods of mathematical physics (in press). Semicentennial Publications of the University of California.

The reflecting power in percentages is given below:

	Methyl, 13°.	Violet, 45°.	Iron, 45°.	Zinc, 45°.
Silicon	25	31	35	44
Platinum	20	21	—	—
Speculum metal	9	10	11	18
Ordinary glass	13	13.5	11	10
Dense glass	10	—	—	—
Calcite	13	14	11	—
Quartz	9	10	9	—
Fluorite	7.5	8	8	—
Rock salt	8	9	9	—

Hulburt showed that the reflecting power of speculum metal is very low in this region, especially for old mirrors, even when repolished. For gratings which can not be repolished the efficiency must be very low, unless the reflectivity increases in the direction of short wave-lengths. The significant feature of the figures given above is that the reflectivity of glass exceeds that of speculum metal, and is still rising. It seems very probable that glass gratings will be found much better than those of speculum metal for the extreme ultra-violet, if difficulties of ruling can be overcome.

Calcite shows a high reflectivity, and, as we might expect, that of fluorite, quartz and rock salt is small. But even for these substances it exceeds that calculated from the index of refraction at λ 1850, indicating that absorption is rapidly increasing and metallic reflection beginning. Platinum, silicon and aluminum, continue to show greater reflectivity than glass, but the curve given by Hulburt falls off so rapidly that it is doubtful whether their supremacy over glass continues for any great distance in the extreme ultra-violet.

A methyl violet mirror showed a reflecting power of 17 per cent., indicating metallic reflection for the rays to which it is photoelectrically sensitive, as we might expect.

UNIVERSITY OF CALIFORNIA,
June, 1919.

DETERMINATION OF THE NORMAL TEMPERATURE BY MEANS OF THE EQUATION OF THE SEASONAL-TEMPERATURE VARIATION AND OF A MODIFIED THERMOGRAPH RECORD.[1]

BY FRANK L. WEST, N. E. EDLEFSEN AND S. P. EWING.

THE air temperature at a place of light rainfall is a periodic function of the time, there being two prominent periods, a twenty-four-hour period and an annual period. That part of the United States between the Rocky and the Sierra Nevada Mountains has a humidity of 50 per cent., a rainfall of from 10 to 20 inches, with more than 300 days of the year without rain.

The equation of the curve that represents the mean daily temperature in terms of the time of year for Utah, is

$$T = 48.5-22.2 \cos (\theta-19^\circ-54')-2.7 \cos 2 (\theta-149^\circ-5') -1.0 \cos 3 (\theta-17^\circ-3'), \qquad (1)$$

the time θ being expressed in degrees. The equation is of rather general application because the first term is the mean annual temperature for the place considered (a function of latitude and elevation) and simply moves the curve up or down the page while the shape or amplitude is determined by the difference in temperature between summer and winter and varies at different places in the interior of the United States from the Salt Lake value by from one to four degrees.

The curve representing the twenty-four-hour temperature change modifies its shape gradually each day, flattening out as winter approaches, but the ratio of the hourly temperature to the mean daily temperature is nearly constant on any day of the year (for Fahrenheit scale only). The equation giving the temperature at any hour expressed as the percentage of the mean is as follows:

$$P = 97.3-25.2 \cos \theta-67^\circ-10') + 3.7 \cos 2 (\theta-38^\circ) -1.5 \cos 3 (\theta-23^\circ-16') \qquad (2)$$

The two equations are empirical and were obtained by the method of the Fourier Series from the U. S. Weather Bureau data.

Variable diathermancy or cloudiness, evaporation or cloud formation, south or north winds are the main factors that cause variations from the normal. In short, cyclonic storms or rain cause the departures.

Applying these equations and then checking on the actual temperature experienced, the differences were from 1 to about 15 degrees with a mean departure of about 7 degrees. The chances in the arid West are one in six that the departure will be less than 2 degrees, two in five that it will be as much as 5 degrees, one in four that it will be as much as 10 degrees and one in seven that it will be 15 degrees. Very rarely is it more than 15 degrees. The probable temperature can be determined by modifying the normal temperature as determined above after consultation with the Weather Bureau as to whether the day in question is to be warmer or colder (as determined from their weather maps) than usual.

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[1] Abstract of a paper presented at the Pasadena meeting of the American Physical Society, June 19, 1919.

By long time averages, normals for climates not as dry as here in the West can be obtained, but rains and variable cloudiness cause rather wide departures from these normals. The above method would give a suggestion of what might be expected even in these places.

Physics Department,
Utah Agricultural College,
Logan, Utah, June 1, 1919.

Formula for the Wave-Lengths of *M*-Radiation.[1]

By Fernando Sanford.

It is known that the Einstein photoelectric equation $Ve = \frac{1}{2}mv^2 = h\nu$ applies to the energy of the inducing electrons and the frequency of the x-radiation in most cases of x-ray emission. The writer has previously shown that for cases where this law applies if the radiating electrons are in orbital motion about a central positive charge, so that $Qe/R^2 = mv^2/R$, where Q is the magnitude of the central positive charge and R the orbital radius, the magnitude of the central charge is given from the equation

$$Q = \frac{2^{1/2}h^{3/2}c^{1/2}}{\pi e m^{1/2}\lambda^{1/2}}$$

Assigning the usual values to e, m and h, this gives

$$Q = \frac{2.882.1^{-12}}{\sqrt{\lambda}}.$$

It has also been shown that the same value of Q may be calculated from the Moseley equation, where for the shortest wave-lengths in the K-radiation band $Q = 2e(N-3.6)$ and for the shortest wave-lengths in the L-radiation band $Q = e(N-15.4)$, where N is the serial number of the element from which the X-rays are derived.

In a paper by W. Stenström[2] is given a table of the wave-lengths of the characteristic M-radiation of a number of elements from N–66 to $N = 92$. If Q be calculated for these lines by the above formula and be introduced into the Moseley equation, it gives $Q = 2.882 \cdot 10^{-12}/\sqrt{\lambda} = 2(N-19.47) \cdot 10^{-10}$ for the α-line of the M-radiation band. The wave-lengths of the α-lines of the M-radiation band may then be calculated from the equation

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[1] Abstract of a paper presented at the Pasadena meeting of the American Physical Society, June 19, 1919.

[2] Ann. d. Phys., 57, 372.

$$\lambda = \left(\frac{2.882\cdot 10^{-12}}{2(N-19.47)\cdot 10^{-10}}\right)^2.$$

The following table gives the values of λ as observed by Stenström and the values of Q and λ as calculated from the above formulæ.

TABLE I.

Element.	N	λ, Obs.	$Q\cdot 10^{10}$	λ, Calc.
U	92	$3.919\cdot 10^{-8}$	145.5	$3.46\cdot 10^{-8}$
Th	90	4.131	142	4.16
Bi	83	5.125	127	5.13
Pb	82	5.302	125.5	5.29
Tl	81	5.471	123.2	5.47
Au	79	5.847	119	5.85
Pt	78	6.058	117	6.05
Ir	77	6.276	115	6.27
Os	76	6.508	113	6.50
W	74	7.007	108.7	6.97
Ta	73	7.272	106.6	7.27
Cp	71	7.856	102.9	7.84
Ad	70	8.162	101	1.15
Er	68	8.813	97	8.82
Ho	67	9.168	95	9.18
Dy	66	9.556	93.2	9.55

It will be seen that with the exception of the uranium line the two values of λ differ by less than the probable error of measurement. Stenström finds a line in uranium with a wave-length of 3.487, which is in better agreement with our calculated value for the α-line.

STANFORD UNIVERSITY,
June 9, 1919.

THE MOVEMENT OF MOISTURE IN SOIL BY CAPILLARITY.[1]

BY WILLARD GARDNER.

FOR the motion of ground water through soils under hydrostatic pressure, it has been assumed, with considerable experimental support, that the velocity is proportional to the pressure gradient, Slichter[2] has observed that the pressure satisfies Laplace's equation and he has made some extended theoretical mathematical calculations for the flow of underground water.

The essential distinction between the case considered by Slichter and the case of capillary flow is that the density function is dependent upon the space coordinates and also upon the time; also that the pressure is a function of the density. For the pressure gradient, a tentative function of the density and density gradient has been substituted, and the velocity is expressed in terms of the density and density gradient. A substitution of this value in the equation of continuity for the case of one-dimensional flow gives a differential equation involving moisture content, one-space coördinate, and the time. This equation is solved for the case of steady motion and experimental results are given for this case. It is pointed out that the equation for steady motion is parabolic in character, the value of the exponent depending upon the validity of the assumptions made. The experimental results indicate that the assumptions are approximately correct.

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[1] Abstract of a paper presented at the Pasadena meeting of the American Physical Society, June 19, 1919.

[2] Slichter, Charles S., Theoretical Investigation of the Motion of Ground Water. 19th Annual Report of the U. S. Geological Report of the U. S. Geological Survey, 1898, Part II., p. 330.

Utah Agricultural College,
Logan, Utah.

BOOK REVIEWS.

The Principles Underlying Radio Communication. Radio Pamphlet No. 40. Signal Corps, U. S. Army. Washington, Government Printing Office, 1919. Pp. 355. Price 55 cents.

Just thirty-one years intervened between Feddersen's observation of the oscillatory discharge in 1857 and Hertz's production of electric waves. Another period of thirty-one years has elapsed between the experiments at Karlsruhe and the appearance of this volume. But on comparison of these two equal periods one is impressed by the small advance in alternate-current electricity made during the earlier period, while in the later period the development has gone by leaps and bounds, until now we have a volume of more than three hundred pages devoted to the *principles* alone of a single branch of alternate-current science, not to mention still larger volumes such as Fleming's, devoted to the whole subject, or still other volumes devoted to particular chapters such as Bucher's "Vacuum Tubes." The explanation of this marked difference appears to lie in the fact that the first thirty years were needed to establish units and standards and to enlarge certain electrical concepts. It was during this earlier period that the portable voltmeter, ammeter and watt-meter were perfected, and that the idea of *resistance* was generalized into that of *impedance* for the electric circuit and of *reluctance* for the magnetic circuit; in brief the earlier period was spent in preparing the ground for the rapid growth which followed the introduction of high-frequency currents in 1888, and for the results which are described in this volume.

The authorship is composite, the manuscript having been prepared by Dr. J. H. Dellinger and six collaborators. The reader will however hardly suspect this fact from the uniformly clear and easy English style which characterizes every page; in fact, one would rather incline to the opposite view, namely, that the composition is essentially that of one man.

This collection of principles is intended for readers whose mathematical equipment is not large. The authors have undertaken the task of maintaining a clear and correct exposition while denying themselves the usual channels of communication, namely, differential equations; and these aims they have accomplished admirably. They have set forth the principles of high-frequency currents and electromagnetic radiation in words of one syllable, so to speak; they have given the physics and have avoided the more complicated mathematical expressions. Nevertheless, they have maintained a quantitative point of view, have gone on to the advanced parts of the science, and have left very little for the student to unlearn. Taken in conjunction with Circular

No. 74 of the Bureau of Standards ("Radio instruments and Measurements") to which constant reference is made one has in this little book a fairly complete treatise adapted alike to the needs of the advanced student of physics and of the radio-engineer. Any treatment along these lines suffers, of course, from the defects of its qualities which here means that the book contains, along with its advanced discussions, a considerable quantity of elementary material.

Among its excellent features must be noted the constant use of analogies; the lucid presentation of resonance (pp. 185–193), of "skin effect" (pp. 208–211), of the performance of the Mg and Zn sparks (p. 257), of Ohm's law in its generalized form; the triple point of view for the action of receiving antennæ (p. 233); and the gradual manner in which the various radio circuits are approached. The number of the various circuits described is large and includes designs adapted to almost every need and circumstance of the telegrapher.

When the next printing is called for it is to be hoped that the authors will realize that their admirable table of contents is hardly a substitute for a good index. Nearly all the figures are apt and satisfactory, but in many of them the lettering is bad; some of them, *e. g.*, the important Fig. 158, illustrating Hertzian electric fields, should be entirely replaced by figures of larger scale and greater legibility. The treatment of vacuum tubes is somewhat scant, likewise that of wireless telephony; however, the aim of the book must be kept in mind—the presentation of principles and not the description of apparatus. Bearing in mind this limitation, one wonders whether, anywhere in the book, due prominence has been given to the two fundamental principles of Oersted and Faraday. On p. 115 they are numbered "3" and "1" respectively and are used in the exposition of dynamo-electric machinery. Since however these two discoveries form the basis of all electromagnetic theory and since they are quantitatively expressed by Maxwell's two vector equations, one feels that they ought to be bracketed together, not only on the printed page but also in the mind of the student. The laws of Lenz, Joule and Ohm are important, but they play a subordinate rôle in electromagnetics. At the beginning of the fourth chapter three important methods of transferring energy are mentioned; but it may be fairly questioned whether more than two are known, namely, (1) *by means of an elastic push or pull, including wave-motion, and* (2) *by means of projectiles—the actual transfer of matter*. Considered from the energy view-point the essential feature of wave-motion is not its kinematics, but rather the elastic properties of the medium which enable that medium to store up and hand on energy, much as the twist of a mill shaft is handed on from the driving pulley to the driven pulley.

The substitution of the word "directly" for "inversely" on p. 42 in the statement that "the force [between two charged pith balls] is also proportional directly to the value of the dielectric constant" is, of course, a mere inadvertence.

This volume which should find a place on the shelf of every student of physics forcibly recalls the remark of the late Lord Rayleigh in his presidential address before the Montreal Meeting of the British Association, in 1884, that "the introduction of powerful alternate current machines by Siemens, Gordon, Ferranti and others is likely also to have a salutary effect in educating those so-called practical electricians whose ideas do not easily rise above ohms and volts." To realize how far these practical electricians *have* risen above ohms and volts, one has only to turn the pages of this volume and find there discussed such topics as effective resistance, radiation resistance, reactance, skin effect, radio and audio frequencies, tuning, coupling, quenching, etc., most of which have made their appearance since the Montreal meeting.

The wise policy of the Bureau of Standards and the Signal Corps which places so helpful a volume as this in the hands of American students is greatly to be commended. H. C.

Hindu Achievements in Exact Science. By BENOY KUMAR SARKAR. New York: Longmans, Green and Co. Pp. xiii + 82. Price, $1.00.

This little, well-printed primer reminds one of a beginner's book in chess. Its object is evidently to stimulate interest in the subject of which it treats. The preface states "the main object of this little book is to furnish some of the chronological links and logical affinities between the scientific investigations of the Hindus and those of the Greeks, Chinese and Saracens. . . . It has been sought to present a comprehensive, though very brief account of the entire scientific work of ancient and mediæval India in the perspective of developments in other lands." The book appears to be largely made up of transcriptions from some of the more comprehensive treatises which are referred to in a bibliography containing 79 titles. The chapter on physics, one of the longest, covers seven pages and consists mainly of a series of statements arranged in syllabus form. Metallurgy and the chemical arts are covered in three pages which give but a very slight idea of metallurgy in India, no adequate reference being made to the wonderful metallurgical development of the Hindus antedating the Christian ear by several centuries. The work of Euro-European writers, such as Hadfield and others, is ignored and no reference is made to the splendid contributions by Panchanan Neogi to the history of iron and copper metallurgy in India.

G. K. BURGESS

Second Series *October, 1919* *Vol. XIV., No. 4*

THE

PHYSICAL REVIEW.

STUDIES OF THE ADSORPTION OF GASES BY CHARCOAL. I.

BY HARVEY BRACE LEMON.

VARIATIONS DUE TO HEAT TREATMENT, PRE-EQUILIBRIUM EFFECTS.[1]

SYNOPSIS.

Problem.—By varying the heat treatment of a given specimen of cocoanut shell charcoal both the rate at which it adsorbs air and the total amount it can adsorb from a given small mass of air can be varied over a wide range. These variations are studied for the case in which the mass of air used is less than that required to saturate the charcoal.

Activation.—An increase in the adsorptive power, "activation," is produced by repeated evacuations at 650° C., each evacuation being followed by an adsorption of air at the temperature of liquid air.

De-activation.—A decrease in the adsorptive power can be produced by increasing the evacuation temperature to between 800° and 900° C.

Hypotheses.—Hypotheses advanced to explain these phenomena are discussed.

EARLY in 1915, while making up some demonstration apparatus for the purpose of showing the presence of the noble gases in the atmosphere using the method of Gehlhoff,[2] large differences were observed in the efficiency of the carbon employed on different occasions. Reference to the literature showed a very great bibliography on the subject of carbon with especial reference to its use for adsorption of gases but no light was shed on the particular question at issue which was why different samples of charcoal made from the same material could show variations in the ratio of 1 : 10,000 in the amount of gas that they would adsorb in a given time. Experiments were begun therefore which resulted in two reports on the subject, the latter of which was presented at the

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[1] This article is published with the approval of Major General William L. Sibert, Director Chemical Warfare Service, U. S. A.

[2] Phys. Zeit., 14, 838, 1913.

Chicago meeting of the American Physical Society, December 2, 1916, and published in the Proceedings in April.[1] In May, 1917, at the request of Mr. Bradley Dewey, then assistant in charge of the Investigation of Gases in Warfare, further publicity of this work was indefinitely postponed. Experiments have been continued on the subject ever since as time has permitted. The departure of every one of the senior members of the staff of this laboratory into the service of the Navy or the Army left the writer with but little time for research and later a call for his services in the Ordnance Department took him away from it altogether. Many others have in the meantime become interested in this subject and a great amount of valuable information has been obtained, especially by some whose previous experience made them especially fitted for work of this sort.

The present paper deals with the early experiments on the subject most of them prior to January, 1918.

The method of experimentation adopted was one which made possible the determination of the time rate of adsorption under the conditions of constant volume. From a practical point of view this seemed desirable. Experimentation with this method falls into two broad categories. (*a*) By using a relatively large amount of gas, conditions of saturation can easily be obtained, and the time necessary to produce them observed. (*b*) By using a relatively smaller amount of gas so that saturation may never be reached very striking differences in the rate of adsorption of different samples or of the same sample under different conditions become apparent, and it is hoped that this may throw further light on the nature of the process. This latter method of use is herein described under the title of "pre-equilibrium effects."

The apparatus is simple and the method of experimentation may be described with reference to Fig. 1. A McLeod Gauge, *M*, having three different ratios (18,920–806.1–107.5) connected to a mercury manometer, *H*, formed a convenient method of observing pressures over the range covered, *i.e.*, 0.00001 cm. to 100 cm. The charcoal was contained in tubes, *C*, either of iron, quartz or Pyrex glass, depending on the temperature to which it was desired to heat it. Small Geissler tubes were attached to the charcoal bulbs for the convenience of identifying the gas content and affording a quick qualitative indication of the pressure. A volume, *B*, closed from the tubes by carefully selected stopcocks, served as a reservoir to hold the gas whose rate and total adsorption was measured by the gauges *M* and *H*. The system was evacuated either by Gaede pump (drum rotating in mercury) or by tandem mercury aspirator and diffusion pump, backed by water aspirator.[1] The three charcoal tubes could be heated and evacuated, "outgassed," simultaneously by an electric furnace. Temperatures were measured by a Leeds & Northrup potential point resistance thermometer with bridge calibrated in degrees C.

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[1] PHYS. REV., 9, 336, 1917.

The experimental method was to outgas the charcoal for measured time and measured temperature after which residual pressure was observed both at the outgassing temperature and at 500° C., at 400° C. and after cooling to room temperature. Connections to the pumps were shut off as soon as the charcoal began to cool and the outgassing considered complete if the pressure was unreadable on the highest ratio of the gauge after cooling to room temperature. The bulb *B* and the gauge space was then filled with dry air at a pressure of about 90 cm. (This varied somewhat with the room temperature in order that the mass of gas should be constant = 1.2 gms.) The mass of charcoal used in each tube was 25.7 gms. This weight was taken with the material saturated with dry air at 29° C. and 75 cm. pressure.

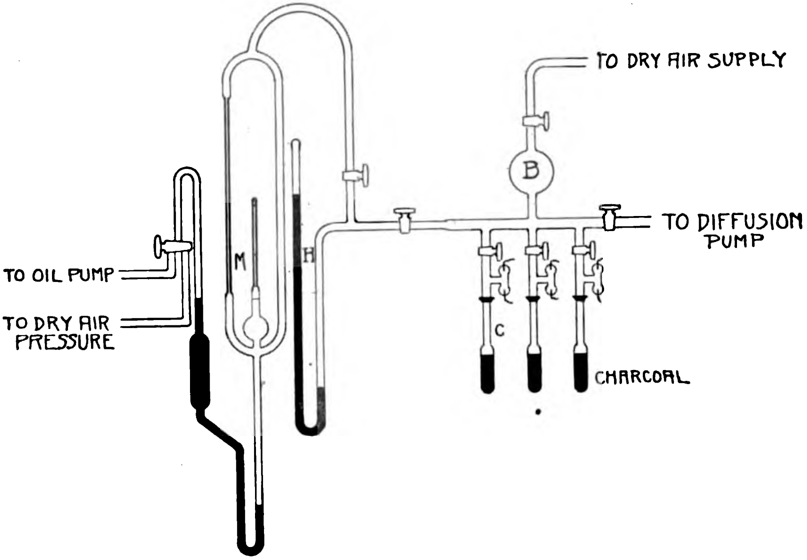

Fig. 1.

After being outgassed one of the three charcoal tubes was immersed in liquid air for about half an hour to assure temperature equilibrium, then the stop cock connecting to *B* was opened and the pressures read at intervals while the contents of B was being adsorbed. Corrections were made for the partial pressure of the residual neon when this began to introduce an error, *i.e.*, at pressures below 0.001 cm. It was assumed that the neon was not adsorbed and a constant amount subtracted. This assumption is obviously not true since pressures frequently are observed which are very much less than the neon partial pressure so that the correction made in this manner leaves less than nothing for the pressure. The amount of neon adsorption has not yet been measured to the writer's knowledge and pressures lower than 0.0005 are not regarded as at all accurate and not plotted. The interesting phenomena can be produced at very much larger pressures than this.

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[1] The mercury pumps were obtained through the courtesy of Mr. C. E. Skinner, Westinghouse Electric and Manufacturing Co., East Pittsburgh. They have been described in the literature by Shrader, Phys. Rev., 12, 70, 1918.

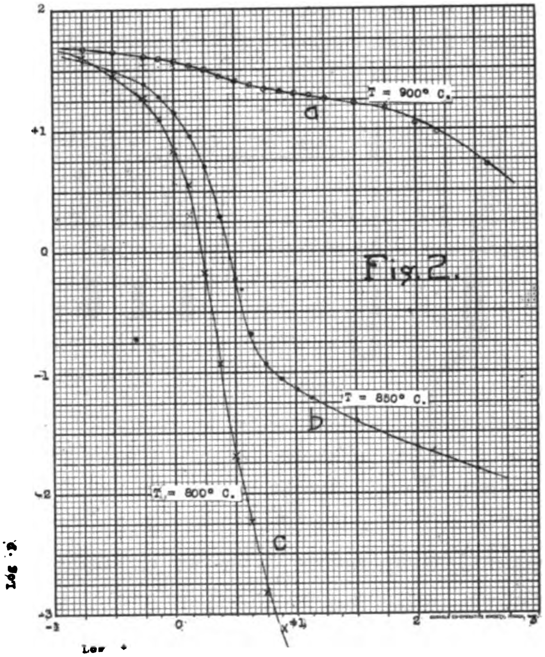

Fig. 2.

For the sake of condensation the results are here given graphically. The logarithm of the pressure in centimeters is plotted against the logarithm of the time, measured from the moment of opening the cock, in minutes. This results always in the same characteristic family of curves of which Fig. 2 is a typical example.

The relative amount of gas is not sufficient in any case to produce saturation of the charcoal and the pressure falls throughout the course of the observations, *i.e.*, for over five hours in the slower samples.

Curves *a*, *b*, *c* are three different specimens of charcoal differing only in their carbonization temperatures which were 900°, 850° and 800° respectively. The outgassing took place at 425° and was continued about six hours. The enormous difference in effectiveness of these three samples is striking. The pressure initially being about 90 cm. is reduced in 10 minutes to 20 cm. in the case of *a*, to 0.071 cm. in the case of *b*, and to about 0.003 cm. in the case of *c*. These curves are by no means unique ones. Each type has been reproduced repeatedly.

Successive runs on the same sample are shown in Fig. 3 for sample 12.

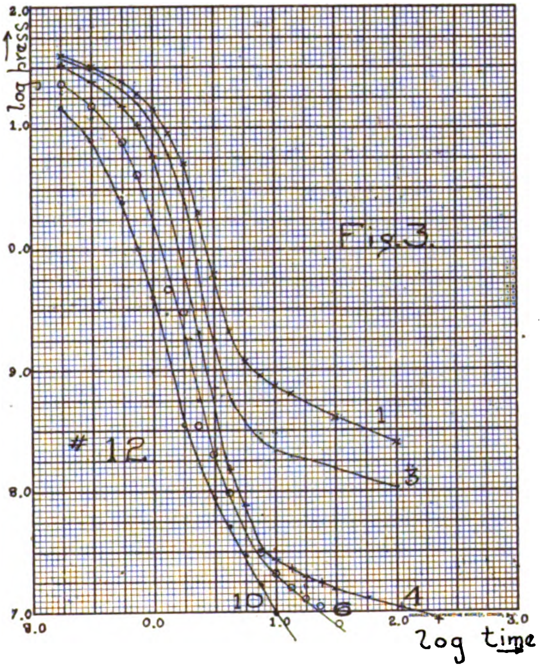

Fig. 3.

This material was carbonized at 850° for 1 hour. Outgassings were at 600° and were never less than 4½ hours which was later adopted as entirely sufficient for complete evacuation. This means only such evacuation as will leave no readable pressure on the highest ratio of the gauge after cooling to room temperature. Residual pressures after such outgassing were at 600° C., 8 x 10^{-3} cm.; at 400° C., 8 x 10^{-6} cm. Systematic improvement, *i.e.*, increase of rate, in the sample after ten consecutive runs occurs and after this much use the sample is practically as good as that one shown in Fig. 2, curve *c*, which was carbonized at a lower temperature. Fig. 4, sample 14, shows a still more extreme case of this variation in a single sample with use. Carbonization was here made at 875° for 1½ hrs. outgassings, at about 600° for 4½ hrs., as in Fig. 3. The entire range of behavior from that of a very poor sample to that of a very good sample is covered in 12 consecutive runs. Curve 5 is irregular and due to a leaking stop-cock as is also the latter part of curve *10*.

It was observed if at any time any sample was heated to 1200° its adsorptive properties were completely ruined. This is illustrated in Fig. 4, *A*, which is a continuation of work on sample 14 of Fig. 4. The volume of gas admitted was here increased from 873 c.c. used in all previous curves to 1,707 c.c. and the first curve *13* shows saturation obtained in about 34 minutes. The sample was now put in a transparent quartz tube. Several outgassings were made at 600° and slight improvement resulted, the saturation value falling considerably, the seventeenth run being the lowest. Between the nineteenth and twentieth runs the temperature was raised to 1200° for 20 minutes. The twentieth run is at the top of the sheet showing the almost complete loss of adsorptive power. On reducing the volume of gas admitted to 351 c.c. and taking several more runs the curve remained stationary in this location.

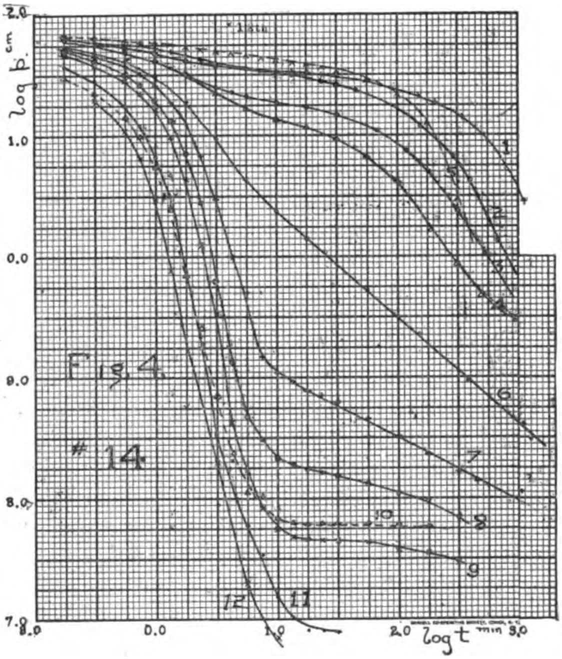

Fig. 4.

The transparent quartz tube was attacked by the carbon at this high temperature and completely altered in its physical characteristics.

It was almost completely devitrified and became of about the appearance of egg shell opaque and brownish in color with little mechanical strength. It was left with sufficient strength however to withstand pressure and was not observed to leak. The transformation is rather peculiar in not becoming visible until the tube cools. On being taken out of the furnace it is a bright cherry red and perfectly transparent, the charcoal in the interior being clearly visible. As it cools in the air it grows opaque. These observations have been repeated on three different tubes with identical results. The change is ascribed to a reaction between the carbon and the quartz although this reaction is commonly supposed to occur only at temperatures considerably higher. The temperature is much too low to allow of the formation of carborundum and it is thought that this compound may be identical with a substance known as silfrax which has been described as having similar physical properties to those here observed and which is found in the cooler portions of the carborundum furnaces. Because of the failure of quartz in this respect iron tubes were adopted in future work at higher temperatures.

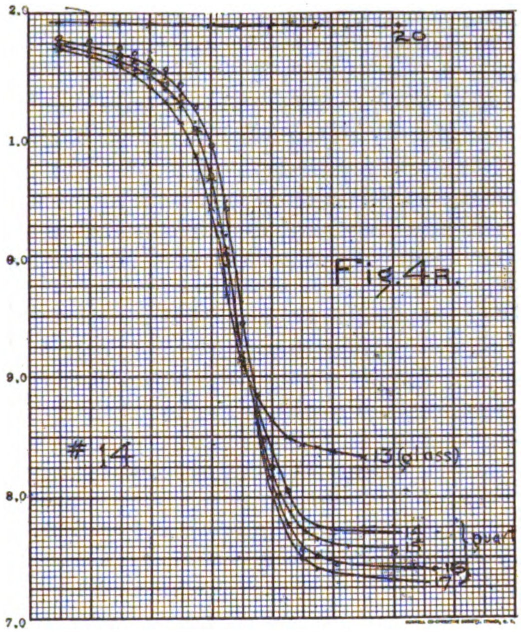

Fig. 4*A*.

It was next found possible to change the adsorptive power of a sample in the reverse of the manner shown above. Fig. 5 of sample 16 is illustrative. This sample was carbonized three hours at 670°, just a trifle above the temperatures to be used subsequently in normal outgassing. The first run indicated an admirable efficiency. The method of reducing the efficiency was to increase the outgassing temperature to 800° or higher and the details of outgassing between each consecutive run are

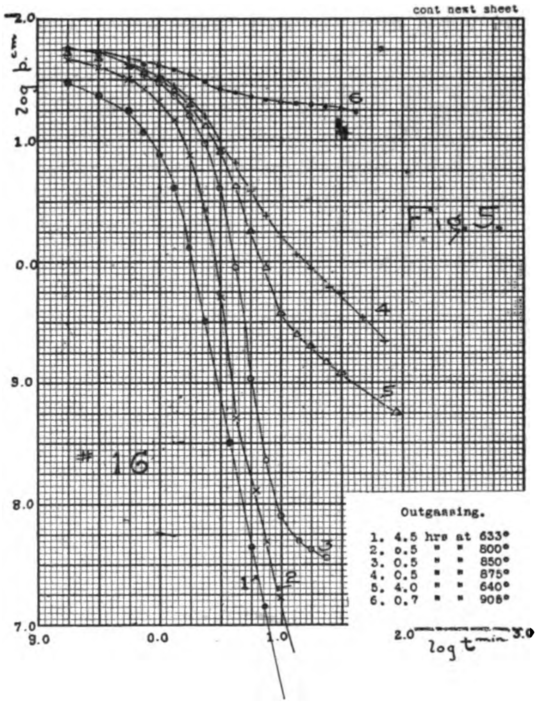

Fig. 5.

given in the insert on the figure. The curves rise for four runs with increasing the outgassing temperature each time. By reducing it to 640° and prolonging the time the sample shows marked recovery. A short 900° outgassing pushes the curve to the top of the page. Fig. 6 is a continuation of the same set of experiments. Here this same sample no. 16 is entirely restored by a series of prolonged low temperature outgassings. The modern term for this process is activation. Fig. 7 shows further continuation of work with the same material in which it is again rendered inefficient (curves for runs 10 to 13) and finally reactivated again (15 to 17). Changes become less easy to produce towards the end of a long series of runs of this sort. Heating samples in iron tubes to 1200° C. was found to destroy the adsorptive power beyond recovery just as well as when the containing tube was quartz.

The most effective charcoal produced up to the present time for the adsorption of air under the conditions outlined above has been the material carbonized at 670° and then put through a series of "runs" and "outgassings" which consist of alternately adsorbing 0.00467 gms. of air per gm. charcoal at – 188° C. and removing it by heating for 4½ hrs. at 600° C. Two hypotheses have been advanced for the interpretation of this fact. The first is that after carbonization there remain considerable amounts of heavy hydrocarbons "clogging the pores" of the material and that these are gradually removed by repeated flushing out with gas. The second has been suggested by Sgt. H. C. Thompson, after reflection on the point of view advanced by Miss Ida Homfray, who has done under the direction of Professor Ramsay some of the best quantitative work yet published on the pressure-absorbed volume-temperature relations of a particular sample.[1] From similarity of the empirical relations discovered to those known to hold for concentrated solutions, the process is visualized as if it were a saturated solution of carbon in essentially liquefied gas. Repeated solution and subsequent evaporation (here taking the form of adsorption at low temperature and

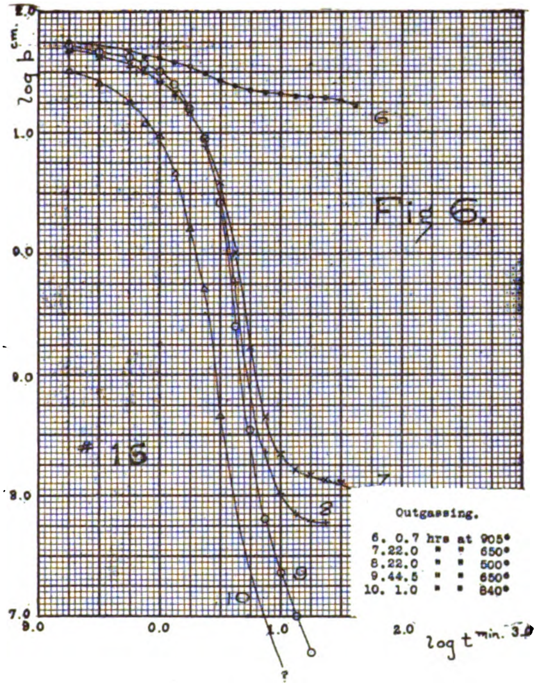

Fig. 6.

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[1] Zeit. Phys. Chem., 74, 139, 1910.

outgassing at moderately high temperature) may result in the gradual alteration of the character of the carbon with respect to its fineness of division, something similar to precipitation taking place. As a test of the hydrocarbon hypothesis attempts were made to improve the behavior of the material by treating it with the lighter liquid hydrocarbons, ligroin, alcohol, and acetone. Through the courtesy of Professor Julius Stieglitz this work was carried out in Kent Chemical Laboratory by Mr. M. C. E. Hanke. The liquids were carefully distilled and redistilled to ensure purity. 50 gms. of the charcoal was then boiled in 500 c.c. of the solvent in a reflux condensor in a water-bath for four to five hours when the solvent was filtered off, redistilled and the residues examined. The following residues were obtained:

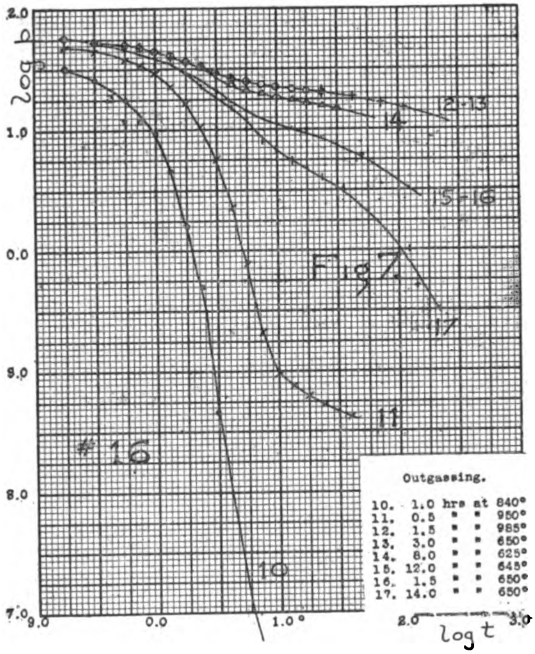

Fig. 7.

Sample 21.

Ligroin 0.005 gm.
Alcohol 0.212 gm.
Acetone 0.38 gm. { 0.35 soluble in acetone.
0.03 soluble in water.

Sample 19.

Ligroin 0.004 gm.
Alcohol 0.294 gm.

$$\text{Acetone } 0.620 \text{ gm.} \begin{cases} 0.570 \text{ soluble in acetone.} \\ 0.050 \begin{cases} 0.040 \text{ soluble water.} \\ 0.010 \text{ insoluble in both.} \end{cases} \end{cases}$$

30 gms. of sample 19 extracted with water gave a residue of 0.4 insoluble in a small amount of water.

These charcoals were then subjected to adsorption tests and both showed marked loss of efficiency rather than gain as might have been expected if the heavy clogging materials had been somewhat removed.

Earlier work in which charcoal was extracted with hot hydrochloric acid to remove ash showed similar injury to the adsorptive power. In this case it had been found necessary to wash the material for nearly eight months to get rid of all traces of acid. In all cases of liquid treated charcoal the adsorptive power rapidly recovers with use when the successive outgassings are kept at not higher than about 600° C. as indicated in general above. These experiments therefore must yet be regarded as inconclusive not only because of the relative small amounts of impurity extracted but because the subsequent loss of efficiency of the material can readily be interpreted as a secondary result due to the difficulty of freeing the pores from the liquids used.

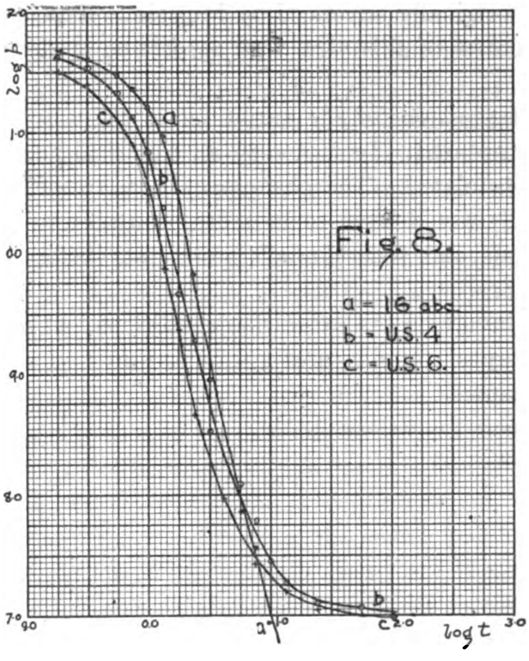

Fig. 8.

Other methods of activation based on the hydrocarbon hypothesis and using gaseous reagents have been developed elsewhere with notable success, especially for field conditions in which the adsorption of a complex vapor is required and where the adsorbing material is already saturated with air, water, vapor, etc. For the rapid production of high vacua, charcoal activated thus is *not* notably better than that discussed above.

A comparison is given in Fig. 8 of the best material produced here by the method of continuous use and that which has been pronounced the best for field conditions in adsorbing noxious vapors and which has been produced by these other methods of activation. The laboratory charcoal, 16 abc, is less active initially but runs to a lower ultimate pressure than does the field material US4 and US6. It is thought likely that there may be a purely surface condensation as well as a slower diffusion into the interior or to less readily accessible surface in accordance with a point of view originally advanced by McBain. Experiments to determine the correctness of this view with respect to modern charcoals are in progress.

No rational expression for the variations in the adsorption described above has yet appeared. In fact even an empirical analytic expression has not been formulated. The phenomena here are of such complexity by nature that it would seem to be more profitable to simplify experimental conditions and secure much more data before attempting to formulate a theory.

Experiments involving the saturation conditions and also experiments on relative adsorption of mixtures are important and will be discussed in subsequent papers.

RYERSON PHYSICAL LABORATORY,
March, 1919.

FLUORESCENCE AND ABSORPTION OF THE URANYL SULPHATES.

By E. L. Nichols and H. L. Howes.

Synopsis.

The spectra of the uranyl sulphates, when resolved by excitation at the temperature of liquid air, exhibit fluorescence series whose frequency intervals range from 85.7 in cæsium uranyl sulphate to 83.0 in potassium uranyl sulphate. The shift towards the violet with increasing molecular weight of the compound is unusually large. The absorption bands extend into group 7 of the fluorescence spectrum without break of interval and there are many reversals where fluorescence and absorption overlap.

URANYL sulphate ($UO_2SO_4.3H_2O$) and the double uranyl sulphates of the alkaline metals are among the most brilliant of known fluorescent substances. Their spectra are characterized by an unusual complexity of narrow bands brought out by cooling to the temperature of liquid air. The group structure is by no means so obviously uniform as in the case of the compounds considered in previous papers nor is there the marked similarity between the spectra of the double sulphates which has been noted in the discussion of the fluorescence and absorption of the chlorides, nitrates and acetates.

There are however certain characteristics common to all the sulphates thus far examined, *i.e.*,

1. Fluorescence at $-185°$ vanishes with the group 7 (frequency 2,000 to 2,070), which is the reversing region for this family of salts, and the eighth group lies entirely within the absorption region.

2. Absorption of the type having the usual 70-frequency interval, extends *without change of interval into* group 7. What we have called the heads of the prominent absorption series lie in the region between 2,040 and 2,060 instead of at 2,170 as in the spectra of the acetates.

3. The fluorescence groups are distinguished by a strong pair of bands fairly dominant in all the spectra excepting that of the sodium salt. The series formed by the members of shorter wave-length of these pairs was terminated towards the violet, where it meets the head of the corresponding absorption series mentioned above.

4. The location of the fluorescence groups in the spectrum of the sulphates, is not approximately the same, for the different salts do not approximate coincidence as is the case with the corresponding acetates. On the contrary, there is, in general, a shift toward the violet with in-

creasing molecular weight, as may be seen from Fig. 1 in which Group 5 of the six sulphates under consideration in the present paper is depicted. This shift is larger than that observed in the double nitrates but not quite so systematic.

If the above groups are aligned by bringing band F into vertical registration as in Fig. 2 it will be seen that the apparent dissimilarity in the composition of the group in the various salts is due rather to the occurrence of various weak bands than to the arrangement of the stronger bands which while not identical approximates to identity almost as closely as in the acetates or the nitrates. As in previous diagrams (see our papers[1] dealing with the spectra of the chlorides, nitrates and acetates) the vertical lines indicate the position of the crests of the bands and, qualitatively only, their relative intensities.

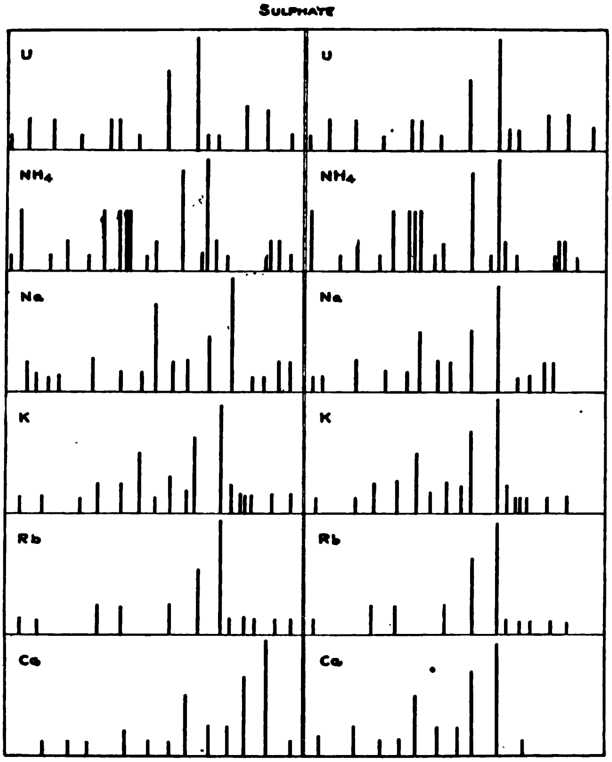

Fig. 1. Fig. 2.

They are estimated in making observations merely as *very strong* (vs), *strong* (s), *medium* (m), *dim* (d), *very dim* (vd), *and very very dim* (v vd) respectively. No attempt is made in the diagrams to indicate the width of the bands.

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[1] PHYS. REV. (2), VI., p. 358; VIII., p. 364; IX., p. 113; X., p. 348; XI., p. 285.

The spectrum of the single sulphate resembles those of the double sulphates much more nearly than is the case with the single and double salts of the other acids.

Wave-lengths, frequencies and relative intensities of the bands observed in the fluorescence spectra of uranyl sulphate, and the double sulphates of ammonium, sodium, potassium, rubidium and cæsium are given in Tables I. to VI. The determinations of wave-lengths were made by the visual and photographic methods described in the papers just cited.

Table I.

Uranyl Sulphate, $UO_2.SO_4 \cdot 3H_2O$.

Prepared by extracting an excess of Uranium Oxide (U_3O_8) with sulphuric acid and oxidizing the solution to $UO_2.SO_4$ by means of H_2O_2. This neutral solution was evaporated to crystallization. The crystals were needles, some being 1 x 2 x 5 mm. in size, apparently orthorhombic with three good pinacoidal cleavages. The angle of the optical axes is very nearly 90° and the double refraction is positive.

Fluorescence at − 185°.

Group.	Series.	μ.	$1/\mu \times 10^3$.	Int.	Group.	Series.	μ.	$1/\mu \times 10^3$.	Int.
2	E	.6254	1599.0	d	6	A	.5256	1902.6	d
	F	.6223	1606.9	d		B	.5237	1909.5	vd
3	B	.6046	1654.0	vd		B′	.5228	1912.7	vd
	C	.6009	1664.2	vd		C	.5210	1919.4	d
	D	.5976	1673.4	d		C′	.5203	1921.9	d
	E	.5942	1682.9	vd		D	.5182	1929.8	d
	F	.5914	1691.0	d		E	.5157	1939.2	m
	H	.5857	1707.4	vd		F	.5135	1947.5	s
	I	.5827	1716.1	vd		F′	.5123	1951.8	m
4	B	.5740	1742.2	vd		H	.5091	1964.4	m
	C	.5716	1749.4	d		I	.5071	1972.0	d
	D	.5686	1758.8	d		J	.5054	1978.6	d
	E	.5657	1767.6	m	7	A′	.5027	1989.2	d
	F	.5630	1776.1	m		B	.5014	1994.4	vd
	G	.5600	1785.7	vd		B′	.5004	1998.5	m
	H	.5574	1794.1	d		C	.4990	2004.0	m
	I	.5551	1801.3	vd		C′	.4981	2007.6	vd
	J	.5534	1807.0	vd		C″	.4978	2008.8	vd
5	A	.5506	1816.5	d		C‴	.4974	2010.5	vd
	B	.5478	1825.6	vd		D	.4964	2014.3	vd
	C	.5450	1834.9	d		D′	.4955	2018.2	vd
	C′	.5441	1837.9	d		E_1	.4941	2023.9	vd
	D	.5423	1843.9	vd		E	.4938	2025.3	m
	E	.5394	1853.8	m		E′	.4933	2027.2	d
	F	.5369	1862.4	s		F_1	.4926	2030.0	vd
	F′	.5357	1866.7	vd		F	.4917	2033.9	s
	G	.5346	1870.7	vd		F′	.4912	2035.8	vd
	H	.5321	1879.2	d		F″	.4905	2038.7	m
	I	.5301	1886.4	d		H	.4878	2049.9	m
	J	.5280	1893.9	vd		J	.4843	2064.8	vd

TABLE II.

Uranyl Ammonium Sulphate, $(NH_4).UO_2.(SO_4)_2.2H_2O$.

Prepared by crystallizing a solution of the two component salts in the proportions of the double salt. The composition has been determined by Rimbach (Ber. d. d. Chem. Ges., 37, 479, 1904; the crystallization by de la Provastaye (Ann. Chem. Phys. (3), 5, 51, 1842, who described it as being monoclinic.

The preparation consisted of square and rounded plates of diameter from .025–.050 mm. The needle-like crystals showed distinct pleochroism from colorless to yellow, the greatest absorption being in the direction of greatest index.

Fluorescence at – 185°.

Group.	Series.	μ.	$1/\mu \times 10^3$.	Int.
2	E	.6214	1609.2	d
	F	.6185	1616.7	d
	H	.6100	1639.3	d
3	B	.6007	1664.7	d
	C	.5977	1673.1	d
	D	.5941	1683.2	d
	E	.5911	1691.8	s
	F	.5883	1699.7	s
	H	.5809	1721.6	vd
4	A	.5755	1737.6	vd
	B	.5717	1749.2	d
	C	.5697	1755.3	d
	C′	.5689	1757.8	d
	D	.5663	1765.8	m
	E	.5632	1775.5	s
	F	.5608	1783.3	s
	G	.5579	1792.3	vd
	H_1	.5549	1802.1	vd
	H	.5539	1805.5	m
	J	.5508	1815.5	vd
5	A	.5491	1821.2	d
	B_1	.5472	1827.5	vd
	B	.5456	1833.0	m
	C_1	.5440	1838.2	m
	C	.5435	1840.1	m
	C′	.5430	1841.6	m
	D_1	.5415	1846.7	vd
	D	.5406	1849.8	d
	E	.5380	1858.8	s
	F_1	.5361	1865.3	vd
	F	.5354	1867.6	s
	F′	.5346	1870.6	d
	G_1	.5337	1873.7	vd
5	H_1	.5300	1886.8	vd
	H	.5295	1888.7	d
	H′	.5290	1890.4	d
	I	.5280	1839.3	vd
	J	.5265	1899.3	vd
6	A	.5249	1905.2	d
	B_1	.5230	1912.0	vd
	B	.5214	1917.8	d
	C_1	.5198	1923.8	m
	C	.5194	1925.4	m
	C′	.5190	1926.8	m
	D_1	.5177	1931.6	d
	D	.5169	1934.6	d
	E_1	.5152	1941.0	vd
	E	.5147	1943.0	s
	F_1	.5128	1950.1	vd
	F	.5123	1951.9	vs
	F′	.5116	1954.7	vd
	G_1	.5106	1958.6	vd
	H	.5073	1971.2	vd
	H′	.5066	1974.0	d
	J	.5038	1984.9	vd
7	A_1	.5025	1990.0	d
	A	.5015	1994.0	vd
	B	.4998	2000.9	d
	C	.4975	2010.0	m
	D_1	.4960	2016.1	d
	D	.4955	2018.3	d
	E	.4933	2027.1	s
	F_1	.4916	2034.2	vd
	F	.4912	2036.0	vs
	F′	.4905	2038.7	vd
	G_1	.4893	2043.7	vd

TABLE III.

Uranyl Sodium Sulphate, $Na_2.UO_2.(SO_4)_2.2H_2O$.

Prepared by crystallizing a solution containing the two component salts in the proportions of the double salt. (See O. de Coninck, Chem. Centralblatt, IX., I., 919, 1905). The preparation consisted of crystalline grains about .5 mm. in diameter with much mother liquor or deliquescence. The crystals are apparently monoclinic with positive double refraction.

Fluorescence at − 185°.

Group.	Series.	μ.	1/μ × 10³.	Int.
2	B	.6296	1588.3	vd
	C	.6255	1598.7	vd
	E	.6182	1617.6	d
	F	.6151	1625.8	m
	G	.6122	1633.5	vd
	H	.6093	1641.2	vd
3	B	.5976	1673.3	d
	B′	.5963	1677.0	vd
	C	.5945	1682.1	d
	D	.5908	1692.6	d
	E	.5880	1700.7	d
	F	.5851	1709.1	s
	G	.5828	1715.9	d
	H	.5797	1725.0	d
4	A	.5729	1745.5	vd
	B	.5698	1755.1	m
	C_1	.5677	1761.5	vd
	C	.5665	1765.1	m
	D_1	.5642	1772.4	d
	D	.5629	1776.5	d
	E	.5607	1784.1	d
	F	.5579	1792.3	m
	G	.5560	1798.6	vd
	H	.5532	1807.7	d
	H′	.5522	1810.9	vd
	I	.5509	2815.9	vd
	J	.5501	1817.9	vd
5	A	.5468	1828.8	d
	B	.5439	1838.4	vd
	C_1	.5418	1845.7	vd
	D	.5406	1849.9	m
	D_1	.5388	1856.0	d
5	D	.5374	1860.8	d
	E	.5355	1867.3	d
	F	.5330	1876.0	m
	G	.5311	1882.7	vd
	G′	.5301	1886.4	vd
	H	.5287	1891.4	d
	H′	.5278	1894.7	d
	I	.5260	1901.1	d
6	A	.5226	1913.5	d
	B	.5197	1924.2	vd
	C_1	.5181	1930.1	vd
	C	.5167	1935.4	m
	D_1	.5152	1941.0	d
	D	.5141	1945.1	vd
	E	.5123	1951.8	d
	F_1	.5110	1956.9	vd
	F	.5101	1960.4	s
	G_1	.5087	1965.8	vd
	G	.5077	1969.5	vd
	H_1	.5061	1975.9	d
	H	.5050	1980.2	m
	I	.5038	1984.9	vd
7	A	.5006	1997.6	vd
	B_1	.4977	2009.2	vd
	B	.4965	2013.9	s
	C_1	.4955	2018.2	m
	C	.4943	2022.9	m
	E	.4910	2036.5	m
	F	.4890	2045.0	d
	G	.4873	2052.3	vd
	H	.4857	2058.9	d
	H′	.4847	2063.1	vd

TABLE IV.

Uranyl Potassium Sulphate, $K_2.UO_2(SO_4)_2.2H_2O$.

Prepared by crystallizing a solution of the two component salts in the proportions of the double salt. The composition has been determined by Rimbach (Ber. d. d. Chem. Ges., 37, 478, 1904. The crystals obtained in this laboratory were orthorhombic. The preparation consisted of six-sided plates and rounded grains about .045 mm. in diameter, the plane of the optical axis being *a* (100) and *b* the acute bisectrix. Double refraction positive.

Group.	Series.	μ.	$1/\mu \times 10^3$.	Int.	Group.	Series.	μ.	$1/\mu \times 10^3$.	Int.
2	C	.6267	1595.7	vd	5	F′	.5332	1875.5	d
	D	.6229	1605.9	vd		G_1	.5324	1878.3	vd
	E	.6188	1616.0	d		G	.5319	1879.9	vd
	F	.6164	1622.3	m		G′	.5314	1881.8	vd
	F′	.6150	1625.9	vd		H	.5295	1888.6	vd
	G	.6129	1631.6	vd		I	.5276	1895.2	vd
3	B	.6010	1663.9	vd	6	A′	.5240	1908.4	vd
	C_1	.5981	1672.0	d		B_1	.5235	1910.2	vd
	C	.5957	1678.8	d		B	.5226	1913.4	d
	D_1	.5941	1683.2	vd		C_1	.5199	1923.4	vd
	D	.5921	1688.0	vd		C	.5189	1927.2	m
	E	.5886	1698.9	vd		D_1	.5173	1933.1	vd
	F	.5859	1706.7	m		D	.5164	1936.5	d
	F′	.5851	1709.1	d		D′	.5157	1939.1	vd
	G	.5830	1715.3	vd		E_1	.5144	1944.0	vd
	H	.5804	1723.0	vd		E	.5137	1946.7	m
4	B	.5718	1748.8	vd		F	.5115	1955.4	s
	C_1	.5697	1755.3	d		F′	.5107	1958.1	d
	C	.5680	1760.7	d		G_1	.5097	1961.9	vd
	D_1	.5662	1766.3	vd		G	.5093	1963.4	m
	D	.5644	1771.9	d		G′	.5088	1965.4	vd
	E	.5616	1780.5	m		H	.5069	1972.6	d
	F	.5589	1789.1	s		I	.5054	1978.6	d
	F′	.5583	1791.3	d	7	A′	.5023	1991.0	m
	G	.5563	1797.5	vd		B_1	.5015	1994.0	vd
	H	.5539	1805.5	vd		B	.5007	1997.3	d
	I	.5519	1811.9	vd		C_1	.4988	2004.7	vd
5	A′	.5481	1824.5	vd		C	.4973	2010.9	s
	B	.5461	1831.0	d		D_1	.4959	2016.5	vd
	C_1	.5438	1838.9	d		D	.4951	2019.7	m
	C	.5424	1843.8	m		E′	.4935	2026.5	m
	D_1	.5405	1850.1	vd		E	.4923	2031.3	d
	D	.5391	1854.9	d		F	.4906	2038.5	s
	E_1	.5374	1860.8	vd		G_1	.4894	2043.3	vd
	E	.5366	1863.5	m		G	.4889	2045.2	d
	F	.5342	1871.8	vs		G′	.4884	2047.5	vd

TABLE V.

Uranyl Rubidium Sulphate, $Rb_2.UO_2.(SO_4)_2.2H_2O$.

Prepared by crystallizing a solution containing the two component salts in the proportions of the double salt. The composition has been determined by Rimbach (Ber. d. d. Chem. Ges., 37, 479, 1904). The crystallization is in every way like the potassium salt although the solubility is less and the crystals smaller. The preparation consisted of six-sided plates about .02 x .04 mm. in size.

Fluorescence at − 185°.

Group.	Series.	μ.	1/μ × 10³.	Int.
1	F	.6485	1542.0	vd
2	C	.6269	1595.1	d
	D	.6225	1606.4	d
	E	.6187	1616.4	vd
	F	.6157	1624.1	d
3	B	.6004	1665.5	vd
	C	.5975	1673.7	d
	C′	.5954	1699.5	d
	D	.5919	1689.5	d
	E	.5887	1698.7	m
	F	.5860	1706.5	s
4	B	.5719	1748.5	vd
	C_1	.5692	1757.0	vd
	C	.5679	1760.9	vd
	D	.5643	1772.2	d
	E	.5616	1780.6	m
	F	.5591	1788.7	s
	F′	.5580	1792.0	vd
	G	.5560	179816	vd
	H	.5538	1805.7	vd
	I	.5520	1811.6	vd
5	B	.5461	1831.2	d
	C_1	.5438	1838.8	d
	C	.5424	1843.7	d
	D	.5392	1854.6	d
	E	.5364	1864.3	m
	F	.5342	1871.8	vs
5	F′	.5332	1875.3	vd
	G_1	.5320	1879.7	vd
	G	.5310	1883.2	vd
	H	.5292	1889.5	vd
	I	.5276	1895.4	vd
6	A′	.5240	1908.4	vd
	B	.5223	1914.5	d
	C_1	.5198	1923.8	d
	C	.5187	1927.8	m
	D	.5163	1937.0	d
	E	.5136	1946.9	m
	F	.5115	1955.2	vs
	F′	.5105	1958.7	vd
	G_1	.5096	1962.3	vd
	G	.5087	1965.8	vd
	H	.5068	1973.3	vd
	I	.5053	1979.0	vd
7	A′	.5021	1991.8	vd
	B	.5008	1996.7	d
	C_1	.4987	2005.2	d
	C	.4973	2010.9	m
	D	.4953	2018.9	d
	E_1	.4930	2028.4	d
	E	.4922	2031.8	d
	F	.4906	2038.4	s
	G_1	.4890	2045.1	vd
	G	.4880	2049.2	vd

TABLE VI.

Uranyl Cæsium Sulphate, $Cs_2UO_2(SO).2H_2O$.

Prepared by precipitating uranyl sulphate by adding cæsium sulphate in calculated amount to form the double salt which is very insoluble. The composition of the crystals is given as above by A. de Coninck (Chem. Centralblat, IX., 1, 1306, 1905). The preparation consisted of very small square plates about .01 mm. on a side, the largest of which showed an apparently uniaxial negative figure. The crystals are therefore presumably tetragonal.

Fluorescence at − 185°.

Group.	Series.	μ.	$1/\mu \times 10^3$.	Int.
2	F	.6129	1631.8	d
3	A	.5989	1669.7	vd
	B	.5964	1676.7	vd
	C_1	.5939	1683.8	d
	C	.5916	1690.3	d
	D	.5894	1696.6	vd
	E_1	.5869	1703.9	vd
	E	.5850	1709.3	d
	F	.5825	1716.6	m
4	A	.5695	1755.9	vd
	B	.5671	1763.4	vd
	C_1	.5652	1769.2	d
	C	.5636	1774.3	d
	D	.5612	1782.0	d
	E_1	.5593	1788.0	d
	E	.5574	1794.0	m
	F	.5550	1801.7	s
	G	.5517	1812.5	vd
	H	.5490	1821.4	vd
	I	.5472	1827.4	vd
5	A	.5434	1840.3	d
	B	.5410	1848.3	vd
	C_1	.5390	1855.2	d
	C	.5374	1860.8	m
	D	.5353	1868.0	d
	E_1	.5336	1874.2	d
	E	.5321	1879.3	m
	F	.5299	1887.0	s
	G	.5276	1895.4	vd
	G′	.5261	1900.8	vd
	H′	.5239	1908.6	d
	I	.5228	1912.9	vd
6	A	.5189	1927.2	d
	B	.5168	1935.0	vd
	C_1	.5148	1942.5	vd
	C	.5140	1945.7	m
	D	.5118	1953.7	vd
	D′	.5110	1956.9	vd
	E_1	.5099	1961.0	d
	E	.5088	1965.5	m
	F	.5067	1973.6	vs
	G	.5047	1981.4	vd
	G′	.5035	1986.1	vd
	H	.5019	1992.6	d
	I	.5003	1999.0	m
7	A	.4970	2012.1	d
	C	.4920	2032.6	m
	D	.4902	2040.0	vd
	E_1	.4888	2045.8	d
	E	.4874	2051.6	m
	F	.4858	2058.6	s

FREQUENCY INTERVALS OF THE FLUORESCENCE SERIES.

The average frequency intervals of the various series, as derived from the foregoing tables are given in Table VII. together with the weighted average for each salt. It will be noted that the intervals of the single sulphate and the double salt of cæsium are distinctly greater than the intervals of the other four sulphates. There is nothing fortuitous about these differences for as will be seen from the table the different series for each salt have intervals within one frequency unit of the general average for that salt, with three exceptions.

These exceptions are Series C_1 in the ammonium and sodium double salts and G_1 in the ammonium salt. Such occasional apparent discrepancies, are not uncommon in the fluorescence spectra of the uranyl salts. They are not due to accidental errors but are probably ascribable to the complexity of bands having overlapping components

Table VII.

Average Frequency Intervals in the Fluorescence Spectra of the Sulphates.

Series.	Intervals.		Weighted Averages.			
	$UO_2SO_4 + 3H_2O$.	$(NH_4)_2UO_2(SO_4)_2 + 2H_2O$.	$Na_2UO_2(SO_4)_2 + 2H_2O$.	$K_2UO_2(SO_4)_2 + 2H_2O$.	$Rb_2UO_2(SO_4)_2 + 2H_2O$.	$CS_2UO_2(SO_4)_2 + 2H_2O$.
A		84.1	84.0			85.6
A′				83.3		
B_1		84.5				
B	85.1	83.5	84.2	83.3	82.8	86.1
B′	85.8		84.2			
C_1		85.6	85.6	83.2	82.7	86.2
C		84.2	84.8	83.1	83.2	85.6
C′	84.9	84.5				
D_1		83.2	84.3	83.3		85.8
D	85.2	83.8	84.2	82.8	82.5	
E_1				83.2		85.7
E	85.0	83.6	83.8	82.1	83.1	85.6
F_1		84.4				
F	85.4	83.9	83.8	83.3	82.7	85.4
F′	84.6	84.0		83.0	83.3	
G_1		85.0		82.5		
G	85.2		84.4	82.7	83.5	86.0
G′				82.9		85.3
H_1		84.6				
H	85.5	83.0	84.4	83.2	83.8	85.6
H′		83.6	84.1			
I	85.2		84.9	83.3	83.7	85.8
J_1	85.2					
J		84.7				
Average	85.2	83.7	84.3	83.0	83.2	85.7

the relative intensity of which in different portions of the spectrum varies progressively. Many such cases are known. A doublet ill-resolved and appearing as a single hazy band; the component of longer wave-length being much stronger in the bands towards the red and dying away gradually in subsequent bands as we approach the blue while the other component steadily increases, will give the effect of an increased frequency interval for the series. The increase might easily be of the general order observed in this case.

There is also always the possibility of the presence of a trace of another uranyl compound one or two of the strongest series in whose spectra appear. Such cases, for example, are not uncommon in the study of the acetates where an admixture of the single acetate occurs.

ABSORPTION SPECTRA.

The difficulties in obtaining a complete record of the absorption bands of the uranyl sulphates are similar to those described in the previous papers already cited. The transmission, like that of the other uranyl salts ranges progressively from almost complete transparency in the red, yellow and green, to a high degree of opacity in the ultra violet.

Large clear crystals of the sulphates are not obtainable and therefore it is not possible to use very thick layers and thus to follow the selective absorption far beyond the reversing region towards the red as has been

TABLE VIII.

Absorption Spectra of Uranyl Sulphates at — 180°.

Salt.	Series.	Frequencies.	Average Interval.
$UO_2SO_4 + 3H_2O$	a_1	2056.8 2128.1 2399.8	68.6
	a	2060.6 2202.2	70.8
	b′	2068.7 2140.5 2209.2	70.1
	d_1	2218.3	......
	d	2016.1 2081.2 2152.9	68.4
	e	2093.6 2160.3 2229.5	68.0
	f_1	2029.9 2170.0	70.1
	f	2102.6 2236.9 2373.0	70.1
	f′	2035.8 2102.6	......
	h_1	2045.8 2115.9 2186.7 2327.2	70.3
	h	2260.4	......
General average			69.6
$(NH_4)_2UO_2(SO_4)_2 + 2H_2O$	a	2061.2 2135.3 2205.8 2275.3	71.3
	b	2072.5 2212.4	69.9
	c	2082.8 2155.1 2226.2 2295.2 2510.0	71.2
	d_1	2016.5 R, 2199.8 2263.0 2409.6 (R)	69.9
	e	2096.0 2236.2 2305.7 2444.4	69.6
	e′	2031.8 2448.6	69.5
	f_1	2244.4 2315.6 2383.3 2524.0 2595.4	70.2
	f	2107.6 2178.7 2250.0 2454.3	69.3
	f′	2039.6 2253.8 2323.8 2392.0 2464.0 2532.8	70.4
	g_1	2044.0 2116.0 2187.1	71.5
	g	2330.5 2401.0 2541.0	70.2
	h	2127.7 2198.8 2236.0 2409.6 2477.1 2549.4	70.3
General average			70.3

TABLE VIII.—*Continued.*

Salt.	Series.	Frequencies.	Average Interval.
$Na_2UO_2(SO_4)_2 + 2H_2O$	a	2069.5	......
	a′	2144.5 2214.7	70.2
	b	2080.8 2153.3	72.5
	c	2093.7 2306.8 2375.9	70.5
	d_1	2167.3	......
	d	2172.5 2243.8 2314.8 2385.5	71.0
	e	2035.8 2107.5 2250.1	71.4
	e′	2039.2	...—
	f	2043.9 2114.3 2184.5	70.3
	g_1	2050.0 2120.3 2260.4	70.1
	g	2190.1	......
	g′	2054.7	......
		2055.2	......
	h	2063.4 2128.6 2199.9	71.3
	h′	2135.2 2207.7	72.1
General average			71.3
$K_2UO_2(SO_4)_2 + 2H_2O$	b_1	2063.1 2412.3	69.8
	b	2064.4 2136.4 2204.6 2276.9	70.8
	b′	2065.7 2341.0	68.8
	c_1	2071.9 2353.5	70.4
	c	2078.6 2289.4	70.2
	d_1	2017.3 2444.1	71.1
	d	2375.3	......
	d′	2158.2	......
	e_1	2174.	......
	e	2095.5 2307.0 2379.8	71.1
	e′	2241.7 2312.1 2385.0	71.6
	f_1	2035.0 2106.8 2246.7	70.6
	f	2039.6 2109.3 2179.4 2248.2 2390.6	70.2
	f′	2250.7	......
	g_1	2293.5	......
	g	2116.4 2188.5 2256.7 2323.7	69.1
	g′	2047.9	......
	h	2056.8 2266.3	69.8
General average			70.3

done in the case of the chlorides.[1] The bands which we were able to locate lie approximately between 2,000 and 2,600 frequency units. They belong almost exclusively to the system having the shorter frequency interval of 70 ±. A few end members of the reversing system, which presumably extends throughout the fluorescence region were discernible. Determinations were made in part by photographing the spectrum of the light transmitted by thin layers, in part by the method of reflection.

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[1] H. L. Howes, Phys. Rev. (2), XI., p. 66, 1918.

TABLE VIII.—*Continued.*

Salt.	Series.	Frequencies.	Average Interval.
$Rb_2UO_2(SO_4)_2$ + $2H_2O$	b_1	2065.6 2136.6 2205.7 2275.1	69.8
	b	2071.2	......
	c_1	2218.8	......
	c	2434.3	......
	d_1	2014.9	......
	d	2230.2 2301.0 2443.2	71.0
	d	2370.5	......
	e_1	2028.4 2096.2 2375.1 2449.8	70.2
	e	2241.2 2307.3 2378.7	68.8
	e′	2312.2	......
	f_1	2106.6 2174.5 2249.2	71.3
	f	2037.9 2107.6	69.7
	f′	2040.3 2109.7 2179.4 2385.7	69.1
	g_1	2045.4 2322.0 2391.3	69.2
	g	2048.5 2117.7 2187.7 2256.7 2325.8	69.3
	h_1	2193.9	......
	h	2085.5 2267.3 2340.8 2408.5	70.0
General average......			69.8
$Cs_2UO_2(SO_4)_2$ + $2H_2O$	a	2096.0 2165.4 2236.9 2375.9 2445.0 2517.7	70.3
	a′	2311.9	......
	c	2103.5	......
	c′	2035.8 2106.4	70.6
	e_1	2115.1	......
	e	2120.0 2261.0 2329.6 2399.8 2471.0 2542.8 2613.4	70.5
	f	2061.2 2130.8 2200.6	69.7
	f′	2133.4	......
	g	2065.5 2206.5 2279.5	71.3
	g′	2071.9 2143.6 2215.8 2353.1 2426.9 2498.3 2567.2	70.8
	h	2077.2	......
	i	2084.6 2294.1	69.8
General average......			70.4

In Table VIII. the frequencies of the bands in the spectra of the six sulphates studies are arranged by series. Each series, as usual, is designated by a small letter corresponding to the capital letter which denotes the fluorescence series to which it is related.

As is frequently the case in these absorption spectra, one or more bands of a given series are commonly missing or at least not discernible in the negatives. On the other hand, nearly all the bands are found to be members of a series, which is definitely related to a fluorescence series and has the proper frequency interval. The occasional isolated bands, moreover are so located that they may be definitely associated to a fluorescence series and may reasonably be classed as the sole visible member of an absorption series the remainder of which fails to appear in our photographs.

The average frequency intervals for the six sulphates are:

UO_2SO_4.	$(NH_4)_2UO_2(SO_4)_2$.	$Na_2UO_2(SO_4)_2$.	$K_2UO_2(SO_4)_2$.	$Rb_2UO_2(SO_4)_2$	$Cs_2UO_2(SO_4)_2$.
69.6	70.3	71.3	70.3	69.8	70.4

These show no systematic departure from the general average 70.3 for the entire group. Lying as they do within one frequency unit of the average we may fairly conclude that within the errors of observation, which are rather large on account of the lack of definition and incomplete resolution of these absorption groups, the various sulphates have a common frequency interval.

The frequency intervals of the various series of a given salt depart somewhat more widely from the average for that salt but again there is no systematic variation and it is probable that all the series would be found to have the same interval were it possible to locate the bands with greater certainty.

Summary.

1. The fluorescence spectrum of the uranyl sulphates consists of eight equidistant bands, the first and eighth of which disappear at the temperature of liquid air.

2. The remaining bands are resolved into groups of narrow line like bands the homologous members of which form series having constant frequency intervals, ranging from 85.7 in cæsium uranyl sulphate to 83.0 in potassium uranyl sulphate.

3. The fluorescence groups are distinguished by a strong pair of bands about eight frequency units apart and several weak bands some of which are doublets.

4. There is a shift of all bands towards the violet, with increasing molecular weights of about fifteen frequency units in passing from the spectrum of uranyl sulphate to that of cæsium uranyl sulphate.

5. The absorption spectra of the sulphates is made up of series of bands having a frequency interval of 70.3 (general average).

6. These absorption series extend into group 7 of the fluorescence without break of interval. There are many reversals where fluorescence and absorption overlap. The reversing region is therefore one group further towards the red than in most spectra of the uranyl compounds.

Physical Laboratory of Cornell University,
June, 1919.

A CRITICAL THERMODYNAMIC DISCUSSION OF THE VOLTA, THERMO-ELECTRIC AND THERMIONIC EFFECTS.

BY P. W. BRIDGMAN.

SYNOPSIS.

In this paper the various relations between the Volta effect, thermo-electric effects, and thermionic effects are critically discussed from the general view point of thermodynamics, avoiding assumptions involving special mechanisms. The ordinary concept of an impressed E.M.F. is found inadequate, and a general definition is proposed, competent to include systems in which the force driving electricity has not the character of a spatially distributed field of force. A thermodynamic proof, dispensing with all special assumptions, is given for a formula for the temperature coefficient of the Volta effect originally given by Lorentz and Kelvin. This temperature coefficient involves a surface heat, the existence of which has not yet been established experimentally, and the possibility of which is usually overlooked. It is shown that Richardson has neglected this surface heat, and that the formula of Lorentz and Kelvin may be deduced also from the phenomena of thermionic emission when the surface heat is taken into account. Formulas are deduced connecting the latent heat of vaporization of electrons, surface heat, surface E.M.F., surface potential jump, Thomson heat, E.M.F. and potential gradient. It is shown that in general local heat, potential difference, and E.M.F. cannot be equal to each other. An expression is found from thermionic considerations for the Volta difference of potential between two parts of the same unequally heated metal, and the possibility suggested of using this effect in determining the hypothetical surface heat. It is shown that it is almost certain that in an unequally heated metal there are currents continuously flowing in closed circuits through the metal and the surrounding electron gas. It is shown that the Volta law of tensions must be capable of extension to include the local potential jumps and E.M.F.'s, although these are not susceptible of direct measurement. Finally, expressions are deduced for the effects of pressure and change of state on the Volta contact difference of potential. These involve the change of volume with surface charge and the effect of surface charge on melting pressure; effects for which no search has been made yet experimentally.

INTRODUCTION.

IN a recent paper I have described the effect of pressure on the thermoelectric properties of metals. In an effort to obtain as much information as possible from the results I later undertook to discuss the relations of thermo-electric phenomena to the Volta effect and the phenomena of thermionic emission. This discussion proved to be difficult because of the great differences of opinion prevalent with respect to fundamental ideas. The old controversies as to the location of the E.M.F. of a cell, the nature of the Volta potential difference, and a locally absorbed heat are all involved and discouragingly confuse the issues. Nearly all previous discussions of these matters have involved special assumptions as to the relations of these phenomena. It is a question to what extent the correctness of the previous results depends on the special assumptions. In order that the discussion of the effects of high pressures might be free from such elements of uncertainty, I have started at the beginning and examined the whole field from a single point of view. In this examination I have tried to avoid every special assumption, for instance assuming neither that the jump of potential at two metals in contact is equal to the Volta contact difference, as does Kelvin, nor that it is equal to the Peltier heat, as do Heaviside and Richardson. The methods employed in this examination are largely the general methods of thermodynamics; in this way the assumption of special mechanisms is avoided. The results of this critical examination constitute this paper.

It turns out that a number of the previous results are unaffected by the special assumptions under which they were developed. In Richardson's work, however, I believe that I have discovered an effect which he has neglected, and which will bring his formula for the temperature coefficient of Volta contact difference of potential into agreement with a formula previously given by Lorentz and later by Kelvin. The effect neglected is a hypothetical surface heat, whose existence has not yet been searched for experimentally and whose magnitude may well be, for any information we have at present, large enough to essentially modify many of the thermionic formulas. In the following a number of thermionic formulas are given as modified by this effect. In addition, the specific relations are developed between the local E.M.F.'s, local potential jumps, and local heats which express the necessary relations when these quantities are not assumed respectively equal, as has frequently been done previously. A connection is found between the surface heat and the Volta difference of potential between two parts of the same unequally heated metal. Finally the effects of pressure and change of state on the various effects are discussed from the point of view of thermodynamics.

The Concept of Impressed E.M.F.

It is in the first place necessary to examine the concept of an impressed E.M.F., because with the discovery of new facts, such as those of electron emission, the old concepts have become inadequate. As commonly understood, a part of the E.M.F. between two points in a stationary medium is due to forces of electro-magnetic origin, and is given by the line integral of the "electric" force. The electric force is to be computed in the classical way from the field equations. In addition to forces of electro-magnetic origin, however, it is necessary to recognize the presence of non-electro-magnetic forces acting on electricity. For instance, in an electron gas in which there is a pressure gradient, the pressure gradient gives rise to a non-electro-magnetic forec acting on electricity. Such forces may be specifically introduced into our equations and are called "impressed forces," in the same way that impressed forces are introduced into mechanics. In an electrical system in which there are impressed forces it is usual to recognize the presence of an impressed E.M.F.; it is this concept of impressed E.M.F. and its relation to that of impressed force which requires examination.

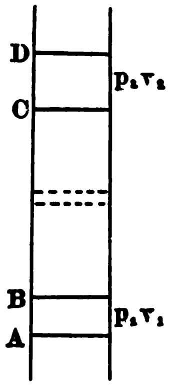

Fig. 1.

It is usual to connect in a simple way the impressed forces with the impressed E.M.F. A careful exposition of this point of view will be found in Abraham (2), for example. Consider a conductor in which the electricity is in equilibrium under the action of the electric and the impressed forces. We may denote the electric force by E^e and the impressed force by E^i. Then since there is equilibrium, $E^e+E^i=0$. The impressed E.M.F. between two points in such a system is usually defined as $\int E^i ds$. This analysis of the situation is inadequate, however, as the following considerations will show.

Consider the behavior of a gas in a field of force, such as an ordinary gas in a gravitational field or an electron gas in a potential field. Let a portion of the gas originally at *AB* (Fig. 1) rise isothermally under equilibrium conditions to *CD*. The work done by the pressures acting across the boundaries in this displacement is zero. This may be proved by a direct integration, or may be simply seen as follows: Consider the work done across the surfaces during a displacement shown by the dotted lines. The work done by the top surface in moving through this displacement will be exactly neutralized at some later instant by the work received by the bottom surface in moving through the same displacement. Hence no net work will be done by the top and bottom surfaces together in the region *CB*. The total work of the operation will be that received by the bottom surface in moving from *A* to *B* minus that done by the top surface in moving from *C* to *D*. The first is p_1v_1, and the second is p_2v_2. But $p_1v_1 = p_2v_2$ (isothermal). Hence no net work is done by the pressures acting across the boundaries in a displacement under equilibrium conditions. Work, however, is done against the gravitational or electrical forces during the displacement considered.

We are here confronted with a paradox, the gravitational force being always equilibrated by other forces, but the gravitational force doing work during a given displacement while the others do none. (There is here no contradiction of the first law of thermodynamics, the energy required to increase the potential energy of position being provided by heat inflow as the gas expands isothermally.) The solution of the paradox lies in the observation that the two forces concerned, the gravitational force and the equilibrating forces, are entirely different in nature. One is a body force, and the other a differential pressure, of the nature of the stresses in an elastic solid. One would never think of saying that a gravitational force and the stresses called into play by it in an elastic solid were equal to each other. Still less can one speak of the line integral of the equilibrating forces; such an expression can have only a formal meaning and cannot be equal to the work done by such forces during the given displacement.

It follows that the concept of an impressed E.M.F. as the line integral of a non-electric force has a chance of being correct only in those cases in which the non-electric force is in its nature a body force, like the forces of the electrostatic field.

Before defining precisely what we shall mean by an impressed E.M.F. it is to be noticed that the idea is a relative one, as in all cases of energy transformation. Consider a closed circuit, for example, in which every one is pretty well agreed in calling the impressed E.M.F. the work done when unit quantity of electricity flows around the circuit. But work done on what? If the circuit consists of a dynamo and a motor, shall we mean the total work done by the motor, or the net work of dynamo and motor together? It must be obvious that to give impressed E.M.F. a precise meaning we must divide our universe into two parts; one is the part in which the action takes place which we specify as E.M.F., and the other we shall call the "outside" part, on which the action of the impressed E.M.F. is expended. We could therefore say that the impressed E.M.F. of a closed circuit is the energy, including heat, delivered to "outside" agencies when unit quantity of electricity flows around the circuit. In the example above, the motor may be taken as the outside agency to which energy is delivered by the E.M.F. of the dynamo when unit quantity flows.

With this recognition that impressed E.M.F. involves the specification of some outside agency to receive energy, and that the magnitude of the E.M.F. will depend on the choice of the outside agency, we define as follows the impressed E.M.F. operative between any two points of a stationary system. "The impressed E.M.F. between two points *A*, *B* in the direction B to A is the energy delivered to outside agencies per unit quantity when positive electricity passes from B to A, plus the increase of energy of the electricity, including in this both increase of energy in the electro-magnetic field and intrinsic energy (as for example in an electron gas)." This reduces to the ordinary definition for a closed circuit, and for a battery on open circuit with terminals of the same metal reduces to the electrostatic potential difference of the terminals, as it should. By "energy per unit quantity" as used in the definition is to be understood the limit of the ratio of energy to quantity for quantities sufficiently large. That is, the distinction emphasized by Lorentz between "mathematical" and "physical" infinitesimals is to be kept in mind, and in the limiting process contemplated in this definition the "physical" infinitesimal quantity of electricity is large enough to include many electrons.

The discussion of this paper will be largely concerned with thermo-electric processes, in which heat energy is converted into electrical energy. This point of view demands that all the heat processes connected with this conversion, except the irreversible Joulean heat, be ascribed to the internal part of the system whose E.M.F. is discussed, as opposed to the outside agencies which receive the converted energy. In particular, the reversible Peltier heat in a thermo-electric circuit is not to be treated as energy delivered to an "outside" source.

These views of impressed E.M.F. are not in agreement with those commonly held. Consider, for instance, the example of the electric double layers which may exist on the surface of separation of metal and ether. The existence of such layers demands a jump in the electrostatic potential. It is common to say that in this double layer there is an equal and opposite impressed E.M.F.[1] The argument is that otherwise the electricity in the double layers could not be in equilibrium. But the argument is not valid if the non-electrical forces are different in nature from the electrostatic forces. We have seen in one special case that the non-electrostatic forces may be like the stresses of an elastic body, instead of like body forces. In the case of double surface layers, we have the further possibility that there may be forces of the nature of kinetic reactions due to bombardment by flying electrons. Such forces are certainly not like body forces in nature. Furthermore, the equilibrium argument demands an inexhaustible supply of freely movable electricity in the surface layer; we have no assurance that this electricity is present. In this paper my position is that it is not justifiable to put local impressed E.M.F.'s equal to local potential jumps. I shall use separate letters for these quantities and keep them distinct.

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[1] See, for example, Heaviside, Electrical Papers, Vol. I, p. 343.

The Volta Effect.

The Volta effect next concerns us. There has been continuous discussion over this without yet any final agreement. There is, however, no question regarding the fundamental experimental facts. If two pieces of metal, say Cu and Zn, are brought into metallic contact, it is found that points in the surrounding medium, gas or vacuum, immediately outside the surfaces of the two metals, are at a difference of potential characteristic of the metals and the medium. When the medium is a perfect vacuum we define the potential as the true Volta potential difference. This difference may be measured in the regular way as the limit of the ratio of the work done on an electrostatic charge to the charge as the charge is made smaller.

The existence of a Volta potential difference follows as a matter of necessity if we suppose jumps of potential between the interior of Zn and ether, interior of Cu and ether, and between interior of Zn and Cu. The Volta jump Zn-Cu is the sum of the jumps ether-Zn, Zn-Cu, and Cu-ether. These jumps of potential demand the existence of corresponding double layers at the corresponding surfaces of separation. The precise strength of each double layer must remain conjectural as long as we have no method of determining the potential of points inside the metal. On the surfaces of Zn and Cu there is, in addition to the double layers, such a distribution of true electricity that under its action and that of the double layers, as computed by the law of the inverse first power, the space immediately outside the surface of each conductor is at constant potential, the potentials outside each conductor differing by the characteristic difference. If the geometrical configuration of Zn and Cu is changed, the single charges redistribute themselves on the surface so as to continue to satisfy the conditions. In particular, if Zn and Cu are made in the shape of plates, we may get the well known condenser action as their distance apart is varied.

The Volta potential difference is maintained automatically by some mechanism in the metal. A consequence of this is that if two pieces of Zn and Cu are charged out of contact with each other to a potential difference V (as measured between points in the surrounding ether immediately outside each metal), and are then brought into contact with a drop of potential difference to V_{zc} (where V_{zc} is the Volta difference), and passage of an amount of electricity E, the amount of electrical energy yielded by the system available to outside agencies is $E[(V - V_{zc})/2]$.

It may be shown by detailed analysis of any cyclic process involving the Volta effect that such a mechanism, automatically maintaining a constant difference between the metals, does not involve a violation of the first law of thermodynamics.

Up to the present no way has been discovered of isolating the jumps of potential involved in the Volta effect. There have, however, been two principal points of view. One is that between points in different metals on opposite sides of a surface of separation there is only a very small potential jump, numerically equal to the Peltier heat, and that the characteristic jumps are located between the metal and the ether; while the other point of view is that there is no jump between metal and ether, but that the entire Volta jump takes place between points within the metals, on opposite sides of their surface of separation. In the following I shall assume neither of these extreme positions, but shall assume potential jumps and the corresponding double layers both between metal and ether and between metal and metal, subject to the single restriction that the sum of the jumps gives the observed Volta jump.

As already mentioned I shall not assume that there are impressed E.M.F.'s at the surfaces equal and opposite to the potential jumps. Neither shall I make the assumption, to me entirely unjustifiable, that at the surface of separation of two metals here is an impressed E.M.F. equal to the Peltier heat. Electricity must now be recognized as a substance capable of possessing kinetic and potential energy. Under these conditions there is no necessary connection between merely the heat and work absorbed when electricity passes from one locality to another.

THE THERMO-ELECTRIC CIRCUIT.

We are now in a position to discuss the ordinary thermo-electric circuit. In order to have the entire situation immediately before us it will pay to reproduce the usual analysis. Consider a circuit of two metals A and B with junctions at t and $t + \Delta t$. The thermo-electric action shall be in such a direction that current flows from A to B at the hot junction. In practise the current usually attains such a value that the energy input is all used in overcoming Ohmic resistance, but we might, if we pleased, insert an electromagnetic engine in the circuit and obtain useful mechanical work. For the present we disregard the irreversible aspects of this process, and assume that the circuit may be treated like a perfect thermodynamic engine. The first law of thermodynamics states that in any closed cycle the energy input equals the energy output. We will take the closed cycle to be constituted by the flow around the circuit of unit quantity of electricity; this extension of the idea of cycle is justified by the fact that after completion of the process every part of the system has returned to its original condition, and so there has been no change of internal energy. The energy output of the cycle is that which might be obtained from the current by an electro-magnetic engine, and for unit quantity of electricity is simply equal the E.M.F. of the circuit measured in appropriate units. We denote this E.M.F. by ΔE_0, where E_0 is the E.M.F. of the cycle between the fixed lower temperature t_0 and a variable upper temperature. The energy input of the cycle is heat input and occurs in four places: at the two junctions and in the two metals A and B. At the junction the reversible heat generated is called the Peltier heat. We denote by P_{AB} the heat absorbed by unit quantity of positive electricity in flowing from A to B. In a steady state, heat absorbed by the current is provided by an inflow of heat from the surroundings. In virtue of reversibility, $P_{AB} = -P_{BA}$. In the wires themselves there is also the reversible Thomson heat. We denote by σ_A the heat absorbed by unit quantity of positive electricity in flowing in the metal A from a lower temperature to a temperature 1 degree higher. σ corresponds to the "specific heat" of electricity. P_{AB}, σ_A, and σ_B are all functions of the temperature. It is a matter of experiment that σ does not depend on the temperature gradient.

There have been in the past numerous attempts to show that σ does depend on the temperature gradient, but the overwhelming concensus of opinion has been that all such supposed effects are to be explained by local inhomogeneities in the metal. Recently, however, Benedicks (3) has reopened the question, claiming to have established the existence of a legitimate effect. As far as his work on solids goes, a paper by Foote and Harrison (4) seems to me to fully meet the situation. With regard to liquid mercury I had, before knowing of Benedick's work, found negative results (5). I had shown that positive results will be obtained unless the apparatus is symmetrically located in the gravitational field. Benedicks does not mention any precaution of this nature, and I believe that his positive results are to be explained in this way. I shall assume in this paper that the effect does not exist.

The first law applied to the cycle now gives at once

$$\Delta E_0 = P_{AB}(t + \Delta t) - \sigma_B \Delta t - P_{AB} + \sigma_A \Delta t,$$

or

$$\frac{dE_0}{dt} = \sigma_A - \sigma_B + \frac{dP_{AB}}{dt}. \qquad (1)$$

In virtue of the experimental fact that the E.M.F. of a circuit from t_1 to t_2 plus that of one from t_2 to t_3 is equal that of a single one from t_1 to t_3, dE_0/dt is independent of t_0, and we write simply dE/dt, where the t_0 from which E is measured need not be specified, and in particular has no relation to the temperature at which the σ_A, σ_B, and dP_{AB}/dt of the equation are taken.

The second law may also be applied, giving

$$\frac{P_{AB}(t+\Delta t)}{t+\Delta t} - \frac{\sigma_B \Delta t}{t+\Delta t/2} - \frac{P_{AB}}{t} + \frac{\sigma_A \Delta t}{t+\Delta t/2} = 0.$$

This may at once be rewritten in the form

$$\Delta\left(\frac{P_{AB}}{t}\right) + \frac{\sigma_A - \sigma_B}{t+\Delta t/2}\Delta t = 0,$$

and gives in the limit

$$\frac{d}{dt}\left(\frac{P_{AB}}{t}\right) + \frac{1}{t}(\sigma_A - \sigma_B) = 0. \tag{2}$$

We may now eliminate $\sigma_A - \sigma_B$ or P_{AB} between 1 and 2, obtaining

$$P_{AB} = t\frac{dE_{AB}}{dt} \tag{3}$$

$$\sigma_B - \sigma_A = t\frac{d^2E_{AB}}{dt^2}. \tag{4}$$

In order to show that the E.M.F. is in such a direction that the current flows from A to B at the hot junction, we write E with the subscripts A and B.

Two aspects of these equations require discussion; first the irreversible aspects of the process, and second the location of the E.M.F.

There are two irreversible processes always involved in any thermo-electric circuit; generation of Joulean heat by the current in overcoming the resistance of the wire, and conduction of heat from the hotter to the colder end of the wire. In the early days of the subject it did not seem very objectionable to entirely disregard these effects, because of the probability that they were due to entirely unrelated parts of the mechanism, so that it would be conceivable that a metal might exist with the same thermo-electric properties as any actual metal, but with negligible thermal conduction and Joulean heat loss. But with the rise of electron theories of metals it became exceedingly probable that all these effects, reversible and irreversible, are tied together by the same mechanism, so that we cannot, at least without justification, assert that any aspect is unimportant. It was this feeling which inspired Lorentz's article in the Wolfskehl conference collection. (6) Thomson also always regarded the thermodynamic argument as by itself unsatisfactory, and looked on the equations obtained by the method above merely as suggestive relations to be tested by experiment.

It is not unusual in elementary text books to find the statement that by properly choosing the dimensions of the circuit the irreversible aspects may be made vanishingly small. An argument by Boltzmann (7) in a much neglected paper shows, however, that this is not true, but that the ratio of irreversible to reversible heat can be reduced only to a minimum, not zero, by a proper choice of the constants of the circuit. Work is extracted from the circuit reversibly by an electro-magnetic engine of properly chosen back E.M.F. The dimensions of the parts of the circuit and the magnitude of the back E.M.F. are the only variables under our control. Suppose now that the best construction possible has been given to the circuit by a proper choice of all the independent variables. We may write down the thermodynamic inequalities that hold for an irreversible process, and obtain a necessary condition between the electrical and thermal conductivities and the thermo-electric constants. This condition is given by Boltzmann and is

$$t\frac{dE}{dt} - P \leqq 2A\sqrt{t}, \qquad (5)$$

where

$$A = R_A^{1/2}K_A^{1/2} + R_B^{1/2}K_B^{1/2}$$

and R_A = specific electrical resistance of A,
K_A = specific thermal conductivity of A,

with corresponding letters with subscripts B for the corresponding properties of B.

The necessary condition written down above on the assumption that the reversible and irreversible aspects of the thermo-electric circuit are inextricably tied together to form one essentially irreversible process is not one which in practice imposes any useful restriction on the constants. $2A\sqrt{t}$ is in nearly every known case greater than $t \cdot dE/dt$ alone, which may always be made positive by a proper arrangement of the metals, and is thus necessarily greater than $t \cdot dE/dt - P$, because in every known case P is also positive when $t \cdot dE/dt$ is made positive. Experimentally, $t \cdot dE/dt$ is fairly easy to determine accurately, the difficulty being with P. So long, therefore, as $t \cdot dE/dt$ alone is less than $2A\sqrt{t}$, no amount of experimental inaccuracy would vitiate the inequality, which thereby loses all interest.

Many direct experimental attacks have been made on the question of the equality of $t \cdot dE/dt$ and P, but the experimental difficulties are great, and the experimental verification is still far from complete. Thus it was not until 1906 (8) that the reversibility of the Peltier heat was shown with any accuracy, the reversibility usually being assumed in experimental work and used as a means of eliminating the Joulean heat.

So far as I know, there is no evidence that $t \cdot dE/dt$ and P are not equal, and with increasing accuracy of experimentation the verification becomes closer. Gottstein (9) has published data on silicon, molybdenite and graphite, in which the effects are large, and which other considerations suggest as the most likely field for discrepancies, and has obtained verification within the limits of accuracy of P, which are about 8 per cent. Caswell (10) has also recently published data on several Bi-Sn alloys, in which the effects are also large, and obtained verification within 6 per cent. He states that the accuracy of the Peltier heat measurements is 2 per cent., but also explicitly emphasizes that his measurements have proved the thermodynamic relation.

Mention should be made in this connection of a theorem of Baedeker (11) to the effect that if the relation

$$\frac{dE_{AB}}{dt} = \frac{1}{t} P_{AB}$$

is true for a single pair of substances it is true for every pair. The proof given by Baedeker does not justify so general a statement as this, however, and presumably the theorem itself does not hold. Baedeker's proof is as follows. The relation (2) above becomes, if extended to include irreversibility,

$$\frac{d}{dt}\left(\frac{P_{AB}}{t}\right) + \frac{1}{t}(\sigma_A - \sigma_B) \geqq 0.$$

Combining this with equation 1 gives

$$\frac{dE_{AB}}{dt} \geqq \frac{1}{t} P_{AB}.$$

The equality sign holds for perfect reversibility. Suppose now that the equality sign holds for a special pair of metals, so that

$$\frac{dE_{AB}}{dt} = \frac{1}{t} P_{AB}.$$

Consider now the couples AC and CB, where C is any third metal. Because of the addition theorem we have

$$\frac{dE_{AC}}{dt} + \frac{dE_{CB}}{dt} = \frac{1}{t}(P_{AC} + P_{CB}).$$

Now if

$$\frac{dE_{AC}}{dt} > \frac{P_{AC}}{t},$$

we must evidently have

$$\frac{dE_{CB}}{dt} < \frac{P_{CB}}{t},$$

which, Baedeker says violates the second law, and hence the theorem. Baedeker failed to notice, however, that in the above inequality the metals must be arranged in such an order that dE_{AB}/dt is positive. His proof therefore holds only when dE_{AC}/dt and dE_{CB}/dt are both positive. The correct theorem is that if the relation of perfect reversibility holds for any pair of metals A and B (dE_{AB}/dt being positive), it also holds for all pairs of metals between A and B that may be chosen from the series of metals arranged in ascending order of dE_{Ax}/dt.

In practise, however, an equation can only be shown to hold within limits. So that instead of a perfect equality we should only have

$$\frac{dE_{AB}}{dt} = \frac{1}{t} P_{AB} + \epsilon,$$

where ϵ is some positive quantity. The argument above now shows that if C is any third metal intermediate between A and B, dE_{AC}/dt cannot exceed P_{AC}/t by more than ϵ. But dE_{AC}/dt is numerically less than dE_{AB}/dt, so that the percentage verification of the formula for the pair AC is not so good as for the pair AB.

If therefore a method is available in which the limits of accuracy are percentage limits, this theorem, contrary to what Baedeker supposed, does not allow us to dispense with an experimental verification of the formula for every pair of metals.

There is no inequality corresponding to

$$\frac{dE_{AB}}{dt} \geqq \frac{P_{AB}}{t}$$

connecting the Thomson heats with the electromotive force of the circuit. The analytical reason is that the two sides of an inequality may not be differentiated. The experimental verification of the formula for the Thomson heats is not yet as complete as that for the Peltier heat.

I shall assume in the following, as most probable from the present data, that the thermodynamic relations obtained by neglecting irreversibility are strictly true, but in my opinion the question cannot yet be regarded as closed.

Consider next the location of the E.M.F. A glance at the thermodynamic arguments shows that we have said nothing about the intensity of the forces at any locality; we have merely equated the total work done in the circuit by the E.M.F. to the total energy inflow in the form of heat, and our justification is the experimental fact that in the complete cycle no other forms of energy have any net effect. The thermodynamic results are entirely unaffected by adding to any possible physical distribution of E.M.F. any other distribution which integrates to zero around the complete circuit. Any argument for a localized E.M.F. must use other kinds of experimental fact than those which we have up to the present used. There has, nevertheless, been an attempt by many physicists to identify the reversible heats with exactly equal E.M.F.'s at the same localities, and conversely. Maxwell does this, as did also Heaviside (12), who speaks with impassioned irritation of those who would "make a force do work where it is not." All the objections of Heaviside as to energy manifestations at the junctions seem to me to be fully answered by a careful consideration of the nature of the Poynting vector, together with a recognition of the fact that within the metal electricity may possess kinetic and potential energies characteristic of the metal. Whatever assumption one cares to make about the location of the E.M.F. will be found to involve such a distribution of the Poynting vector as to automatically take care of any unusual energy transformations introduced by the assumptions. However, in accordance with the mode of thought of Maxwell and Heaviside it is usual to speak of a Peltier E.M.F. and a Thomson E.M.F. (*e.g.*, Caswell (10), 1916) which are precisely equal to the corresponding reversible heats, and to assume that these are the total E.M.F.'s at the junction or in the unequally heated metal. This, it seems to me, is entirely unjustified, and my position in this paper is that we know nothing about these localized E.M.F.'s from experiments such as these on closed circuits. When in the following I speak of a Peltier E.M.F., I mean whatever E.M.F., if any, there may be at the junction, but shall assume no relation with the heat at the junction. Similarly by the Thomson E.M.F. I mean the E.M.F. distributed throughout the metal in virtue of the temperature gradient, but again shall assume no relation with the Thomson heat.

A second point of view, that other specifiable E.M.F.'s than those corresponding to the local heats are involved has been maintained by many. Among the prominent early holders of this view was Lord Kelvin, who believed that at the junction of two metals there is an E.M.F. equal to the ordinary Volta contact potential difference. Adopting this point of view, Kelvin in one of his later papers (13) deduced a relation between Volta potential difference and a quantity analogous to the Peltier heat by a purely isothermal process, thus avoiding the difficulties of irreversibility. The same relation had also been given nine years previously by Lorentz (14) in a paper corrected by him to meet the criticisms of Budde (15). Both the papers of Kelvin and Lorentz seem to have been entirely overlooked by subsequent writers. It is now becoming evident, however, from such work as that of Richardson (16) on thermionic emission, that Kelvin's view is probably no more correct than the other, and that there may be a potential jump between the surface of the metal and the surrounding ether. I shall show from an examination of the phenomena of thermionic emission that there are certain relations between the E.M.F. of a circuit, Peltier heat, Volta potential difference, etc., irrespective of what hypothesis one adopts about the location of the potential jumps. Richardson himself makes the assumption that the local E.M.F. is equal the local heat, and with this assumption obtains a relation similar to Kelvin's.

TEMPERATURE COEFFICIENT OF VOLTA EFFECT.

The following deduction of a relation involving Peltier heat and Volta difference is considerably simpler than that of either Lorentz or Kelvin, and furthermore makes no assumptions as to the location and magnitude of the surface potential jumps accompanying the Volta potential difference. The method also permits somewhat more general conclusions.

Imagine an infinite plate condenser composed of two different metals A and B, all parts of which are maintained at the same temperature, and the distance apart of which is variable. The two plates may be connected by a wire. We imagine this wire to be half of A and half of B, so that when charge is transferred from one plate to the other through the wire the ordinary Peltier heat is developed at the junction.

As the independent variables fixing the state of the system we choose:

t = absolute temperature,
ρ = positive charge per unit area on A (there is of course an equal negative charge on B),
c = capacity per unit area (this depends only on the distance apart of the plates).

The following dependent variables are to be considered:

V_{BA} = Volta contact difference of potential,
u = internal energy per unit area,
dW = work done by the system in any infinitesimal change,
dQ = heat absorbed in any infinitesimal change.

Now the second law states that $(du+dW)/t$ is an exact differential for any reversible change. We accordingly have to find dW and du. For dW we have the equation

$$dW = \left(\frac{\partial W}{\partial T}\right)_{\rho c} dt + \left(\frac{\partial W}{\partial \rho}\right)_{tc} d\rho + \left(\frac{\partial W}{\partial c}\right)_{t\rho} dc.$$

The partial derivatives may be evaluated as follows. Obviously no work is done if the temperature of the system is changed, keeping the distance of separation of the plates constant, if at the same time no charge is transferred from plate to plate. The analytic statement of this is

$$\left(\frac{\partial W}{\partial t}\right)_{\rho c} = 0.$$

Again, if work is obtained by letting current flow from plate to plate the effective driving force is the difference of potential minus the Volta difference. The work done during this passage of current may be extracted by a reversible electro-magnetic engine in the wire between the plates. This gives

$$\left(\frac{\partial W}{\partial \rho}\right)_{tc} = -\left(\frac{\rho}{c} - V_{BA}\right) d\rho.$$

This is independent of assumption as to the location of the Volta jump, whether it is a surface affair between the metal and the surrounding electron gas, or between the two metals, or both. The conclusions are therefore independent of special hypothesis. Finally, if the distance apart of the plates is changed, thus changing the capacity, but keeping temperature and charge constant, we have the mechanical work

$$\left(\frac{\partial W}{\partial c}\right)_{T\rho} dc = \frac{1}{2}\frac{\rho^2}{c^2} dc.$$

This is an immediate consequence of the familiar expression for the work of collecting an electrical distribution from infinite subdivision, $\frac{1}{2}\rho \times$ pot. diff. ($= \frac{1}{2}(\rho^2/c)$ in this case).

We may also treat du as we have dW, writing

$$du = \left(\frac{\partial u}{\partial t}\right)_{\rho c} dt + \left(\frac{\partial u}{\partial \rho}\right)_{tc} d\rho + \left(\frac{\partial u}{\partial c}\right)_{t\rho} dc.$$

Substituting now above, we have

$$\frac{1}{t}\left\{\left(\frac{\partial u}{\partial t}\right)_{\rho c} dt + \left[-\frac{\rho}{c} + V_{BA} + \left(\frac{\partial u}{\partial \rho}\right)_{tk}\right] d\rho + \left[\frac{1}{2}\frac{\rho^2}{c^2} + \left(\frac{\partial u}{\partial c}\right)_{t\rho}\right] dc\right\}$$

is a perfect differential. This statement will give us three equations, because of the three independent variables, instead of only one equation, as in the examples of elementary thermodynamics, where we are concerned with only two independent variables.

In the first place, from the coefficients of dt and dc, we obtain

$$\frac{\partial}{\partial c}\left\{\frac{1}{t}\left(\frac{\partial u}{\partial t}\right)_{\rho c}\right\}_{t\rho} = \frac{\partial}{\partial t}\left\{\frac{1}{t}\left[\frac{1}{2}\frac{\rho^2}{c^2} + \left(\frac{\partial u}{\partial c}\right)_{t\rho}\right]\right\}_{c\rho},$$

whence

$$-\frac{1}{2}\frac{1}{t^2}\frac{\rho^2}{c^2} - \frac{1}{t^2}\left(\frac{\partial u}{\partial c}\right)_{t\rho} + \frac{1}{t}\left(\frac{\partial^2 u}{\partial c \partial t}\right)_{\rho} = \frac{1}{t}\left(\frac{\partial^2 u}{\partial c \partial t}\right)_{\rho}$$

giving

$$\left(\frac{\partial u}{\partial c}\right)_{tp} = -\frac{1}{2}\frac{\rho^2}{c^2}.$$

But we have already seen that

$$\left(\frac{\partial W}{\partial c}\right)_{tp} = \frac{1}{2}\frac{\rho^2}{c^2},$$

so that

$$\left(\frac{\partial Q}{\partial c}\right)_{tp} = 0. \tag{6}$$

Therefore no heat is absorbed when the plates are moved with respect to each other at constant temperature and charge. This states a fundamental assumption of electrostatics to the effect that charged bodies may be moved relatively to each other with no thermal effects.

Secondly, from the coefficients of $d\rho$ and dc we get

$$\frac{\partial}{\partial c}\left\{\frac{1}{t}\left[-\frac{\rho}{c} + V_{BA} + \left(\frac{\partial u}{\partial \rho}\right)_{tc}\right]\right\}_{tp} = \frac{\partial}{\partial \rho}\left\{\frac{1}{t}\left[\frac{1}{2}\frac{\rho^2}{c^2} + \left(\frac{\partial u}{\partial c}\right)_{tp}\right]\right\}_{ct},$$

which expands to

$$\frac{\rho}{c^2} + \left(\frac{\partial V_{BA}}{\partial c}\right)_{tp} + \left(\frac{\partial^2 u}{\partial \rho \partial c}\right)_t = \frac{\rho}{c^2} + \left(\frac{\partial^2 u}{\partial \rho \partial c}\right)_t,$$

or

$$\left(\frac{\partial V_{BA}}{\partial c}\right)_{tp} = 0, \tag{7}$$

which states that merely changing the distance between the plates does not change the Volta difference. We have not thereby proved that the Volta difference cannot depend on the distance apart of the plates; we have merely proved that such independence is a logical consequence of the statements that we made about the nature of the Volta effect.

Formulas 6 and 7 are not given by Kelvin or Lorentz.

Finally, we obtain from the coefficients of dt and $d\rho$,

$$\frac{\partial}{\partial \rho}\left\{\frac{1}{t}\left(\frac{\partial u}{\partial t}\right)_{\rho c}\right\}_{tc} = \frac{\partial}{\partial t}\left\{\frac{1}{t}\left[-\frac{\rho}{c} + V_{BA} + \left(\frac{\partial u}{\partial \rho}\right)_{tc}\right]\right\}_{\rho c}.$$

Expanding,

$$\frac{1}{t}\left(\frac{\partial^2 u}{\partial \rho \partial t}\right)_c = -\frac{1}{t^2}\left[-\frac{\rho}{c} + V_{BA} + \left(\frac{\partial u}{\partial \rho}\right)_{tc}\right] + \frac{1}{t}\left[\left(\frac{\partial V_{BA}}{\partial t}\right)_{\rho c} + \left(\frac{\partial^2 u}{\partial t \partial \rho}\right)_c\right].$$

$(\partial^2 u/\partial \rho \partial t)_c$ cancels. $(\partial V_{BA}/\partial t)_{\rho c}$ we may take as the ordinary temperature derivative of the Volta contact difference of potential. Experimentally there is yet no evidence for a variation of V with ρ, and we have shown that it does not depend on c, so that we might write simply

dV_{BA}/dt, but the thermodynamic distinction between the derivatives should not be forgotten. It might be proved experimentally at some later time that V_{BA} does depend on ρ, and in fact there are some plausible reasons for expecting an effect of this nature. I am not aware that any very careful search has been made for a variation of V with ρ.

The only unknown now remaining in the above equation is $(\partial u/\partial \rho)_{tc}$. Under all conditions we have $du = dQ - dW$. Now if the temperature is constant, and the distance apart of the plates is constant (c = constant), the only mechanical work is that which might be got out of the electrical current when ρ changes, and the heat absorbed is the heat involved in the transfer of electricity from one plate to the other. Write this for the moment as P_{BA}', this being the heat absorbed when unit quantity of electricity passes from B to A. Then

$$\left(\frac{\partial u}{\partial \rho}\right)_{tc} d\rho = P_{BA}' d\rho + \left(\frac{\rho}{c} - V_{BA}\right) d\rho.$$

Substitute this above and we get

$$O = -\frac{P_{BA}'}{t} + \frac{dV_{BA}}{dt},$$

or

$$\frac{dV_{AB}}{dt} = \frac{P_{AB}'}{t}. \tag{8}$$

This is the equation deduced by Lorentz (14) and Kelvin (13). It is to be noticed that all parts of the system have been at the same temperature, so that during the cycle there have been no irreversible heat transfers, and therefore there is no question of the validity of the conclusions, as there was in the case of the ordinary thermoelectric circuit. Kelvin hoped that an experimental confirmation of this formula would make more probable the correctness of the ordinary formulas for the thermo-electric circuit. In attempting the confirmation he at first identified, as one naturally would, the heat P_{BA}' with the ordinary Peltier heat P_{BA}. This amounts to assuming that all the thermal effects involved in the transfer of charge from B to A are to be found at the surface of separation of A and B. Making this identification of P' with P, the experimental verification failed by a thousand fold. Now the ordinary thermodynamic formulas hold within much narrower limits. So that not only did this attempt of Kelvin's to place the ordinary thermodynamic reasoning on a firmer basis fail, but in order to explain the discrepancies between the new formula and experiment, he was driven to postulate the existence of another surface thermal effect, in much the same way that he had previously postulated the existence of the Thomson heat. The possibility of such a surface heat was also explicitly recognized by Lorentz (14), after it was suggested to him by Budde (15).

The new surface heat that Kelvin had to postulate was a reversible generation or absorption of heat at the surface of a conductor when the surface charge changes. So far as I am aware the existence of the effect has never been demonstrated; in fact the papers of Lorentz and Kelvin seem not to be generally known, and apparently no serious attempt has been made to discover the effect. I shall return to this matter later in connection with a deduction of the same formula by a method similar to that of Richardson.

THERMIONIC EFFECTS.

This is probably as far as we can get by an application of the two laws of thermodynamics to these two phenomena. Further progress demands different types of experiments from those contemplated in the applications above. Now this new order of experimental fact has been supplied in the last few years, in principal part by the work of Richardson (16) on thermionic emission. In this work the energy relations and transformations are discussed when electricity is taken directly through the surface of the metal from metal to ether. In the previous work of this paper we have been restricted in transferring electricity from conductor to conductor to the staid and classical methods of motion over the metallic surfaces or through the body of the metal. We may now extend our thermodynamic processes to cycles in which electricity is transferred from one conductor to another through the surface and the surrounding ether.

Richardson has developed by thermodynamic arguments many of the formulas connecting thermionic effects with those already discussed, but he has frequently made specific assumptions as to localized E.M.F.'s, and has further, it seems to me, confused or used interchangeably the thermodynamic quantities, quantity of energy, quantity of heat, and work, and has not sharply specified the conditions to which some of his quantities, notably the heat of vaporization of an electron, apply. A critical examination of his methods will suggest changes in his formulas.

The starting point is the observation that at high temperatures all metals emit electrons, and therefore, when in a state of equilibrium, are surrounded by an electron atmosphere. The density of this atmosphere becomes rapidly lower at lower temperatures, but we assume that it essentially exists at all temperatures, and obtain what information we can from the fact of its existence. The density of this gas is so small under ordinary conditions that the charge it carries does not affect its behavior, and it accordingly obeys the perfect gas laws. Richardson has given a qualitative argument for this, or it may be easily proved by writing down the equations of equilibrium of a gas in which there is a distributed volume charge proportional to the density, and finding the explicit solution for small densities.

The process of a metal surrounding itself with an electron atmosphere is analogous to the process of evaporation of a solid, and the same thermodynamic arguments apply to it. Any purely electrostatic action due to the isolation of the negative charge in the gas and a positive charge on the metal can be eliminated by evaporating only a small quantity of electrons, the electrostatic energy being proportional to the square of the charge. Assuming now the perfect gas law, we first find the relation between electron gas density and temperature.

Clapeyron's equation applies to this and gives

$$\frac{dt}{dp}=\frac{t\Delta v}{H},$$

where Δv is the change of volume when 1 gm. of electrons evaporates reversibly, and H is the latent heat of evaporation. Professor Hall has objected to the use of this equation, or the equivalent one by Richardson (Eq. 1, p. 28 of Richardson's book), and has published a statement to that effect in a recent number of the Proceedings of the National Academy of Sciences (17). He has later recognized, however, that the objection to Richardson's argument is not to be found at this stage, and is stating his revised position in a forthcoming note to the National Academy.

Returning now to Clapeyron's equation, since the perfect gas law holds, $p = nkt$, where n is the number of electrons per c.c., and k is the gas constant. The gas law now gives

$$dp = ktdn + nkdt.$$

Δv is obviously approximately equal to v, the volume of 1 gm. of electron gas. Making these two substitutions in Clapeyron's equation, we get

$$dt = \frac{tv}{H}(ktdn + nkdt),$$

which may be rewritten

$$dt = \frac{kt^2}{\frac{H}{nv}}\left(\frac{dn}{n}+\frac{dt}{t}\right).$$

Now nv is the total number of electrons per gm. of gas, and hence H/nv is the latent heat per electron. Denote this by η, and the equation becomes

$$\frac{\eta dt}{kt^2}=\frac{dn}{n}+\frac{dt}{t}. \tag{9}$$

An integration of this equation gives

$$n = At^{-1}e^{\int\frac{\eta dt}{kt^2}}, \tag{10}$$

in which A is not a function of t. This is the same as Richardson's equation 7, p. 29, noting that his φ is the energy change in the evaporation of one electron, so that $\eta = \varphi + kt$.

It is necessary to scrutinize pretty closely the conditions under which the η of this formula is to be determined. Clapeyron's equation, as used above, applies to an isolated system of metal with its electron vapor. We may imagine this system enclosed in a box provided with a piston working on the electron vapor, and so able to interchange work with the surroundings. The η in the formula above is the heat per electron which must be communicated to the box from outside during the evaporation of electrons and accompanying motion of the piston at constant pressure. Now during this process in which an electron is evaporated, a positive charge is left behind on the surface of the metal, so that during this process the surface charge is changing. We may split η into two parts. The first, η_ρ, is the latent heat which would be absorbed if the evaporation took place at constant surface charge. The second part of the heat is a heating effect due to the appearance of a positive charge on the surface; this is merely Lorentz and Kelvin's surface heat. If we put P_s for the heat absorbed by the system when unit positive charge is imparted to the surface, we have

$$\eta = \eta_\rho - \epsilon P_S, \tag{11}$$

where ϵ is the electronic charge, taken as a negative number.

These results may now be applied to contact difference of potential. Imagine two metals A and B together in a region at temperature t, in metallic contact. Each metal will sourround itself with an atmosphere of electrons by spontaneous emission, and these two atmospheres will in general be of different densities and pressures. The gas equilibrium is maintained by an electrostatic potential gradient accompanying the pressure gradient. Notice that this equilibrating electrostatic field, for gases of small densities, is due to charges outside the gas itself. The ordinary gas equations show that equilibrium demands the equation

$$\frac{n_A}{n_B} = e^{-\frac{\epsilon v}{kt}BA}, \tag{12}$$

where ϵV_{BA} is the work done against electrostatic forces in taking an electron from a point immediately outside B to a point immediately outside A, n_A and n_B are the number of electrons per c.c. immediately outside A and B respectively.

Under ordinary conditions it is doubtful whether two pieces of metal ever reach equilibrium in this way, because the emission of electrons at ordinary temperatures is so excessively slow. From data on page 69 of Richardson it may be computed that a sphere of tungsten 1 cm. in radius requires something of the order of 10^{55} years to charge itself to a potential of 1 volt by spontaneous emission of electrons at 0° C. It is to be considered, therefore, whether the Volta potential difference as measured under ordinary conditions is the same as it would be if measured after a time long enough for the attainment of equilibrium conditions. Now it is reasonable to suppose that the potential jumps at the surface of metal and ether or metal and metal are determined solely by forces operative at the surfaces, little if any affected by the presence of an electron gas outside. In a system of two metals in contact there may be jumps of potential at four places; at the three surfaces of separation of metal-ether or metal-metal, and through the ether between points outside the two metals. The sum of the four drops is zero, because the electrostatic field is conservative. Three of the drops are the same, probably, whether equilibrium is reached or not. The fourth drop, which is the Volta drop, is therefore also probably the same whether equilibrium is reached or not. We may therefore apply our thermodynamic reasoning to systems in which equilibrium has been attained with high probability that the Volta drop appearing in the formulas is the same as that measured under ordinary conditions.

We are now in a position to obtain V. Taking logarithms of the equation (12) above,

$$V_{BA} = \frac{kt}{\epsilon} \log \frac{n_B}{n_A}.$$

Substituting values of n

$$V_{BA} = \frac{kt}{\epsilon} \left\{ \log \frac{A_B}{A_A} + \int \frac{\eta_B - \eta_A}{kt^2} dt \right\}.$$

Now A_A and A_B are independent of t. Hence, differentiating the above equation,

$$\eta_A - \eta_B = \epsilon V_{BA} - \epsilon t \frac{dV_{BA}}{dt}. \tag{13}$$

It is to be noticed that during this differentiation all the other variables on which the quantities might conceivably depend are to be kept constant. In particular, if V_{BA} is possibly a function of the total charge on the surface, we should have to keep the surface density constant during the differentiation, and dV_{BA}/dt would become $(\partial V_{BA}/\partial t)_p$, as in the previous deduction by the condenser method.

We may obtain another relation by taking a gram of electrons around a closed cycle between A and B. This cycle may be performed in the following way. Imagine separate enclosures about A and B, containing the electron gas, and connected by a pipe in which there is a piston (see Fig. 2). The piston is to be pushed toward the enclosure about B and withdrawn by the corresponding amount from that about A. This transfers electrons from the gas about B to the gas about A. During the process of transfer of electrons from the atmosphere about B, through the metal, to the atmosphere about A, there has obviously been no change in the surface charges on A or B. The latent heats of vaporization involved in this process will therefore be the heats at constant charge. Now the process outlined above is isothermal and reversible, and therefore the total heat absorbed is zero. This gives

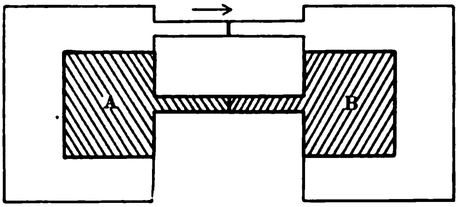

Fig. 2.

$$\eta_{pA} - \eta_{pB} + \epsilon P_{BA} + Q = 0.$$

P_{BA} is the ordinary Peltier heat at the junction. Q is the heat absorbed by the gas per electron in passing under equilibrium conditions from the gas about A to that about B. In this transfer the gas remains isothermal, and pressure and potential change simultaneously as equilibrium demands.

Now the analysis of the introduction shows that when a gas is displaced in this way in a potential field the mechanical work done by the pressures acting across the boundaries is zero. The first law of thermodynamics demands, therefore, that the heat inflow shall equal the change of energy. This change is composed of two parts, energy of position in the field, and intrinsic energy. But now the intrinsic energy of a gas does not change at constant temperature. Hence

$$Q = \epsilon V_{AB}.$$

We may apply our definition of E.M.F. given in the introduction and find the E.M.F. between two points in the electron gas outside A and B to be V_{AB}. In this particular case, therefore, electrostatic potential difference, local E.M.F., and heat locally absorbed are numerically equal.

The reason is that in this particular case (electron gas) the intrinsic energy of the electrons is a function of temperature only. We shall prove later that this equality cannot hold in general.

The Q found as above is the same as that which would be found from the ordinary formulas for a substance not in a field of force. This calculation gives

$$Q = \int_{p_A}^{p_B} \left(\frac{\partial Q}{\partial p}\right)_t dp. = - \int_{p_A}^{p_B} t \left(\frac{\partial v}{\partial t}\right)_p dp.$$

For a perfect gas this becomes

$$Q = - kt \int_{p_A}^{p_B} \frac{dp}{p} = kt \log \frac{p_A}{p_B} = kt \log \frac{n_A}{n_B}.$$

But we have seen that the condition of equilibrium in a potential field is

$$\log \frac{n_A}{n_B} = - \frac{\epsilon V_{BA}}{kt}.$$

Substituting,

$$Q = \epsilon V_{AB},$$

as before.

We would not be justified, however, in assuming that the other formulas of thermodynamics are not affected by the presence of the potential field. In particular, the work done by a gas in expanding isothermally against the pressures exerted across its boundaries is Q when there is no potential field. We have just seen that this work is zero in a potential field. Work in a potential field is therefore not equal to $\int p dv$.

Instead of saying as above that the heat absorbed goes directly toward increasing the energy of position, we may, if we prefer, describe the phenomena by saying that the heat absorbed goes to the work which would normally be done by the gas in expanding out of a potential field, and that besides this work, additional work is received across the boundaries when expanding in a potential field, which goes to increase the energy of position.

Now substituting the value found for Q gives

$$\eta_{\rho A} - \eta_{\rho B} = \epsilon(V_{BA} - P_{BA}). \tag{14}$$

An equation similar in appearance was given by Richardson, in which $\eta_{\rho A} - \eta_{\rho B}$ is replaced by $\varphi_A - \varphi_B$. He obtained the equation by identifying φ_A, φ_B and P_{AB} with the local E.M.F.'s, and by disregarding the difference between the heats of vaporization at constant surface density and with variable density.

In a recent paper, Langmuir (18), essentially following Richardson's point of view, has also failed to recognize the difference between the heat of vaporization at variable surface density, which appears in the thermodynamic analysis, and the heat under constant surface density, which may be measured experimentally. Langmuir's calculation of the magnitude of the heat of vaporization from the work done by an electron in escaping from the attraction of its image in the surface applies to vaporization with variable surface density, and not to the experimental conditions of vaporization at constant density. His argument for the probable non-existence of the double layer on the surface metal-ether would therefore seem to me to lose much of its force. This failure to distinguish between the two heats of vaporization would also seem to be involved in Langmuir's formulation (on page 172 of his paper) of the distinction between a potential difference and an E.M.F.

Combining the two equations (13), and (14), for $\eta_{\rho A} - \eta\rho_{\rho B}$, with (11) gives,

$$t\frac{dV_{BA}}{dt} = P_{BA} + P_{AS} - P_{BS}. \tag{15}$$

Now if positive charge is passed from the surface of B to A through the metal, as in the condenser analysis previously given, negative charge appears on B and positive on A, and there may be accompanying heating effects in three places, at the two surfaces of separation of metal from ether, and at the surface metal-metal. The sum of these three heats is what was previously called P_{BA}'. Obviously

$$P_{BA}' = P_{BA} + P_{AS} - P_{BS}, \tag{16}$$

and we have

$$t\frac{dV_{BA}}{dt} = P_{BA}', \tag{17}$$

which is the equation (8) of Lorentz and Kelvin previously given. As already stated, it would be more correct, until more definite experimental evidence is at hand to write $\partial V_{BA}/\partial t$ instead of dV_{BA}/dt.

The deduction of this same equation by methods so different is pretty good presumptive evidence of its correctness.

The equation corresponding to the above given by Richardson is

$$t\frac{dV_{BA}}{dt} = P_{BA},$$

where P_{BA} is the ordinary Peltier heat. The surface heats do not appear in his equation, because, as already explained, he has neglected to distinguish latent ~~at~~ heats at constant from those at variable surface charge.

Assuming Richardson's formula to be correct, we may combine with the formulas for a thermo-electric circuit, obtaining the relation

$$\frac{dV_{AB}}{dt} = \frac{dE_{AB}}{dt}.$$

This consequence of Richardson's relation is frequently supposed to be correct,[1] but obviously is not correct if there is a surface heat.

The formula above as given by Richardson has never been verified experimentally. The failure of experimental verification may very well be due in large part to experimental difficulties. Recent work of Langmuir (19) has shown the enormous effect on a metal surface of a layer of adsorbed gas only one molecule deep. No experiments on this subject have ever been made in which this layer has been removed. Perhaps the most recent work is that of Compton (20), in which there is a large possible effect from residual gas, as he himself explicitly states. The experimental discrepancy has been reduced by this latest work of Compton from the thousand fold of Kelvin to fifty fold. There is still, however, as far as the best experimental evidence goes, ample room for the existence of the surface heat.

In view of the important rôle which the equation of Richardson plays in the subject of thermionic emission, the question of the existence of the surface heat assumes considerable interest. A direct experimental attack on the problem does not appear promising. Assuming in the most favorable case that the entire outstanding discrepancy in the formula

$$t\frac{dV_{BA}}{dt} = P_{BA}$$

is due to the hypothetical surface heats, the rise of temperature of a conductor on imparting to it a surface charge may be computed approximately. The greatest rise of temperature will be produced on thin conductors, such as wires or foil. But under these conditions the greatest effect to be expected is entirely overwhelmed by the heating effect due to the mechanical stress produced in the conductor by the mutual repulsion of the charges on the surface. (It is to be remarked that the η above does not include this electrostriction effect, which may be made vanishingly small by working with large enough conductors.) The best chance of detecting the existence of the surface heat is by an experimental determination of dV_{BA}/dt, improving the vacuum conditions as much as possible, or by a method to be suggested later.

Professor Hall in a recent paper (17) has stated his belief in the existence of the surface heats. He argues for their existence by giving such a picture of the mechanism involved in a redistribution of surface charges as to make a heating effect seem probable. His conception is that at every point within and on the surface of a metal there is a dissociation of atoms into positive and negative ions, whose relative number is determined by the mass law. A disturbance of equilibrium by the removal of electrons from the surface layer is followed by such a change in the surface dissociation as to again satisfy the mass law. This new dissociation, it is most natural to suppose, is accompanied by heating effects.

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[1] See, for example, Langmuir, *loc. cit.*, page 182.

The introduction of the idea of a dissociation determined by the mass law is, I believe, a valuable addition to our stock of concepts of what may be taking place within a metal, and its application to the surface effects is one way of avoiding one very real difficulty. This difficulty is encountered in trying to picture the processes involved in giving a metal surface a positive charge. Let us suppose the surface initially uncharged, without surface layers. Then the surface may be given a negative charge by the mere addition of electrons, but to produce a positive charge, electrons must be removed from the interior of the atoms. These two processes are essentially different in character, and the last must almost of necessity involve a heating effect, which the first need not. There is thus a dissymmetry in the action of a positive and a negative charge which we are not willing to admit. The assumption of a continual supply of positive and negative ions in the surface, as Professor Hall suggests, avoids this dissymmetry, and to that extent has intrinsic probability. Nevertheless the extension of the concept of mass action to a surface layer seems questionable, and I believe the difficulty can be met in another way. If we suppose the surface is covered with a double layer, the electrons being on the side away from the metal, then a negative charge is added by adding electrons to the outer layer, and a positive charge is added by removing electrons from the outer layer. There is thus no difficulty from dissymmetry. I prefer, therefore, to regard the question of the existence of this surface heat as still open, to be settled by new experimental evidence, but incline to the opinion that it does exist.

We may now combine these thermionic formulas with those of the thermo-electric circuit. We introduce the following new quantities in addition to those already considered.

S_{AE} = potential jump at surface of separation of A and ether,
S_{BE} = potential jump at surface of separation of B and ether,
S_{AB} = potential jump at surface of separation of A and B,
Σ_{AE} = E.M.F. at surface of separation of A and ether,
Σ_{BE} = E.M.F. at surface of separation of B and ether,
Σ_{AB} = E.M.F. at surface of separation of A and B,
σ_A' = Thomson E.M.F. in A,

σ_B' = Thomson E.M.F. in B,
σ_A'' = Thomson potential gradient in A,
σ_B'' = Thomson potential gradient in B.

It may of course be questionable whether there is any E.M.F. at the surface of separation of A and ether, and it is any event exceedingly improbable that the surface action can be completely represented by an E.M.F. It may be, however, that part of the action is properly so representable. We denote this part of the action by the letters above, and find what conditions it has to satisfy.

With regard to the Thomson potential gradient it is to be emphasized that this is entirely different in character from the surface jumps S_{AE} or S_{AB}. A jump of the potential at the surface necessarily involves a double layer at the surface; the jump is entirely determined by the double layer, and is unaffected by charges elsewhere. The surface jump is therefore probably determined by forces at the surface, which are probably characteristic of the surface and independent of the state of other parts of the system. The Thomson potential gradient (*i.e.*, the potential gradient in an unequally heated metal) is, however, affected by the distribution of charge throughout the system, and can therefore be characteristic of the system only under specified conditions. We shall in the following understand by σ'' the potential gradient in open circuit. The Thomson E.M.F. and Thomson heat are, on the other hand, probably determined merely by the local forces, independently of other parts of the system. We shall assume then to be of this character in the following.

We now have the following equations;

$$V_{BA} = \frac{1}{\epsilon}(\eta_{pA} - \eta_{pB}) + P_{BA}, \tag{18}$$

$$V_{BA} = S_{AE} - S_{BE} + S_{BA}, \tag{19}$$

$$V_{BA} = \Sigma_{AE} - \Sigma_{BE} + \Sigma_{BA}. \tag{20}$$

These three depend on the fact, already proved, that in the electron gas potential difference, E.M.F., and heat are locally equal. These three equations are obtained from the isothermal system by describing a closed path out of the ether into B, across the surface of separation of B and A, out into the starting point in the ether. By describing a closed path entirely within the metal, maintaining now the junctions of A and B at different temperatures, we may obtain the following three additional equations

$$\frac{dE_{BA}}{dt} = \frac{d\Sigma_{BA}}{dt} + \sigma_B' - \sigma_A', \tag{21}$$

$$\frac{dE_{BA}}{dt} = \frac{dP_{BA}}{dt} + \sigma_B - \sigma_A, \tag{22}$$

$$\frac{dE_{BA}}{dt} = \frac{dS_{BA}}{dt} + \sigma_B'' - \sigma_A''. \tag{23}$$

The last of these is the expression for the difference of potential between the ends of A and B on open circuit, assuming that the E.M.F. of a thermo-electric circuit is the same on open and closed circuit. This is matter for experimental examination; I do not know how carefully this examination has been made. Certainly the equality of E.M.F. on open and closed circuit is usually assumed without question, and in the absence of experimental evidence to the contrary, we shall make the same assumption here.

We also have the additional equations

$$\frac{P_{BA}}{t} = \frac{dE_{BA}}{dt} = \frac{dV_{BA}}{dt} - \frac{1}{t}[P_{AS} - P_{BS}]. \tag{24}$$

Now differentiating (18) and combining with (22) and (24) gives

$$\frac{d\eta_{\rho A}}{dt} + \epsilon\sigma_A - \frac{\epsilon}{t}P_{AS} = \frac{d\eta_{\rho B}}{dt} + \epsilon\sigma_B - \frac{\epsilon}{t}P_{BS}. \tag{25}$$

Similarly (20), (21), and (24) give

$$\frac{d\Sigma_{AE}}{dt} + \sigma_A' - \frac{P_{AS}}{t} = \frac{d\Sigma_{BE}}{dt} + \sigma_B' - \frac{P_{BS}}{t}, \tag{26}$$

and (19), (23), and (24) give

$$\frac{dS_{AE}}{dt} + \sigma_A'' - \frac{P_{AS}}{t} = \frac{dS_{BE}}{dt} + \sigma_B'' - \frac{P_{BS}}{t}. \tag{27}$$

Hence we have proved that

$$\frac{d\eta_\rho}{dt} + \epsilon\sigma - \frac{\epsilon P_S}{t}, \qquad \frac{d\Sigma_E}{dt} + \sigma' - \frac{P_S}{t}, \qquad \text{and} \qquad \frac{dS_E}{t} + \sigma'' - \frac{P_S}{t}$$

are all independent of the metal.

We may now obtain the universal value of $d\eta_\rho/dt + \epsilon\sigma - \epsilon P_S/t$ by an argument similar to that of Richardson. In this deduction we shall assume that the irreversible processes going on at the same time that we describe the various cycles are without effect.

Surround the two ends of the bar at difference of temperature by two chambers to contain the electron atmosphere (see Fig. 3), with pistons as shown. Electrons are first to be conveyed from one chamber to the other by operating the pistons as shown. The cycle then is to be completed in two steps. First, operate on the transferred gas under potential $V + \Delta V$, allowing it to expand to temperature t and to that pressure, $p + \Delta p'$, which is in equilibrium at t with gas at pressure p at a potential V. The second step consists in the transfer of gas at t from $V + \Delta V$ and $p + \Delta p'$ to V and p. During this second step V and p shall change together in such a way that they always have the simultaneous values to be found in an isothermal gas in equilibrium extendingfrom p, V to $p + \Delta p'$, $V + \Delta V$. To the first of these two steps all the ordinary formulas of thermodynamics apply, because the process is described at constant potential. During the second process, however, the potential is changing, and all of the ordinary formulas do not apply. In particular, we have shown that the work cannot be computed in the regular way, but is zero. We have, however, shown that the heat absorbed, and therefore the change of entropy, is not affected by the fact that the process is described in a varying potential field. The change of entropy may therefore be computed for these two steps together in the regular way, and therefore is independent of the precise way of passing from the initial to the final state.

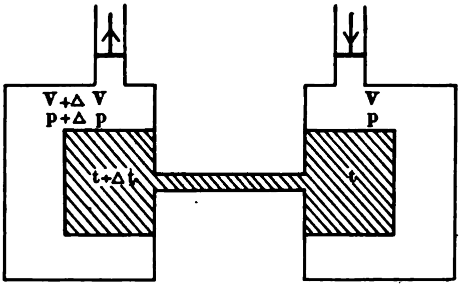

Fig. 3.

Applying the second law to the cycle now gives

$$\frac{1}{\epsilon}\frac{\eta_p}{t} + \frac{\sigma\Delta t}{t + \frac{\Delta t}{2}} + \frac{1}{\epsilon}\,\frac{\eta_p + \Delta t\frac{d\eta_p}{dt}}{t + \Delta t} + \frac{1}{\epsilon}\int d\psi = 0,$$

where $\int d\psi$ is the change of entropy of the gas per electron on being carried from $(t + \Delta t,\ p + \Delta p)$ to $(t,\ p)$, and may be computed by the ordinary thermodynamic formulas. Now

$$d\psi = \left(\frac{\partial\psi}{\partial t}\right)_p dt + \left(\frac{\partial\psi}{\partial p}\right)_t dp = \frac{C_p}{t}dt - \left(\frac{\partial v}{\partial t}\right)_p dp.$$

For a perfect gas, satisfying the equation $pv = kt$,

$$\left(\frac{\partial v}{\partial t}\right)_p = \frac{k}{p},$$

and

$$C_p = \frac{k\gamma}{\gamma - 1},$$

where γ is the ratio of C_p to C_v. Hence

$$\int d\psi = \frac{k\gamma}{\gamma - 1}\int \frac{dt}{t} - k\int \frac{dp}{p}.$$

Substituting in the equation above and taking the limit,

$$\frac{1}{\epsilon}\frac{d}{dt}\left(\frac{\eta_p}{t}\right) + \frac{\sigma}{t} - \frac{1}{\epsilon}\cdot\frac{k\gamma}{\gamma - 1}\cdot\frac{1}{t} + \frac{1}{\epsilon}\cdot\frac{k}{p}\cdot\frac{dp}{dt} = 0.$$

But we have already found that

$$\frac{1}{p}\frac{dp}{dt} = \frac{\eta}{kt^2} = \frac{\eta_p - \epsilon P_S}{kt^2}.$$

Hence the desired relation

$$\frac{d\eta_p}{dt} + \epsilon\sigma - \frac{\epsilon P_S}{t} = \frac{k\gamma}{\gamma - 1}. \qquad (28)$$

Instead of this equation Richardson gives

$$\frac{d\varphi}{dt} + \epsilon\sigma = \frac{k}{\gamma - 1}.$$

It differs from that deduced above in the absence of the surface heat. Richardson uses the value of φ as given by this equation to get the thermionic current. His formula checks with experiment, but is of such a form that only very wide changes in the form of φ could be detected. Thus, using the value he gives for φ, there is a factor t^2 in the value of current (formula 17, page 33), but he states later that equally good agreement is obtained if a factor $t^{1/2}$ replaces t^2. Richardson's data do not, therefore, afford a check on his value for φ. The neglect of the surface heat in Richardson's equation would therefore seem to be indefensible, and until the order of the effect is known, we cannot tell whether Richardson's equation is even approximately correct. It is to be noticed that Richardson's formula would become the same as that developed above in case P_S should be proportional to t. The φ of his formula must then be understood to be the change of energy on evaporation at constant surface charge.

We may now obtain further information by applying the first law to the cycle, writing total work done in the complete cycle equal heat absorbed. Work is done in the cycle when the piston is pushed into the right hand chamber, when it is pulled out from the left hand chamber, and when the gas expands in the first of the two processes mentioned above, but no work is done in the second of the two steps. Heat is absorbed in all of the operations. The first law gives

$$\frac{d}{dt}(pv)\Delta t + \int_{t-\Delta t}^{t}\left(\frac{\partial W}{\partial t}\right)_p dt + \int_{p+\Delta p}^{p+\Delta p'}\left(\frac{\partial W}{\partial p}\right)_t dp$$

$$= \frac{d\eta_p}{dt}\Delta t + \epsilon\sigma\Delta t + \int_{t+\Delta t}^{t}\left(\frac{\partial Q}{\partial t}\right)_p dt + \int_{p+\Delta p}^{p+\Delta p'}\left(\frac{\partial Q}{\partial p}\right)_t dp - \epsilon\Delta V,$$

where

$$\int_{t+\Delta t}^{t}\left(\frac{\partial W}{\partial t}\right)_p dt = \int_{p+\Delta p}^{t} p\left(\frac{\partial v}{\partial t}\right)_p dt = -k\Delta t,$$

$$\int_{p+\Delta p}^{p+\Delta p'}\left(\frac{\partial W}{\partial p}\right)_t dp = \int_{p+\Delta p}^{p+\Delta p'} p\left(\frac{\partial v}{\partial p}\right)_t dp = \frac{\Delta p - \Delta p'}{n},$$

$$\int_{t+\Delta t}^{t}\left(\frac{\partial Q}{\partial t}\right)_p dt = \int_{t+\Delta t}^{t} c_p dt = \frac{-k\gamma}{\gamma - 1}\Delta t,$$

$$\int_{p+\Delta p}^{p+\Delta p'}\left(\frac{\partial Q}{\partial p}\right)_t dp = -\int_{p+\Delta p}^{p+\Delta p'} t\left(\frac{\partial v}{\partial t}\right)_p dp = \frac{\Delta p - \Delta p'}{n}.$$

Substituting these values, the left-hand side cancels out, leaving,

$$0 = \frac{d\eta_p}{dt} + \epsilon\sigma - \frac{k\gamma}{\gamma - 1} - \epsilon\frac{dV}{dt}.$$

But we found that

$$\frac{d\eta_p}{dt} + \epsilon\sigma - \frac{k\gamma}{\gamma - 1} = \frac{\epsilon P_S}{t}.$$

Substituting,

$$\frac{dV}{dt} = \frac{P_S}{t}. \qquad (29)$$

This equation was also obtained by Lorentz (14) by an entirely different method.

Now dV/dt is the Volta difference per degree between different parts of the same unequally heated metal, provided that free circulation of the electrons from hot to cold through the electron gas is prevented. This is the Volta difference which would be observed at temperatures so low that the emission of electrons is negligible. We have here, therefore, a possible experimental method of detecting the existence of the surface heat P.

At temperatures so high that electron emission is not negligible there is no reason to expect that the free electron vapor is in equilibrium with an unequally heated metal, but there is a possibility that electricity may flow in closed circuits, as shown in Fig. 4, through the electron gas and the metal. I am not aware that the existence of such currents has been detected or suggested. If it should happen that such currents do not exist, we may obtain an additional relation. The electrons spontaneously emitted by different parts of the surface must in this case be of such densities as to be in equilibrium under the potential gradient given by $dV/dt = P_S/t$. This gives, together with (9),

$$\frac{\eta}{kt^2} = -\frac{\epsilon}{kt}\frac{dV}{dt},$$

or

$$\eta_p - \epsilon P_S = -\epsilon P_S,$$

or

$$\eta_p = 0.$$

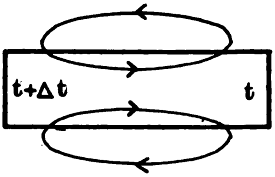

Fig. 4.

Now this condition is almost certainly not satisfied, so that it is exceedingly probable that closed circuits like those indicated do exist in an unequally heated metal.

Having found the value of dV/dt, we may now find the universal value of $d\Sigma_E/dt + \sigma' - P_S/t$ and $dS_E/dt + \sigma'' - P_S/t$. We may obtain the latter immediately by noting that the total change of potential in a closed circuit is zero. This gives

$$\frac{dS_E}{dt} + \sigma'' - \frac{dV}{dt} = 0.$$

Substituting for dV/dt,

$$\frac{dS_E}{dt} + \sigma'' - \frac{P_S}{t} = 0, \qquad (30)$$

the desired relation.

Next to obtain the value of $d\Sigma_E/dt + \sigma' - P_S/t$, we may write down the expression for the total E.M.F. encountered in going from the electron gas surrounding the metal at t, through the metal, to the electron gas at $t + \Delta t$. Our definition of E.M.F. is total change of energy plus work delivered to outside agencies. The change of energy of the electron gas is an intrinsic change ($= k\Delta t$) plus the change of electrostatic energy ($= \epsilon\Delta V$). The work delivered is $d/dt \cdot (pv)\Delta t$. Hence

$$\epsilon\left[\frac{d\Sigma_E}{dt}\Delta t + \sigma'\Delta t\right] = \epsilon\Delta V + k\Delta t + \frac{d}{dt}(pv)\Delta t.$$

Now

$$\frac{d}{dt}(pv)\Delta t = k\Delta t.$$

Hence

$$\frac{d\Sigma_E}{dt} + \sigma' = \frac{dV}{dt} + \frac{2k}{\epsilon},$$

or

$$\frac{d\Sigma_E}{dt} + \sigma' - \frac{P_S}{t} = \frac{2k}{\epsilon}, \qquad (31)$$

the desired relation.

It is to be especially emphasized that the form of the formulas just obtained essentially modifies the point of view hitherto maintained. Our position up to this point in this paper has been one of agnosticism; we have been obliged to retain separate expressions for local E.M.F., heat, and potential difference because there seemed no necessity that these should be equal, although such equality is usually assumed. But now the fact that the formulas obtained for

$$\frac{d\eta_P}{dt} + \epsilon\sigma - \frac{\epsilon P_S}{t}, \qquad \frac{d\Sigma_E}{dt} + \sigma' - \frac{P_S}{t}, \qquad \text{and} \qquad \frac{dS_E}{dt} + \sigma'' - \frac{P_S}{t}$$

are of different form constitutes positive proof that local E.M.F., heat, and potential difference cannot be mutually equal, and that the assumption of such equality is in general positively incorrect. The assumption can be valid only in certain special cases, as in an electron gas.

Richardson has frequently set local E.M.F. and local heat equal to each other. This is in general incorrect, but will not necessarily lead to error in those processes in which a complete cycle is described.

Accepting now as proved that in general local heat, potential difference, and E.M.F. cannot be mutually equal, we will, in the next section, deduce additional relations between them.

THE GENERALIZED VOLTA LAW OF TENSIONS.

In a complete circuit composed of three metals in equilibrium at uniform temperature we have the following four relations:

$$V_{AB} + V_{BC} + V_{CA} = 0 \qquad (32)$$

$$S_{AB} + S_{BC} + S_{CA} = 0 \qquad (33)$$

$$\Sigma_{AB} + \Sigma_{BC} + \Sigma_{CA} = 0 \qquad (34)$$

$$P_{AB} + P_{BC} + P_{CA} = 0. \qquad (35)$$

The first of these is a statement that the total change in electrostatic potential in a closed path outside the metals is zero; the second is the same statement for a closed path within the metals; the third states that the total E.M.F. in a circuit in the metal is zero; and the fourth that the total heat absorbed in a cycle by electricity is zero. The first two are a mathematical consequence of the properties of an electrostatic distribution, and the last two are a consequence of the law of the conservation of energy, because in a system of solids all at one temperature there is no permanent source of energy. It is not possible that these relations should not be satisfied; the system must of necessity so adjust itself that they are.

Of the four relations above, the first is the ordinary Volta law of tensions. The point of the Volta law is not that the laws of electrostatics or of the conservation of energy are satisfied, but that each of the quantities in the above equations, V_{BC} for example, depends only on the two metals B and C, and is unaffected by the presence in the circuit of the third metal A. The proof of this fact is given by experiment.

Of the four equations, the quantities in only two of them, the first and the last, are susceptible of direct observation. It is proved experimentally that P_{BC}, for example, as well as V_{BC}, is independent of the presence in the circuit of the third metal. Therefore the Peltier heats satisfy the same conditions as the Volta difference of potential.

By the "generalized" Volta law of tensions we shall understand the correctness of the four equations above, and in addition a statement that any of the four types of action at the surface of two meals is independent of the presence of a third.

The quantities entering the second and third equations above are not directly measurable, and it is therefore pertinent to inquire whether the generalized Volta law holds; that is, whether S_{BC}, for example, is independent of the presence of A. The answer to this question is yes. Although a direct experimental proof is at present impossible, a theoretical proof may be given, because of certain relations holding between S and Σ and V.

It may be proved mathematically that the ordinary Volta law as partly expressed in the first of the above equations demands that each of the quantities may be split up into two quantities, each depending on only one of the metals at the junction. That is, it is possible to write $V_{BC} = V_C - V_B$, where V_B depends only on the properties of B and V_C only on those of C. This analysis is physically significant because it represents the action at the surface of B and C as the joint result of independent actions by B and C, each unaffected by the presence of the other. The proof of the generalized Volta law will be given by similarly splitting each of the quantities above into the difference of two quantities, each depending only on the properties of one of the metals at the junction.

This analysis is immediately effected from the expression for V_{AB}.

We have

$$\frac{n_A}{n_B} = e^{-(\epsilon V_{BA})/kt}.$$

Whence

$$V_{AB} = \frac{kt}{\epsilon}\log\frac{n_A}{n_B},$$

and we write at once

$$V_{AB} = V_B - V_A,$$

where

$$V_A = -\frac{kt}{\epsilon}\log n_A. \qquad (36)$$

It is to be noticed that this analysis is not unique. We might equally write, for example,

$$V_A = -\frac{kt}{\epsilon}\log p_A,$$

which differs from the above by a quantity independent of A, but depending on the temperature.

We may now split up the other quantities with the help of equations (18), (19) and (20).

$$P_{BA} = V_{BA} + \frac{1}{\epsilon}(\eta\rho_B - \eta\rho_A) = P_A - P_B,$$

and

$$P_A = -\frac{kt}{\epsilon}\log n_A - \frac{1}{\epsilon}\eta_{\rho A} \qquad (37)$$

$$S_{BA} = V_{BA} - S_{AE} + S_{BE} = S_A - S_B,$$

and

$$S_A = -\frac{kt}{\epsilon}\log n_A - S_{AE} \qquad (38)$$

$$\Sigma_{BA} = V_{BA} - \Sigma_{AE} + \Sigma_{BE} = \Sigma_A - \Sigma_B,$$

and

$$\Sigma_A = -\frac{kt}{\epsilon}\log n_A - \Sigma_{AE}. \qquad (39)$$

With regard to the last two it is to be said that it is mathematically conceivable that S_{AE} and Σ_{AE} should depend on the presence of the other metal, but physically it appears excessively improbable. Granting this assumption, this analysis shows that the generalized Volta law of tensions applies to the unmeasurable quantities S and Σ as well as to V and P.

Instead of the quantity Σ, we may, if we prefer, introduce the intrinsic energy of electricity, u, in the conductor. Taking the energy of a perfect gas to be zero at absolute zero of temperature, we obtain, on applying our definition of impressed E.M.F. between two points

$$\epsilon\Sigma_{AE} = \tfrac{3}{2}kt - \epsilon u_A + \epsilon S_{AE} + kt,$$

whence

$$\Sigma_{AE} = \tfrac{5}{2}kt - u_A + S_{AE}. \tag{40}$$

No simplification of the formulas is obtained on replacing Σ in terms of u. For theoretical discussions of the electron mechanism, however, it may be preferable to make this substitution.

THE EFFECT OF HYDROSTATIC PRESSURE ON THE VOLTA EFFECT.

The condenser analysis by which the effect of temperature on the Volta effect was determined may be modified to include the effects of hydrostatic pressure. The system of two metals A and B shall be characterized by the following variables.

t = absolute temperature,

ρ = positive charge per unit area on A (as before there is an equal opposite charge on B),

c = capacity per unit area,

π = hydrostatic pressure per unit area on the plate A (the symbol π is used for pressure to avoid confusion with p, the vapor pressure of the electron atmosphere).

There is no essential restriction in discussing the effect of a pressure applied to the plate A alone. After the formulas for this case have been developed, the effect of pressure on B alone may be found simply by interchanging the letters in the analysis, and then the effect of independently variable pressure on both A and B by adding the two effects.

The following dependent variables are to be discussed:

V_{BA} = Volta rise of potential on passing from B to A (see previous discussion for more detailed specification),

u = internal energy of system per unit area,

dW = work done by the system in any infinitesimal change,

dQ = heat absorbed by the system in any infinitesimal change,

v = volume of A per unit area.

Now, as before, we must have $(du + dW)/t$ an exact differential for reversible changes. We have

$$dW = \frac{\partial W}{\partial t}dt + \frac{\partial W}{\partial \rho}d\rho + \frac{\partial W}{\partial c}dc + \frac{\partial W}{\partial \pi}d\pi$$

and

$$du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial \rho}d\rho + \frac{\partial u}{\partial c}dc + \frac{\partial u}{\partial \pi}d\pi.$$

We are able to completely calculate dW from our knowledge of the mechanics of the system. We have

$$\frac{\partial W}{\partial t}=\pi\frac{\partial v}{\partial t},$$

$$\frac{\partial W}{\partial \rho}=-\left(\frac{\rho}{c}-V_{BA}\right)+\pi\frac{\partial v}{\partial \rho},$$

$$\frac{\partial W}{\partial c}=\frac{1}{2}\frac{\rho^2}{c^2}+\pi\frac{\partial v}{\partial c},$$

$$\frac{\partial W}{\partial \pi}=\pi\frac{\partial v}{\partial \pi}.$$

Substituting above,

$$\frac{1}{t}\left\{\left[\pi\frac{\partial v}{\partial t}+\frac{\partial u}{\partial t}\right]dt+\left[\pi\frac{\partial v}{\partial \rho}-\frac{\rho}{c}+V_{BA}+\frac{\partial u}{\partial \rho}\right]d\rho\right.$$
$$\left.+\left[\frac{1}{2}\frac{\rho^2}{c^2}+\pi\frac{\partial v}{\partial \pi}+\frac{\partial u}{\partial c}\right]dc+\left[\pi\frac{\partial v}{\partial \pi}+\frac{\partial u}{\partial \pi}\right]d\pi\right\}$$

must be an exact differential. This will give us six relations between the coefficients of the six possible pairs of differentials.

The work for finding these derivatives is entirely straightforward; only the results need be given here.

From the coefficients of dt and $d\rho$,

$$\frac{\partial V_{BA}}{\partial t}=\frac{1}{t}\frac{\partial Q}{\partial \rho}=\frac{1}{t}P_{BA}'. \tag{41}$$

From the coefficients of dt and dc

$$\frac{\partial Q}{\partial c}=0. \tag{42}$$

From the coefficients of dt and $d\pi$,

$$\frac{\partial Q}{\partial \pi}=-\tau\frac{\partial v}{\partial t}. \tag{43}$$

From the coefficients of $d\rho$ and dc,

$$\frac{1}{t}\frac{\partial V_{BA}}{\partial c}=0. \tag{44}$$

From the coefficients of $d\rho$ and $d\pi$,

$$\frac{\partial V_{BA}}{\partial \pi}=-\frac{\partial v}{\partial \rho}. \tag{45}$$

From the coefficients of dc and $d\pi$,

$$\frac{1}{t}\frac{\partial v}{\partial c}=0. \tag{46}$$

Of these six relations the three which do not involve $d\pi$, (41, 42 and 44) are the same as those obtained by the previous analysis, except that the partial derivatives now involve the condition of constancy of π, a variable which did not enter in the previous work. Of the three remaining relations, 46 states that there is no change of volume on merely changing the distance between the plates, and 43 gives the ordinary thermodynamic relation for the heat absorbed during a change of pressure, it now being necessary to add the conditions of constant ρ and c to the ordinary condition of constant t during change of pressure. The remaining relation, 45, is the only one of the six which gives any essentially new information. This equation expresses the change in Volta difference between two metals, when one is subjected to pressure, in terms of the change of volume of that metal when unit positive charge is imparted to it. This change of volume is not an electrostriction effect, which varies as ρ^2 and depends on the shape of the metals, but is to be thought of as the effective volume of the electrons added or subtracted. There seems to be no experimental evidence for even the order of magnitude of $\partial v/\partial \rho$.

Within the range in which $\partial v/\partial \rho$ may be regarded as constant, this formula gives the Volta contact difference of potential between compressed and uncompressed metal.

$$V_{0\pi} = -\pi \frac{\partial v}{\partial \rho}. \tag{47}$$

The formula

$$\frac{\partial V_{BA}}{\partial \pi} = -\frac{\partial v}{\partial \rho}$$

may also be obtained by considering the effect of hydrostatic pressure in A on the pressure of the electron vapor in equilibrium with it through a stress resisting membrane. The formulas to be applied are well known; they may, for example, be obtained as special cases of formulas which I have given in a previous paper (21). The applicability of such formulas to such conditions as these becomes at least questionable, however, when one calculates that the density of the electron vapor in equilibrium with tungsten at 0° C., for example, is such that there is one electron in a sphere of radius 350 light years. The method of analysis used above makes no reference to the electron gas, and leads to the same results for those quantities not directly concerned with the vapor.

Effect of Pressure on Thermo-electric Properties.

A certain amount of information may now be obtained regarding the effect of pressure on thermo-electric properties other than the Volta effect. This may be obtained from the formulas previously given, taking as the metals A and B the same metal in two states, one without pressure and the other with it. The same double subscript notation as before may be used. Thus $S_{0\pi}$ denotes the potential jump in passing within the metal through the surface separating uncompressed metal from metal compressed to the pressure π.

For the Peltier heat in passing from uncompressed to compressed metal in terms of the E.M.F. of a couple composed of uncompressed and compressed metal, formula 3 gives

$$P_{0\pi} = t\frac{dE_{0\pi}}{dt}. \tag{48}$$

For the difference between the Thomson heats in the compressed and uncompressed metal, formula 4 gives,

$$\sigma_\pi - \sigma_0 = t\frac{d^2E_{0\pi}}{dt^2}. \tag{49}$$

For the effect of pressure on the density of an electron gas in equilibrium with a compressed metal formula 12, combined with formula 47 for the Volta difference of potential between compressed and uncompressed metal, gives

$$\frac{1}{n}\frac{\partial n}{\partial \pi} = \frac{\epsilon}{kt}\frac{\partial v}{\partial p}. \tag{50}$$

For the effect of pressure on the latent heat of vaporization of electrons under variable surface charge (total charge on metal and in the gas remaining constant) formula 13 gives

$$\eta_\pi - \eta_0 = \epsilon V_{0\pi} - \epsilon t\frac{dV_{0\pi}}{dt}. \tag{51}$$

For the effect of pressure on the latent heat of vaporization at constant surface charge formula 14 gives

$$\eta_{\rho\pi} - \eta_{\rho 0} = \epsilon(V_{0\pi} - P_{0\pi}). \tag{52}$$

For the effect of pressure on the surface heat, we get from 11 and 51 and 52

$$P_S(\pi) - P_S(0) = t\frac{dV_{0\pi}}{dt} - P_{0\pi}. \tag{53}$$

Of the six quantities treated in the last six equations, the first two, namely the effect of pressure on Peltier and Thomson heats, have been determined experimentally (1) from measurements on a "pressure" thermo-couple. The last four, namely the effect of pressure on electron gas density, the two latent heats, and the surface heat, cannot at present be determined numerically, because they involve the "pressure" Volta effect ($V_{0\pi}$) in addition to the "pressure" Peltier heat ($P_{0\pi}$). A determination of the pressure Volta effect, in addition to present knowledge, would give immediately all four of these quantities. Of course there are relations between the effects of pressure on these four quantities which may be obtained by eliminating $V_{0\pi}$ and $P_{0\pi}$ between the several equations.

In addition to the pressure effects 48–53, we may also obtain expressions for the effect of pressure on the various potential jumps and surface E.M.F.'s, but these can no longer be determined in terms only of $E_{0\pi}$ and $V_{0\pi}$, but involve an actual determination of some potential jump or surface E.M.F., which we are not yet in a position to make.

For the potential jumps, formula 19, gives

$$(S_{\pi E} - S_{0E}) + S_{0\pi} = V_{0\pi}. \tag{54}$$

For the surface E.M.F.'s, formula 20 gives

$$(\Sigma_{\pi E} - \Sigma_{0E}) + \Sigma_{0\pi} = V_{0\pi}. \tag{55}$$

For the Thomson potential gradient, formula 23 gives

$$\sigma_\pi'' - \sigma_0'' = \frac{dS_{0\pi}}{dt} - \frac{dE_{0\pi}}{dt}. \tag{56}$$

For the Thomson E.M.F., formula 21 gives

$$\sigma_\pi' - \sigma_0' = \frac{d\Sigma_{0\pi}}{dt} - \frac{dE_{0\pi}}{dt}. \tag{57}$$

Formulas 28, 30 and 31 give nothing new when applied to pressure effects.

With regard to the quantity $V_{0\pi}$ $(= -\pi \cdot \partial v/\partial p)$, we can possibly obtain an idea of the order of magnitude within 100 or 1,000 fold by assuming that all the change of volume on imparting a charge to a metal is the volume of the electrons added. Assuming that the radius of an electron is 1.8×10^{-13}, this gives for a lower limit

$$\frac{\partial v}{\partial p} = -1.7 \times 10^{-19} \text{ c.c.}$$

per coulomb. This is so excessively minute that the actual $\partial v/\partial p$ is probably forever beyond the reach of direct observation.

In view of the extreme smallness of the probable effect of pressure on the Volta potential difference it is natural to inquire whether we may not neglect this effect in comparison with the directly measurable $P_{0\pi}$, and so obtain an approximate idea of the order of the effect of pressure on the quantities given in equations 50–53. It turns out that this is probably not allowable. The experimental data give for molybdenum, for example, the value 1.5×10^{-18} for the effect of 1 dyne pressure on the Peltier heat. For most metals, the effect is of the order of 10 times greater, or 100 times the volume effect as calculated above from the volume of one coulomb of electrons. In view, however, of the extreme uncertainty in this latter value, I do not believe that a margin of 100 fold is sufficient, and that we would be justified in neglecting the effect of pressure on Volta effect in comparison with that on Peltier heat.

EFFECT OF MELTING OR CHANGE OF STATE ON THE VOLTA POTENTIAL DIFFERENCE AND THERMO-ELECTRIC PHENOMENA.

The Volta difference of potential between a metal in two states of aggregation, as for example solid and liquid, may be found by a slight modification of the preceding condenser analysis. This may be done by choosing for the independent variables t, ρ, c and volume, instead of t, ρ, c and pressure. The details of the work are the same as before. Six differential relations are obtained as before. The only essentially new one of these is

$$\left(\frac{\partial V_{BA}}{\partial v}\right)_{t,\rho,c} = \left(\frac{\partial \pi}{\partial \rho}\right)_{t,c,v}. \tag{58}$$

If this equation is applied to melting or other change of state, we have the statement that the change in Volta contact difference of potential between B and A when A melts is equal to the change of volume on melting multiplied by the change in the melting pressure of A when unit quantity of electricity is added to the exterior surface of A at constant temperature, volume, and capacity. In virtue of the Volta law of tensions, this becomes, if we denote the two phases by subscripts 1 and 2,

$$V_{21} = (v_1 - v_2)\left(\frac{\partial \pi}{\partial \rho}\right)_{\tau}. \tag{59}$$

The effect of a surface charge on melting point is one on which there seems to be no experimental evidence. It is not probable that a charge on the surface could change the equilibrium conditions between liquid and solid at points within the metallic mass, for there is no electric field at such points, and there is no real charge on a surface of separation of solid and liquid metal. It is possible, however, that a surface charge should have a surface effect on melting. For instance, the charged surface of a solid metal might possibly melt at a lower temperature than the interior, or conversely, the charged surface of a liquid might solidify at a higher temperature than the interior.

We may obtain partial information as to the effect of a change of state on the other thermo-electric and thermionic quantities by using the subscripts A and B of the first sections of this paper for the same metal in two different states of aggregation. The work is exactly similar to that of the section on pressure effects, and a series of formulas would be obtained parallel to 48–57.

Very little experimental work has been done in this field. So far as I know there are no measurements of the Volta potential difference between solid and liquid metal, so that there is no way at present of telling the order of magnitude of certain of the quantities in the above equations. Experiments have been made, however, on the thermal E.M.F. of closed circuits in which one of the metals passes through the melting point. There has been considerable disagreement about the facts, but the latest work (22) seems definitely to establish that there is a discontinuous change in the direction of the E.M.F. curve on melting. Attempts have been made to deduce from these data information as to the behavior of the Volta effect, etc., but all such attempts have involved some of the special assumptions about equality of local E.M.F.'s, heats, and potential differences which we are unwilling to admit.

Hitherto the electron theory has been unsuccessful in explaining the effects on melting, in most every case giving the wrong sign to the effect.

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THE MAGNETIC PROPERTIES OF SOME RARE EARTH OXIDES AT LOW TEMPERATURES.

BY E. H. WILLIAMS.

SYNOPSIS.

The magnetic properties of several rare earth oxides have been studied at temperatures varying from room temperature to − 150° C. and the results compared with values previously obtained at higher temperatures.

It is found that, while good agreement is obtained at room temperature where the two measurements overlap, the value of θ in the expression $X(T + \theta) = \text{const.}$, is smaller for low temperatures than for high temperatures, thus indicating that Curie's law is followed more closely at low temperature than at high temperature for the oxides investigated. This is opposite to what H. K. Onnes and A. Pirrier found for oxygen.

IN a previous article[1] the author gave the results obtained in the study of the magnetic properties of some rare earth oxides at temperatures above that of ordinary room temperature. Since making the above investigation the apparatus has been reconstructed to adapt it to low temperature measurements and the magnetic properties of the same oxides have been studied down to temperatures of about − 150° C.

The two investigations overlap each other at room temperature measurements so that it is possible to check one against the other. This is important inasmuch as the previous values found by the author for the magnetic susceptibility of the various rare earth oxides were lower than those found by other experimenters, among whom was O. A. Randolph, of this department, who measured the magnetic susceptibility of several of the above oxides at temperatures ranging from room temperature to − 140° C.

The method used was the same as that employed for high temperatures. Low temperatures were obtained by the method described by C. T. Knipp.[2] It was possible to make determinations of the susceptibility at temperatures as low as − 150° C. Determinations of the magnetic susceptibility were made at three or four temperatures between that and room temperature.

The temperatures were measured by means of a copper-advance thermo-couple of No. 40 wire which was calibrated both before and after making the magnetic measurements and was found to have remained practically constant. One junction of the thermo-couple was kept in melting ice and the other was imbedded in the oxide whose temperature was to be determined. It was necessary to correct for conduction of heat along the wires leading to the junction in the oxide since the temperature gradient near this junction was quite large. At − 150° C. the correction was 5.7° ± 0.3°. This was the largest source of error in the temperature determinations.

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[1] PHYS. REV., N.S., Vol. 12, 158, 1918.

[2] PHYS. REV., Vol. 15, 125, 1902.

After reconstructing the apparatus to adapt it to low temperature measurements it was necessary to redetermine all constants so that, experimentally, the values obtained at low temperatures were entirely independent of those obtained at high temperatures.

Measurements were made of the magnetic susceptibility of the oxides of seven of the rare earths, namely, dysprosium, erbium, gadolinium, samarium, neodymium, lanthanum and yttrium. The results for the three oxides having the highest magnetic susceptibility are shown in Tables I., II. and III. The values given in these tables were taken from curves obtained by plotting the observed results. A comparison of the values of the magnetic susceptibility at 20° C. given in these tables with the values found at the same temperature in the previous work shows that, in all cases, the agreement is within experimental error.

TABLE I.

Dysprosium Oxide. 99.5 + Per Cent. Pure.

t.	T.	$\chi \times 10^6$.	$\chi T \times 10^3$.	$\chi (T + 15) \times 10^3$.
20	293	233.3	68.4	71.9
0	273	250.0	68.2	72.0
− 40	233	291.3	67.9	72.2
− 80	193	347.4	67.0	72.3
−100	173	384.2	66.5	72.2
−120	153	430.2	65.8	72.3
−140	133	490.0	65.2	72.5

TABLE II.

Erbium Oxide. 99.6 + Per Cent. Pure.

t.	T.	$\chi \times 10^6$.	$\chi T \times 10^3$.	$\chi (T + 13.5) \times 10^3$.
20	293	188.6	55.3	57.8
0	273	201.7	55.1	57.8
− 40	233	234.8	54.7	57.9
− 80	193	282.3	54.5	58.3
−100	173	314.8	54.5	58.7
−120	153	355.0	54.3	59.1
−140	133	402.8	53.6	59.0

TABLE III.

Gadolinium Oxide. 99 + Per Cent. Pure.

T.	T.	$X \times 10^6$.	$XT \times 10^3$.	$X(T + 12) \times 10^3$.
20	293	129.7	38.0	39.6
0	273	138.2	37.7	39.4
− 40	233	160.7	37.4	39.4
− 80	193	194.2	37.5	39.8
−100	173	217.0	37.5	40.1
−120	153	244.6	37.4	40.4
−140	133	279.0	37.1	40.5

According to Curie's law the susceptibility of paramagnetic bodies times the absolute temperature is equal to a constant. Few substances, however, follow this law exactly but follow a modification of the law, namely, the susceptibility times the absolute temperature plus a constant is equal to a constant, that is, $X(T + \theta) =$ constant, in which each substance has its own value of θ.

The value of θ for dysprosium oxide found by the author in the work referred to above was 15. This gave for an average value of the constant $X(T + \theta) \times 10^3$, for a temperature range from 300° C. to 20° C., 72.1. In the present work with dysprosium oxide, Table I., the same value of θ has been used with the result that the constant increases with decrease of temperature. The same is true in the cases of erbium oxide and gadolinium oxide, Tables II. and III., where the value of θ previously found for these oxides is again used. Neodymium oxide showed the same result. Taking the value of θ equal to 44, as previously found, the value of $X(T + \theta)$ increases by about four per cent. in the temperature range from 20° C. to − 140° C.

In Table IV. is given a comparison of the values of the magnetic susceptibility at 20° C. reported on page 161 of the article referred to above with those corresponding to the same temperature in the present work.

TABLE IV.

Temperature, 20° C.

		$X \times 10^6$.	$X(T + \theta) \times 10^3$.
Dy_2O_3	Table I., new	233.3	71.9
"	" V., old	234.1	72.1
Er_2O_3	Table II., new	188.6	57.8
"	" IV., old	189.1	58.0
Gd_2O_3	Table III., new	129.7	39.6
"	" IV., old	130.1	39.7

In the earlier article the conclusion was drawn that the magnetic susceptibility of samarium oxide does not vary with the temperature. In the present investigation the results show clearly a slight increase in the magnetic susceptibility with decrease in temperature, the value of X being about ten per cent. greater at − 140° C. than at room temperature. However, this is a relatively small increase as the susceptibility of the oxides shown in Tables I., II. and III. more than doubles in value in the same temperature range.

Lanthanum oxide again gave negative values for the magnetic susceptibility. However, the values were so small that it was impossible to determine whether there was a variation or not. Any slight variation which may exist would be masked by the errors due to experiment. In the case of yttrium oxide the magnetic susceptibility was found to increase slightly with decrease of temperature but since the magnitude of the susceptibility is less than 1×10^{-6}, the experimental error is relatively large.

The results shown above lead to the conclusion that θ is not a constant but is some function of the temperature,—a function such that the value of θ decreases with decrease of temperature. If this is true Curie's law is followed more closely, in the case of rare earth oxides, at low temperatures than at high temperatures. It is interesting to note that H. K. Onnes and A. Perrier[1] found that, in the case of oxygen, the deviation from Curie's law becomes greater with decrease of temperature.

Laboratory of Physics,
University of Illinois,
Urbana, Illinois.
May, 1919.

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[1] Konink, Akad. Wetensch. Amsterdam Proc., 16, p. 894, March 26, 1914.

DISPLACEMENT FROM THE APPARENT VERTICAL IN FREE FALL.

BY WILL C. BAKER.

SYNOPSIS.

A simple method is given of obtaining the displacement by operating in fixed space. It is simple and permits one to visualize the southward component. An appreciable difference in the result is obtained according as the trajectory is treated as a parabola or an ellipse, the expression found in the latter case being only two thirds that found for the former.

IN looking over the various treatments of the equatorward deflection of a freely falling body, the method herein described has not been found. It brings into prominence certain features that are perhaps worthy of record. The case dealt with is that of a body falling towards a homogeneous spherical earth that revolves uniformly about a fixed axis. These assumptions are common to most of the methods found, very few taking account of the earth's equatorial protuberance.

Let the figure represent a sphere of radius R, fixed in space; and let the earth be represented by a second sphere, coincident with the fixed sphere at time $t = 0$, but rotating about the axis OC, with the small uniform angular velocity ω. AF represents a material rod of height h, fixed in the rotating sphere at latitude λ, and normal to the surface at the point A. FD is a plumb-line that takes the position shown due to the fact that it is in uniform circular motion about OC. Its direction gives the *apparent vertical* at the point A, as it is at rest with regard to the rotating sphere. The bob D is supposed to hang just above the spherical surface, and in its rotation to trace out the small circle DBK. Consider now a mass m released from the point F at time $t = 0$, and being at the moment of release at rest with regard to F. *In space* this body has at time $t = 0$ an initial velocity

$$(R + h)\,\omega \cos \lambda$$

along FH, which is normal to the plane of the circle DAC, and it will experience in space an acceleration $\bar{g}$ directly towards O. Thus its trajectory will lie in the plane $OFHM$ *which is fixed in space* and which cuts the fixed sphere in the *great circle* ABL. It will appear later, however, that the body under these conditions always falls on the portion BL.

As the plumb-bob is in uniform circular motion in the small circle *DBK*, it has the normal accelerating force $mr\omega^2$, along the radius r of that circle. This force is due to the resultant of the two forces acting on the bob, viz., (1) the tension of the cord mg, along DF, where g is the apparent value of the acceleration due to gravity at the point D; and (2) its *true weight* $m\bar{g}$ along DO where $m\bar{g}$ measures the force with which the bob is attracted towards the centre O. The relation of these vectors is shown in the upper part of the diagram. From this

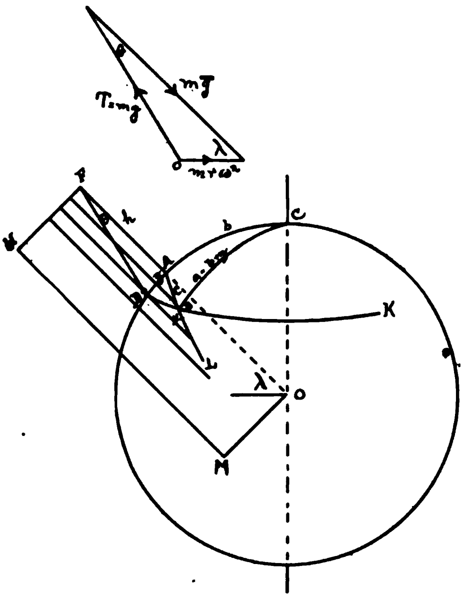

Fig. 1.

$$mg \sin \theta = mr\omega^2 \sin \lambda$$

or

$$\sin \theta = \frac{r\omega^2 \sin \lambda}{g}$$

$$= \frac{R \cos \lambda \; \omega^2 \sin \lambda}{g}.$$

And, since θ is small, the *arc* z (*i.e.*, AD) $= h \sin \theta = \dfrac{hR\omega^2 \sin \lambda \cos \lambda}{g}$.

The *angle* z, subtended by the arc z at O, is given by

$$\frac{h\omega^2 \sin \lambda \cos \lambda}{g}$$

and this has a maximum value for $\lambda = 45°$.

In the spherical triangle ABC $\cos a = \cos (b + z) = \cos c \cos b$ since the angle at $A = \pi/2$,

$$\begin{aligned}\cos c &= \frac{\cos(b+z)}{\cos b}\\ &= \cos z - \sin z \cot\lambda, \text{ for } \cot\lambda = \tan b\\ &= 1 - \frac{z^2}{2!} + \frac{z^4}{4!}\cdots - \left(z + \frac{z^3}{3!}\cdots\right)\cot\lambda.\end{aligned}$$

Now even if we take h as 40,000 cm., which is the greatest value for which I can find[1] any experiments, and if we consider these to have been made in the latitude of greatest southerly deviation (45°), z is found to be 1.07×10^{-7} radians, and the terms in z^2 would appear only in the fourteenth place of decimals. Thus to this degree of approximation

$$\cos c = 1 - z\cos\lambda$$

which on substituting the value of z gives

$$\cos c = \frac{1 - h\omega^2\cos^2\lambda}{g}$$

but from the cosine series

$$\cos c = 1 + \frac{c^2}{2!}$$

to terms in c^2, therefore on comparison

$$c = \omega\sqrt{\frac{2h}{g}}\cos\lambda$$

and the arc c (*i.e.*, AB) $= R\omega\sqrt{\dfrac{2h}{g}}\cos\lambda$, showing that B is the point occupied by the plumb-bob at the instant that the falling body strikes the spherical surface (the angle ABD being small, as it always is) for this is the angle that D moves through in the time of fall ($\sqrt{2h/g}$ when we neglect terms in ω^2).

Consider next the trajectory in the fixed plane $OFHM$. It would appear at first sight that on account of the very small rotation of the earth during the fall no appreciable difference would result from treating this as a parabola or as an ellipse. If we denote the point of impact on the line BL from a parabolic path as N, and that from an elliptical path as N' the two distances AN and AN' will differ only by a very small part of either line, but when we realise that it is the difference BN or BN' that we seek, and that AB is nearly as long as AN or as AN' it becomes evident that our result depends on the *differences* between quantities of the same order of magnitude, and that quantities otherwise insignificant become of importance.[2]

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[1] Rundell, quoted by E. H. Hall, PHYS. REV., Vol. XVII., p. 186, 1903.

[2] Even in the case of the extreme height quoted the distances are as follows: $AB = 295955$ cm., $AN = 295973$cm., $AN' = 295967$ cm., differences of the order of 5/1000 of a per cent. but $AN - AB = 18$ cm., $AN' - AB = 12$ cm., and these differ by 66 per cent.

Taking the line of action of the acceleration as constant, we have a constant horizontal velocity of $(R + h)\omega \cos \lambda$ during the fall $(\bar{t})$ and the resulting value of AB will be $(R + h)\omega\bar{t} \cos \lambda$ and subtracting the value of $AB = \omega\bar{t} \cos \lambda$ we have

$$
\begin{aligned}
BN &= (R + h)\omega\bar{t} \cos \lambda - R\omega\bar{t} \cos \lambda \\
&= h\omega\bar{t} \cos \lambda \\
&= \tfrac{1}{2}\omega g\bar{t}^3 \cos \lambda \text{ since } h = \tfrac{1}{2}gt^2.
\end{aligned}
$$

This is of course along the great circle, and we get the easterly and the southerly displacements by multiplying this by the cosine and the sine of the angle ABD.

$$
\begin{aligned}
ABD &= \sin^{-1} \frac{z}{AB} \\
&= \sin^{-1} \frac{hR\omega^2 \sin \lambda}{g(R + h)\omega t} \text{ or very nearly,} \\
&= \sin^{-1} \frac{h\omega \sin \lambda}{g\bar{t}}.
\end{aligned}
$$

Turning now to the elliptical path, take axes of X and Y respectively as OF and OM. We have for the component acceleration parallel to X

$$\frac{d^2x}{dt^2} = - \frac{gR^2x}{(x^2 + y^2)^{3/2}},$$

also the initial velocity along FH

$$\left(\frac{dx}{dt}\right)_0 = (R + h)\omega \cos \lambda$$

Writing

$$x = x_0 + \left(\frac{dx}{dt}\right)_0 t + \left(\frac{d^2x}{dt^2}\right)_0 t^2 + \left(\frac{d^3x}{dt^3}\right)_0 t^3 \cdots$$

we get

$$\bar{x} = (R + h)\omega\bar{t} \cos \lambda - \frac{g\omega\bar{t}^3 \cos \lambda}{6}$$

where $\bar{x}$ and $\bar{t}$ indicate the values of these two quantities at the instant of impact. Now treating this value $(BN' = \bar{x})$ as in the case of the parabolic path we find for BN'

$$\tfrac{1}{3}\omega g\bar{t}^3 \cos \lambda$$

which is the Gaussian expression usually obtained by a transformation to rotating axes. It differs from the value obtained on the parabolic path simply by the factor two thirds as explained above.

Physical Laboratory,
Queen's University,
Kingston, Ont.,
May 22, 1919.

A NOTE ON THE COMPARISON OF INDUCTANCES, OR OF AN INDUCTANCE AND A CAPACITY BY AN ELECTROMETER METHOD.

BY ALVA W. SMITH.

SYNOPSIS.

A method of comparing inductances is described. This method makes use of an electrometer whose quadrants are connected across a condenser in series with the inductance and whose needle is connected to the middle of a high resistance shunting the condenser and inductance. An equation connecting the deflection of the electrometer with the inductance, capacity and current in the circuit is developed. This equation states that the deflection of the electrometer is proportional to the inductance, to the square of the current and inversely proportional to the capacity. This equation is checked experimentally. By this method several inductances are measured and the results found to be in agreement with those obtained with an A. C. bridge.

IN Fig. 1, LR represents a coil of resistance R and inductance L to be measured, C is a variable condenser, E a quadrant electrometer with its pairs of quadrants connected across the condenser and its needle connected to the middle of the high resistance HR (100,000 ohms), the latter being non-inductive. A is a shunted Duddell thermo-galvanometer used as an ammeter and G, an A.C. generator.

Let v_1, v_2, v_3, v_4 represent the instantaneous values of the potentials at the points indicated on the diagram. The deflection of the electrometer[1]

$$d = \frac{K}{T}\int_0^T (v_1 - v_2)\left(v_3 - \frac{v_1 + v_2}{2}\right)dt, \qquad (1)$$

where K is the electrometer constant and T the period of the alternating current. Since the needle is connected to the middle of HR,

$$v_3 = \frac{v_1 + v_4}{2}. \qquad (2)$$

$$d = \frac{K}{2T}\int_0^T (v_1 - v_2)(v_4 - v_2)dt. \qquad (3)$$

Assume that a harmonic current

$$i = I_m \sin \omega t$$

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[1] F. M. Laws, Electrical Measurement, p. 250.

is following in the circuit. Then

$$v_1 - v_2 = \frac{1}{C}\int i\,dt = -\frac{I_m}{C\omega}\cos\omega t, \tag{4}$$

$$v_2 - v_4 = L\frac{di}{dt} + Ri = L\,I_m\omega\cos\omega t + R\,I_m\sin\omega t. \tag{5}$$

$$d = \frac{K}{2T}\int_0^T\left(\frac{LI_m^2}{C}\cos^2\omega t + \frac{RI_m^2}{C\omega}\sin\omega t\cos\omega t\right)dt = \frac{K}{2}\frac{LI_m^2}{2C}, \tag{6}$$

since average value of $\cos^2\omega t = 1/2$ and average value of $\sin\omega t\cos\omega t = 0$. Expressing the maximum value of the current in terms of the effective value, we have

$$d = \frac{K}{2}\frac{LI^2}{C}. \tag{7}$$

This equation will hold only for an electrometer obeying the general law of equation (1). It is only the wattless components of the voltage across the condenser and coil that are operative. If, however, the condenser has either leakage or absorption, the deflection will be in part determined by the power components and no longer be expressed by the simple equation (7). The validity of this equation does not depend upon the assumption of a simple sine wave form. For with a more complex wave form the deflection is

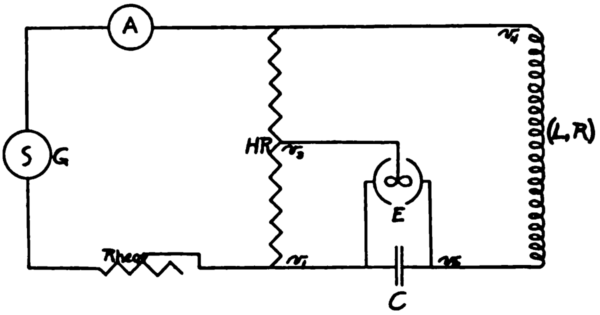

Fig. 1.

$$d = \frac{K}{2}\frac{L}{C}(I_1^2 + I_1'^2 + I_2^2 + I_2'^2 + \cdots)$$

where I_m, I_m', are the amplitudes of the sine and of the cosine terms respectively of the mth-harmonic. The parenthesis is the square of the effective current and is equal to I^2 in equation (7).

To test this equation experimentally, L, C and the frequency were first kept constant and the electrometer deflections read for a series of values of currents. The values of d are plotted against I^2 in graph (1), Fig. 2, and some of the observed values recorded in Table I., (A). The graph shows that the deflections are proportional to the square of the current, if the inductance, capacity and frequency are kept constant.

TABLE I.

Tests on $d = \frac{K}{2}\frac{LI^2}{C}$.

(A) I Variable.		(B) C Variable.		(C) L Variable.	
I amps.	d cm.	C mf.	d cm.	L mh.	d cm.
.052	1.58	.1	9.80	7.0	2.00
.075	3.30	.133	7.35	15.0	4.22
.095	5.14	.143	6.83	20.0	5.70
.115	7.72	.167	5.90	30.0	8.37
.143	11.50	.2	4.95	40.0	11.19
.172	16.35	.25	3.90	50.0	13.97
.197	21.5	.3	3.25	60.0	16.77
.218	26.35	.7	1.39	70.0	19.47
.223	27.6	.9	1.10	75.0	20.97
L = 50 mh. C = 1 mf. f = 590 cycles/sec.		L = 75 mh. I = .150 amps. f = 620 cycles/sec.		I = .125 amps. C = 1 mf. f = 650 cycles/sec.	

Graph (2), Fig. 2, is one obtained by keeping L, I and C constant and varying the frequency. Deflections of the electrometer are plotted

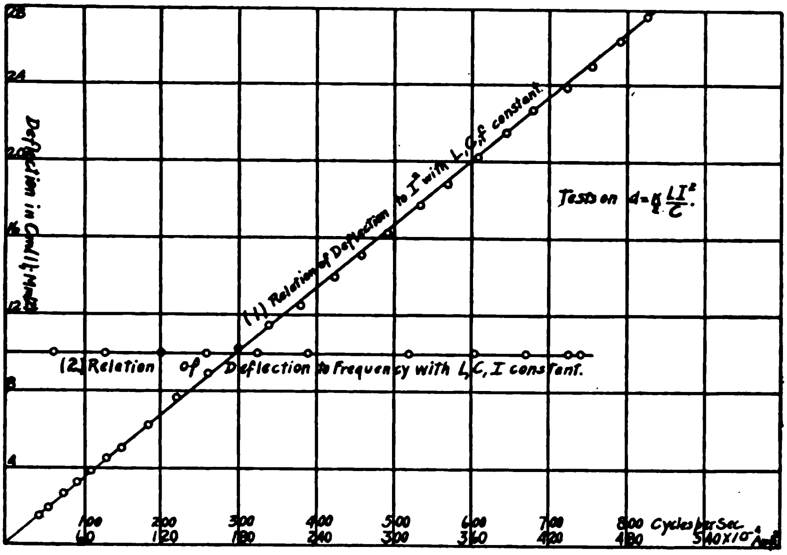

Fig. 2.

against frequency. It shows that the deflections are independent of the frequency over the range of frequencies used.

A further test was made by keeping L, I and the frequency constant and varying C. In graph (3) of Fig. 3 are shown the results obtained. Deflections are plotted against the reciprocals of the capacity. Proportionality between these two variables is shown here.

Tests were then made with I, C and the frequency constant and L variable. Graphs (4), (5), (6) were obtained, different values of current being used. The foregoing graphs confirm equation (7).

In measuring an inductance of unknown value, the deflection of the electrometer is observed with this inductance in circuit and I and C adjusted to give suitable electrometer deflections. An inductance standard is substituted for the unknown, and the deflection read for the same value of current and capacity. The inductances then are proportional to their respective deflections. An alternative solution is to determine the electrometer constant K and apply equation (7) for any convenient values of current and capacity.

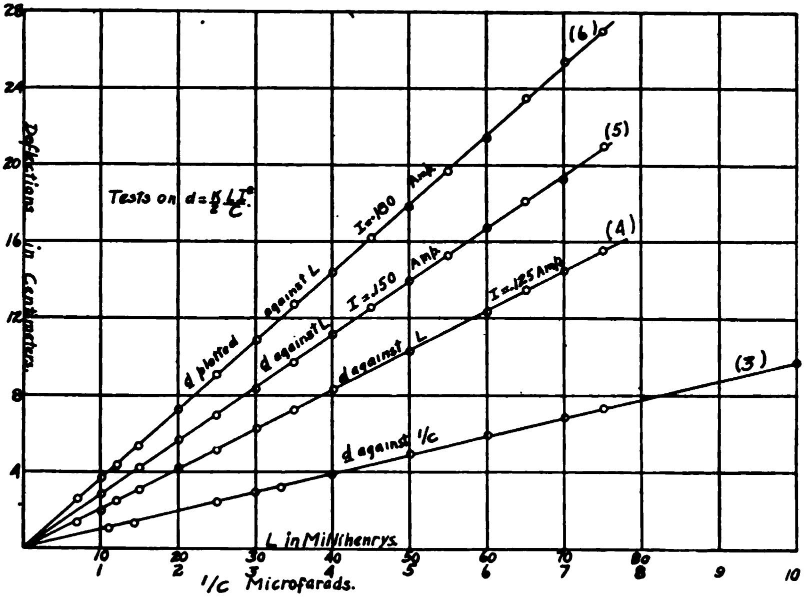

Fig. 3.

Measurements by this method have been compared with those given by the Maxwell bridge method. From such a comparison Table II. is compiled. In this table L is the value of inductance measured by the bridge method and L' is its value measured by the electrometer method.

The average deviation between the values given by the two methods is about .5 per cent.

TABLE II.

Coil.	L in *mh.*	L' in *mh.*	Per Cent. Diff.
1	19.97	20.00	.2
2	94.4	94.6	.2
3	110.4	110.0	.4
4	128.0	127.0	.8
5	154.8	154.5	.2
6	214.0	216.0	.9
7	318.0	321.0	.95

For given values of current, capacity and inductance these methods appear equally sensitive for all frequencies used. In this respect it seems superior to methods involving the use of a telephone receiver or a vibration galvanometer as a detector. Moreover, it does not involve a knowledge of the frequency as is required in the Wilson method.[1] Of course, for a given impressed voltage, that frequency which coincides most nearly with the resonant frequency of the circuit containing L and C will give the maximum current and hence maximum electrometer deflection, thus making the sensibility of the method a maximum.

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[1] F. M. Laws, Electrical Measurements, p. 414.

THE MOST PROBABLE VALUE OF THE PLANCK CONSTANT *h*.

By Raymond T. Birge.

Synopsis.

The most probable value of Planck's Constant *h* has been computed, together with the probable error. The latest value of each of the other constants appearing in the equations has been used; and the probable error in each has been taken into consideration. The mean value from seven distinct methods is (6.5543 ± 0.0025) x 10^{-27} erg. sec., and each of the seven residuals lies within the limits of error. The indicated error in the final mean is practically exclusive of any error in *e* (assumed 4.774). However, by assuming Lewis and Adams' theory of ultimate rational units, it is possible to compute, in a number of different ways, an independent value of *e*, or of *h*. One such computation, using Blake and Duane's X-ray data, yields $e = 4.7705 \pm 0.0046$, $h = 6.550 \pm 0.013$.

In March, 1916, Millikan[1] published the results of a determination of the value of *h* from photo-electric data. The probable error was estimated at 0.5 per cent., and this constituted by far the most accurate determination of *h*, up to that time. Since then, however, all methods of attack have been improved in accuracy until, at the present time, we have seven distinct methods, each accurate to 0.5 per cent. or less. It is therefore of considerable importance to determine the most probable value of *h*, resulting from all known methods.

In carrying out the computations, the author has found it necessary in some cases to recompute previous results. For there are many other constants which appear in the formula for *h*, and the latest values of these constants should be used. Thus Millikan, in his photo-electric work, used 3.000 for the velocity of light (*c*). Most authors at the present time use 2.9986, as given by Kaye and Laby.[2] Since *c* enters as the second power, in the determination of *h*, this change of value produces a change of 0.1 per cent., in *h*,—an amount negligible at the time of Millikan's work, but not at all negligible at the present time.

In the following work I have assumed the author's own estimate of his probable experimental error, when such error is explicitly stated. In other cases the error has been judged as well as possible from the results themselves, together with the author's discussion of them. This final error has then been combined, by the ordinary least square formulæ, with the probable errors in each of the other quantities appearing in the equation for h. The seven values of h, each with its resulting probable error, have then been combined to give the final most probable value, together with the probable error in this value.

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[1] Phys. Rev. (2), 7, 355, 1916.

[2] Physical and Chemical Constants, p. 69, 1918 edition.

The results are as follows:

METHOD 1. FROM THE STEFAN-BOLTZMANN TOTAL RADIATION CONSTANT a, OR σ.

$$E = \frac{ac}{4} T^4 = \sigma T^4, \tag{1}$$

where E = the total radiation from unit surface of a black body (*i. e.*, through solid angle 2π) per unit time.

a is connected with h through the Planck radiation formula, the resulting equation being[1]

$$a = \frac{48\pi\alpha k^4}{c^3 h^3}. \tag{2}$$

But

$$\alpha = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \text{etc.} = \frac{\pi^4}{90}. \tag{3}$$

Therefore

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3}, \tag{4}$$

and

$$h = \sqrt[3]{\frac{8\pi^5 k^4}{15 c^3 a}}, \tag{5}$$

where

$$k = \frac{p_0 V_0 e E}{T S c} = (1.372 \pm 0.0014) \times 10^{-16}, \tag{6}$$ [2]

k being the Boltzmann entropy constant (or, in other words, the gas constant for one molecule) and having practically the same probable error[3] as e (= 4.774 ± 0.005).

The best experimental determination of σ ($= ac/4$) is that of Coblentz,[4] the value being

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[1] See page 171, Masius, "Planck's Heat Radiation."

[2] Millikan, PHYS. REV. (2), 2, 142, 1913.

[3] Millikan, in his evaluation of k used for E (the electro-chemical equivalent of silver) 0.001118, defining the International ampere. This combined with 107.88 for S (the atomic weight of silver), gives 96,494 coulombs for the value of the Faraday. But the best value for E (Kaye and Laby, page 8, 1918 edition) is 0.00111827, giving 96,470 for the Faraday. Millikan likewise uses 1,013,700 for p_0, instead of 1,013,200 (K. and L., page 5). The two errors cancel to within 0.02 per cent. Finally, he uses 273.11 for T, which is still about the best value (K. and L., page 44).

[4] Proc. Nat. Acad. Sci. 3, 504, 1917.

$$\sigma = (5.72 \pm 0.012) \times 10^{-5} \text{ erg. cm}^{-2} \text{ deg}^{-4} \text{ sec}^{-1}. \tag{7}$$

Therefore

$$h = \sqrt[3]{\frac{8\pi^5\{(1.372 \pm 0.0014) \times 10^{-16}\}^4}{15(2.9986 \times 10^{10})^2 \times 4 \times (5.72 \pm 0.012) \times 10^{-5}}} \tag{8}$$

$$= (6.551 \pm 0.009) \times 10^{-27} \text{ erg sec.}$$

Method 2. From the Wien Constant c_2 (= C_2 of Coblentz).

$$\lambda_{\max} T = c_2/\beta = b \;(= A, \text{ of Coblentz}), \tag{9}$$

where $\beta = 4.9651$. (The root of the equation $e^{-\beta} + \beta/5 - 1 = 0$.) c_2 is connected with h by the Planck equation,[1] giving

$$c_2 = b\beta = ch/k. \tag{10}$$

The best value of c_2 is that of Coblentz.[2] The author states that the value of c_2 lies almost certainly between 1.43 and 1.44 cm. degrees and recommends[3] the tentative adoption of 1.433. I therefore use this value and judge from the above maximum variation, a probable error of 0.0025, giving

$$c_2 = (1.433 \pm 0.0025) \text{ cm. degrees.} \tag{11}$$

Therefore

$$h = \frac{(1.433 \pm 0.0025) \times (1.372 \pm 0.0014) \times 10^{-16}}{2.9986 \times 10^{10}} = 6.557 \pm 0.013. \tag{12}$$

Method 3. From the Rydberg Constant N_0, by Bohr's Theory of Atomic Structure.

$$N_0 = \frac{2\pi^2 e^4}{h^3(e/m)}, \tag{13}^1$$

where the term $(1 + m/M)$ has been omitted, since it differs from unity by only one part in 1852.6. The best value of N_0 is[5]

$$109{,}678.6 \times 2.9986 \times 10^{10} = 3.28882 \times 10^{15} \text{ sec}^{-1}.$$

The mean value of e/m, for negative electrons[6] is 1.772×10^7 E.M.U. From the Zeeman effect[3] the mean value is 1.775. The former group of experiments is probably the more accurate, and hence I have assumed

$$e/m = (1.773 \pm 0.002) \times 10^7 \text{ E.M.U.} \tag{14}$$

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[1] Masius, *loc. cit.*

[2] Bulletin Bur. of Standards, 13, 459, 1916.

[3] Private communication to the author. Mendenhall (Phys. Rev. (2), 10, 515, 1917) obtains 1.440 and concludes that there is a real discrepancy between the radiation and electrical values of *h*. The results presented in this paper show no such discrepancy.

[4] Bohr, Phil. Mag., 26, 1, 1913.

[5] Birge, Science 48, 47, 1918.

[6] Kaye and Laby, pp. 98–99, 1918 edition.

Therefore

$$h = \sqrt[3]{\frac{2\pi^2\{(4.774 \pm 0.005) \times 10^{-10}\}^5}{(1.773 \pm 0.002) \times 10^7 \times 2.9986 \times 10^{10} \times 3.2888 \times 10^{15}}} \qquad (15)$$

$$= 6.542 \pm 0.011.$$

METHOD 4. FROM EINSTEIN'S PHOTO-ELECTRIC EQUATION.

$$V \cdot e = h \cdot \nu - p. \qquad (16)$$

Millikan[1] gives for the final value of h, from his photoelectric work, 6.57 ± 0.5 per cent. However, this is the result of the work with sodium only. If the value obtained with lithium (6.584 ± 1.0 per cent.) is included also, we have for the final mean 6.572 ± 0.4 per cent. But this corresponds to a value of $c = 3.000$. Using, instead, 2.9986, we obtain

$$h = 6.578 \pm 0.026. \qquad (17)$$

METHOD 5. FROM THE QUANTUM RELATION, AS APPLIED TO GENERAL X-RADIATION.

$$V \cdot e = h \cdot \nu, \qquad (18)$$

where

$$\nu = \frac{1}{2d \sin \theta}, \qquad (19)$$

and

$$d = \text{calcite grating space.}$$

If V is expressed in volts, and ν in frequency numbers (number of wave-lengths in one cm.) we have

$$h = \frac{2 \cdot d \cdot \sin \theta \cdot e \cdot V \cdot 10^8}{c^2}. \qquad (20)$$

The procedure of Blake and Duane[2] is so accurate that the probable error of the experimental work itself ($V \cdot \sin \theta$) is about 0.02 per cent. (from the results for h on page 634 loc cit.). The authors therefore rightly conclude that the final error in h is due almost entirely to the uncertainty in the values of e and d. Blake and Duane use for d Compton's value[3] of 3.028. Compton, in a later article[4] concludes that the most probable value is

$$d = (3.0281 \pm 0.0010) \times 10^{-8} \text{ cm.} \qquad (21)$$

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[1] PHYS. REV. (2), 7, 355, 1916.

[2] PHYS. REV. (2), 10, 624, 1917. In addition, A. Müller (Phys. Zeits., 19, 489, 1918; Science Abstracts, No. 303, March, 1919) obtains $h = 6.58 \pm 0.07$, while E. Wagner (Ann. d. Physik, 57, 401, 1918; Science Abstracts, No. 440, April, 1919) obtains $h = 6.49$, the accuracy not being stated in the abstract.

[3] PHYS. REV. (2), 7, 655, 1916.

[4] PHYS. REV. (2), 11, 430, 1918.

Uhler[1] considers this estimate of error too small, as it includes only the error in e, which appears here to the one third power, *i.e.*,

$$d = \left(\frac{\frac{1}{2}M_1eE}{\rho_1\phi(\beta_1)Sc}\right)^{1/3}, \tag{22}$$

where M_1 = molecular weight of $CaCO_3$ = 100.075.
ρ_1 = density of the calcite crystal = 2.7116 gr./c.c.
$\phi(\beta_1)$ = volume of the calcite rhombohedron = 1.0963.
E = electro-chemical equivalent of silver = 0.00111827 × 10 gr. per E.M.U.
S = atomic weight of silver = 107.88.

Compton uses Millikan's value of N $(= Sc/eE) = 6.062 \times 10^{23}$, based on $E = 0.001118$. The corrected value of $N = 6.0594$.

Uhler's discussion of the above data indicates that, outside of the error in e, there may be a probable error in d of about 0.06 per cent. In obtaining the final probable error in h, it is not permissible to combine the total probable error in d, with that in e, and in the experimental results. Instead, the error in d, aside from that due to e, must be combined with the total per cent. error in $e^{4/3}$ (of which $e^{1/3}$ appears implicitly in d) and with the experimental error (in $V\cdot\sin\theta$). The error in c^2 is negligible.

Therefore the probable error in h equals

$$\sqrt{0.06^2 + 0.13^2 + 0.02^2} = 0.144 \text{ per cent.} \tag{23}$$

Therefore,

$$h = 6.555 \pm 0.009. \tag{24}$$

Method 6. From Lewis and Adams' Theory of Ultimate Rational Units.

This theory[2] gives directly the value of the Wien constant σ. To obtain h, it is necessary to assume the truth of Planck's formula. Making such an assumption, there results (page 101, *loc. cit.*)

$$h = \frac{16\pi^2e^2}{c}\sqrt[3]{\frac{8\pi^5}{15}}, \tag{25}$$

where h is expressed as a definite function of the charge on the electron, so that the only error in h is that due to the error in e. This gives

$$h = 6.560 \pm 0.014. \tag{26}$$

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[1] Phys. Rev. (2), 12, 39, 1918.

[2] Phys. Rev. (2), 3, 92, 1914.

METHOD 7. FROM THE QUANTUM RELATION, APPLIED TO IONIZATION AND RESONANCE POTENTIALS.

Foote and Mohler[1] give a list of six ionization potentials, and seven resonance potentials. The arithmetic mean of the thirteen resulting values of h is 6.55. In order however to obtain the most probable value of h, it is best to plot voltage against frequency, and to obtain the slope of this straight line. For

$$h\cdot\nu = \frac{e\cdot V\cdot 10^8}{c^2}, \tag{27}$$

where V = potential in volts,
ν = frequency in wave-numbers.

Therefore

$$h = \frac{e}{c^2}\times\frac{dV}{d\nu}. \tag{28}$$

The thirteen values just mentioned have since been supplemented by data on As, Rb, and Cs[2] and on Ca,[3] but these latter results are rather inaccurate and so can well be omitted. The author accordingly used only the thirteen points and obtained, by least square formulæ, the most probable constants of the voltage-frequency curve. Assuming first the more general relation

$$\text{Voltage} = a\times\text{frequency} + b, \tag{29}$$

there results

$$a = (1.2471 \pm 0.0078)\times 10^{-4},$$

giving

$$h = 6.615 \pm 0.041, \tag{30}$$

while

$$b = -\,0.041 \pm 0.037 \text{ volts}. \tag{31}$$

The probable error in b is practically equal to the actual intercept, and hence it seems justifiable to assume that the intercept is zero, *i.e.*, to assume the usual quantum relation (27) or (28). Performing, on this assumption, the least square solution, I obtain

$$h = 6.579 \pm 0.021, \tag{32}$$

a result considerably higher than the arithmetic mean. The error in this latter procedure is due to the equal weighting of all points on the curve, regardless of their distance from the origin. The least square solution gives the proper weighting to each point. Equation (32) includes the error in e (0.1 per cent.) as well as that due to the experimental results (0.3 per cent.).

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[1] Phil. Mag., 37, 33, 1919.

[2] Foote, Rognley, and Mohler, PHYS. REV., 13, 59, 1919.

[3] Foote, personal communication to the author.

The seven values of h are given in Table I. The least square mean of these values is

$$h = (6.5543 \pm 0.0025) \times 10^{-27} \text{ erg sec.} \tag{33}$$

The residuals also are given in Table I. In all cases they are less than, or approximately equal to, the probable error, proving that all seven methods are consistent among themselves. Considering all the circumstances, this is a most surprising result.

TABLE I.

Value of h.	Residual.	Method.	Equation.	Dependence on e.
6.551 ± 0.009	− 0.0033	a	(8)	$e^{4/3}$
6.557 ± 0.013	+ 0.0027	c_2	(12)	e
6.542 ± 0.011	− 0.0123	N_0	(15)	$e^{5/3}$
6.578 ± 0.026	+ 0.0237	photo-elec.	(17)	e
6.555 ± 0.009	+ 0.0007	X-rays	(24)	$e^{4/3}$
6.560 ± 0.014	+ 0.0057	Lewis and Adams.	(26)	e^2
6.579 ± 0.021	+ 0.0247	Ion. Pot.	(32)	e

Mean value of h = 6.5543 ± 0.0025.

It is not to be concluded, from this result, that the probable absolute error in h is only 0.04 per cent. For all seven of the above methods include e to a positive power, and any error in e would therefore affect all of them in approximately the same manner. (The power of e is given in Table I., last column.) But if Lewis and Adams' relation between e and h is a true one, it is possible to eliminate e from each of the other six equations, thus obtaining a value of h independent of Millikan's value of e. Or, by eliminating h, it is possible to obtain an independent value of e.

The most accurate of these six methods is that of Blake and Duane. Accordingly, the elimination will be shown for this one method only. Using equations (20), (22) and (25), we obtain

$$e = \left(\frac{2V\sin\theta\cdot 10^8}{c^2}\right)^{3/2}\left(\frac{1}{2}\frac{M_1E}{\rho_1\phi(\beta_1)Sc}\right)^{1/2}\left(\frac{c}{16\pi^2}\sqrt[3]{\frac{15}{8\pi^5}}\right)^{3/2} \tag{34}$$

and

$$h = \left(\frac{2V\sin\theta\cdot 10^8}{c^2}\right)^{3}\left(\frac{1}{2}\frac{M_1E}{\rho_1\phi(\beta_1)Sc}\right)\left(\frac{c}{16\pi^2}\sqrt[3]{\frac{15}{8\pi^5}}\right)^{2}. \tag{35}$$

Computing backwards from Blake and Duane's results,

$$\left(\frac{2V\sin\theta\cdot 10^8}{c^2}\right) = \frac{h}{e\cdot d} = \frac{6.555\times 10^{-27}}{4.774\times 10^{-10}\times 3.028\times 10^{-8}} \tag{36}$$
$$= 4.5346\times 10^{-10}.$$

Using for the other quantities in (34) and (35) the values already specified, we obtain

$$e = 4.7705 \pm 0.0050, \tag{37}$$

$$h = 6.550 \pm 0.012. \tag{38}$$

The errors in e and h are obtained from the previously assumed error of 0.02 per cent. in $(V \cdot \sin\theta/c^2)$, and of 0.06 per cent. in $(\frac{1}{2}M_1E/\rho_1\phi(\beta_1)Sc)^{1/3}$ The velocity of light appears in e as c^{-2}, and in h as c^{-5}. The error in c is probably not over 0.01 per cent.

This new value of e has therefore the same probable error as Millikan's value. Its correctness depends however on the following assumptions:

1. The truth of Lewis and Adams' theory of ultimate rational units.
2. The truth of the relation between a and h, as given by Planck's radiation formula.
3. The truth of the quantum relation, as applied by Blake and Duane to X-ray data.

Because of the more direct method used by Millikan, his value of e is likely to be more nearly correct. We may therefore conclude that

$$\mathbf{h} = (6.5543 \pm 0.0025) \times 10^{-27} \text{ erg sec.},$$

remembering however that in addition to the probable error, this value assumes the correctness of 4.774 as the value of e, and actually possesses a per cent. error somewhat greater than the per cent. error in e, and in the same sense.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA,
July, 1919.

ON THE CRITICAL ABSORPTION AND CHARACTERISTIC EMISSION X-RAY FREQUENCIES.

By William Duane and Kang-Fuh Hu.

Synopsis.

In the analysis of X-ray line spectra it is of considerable importance to know whether or not the critical absorption, the critical ionization and the highest characteristic emission frequencies associated with any one single X-ray series are exactly equal to each other. Such knowledge must have a marked influence on speculation and theories as to the mechanism of X-radiation and the structure of atoms.

The object of the research reported in this paper has been to measure these three frequencies, as precisely as possible, using the same instruments throughout the work. The measurements have been made with the X-ray spectrometer employed several years ago to determine the value of h. The X-rays came from the steadiest sources at present available, namely, Coolidge tubes excited by currents from a high tension storage battery.

As results of our research we conclude *that the critical ionization frequency equals the critical absorption frequency to within considerably less than 1/10 per cent.;* and that *the critical absorption frequency exceeds the frequency of the γ line in the emission series by 1/4 per cent. or 1/3 per cent.*, these being determined by measuring from the *centers* of the drops and peaks in the curves corresponding to them respectively.

In the analysis of characteristic X-ray spectra we recognize four different kinds of frequency of vibration associated with each series of lines. In the K series of a chemical element, for example, we have: (*a*) several emission frequencies; (*b*) one critical absorption frequency; (*c*) one critical ionization frequency and (*d*) one frequency such that when it is multiplied by Planck's action constant, h, the product equals the minimum energy the electron in the X-ray tube must have in order that it may be able to produce the K emission series.

It is known that the highest frequency in the emission spectrum (the γ line) and the other three frequencies (*b*), (*c*) and (*d*) lie close together, *i.e.*, within a few per cent. of each other.

The object of the research reported in this paper has been to determine whether, within the limits of experimental error, the γ line frequency the critical absorption frequency and the critical ionization frequency, are exactly equal to each other or not.

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[1] A paper presented at the New York meeting of the American Physical Society, April 27, 1918.

Throughout these experiments we used the same X-ray spectrometer (namely that described in the PHYSICAL REVIEW for December, 1917, page 624), and, in order to eliminate some of the possible sources of error, we did not change the general set up of the instrument.

We calculated the wave-lengths of the X-rays reflected from the surface of the calcite crystal by means of the equation

$$\lambda = 6.056 \times \sin\theta \times 10^{-8} \text{ cm.}$$

where θ is the grazing angle of incidence.

The X-rays came from tubes of the Coolidge type. One of the tubes had a tungsten target and the other had a rhodium target.

The X-rays passed through two narrow slits before they reached the calcite crystal. As the third slit, that in front of the ionization chamber, was large enough to admit the entire reflected beam, this method eliminates large corrections for the penetration of the X-rays into the crystal (*l.c.*).

The electrical current (2 milliamperes) through the X-ray tube came from a high potential storage battery. In most of the experiments we held the difference of potential between the electrodes of the tube constant at 37,750 volts, and, if any small unavoidable variation from this value occurred, we corrected for it.

In order to avoid actually determining the zero point on the scale of the instrument we either measured the lines on each side of it, or else found their positions in both the first and the second order spectra. The various sets of measurements agree with each other to within the limits of experimental error.

In order to determine whether or not the critical absorption frequency differs from the critical ionization frequency, we filled the ionization chamber with methyl-iodide and used a thin layer of potassium iodide as the absorber.

The curves in Fig. 1 represent the ionization currents as functions of the readings of the verniers attached to the crystal table. The curves A and A' refer to experiments in which no absorber was used. The sharp breaks in the curves occur at angles corresponding to the critical ionization wave-length of iodine. X-rays of shorter wave-length than this critical wave-length produce more ionization than X-rays of longer wave-length do. Measuring from the center of the drop on one side of the zero to the center of the drop on the other side we get for the double grazing angle 2θ (after subtracting an excentricity correction of $22''$) $2\theta = 7'\ 4'\ 25''$. Whence the critical ionization wave-length λ_i is $\lambda_i = .3736 \times 10^{-8}$ cm.

Curve B represents an experiment in which the X-rays passed through a thin layer of potassium iodide before reaching the spectrometer. The iodine in the salt absorbs X-rays of shorter wave-lengths than the critical absorption wave-lengths to a greater extent than it does longer waves. The magnitude of the change in the absorption depends, of course, upon the thickness of the absorbing layer, and in this experiment the layer was thick enough to more than counteract the increase in ionization for X-rays shorter than the critical ionization wave length. The ordinates in curve *C* are the ratios of those in curve *B* to those in curve *A* respectively. Curve *C* represents, therefore, the true critical absorption effect uninfluenced by the change in ionization at the critical ionization wave-length; and the angle measured to the center of the drop corresponds to the critical absorption wave-length λ_a. It appears from the graphs that the centers of the drops in curve *A* and curve *C* fall sensibly at the same abscissa; that is, the angles corresponding to the critical absorption and critical ionization wave-length do not differ from each other by more than a few seconds of arc. A difference of 10″ of arc could be detected, which means that *the critical absorption and critical ionization wave-lengths are equal to each other to within less than 1/10 per cent.*

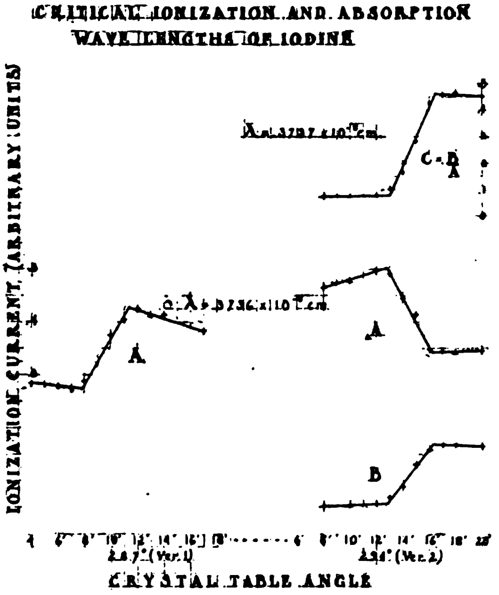

Fig. 1.

In order to determine whether or not the K critical absorption wave-length (λ_a) of a chemical element is the same as that of the γ emission line in the K series we have investigated the emission spectrum of an X-ray tube having a rhodium target.

The curves in Fig. 2 represent the ionization currents as functions of the readings of one of the verniers attached to the crystal table.

For the two α lines readings were taken 15″ of arc apart on both sides of the zero. The peaks corresponding to these two lines are separated from each other, but the separation appears to be better on the right hand side than on the left. Some slight inequality in the structure of the crystal planes may account for this.

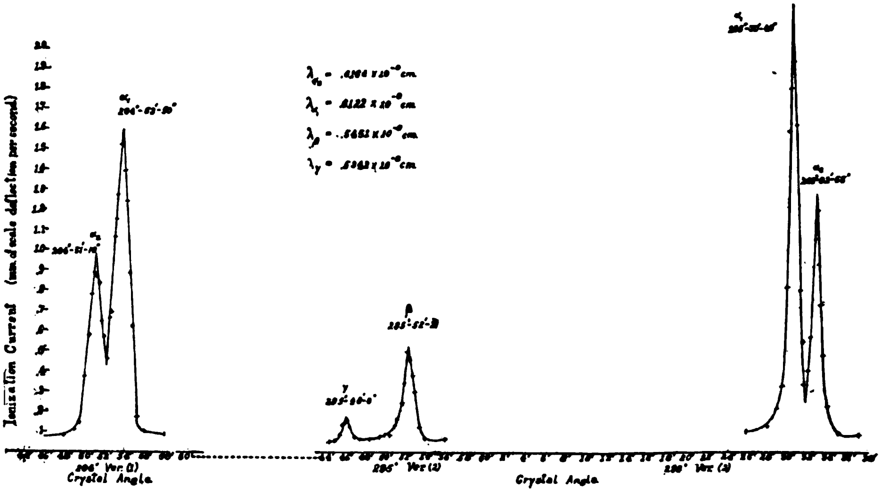

Fig. 2.

As the difference between the wave-lengths of the two α lines amounts to only two thirds of one per cent. this gives an idea of the precision with which the wave-lengths can be estimated. It appears to be possible to determine between which two settings of the instrument a peak lies, and also which of the two settings it lies nearer to. This means that the precision of the measurement is the precision with which the angles can be read by means of the verniers. The verniers' scales are supposed to give readings to 5″ of arc. By estimating from several readings on the two sides of each peak it seems reasonable to suppose that the wave-lengths can be determined with a precision considerably less than one tenth per cent., the grazing angles for these lines being larger than 6°.

The curves in Fig. 2 show also the peaks corresponding to the β and γ lines. The readings for these were taken on one side of the zero only, but the wave-lengths have been calculated from the zero determined by the α line measurements.

The curves in Fig. 3 represent the two α line peaks in both the first and the second order spectra. Passing from the first to the second order spectrum the distance between the two peaks about doubles itself, while the breadth of each peak remains about the same.

The wave-lengths of the α lines calculated from the first and second order spectra, without direct reference to the zero, equal the corresponding wave-lengths calculated from the first order spectra alone.

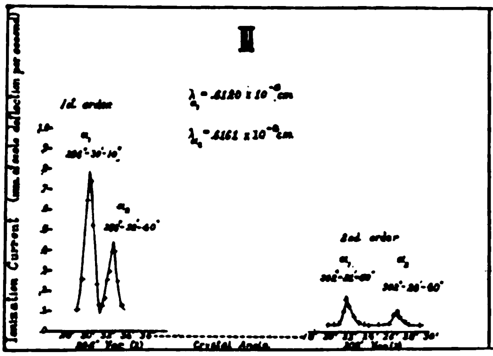

Fig. 3.

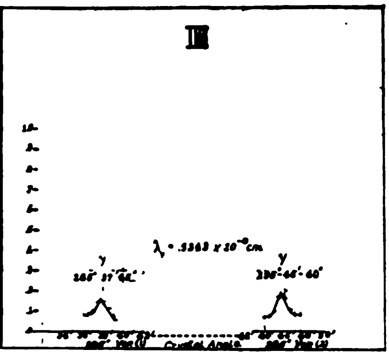

Fig. 4.

The peaks in the curves of Fig. 4 correspond to the γ line in the first order spectrum on each side of the zero.

The curves in Figs. 2, 3 and 4 furnish three independent methods of estimating the position of the zero on the scale.

A correction for eccentricity amounting to about 45″ of arc must be subtracted from the double grazing angle.

The following table contains the wave-lengths of all four lines.

K Series of Rhodium (45). $\lambda \times 10^8$ *cm.*

α_2.	α_1.	β.	γ.
.6164	.6122	.5451	.5342
.6163	.6121	.5453	.5343
.6164	.6120	.5454	.5342

In 1917[1] Prof. F. C. Blake and one of us obtained the value $\lambda = .5324 \times 10^{-8}$ cm. for the critical K absorption frequency of rhodium. As this differs from the values of λ_γ contained in the above table by slightly more than the probable error of the measurement, and as the spectrometer had been partly dismantled and the parts reassembled, the authors of this paper decided to re-measure the critical absorption wave-length.

For this purpose we used the tungsten target X-ray tube and an absorbing layer of rhodium salt placed between the tube and the spectrometer.

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[1] Phys. Rev., December, 1917, p. 702.

Measuring from the centers of the sharp breaks in the curves on the two sides of the zero line, we get for the critical K absorption wave-length of rhodium $\lambda = .5330 \times 10^{-8}$. This value agrees with that obtained in 1917 to within approximately one tenth per cent., but differs from the wave-length of the γ emission line by almost one fourth per cent.

In order to obtain additional evidence as to whether or not this difference really exists, we have made some experiments using the rhodium target tube both with and without the rhodium salt absorber.

The curves in Fig. 5 represent the results obtained. The widths of the slits were somewhat larger than in the previous experiments. They were larger, also, for the measurements on the right hand side of the zero, than on the left. The readings for corresponding parts of the curves with and without the absorbing screen were made on the same day, but the readings for different sections of the curves were made on different days. This is shown by the discontinuities in and overlappings of the curves.

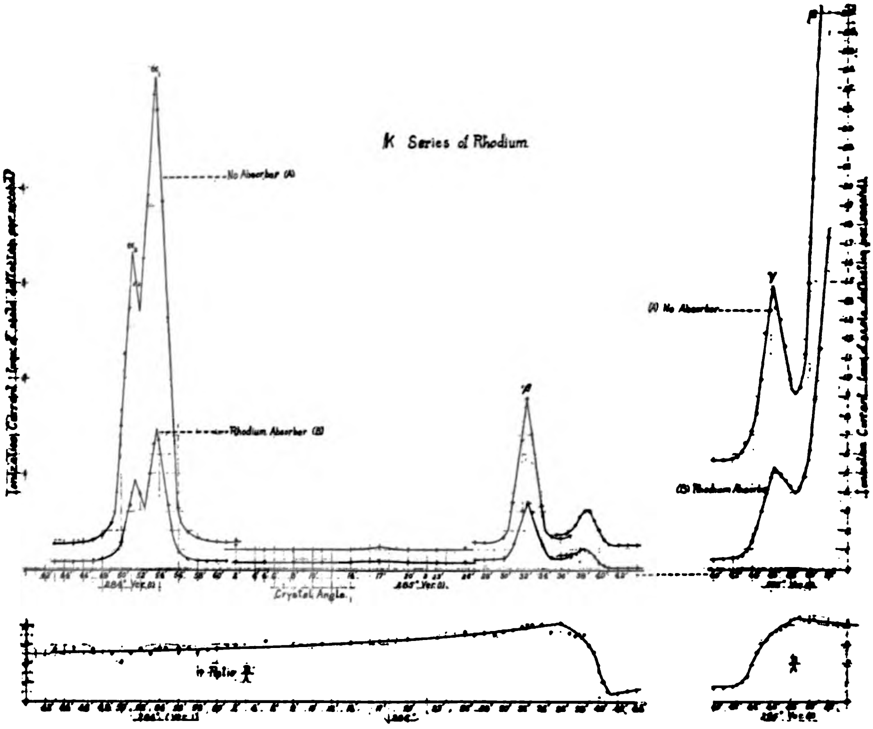

Fig. 5.

Curve A has been platted from the data obtained without the absorbing screen, and curve B, with the absorbing screen. The third curve below represents the ratio of the B to the A ionization currents.

The curves indicate that *the rhodium has no appreciable selective absorption for the X-rays in its own* α_1, α_2 *and* β *emission lines.* The marked increase in absorption occurs, however, close to the peak representing the γ line. Further the wave-length corresponding to the center of the γ line peak is about one third per cent. longer than that corresponding to the center of the absorption drop.

It appears, therefore, from both experiments that, if the wave-length of the emission line corresponds to the center of the peak, and, if the critical absorption wave-length corresponds to the center of the absorption drop, *the critical absorption wave-length is about one fourth per cent. shorter than that of the* γ *emission line.*

PROPAGATION OF SOUND IN AN IRREGULAR ATMOSPHERE.

BY G. W. STEWART.

SYNOPSIS.

Under poor atmospheric conditions, lower frequencies in aëroplane engine sounds become relatively enhanced; under good conditions, frequencies of order of 1,000 d.v. are heard at greatest distances. The former is explained by irregularities in the atmosphere and the latter by characteristics of audition.

Intensity of the sound varies much more rapidly than as the inverse square, crude observations giving much more nearly inverse sixth and fourth powers for maximum ranges under fair and good listening conditions, respectively.

LORD RAYLEIGH'S[1] reference to and explanation of the "highly uneven character of the sound" from aëroplanes in flight, leads the writer to make a record of three additional facts.

During experiments in connection with aëroplane detection and location the observers noticed that under what might be termed "poor listening" atmospheric conditions the sound from the aëroplane when at the greatest hearing distance was limited to the lowest frequencies in the emitted complex sound. These frequencies were for these particular aëroplanes approximately 90, 180, 270, etc., and the most prominent component was the one of lowest pitch. The sound from the same aëroplanes heard at the greatest possible distance under excellent night conditions was distinctly different. The lowest frequencies just named were not noticeable and the sharp cracking sounds of the engine explosions, with prominent components, probably of the order of 1,000, were most distinctly in evidence. The difference in the character of the sound in the two cases may be described as the cutting off of the higher frequencies in the former and of the lower frequencies in the latter. That there may be a more rapid decay of intensity of the higher frequencies is readily understood by a consideration of differences in wave-length. The irregularities in the planity of the strata, for example, would be more effective in scattering, by reflection and refraction, the frequencies having the shortest wave-lengths. In general, the irregularities of the atmosphere would affect the shorter wave-lengths the more.

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[1] Nature, Vol. 101, 1918, p. 284.

The apparent better transmission for the higher frequencies in the second instance is not to be explained by any influence of the medium but rather by the characteristics of audition. The sense of loudness for the different frequencies is not the same, whether the intensities are measured in mechanical units or in terms of minimum audibility. It is the latter unit of measurement that is of interest in the present case, for the nature of the sound heard at a great distance from the source depends upon audibility.

Professor Sabine[1] has proved that a frequency of 64 and of 1,024 appear of equal loudness when the former is 7×10^4 times and the latter 15×10^6 times its own minimum audible intensity. This means that the observer might be situated near a source emitting these two sounds and consider them of equal loudness, and yet with a homogeneous atmosphere and the decrease of intensity of sound according to the inverse square law, he could find that as he recedes from the source he would cease to hear the lower frequency first because its intensity has passed below the minimum audible limit. Thus the explanation of the above-mentioned change of quality with distance from the aëroplane under good listening conditions is to be explained on the varying relative intensities required for equal loudness and minimum audibility. According to Sabine's values, given sounds of equal loudness at a given distance, the one of frequency approximately 1,024 will "carry" the farthest in an ideal medium if the ear is used as the receiving instrument, and the frequencies 64 and 2,048 on either side of this "maximum" will be heard approximately the same distance.

The other experimental fact worthy of record is the rapidity with which the intensity falls off with distance in the atmosphere. The experiments here to be mentioned are crude, but are sufficiently accurate inasmuch as the exact conditions of the atmosphere could not be recorded. Occasions were selected when the temperature gradient at the ground was practically zero and a comparison was made of the maximum distance from the unaided ear and from the listening device, respectively, that a complex sound could be heard. This experiment was repeated many times and in every case the maximum distance with the instrument was approximately ten times that with the unaided ear. Conditions were such that the selectivity of the instrument was believed negligible. Inasmuch as the experiments were performed on a heavily grassed prairie the decrease of intensity with distance must have been even more rapid than as the inverse square. Similar listening experiments were made using the same device and with aëroplanes in flight. On fair sunny days, cumulus clouds forming, with aëroplanes at elevation of one thousand yards, the aëroplanes could be heard only twice as far as with the unaided ear. On days when the atmosphere was obviously more irregular, the decay of intensity was much more rapid. Under good night observing conditions with the aëroplane at an elevation of two thousand yards this maximum distance was increased to three times the distance possible with the unaided ear.

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[1] Contributions, Phys. Laboratory, Harvard Univ., No. 8, 1910.

It thus appears that even when the atmosphere is favorable to sound transmission, there is sufficient irregularity to cause a surprisingly rapid decay of sound from an elevated source.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

Second Series *November, 1919* *Vol. XIV., No. 5*

THE

PHYSICAL REVIEW.

ON THE DISTRIBUTION OF ENERGY IN THE VISIBLE SPECTRUM OF AN ACETYLENE FLAME.

By E. P. Hyde, W. E. Forsythe and F. E. Cady.

Synopsis.

A knowledge of the distribution of energy in the visible spectrum of an acetylene flame has become important within the last few years through the use of this flame, in cylindrical form, in investigations of the visibility of radiation. It can be shown by computation that the data on acetylene published by Coblentz form a curve in the visible spectrum which will not agree with that of a black body at any temperature to better than 7 or 8 per cent. As this would mean that no color match could be obtained and as previous experience of the authors had led to the conclusion that the energy curve of acetylene differed in shape from that of a black body only in the extreme red, a short investigation was undertaken to verify this conclusion.

Tungsten lamps whose current color-temperature relation was carefully determined in this Laboratory, were sent to the Eastman Kodak Company and to the Bureau of Standards with the request that they be compared with the acetylene flame and the current for color match be found. The results gave an average value of 2360° K. ± 10° K., and neither laboratory reported any difficulty in obtaining a match in color. However the Bureau of Standards reported a difference amounting to about 75° K. between the flame as given by the Eastman standard burner and that given by the "Crescent *Aero*" burner, the latter being higher.

The spectral distribution of the flame was measured by means of a spectrophotometer and a spectral pyrometer and the results gave a curve agreeing within the limits of error with that of a black body at 2360° K. In the extreme red, beyond 0.70μ there was indication of a higher emissivity for the acetylene. A photographic method gave results corroborating those just mentioned.

A test of the sensibility of the color-match method to show differences in the spectral energy curve, showed that if two spectral curves matched at 0.5μ and 0.7μ and differed by as little as 4 per cent. in the middle of the spectrum, the two light sources could not be made to match in color.

In conclusion it is recommended that the relative emission intensities of a cylindrical acetylene flame, at least for that type represented specifically by the Eastman standard burner and for the wave-length interval from 0.4μ to 0.7μ should be taken as identical with those of a black body at 2360° K.

THE acetylene flame has frequently been employed in spectroradiometric work, and it has been subjected to investigation with respect to the distribution of energy in its spectrum as well as with respect to its constancy and reproducibility as a standard of luminous intensity. At the present time it demands a special consideration in view of its employment as a standard source in the determination of the visibility of radiation. In the recent investigations of the spectral energy distribution of acetylene a cylindrical flame has been used,—either as produced by a single-jet "Crescent *Aero*" burner as employed by Coblentz,[1] or as given in the standard lamp recommended by the Eastman Kodak Company laboratory, consisting of a 1/4-foot "Bray" tip and a limiting diaphragm restricting the utilizable portion of the flame to 3 mm., as thoroughly described and discussed by Jones.[2] The present note deals exclusively with the cylindrical flame, and the data reported are therefore comparable only with other data obtained with a flame of this form.

In connection with a general survey of the various published investigations of the visibility of radiation, including a careful study of the methods and apparatus employed, the authors observed that the data given by Coblentz for the spectral energy distribution of acetylene showed that the spectral energy curve departed somewhat from that of a black body, indicating that no temperature of a black body could be found for which the distribution of energy in the visible spectrum would be sensibly the same as that of the acetylene flame. This discrepancy is most pronounced in comparing the energy distribution of the two sources in the extreme blue end of the spectrum, when the black-body energy distribution is computed from Wien's equation for a temperature such that at 0.48 μ and at 0.68 μ the emission intensities are relatively the same for the two sources. This comparison is shown in Table I., column 2, where are given the ratios of the emission intensities of a black body at 2434° K. to the published values for acetylene referred to above, the ratios at 0.48 μ and at 0.68 μ arbitrarily being taken as unity. An inspection of this column shows that the relative emission intensity of acetylene at 0.40 μ is approximately 1.6 times that of the black body at 2434° K., and although the deviations in the middle of the spectrum are much smaller,—of a different magnitude,—they would still seem sufficiently large to be indicated by a difference in hue if an attempt were made to match the acetylene and the black body in color, as in the determination of the color temperature of acetylene. It therefore occurred to the authors to obtain the color temperature of the acetylene flame in order to find: (1) Whether any appreciable color difference between the black body and the acetylene flame would be observed when the former was set at the nearest temperature to match the acetylene in color; and (2) whether the color temperature found would indicate a distribution of energy consistent in some region of the spectrum with that given by Coblentz.

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[1] Bul. Bur. of Standards, 13, p. 355, 1916–17.

[2] Trans. Ill. Eng. Soc., 9, p. 716, 1914.

Since a standard acetylene burner was not available in Nela Research Laboratory, two carefully prepared tungsten lamps for which the energy distributions at different voltages were known by comparison with a black body, were sent respectively to Dr. Jones, of the Eastman Kodak Company and to the Bureau of Standards, with the request that the voltage for color match with the standard acetylene flame be determined. No mention of the possible difficulty of obtaining a satisfactory color match was made, the hope being that some statement to this effect would be forthcoming if difficulty were experienced. Previous observations by the authors employing acetylene flames of types other than the standard had shown no such marked selectivity, and consequently it was not expected that such a difficulty would be encountered in the present case.

The two laboratories very kindly made the requested determinations and reported the voltages found for color match. According to the color scale in use in Nela Research Laboratory,[1] these two voltage determinations conduced to the following color temperatures of the standard acetylene flame:

Lamp sent to Bureau of Standards—Voltage 112.3, Color Temp. 2434° K.
Lamp sent to Eastman Kodak Co.—Voltage 102.3, Color Temp. 2352° K.

The difference between these two determinations was so great that it seemed advisable to procure a standard Eastman burner and undertake in Nela Research Laboratory an independent determination of its color temperature. The result of this investigation was a value of 2360° K. ± 10° K. when the lamp was operated at a pressure of 9 cm. and fed with acetylene from a Thorn and Hoddle generator, purified by passage through "Curaze," a material furnished with the generator. A study of the effect of pressure indicated that only very small changes (a few degrees) in color temperature resulted from a change in pressure from 9 cm. to 7.5 cm.,—the pressure employed by Coblentz in his "Crescent *Aero*" burner. Measurements were also made with acetylene obtained from a Prestolite cylinder and passed through a solution of bisulphite of soda to get rid of possible acetone. This experiment gave a somewhat higher color temperature with a maximum difference of 30°. Owing to uncertainty as to the purity of the gas in the tank these values were given no weight.

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[1] These temperatures are based upon Wien's equation with c_2 taken as 14350 μ deg. and upon the temperature of the gold point taken as 1336° K. On this scale the palladium point has been found to be 1828° K. For convenience, a black body held at this temperature is used in calibrating the pyrometers.

During the investigation it was found that for a time after the burner was lighted, when supplied with acetylene from a fresh charge of the Thorn and Hoddle generator, the color temperature was much higher than normal. This may be due to an admixture of air with the acetylene. By permitting the generated acetylene to escape for a while before connecting the supply tube to the burner the effect was eliminated.

In addition to the determinations of color temperature, measurements of the spectral energy distribution were also made with both the spectrophotometer and the spectral pyrometer. With the former instrument readings were made over the range 0.48 μ to 0.68 μ, the results indicating a black-body distribution corresponding to the temperature of 2360° K., the same as found by direct color match. The measurements with the spectral pyrometer extended from 0.45 μ to 0.76 μ and also confirmed the previous results in the region from 0.47 μ to 0.70 μ. The uncertainty of the measurement at 0.45 μ was so great that the observed deviation of 4 per cent. is within the experimental error. Beyond 0.70 μ the measurements indicated that the acetylene flame had a slightly greater emissivity (amounting to about 7 per cent. at $\lambda = 0.75\ \mu$) than that of a black body at 2360° K., consistent with results obtained previously by one of the present authors.[1] Owing to the difficulties of measurement at the extreme wave-lengths too much weight should not be assigned to the observed values in this region; however, all measurements showed the increased emissivity in the red.

It is seen that all the measurements,—those of direct color match and those of spectral energy distribution, conduce to the value 2360° K. as the color temperature of the standard Eastman burner. This result is consistent within experimental errors with the value 2352° K. obtained through the incandescent lamp sent to the Eastman laboratory and compared with a standard burner there through the courtesy of Dr. Jones. It differs, however, by approximately 75° from the value obtained from the incandescent lamp which was matched in color with the "Crescent *Aero*" burner at the Bureau of Standards through the kind collaboration of Messrs. Crittenden and Coblentz.

Another determination through an incandescent lamp was therefore requested of the Bureau of Standards, with a transmitted statement of the results given above. The Bureau courteously undertook a second determination, this time making measurements on both the "Crescent

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[1] Jour. of Frank. Inst., 170, p. 30, 1910.

Aero" burner and on the Eastman standard burner for which Dr. Coblentz had reported[1] the same spectral energy distribution except for a slightly greater emissivity in the extreme red end of the spectrum beyond 0.68 μ. The results of this determination showed a color temperature of 2450° K. as compared with the value 2435° K. previously obtained for the "Crescent *Aero*" burner, and a color temperature of 2370° K. for the Eastman burner. The latter value agrees very well with that (2360° K.) obtained in this Laboratory, but the value found for the other burner, though fairly consistent with the previous determination, indicates clearly that the Eastman burner has an energy distribution distinctly different from that of the "Crescent *Aero*" burner. According to the measurements of Dr. Coblentz the only difference is to be found in the spectral region beyond 0.70 μ, but it is impossible to account for the difference in color temperature of 75° or 80° on this basis. It would also seem inadequate to account for the difference on any such grounds as the quality of acetylene gas used or the effect of atmospheric conditions, since the various determinations on the Eastman burner, made at different places and under different conditions, show a maximum variation of only 18°.

As already stated, the present study was suggested by the deviation of the published results of the energy distribution of the standard acetylene burner from that of a black body. By reference to Table I. in which are given the ratios of the emission intensities of both the "Crescent *Aero*" and the Eastman standard burners, as reported by Coblentz, to those of a black body at 2360° K. and at 2450° K., the ratio at 0.48 μ arbitrarily being taken as unity, it is seen that the reported energy distribution of neither acetylene burner corresponds to that of a black body at any temperature. Not only would it appear that acetylene has an extremely high emissivity in the region of short wave-lengths (0.40 μ to 0.45 μ) but also that in the middle region of the visible spectrum (0.5 μ to 0.7 μ) the deviations are sufficiently large to justify the expectation that no exact color match could be obtained with a black body at any temperature. One would expect, if the published data were accurate, that the acetylene flame would have a slightly purplish color in comparison with which the black body would appear slightly greenish by contrast. Neither of the other two laboratories has reported any such observed effect, and careful observation in Nela Research Laboratory with the phenomenon especially in mind has failed to detect such an effect.

The question naturally arises as to the sensibility of the color-match method. Long experience in this Laboratory has established the conclu-

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[1] *Loc. cit.*

Table I.

Ratio of Emission Intensities of a Cylindrical Acetylene Flame as Found by Coblentz for the "Crescent Aero" and the "Eastman Standard" Burners, to those of a Black Body at Various Temperatures. (Ratio at 0.48 μ taken as unity in each case.)

Wave-length.	Ratios Black Body to "Crescent *Aero*" Burner.			Ratios Black Body to "Eastman Standard" Burner.	
	2434° K.	2360° K.	2450° K.	2360° K.	2450° K.
0.40 μ	0.62	0.58	0.62	0.58	0.62
.42	.74	.70	.75	.70	.75
.44	.86	.83	.87	.83	.87
.46	.94	.93	.95	.93	.95
.48	1.00	1.00	1.00	1.00	1.00
.50	1.04	1.06	1.04	1.06	1.04
.52	1.05	1.09	1.04	1.09	1.04
.54	1.05	1.10	1.04	1.10	1.04
.56	1.06	1.12	1.04	1.12	1.04
.58	1.05	1.12	1.04	1.12	1.04
.60	1.05	1.14	1.03	1.14	1.03
.62	1.04	1.14	1.02	1.14	1.02
.64	1.03	1.14	1.01	1.14	1.01
.66	1.01	1.12	.99	1.12	.99
.68	1.00	1.11	.97	1.11	.97
.70	.99	1.11	.96	1.10	.95
.72	.98	1.10	.95	1.09	.93
.74	.97	1.10	.93	1.08	.91

sion that a relative change in emission intensity at 0.5 μ and at 0.7 μ of 2 per cent., corresponding to a change of approximately 0.5 per cent. in the temperature of a black body in the region of 2400° K. is readily observable. But although this sensibility obtains in matching a black body with another source having approximately a black-body distribution, it tells nothing about the sensibility of the method in comparing with a black body a source which has a relatively high or relatively low emissivity in the middle of the visible spectrum. An important point to determine is the necessary magnitude of this departure from a black-body distribution in order that it should be observed in the color difference as seen in the photometer.

An experiment was arranged to establish this point. A piece of plate glass was selected which by absorption changed the color of the light from a tungsten lamp to a slightly greenish hue. The change in color was so small that close observation was required to detect it. That it was appreciable was indicated, however, by the fact that out of eleven trials an observer judged correctly ten times whether the glass was or was not interposed, and the condition of the experiment was such that no other indication than the greenish tint due to the absorption of the glass could be used as a basis of judgment. Having shown that the color difference between the direct light from the lamp and that transmitted by the glass was appreciable a determination of the spectral transmission of the glass was undertaken. This determination was made using both a spectrophotometer and a spectral pyrometer, the two methods giving relative transmission curves differing by not more than 1 per cent.

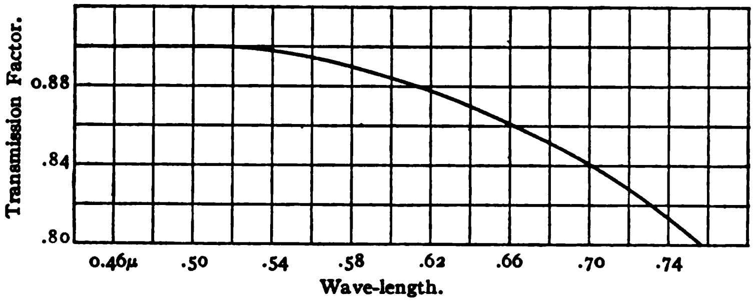

Fig. 1.

Transmission of a Selected Piece of Plate Glass of Slightly Greenish Hue.

The resultant transmission curve for the glass is shown in Fig. 1. It is seen that the glass is slightly bluish as well as having a relatively larger transmission in the green, and consequently the nearest color temperature of light transmitted through the glass would be higher than that of the direct light from the lamp. For the source employed these two color temperatures were found by experiment to be 2414° K. and 2374° K. respectively. If now the computed emission intensities of a black body at 2374° K. are multiplied by the relative transmission factors given in Fig. 1, and the resultant values compared with the computed emission intensities of a black body at 2414° K., the magnitude of the excess in green of the light transmitted by the glass is shown. The results of these computations are given in Table II., computed for every 0.04 μ. It is seen that the relative difference between the two energy distributions which were observed by the color-match method to be appreciably different, is at most 3.8 per cent. between the wave-length limits of 0.48 μ and 0.72 μ, so that it may safely be concluded that corresponding relative differences as large as 6 or 7 per cent., such as would be indicated by comparison of the measurements on acetylene by Dr. Coblentz with the energy distribution of a black body at the nearest color temperature and over the same wave-length interval, should easily be observed.

As stated in a previous paragraph the largest deviation between the published emission intensities of acetylene and the computed emission intensities of a black body at color match (2360° K. for the Eastman standard burner and approximately 2440° K., the average of the two values given by the Bureau of Standards, for the burner used by Coblentz)

TABLE II.

Modification of Black-Body Radiation by Transmission through a Piece of Slightly Selective Plate Glass.

Wave-length.	Transmission Factor of Glass.	(a) Relative Emission Intensities of Black Body at 2374° K. as Modified by Glass.	(b) Relative Emission Intensities of Black Body at 2414° K.	Column (a) — Column (b) in Per Cent.
0.48 μ	0.90	120	120	±0.0
.52	.90	212	208_5	+1.7
.56	$.89_5$	333	326	+2.1
.60	$.88_5$	480	468	+2.6
.64	.87	641	630	+1.7
.68	.85	806	804	+0.2
.72	.83	969	981	−1.2

is to be found in the blue end of the spectrum. The authors' measurements with spectrophotometer and spectral pyrometer could be extended in this region only to 0.45–0.48 μ, so that some other method was required. For this purpose photography was employed, and although its indications as used are not accepted as conducing to accurate quantitative results, they may be depended upon for approximate values.

The method employed consisted in taking a spectrogram of the acetylene flame, and then on an adjacent portion of the plate, a spectrogram with the same exposure time of a standard tungsten lamp operating at the voltage corresponding to the color temperature (2360° K.) of the acetylene burner. From previous measurements of the brightness temperatures of the two sources, it was known to be necessary to interpose a 45° rotating sectored disk in the path of the light from the tungsten lamp in order to bring the two spectra to approximately the same intensities at 0.55 μ. The two spectrograms thus obtained were measured for photographic density at the two wave-lengths 0.55 μ and 0.41 μ, the method of measurement consisting in determining the transmission (corrected for the plate) of the two spectrograms at the two wave-lengths by the use of a pyrometer. A pyrometer with a very small filament was found to be a most convenient instrument for measurements of this kind.

The results of this experiment showed roughly the same relative emission intensities of the Eastman standard acetylene flame and a black body at 2360° K. between the two wave-lengths,—0.55 μ, in the central region of the visible spectrum, and 0.41 μ, well out in the extreme blue end of the spectrum. If Coblentz's values for acetylene were accepted the emission intensity at 0.41 μ for acetylene should be about 75 per cent. more than that of the black body at 2360° K., as compared with the same emission intensities at 0.55 μ, and so the photographic density of the spectrogram for acetylene should be relatively much greater in the short wave-length region.

As a check experiment a series of spectrograms were taken as before, except that the 45° sectored disk was replaced by a 60° disk. For the same exposure time this would expose the plate to 60/45 or 4/3 the quantity of light for the tungsten source, and would show whether an increase of 33-1/3 per cent. in the luminous flux would produce a very appreciable difference. The result was without question the photographic density of the tungsten spectrograms being uniformly greater throughout the spectrum, and yet the difference in exposure for the two spectrograms was less than one half that which would have resulted between the two chosen wave-lengths of the acetylene spectrogram if the relative emission intensities of acetylene were of the order of magnitude found by Coblentz.

It should be noted that the color temperature of acetylene has been assumed to be 2360° K., or that of the Eastman burner, rather than 2440°, that found at the Bureau of Standards, for the burner employed by Coblentz in his principal measurements. But it should also be noted that he found the same emission intensities for the Eastman burner throughout the visible spectrum except in the red beyond 0.68 μ. Moreover, the results would suffer no change in magnitude if the higher color temperature had been assumed.

From these various experiments it must be concluded that the emission intensities published by Coblentz for a cylindrical acetylene flame of the type found in the Eastman standard burner are subject to considerable uncertainty. In the central portion of the visible spectrum the results would seem to be uncertain to the extent of at least 5 per cent., and in the blue end of the spectrum the values given are too high by many per cent. At 0.41 μ for example, the published value would seem to be too high by at least 50 per cent.

In connection with the above experiments measurements were made of the variation of the brightness temperature and of the color temperature for the different parts of the flame seen through the 3 mm. diaphragm used with the Eastman standard burner. There was found a variation of about 25° in the brightness temperature and somewhat more in the color temperature between the top and bottom of the flame. The brightness temperature of the flame at the middle of the diaphragm was found to be 1728° K. at 0.665 μ, which agrees well with the average brightness temperature of the entire portion of the flame exposed by the diaphragm.

Until more accurate data are available the authors would recommend that the relative emission intensities of a cylindrical acetylene flame, at least for that type represented specifically by the Eastman standard burner and for the wave-length interval 0.4 μ to 0.7 μ, should be taken as identical with those of a black body at 2360° K. As stated in the previous paragraph, the brightness temperature at 0.665 μ was found to be 1728° K. From these two sets of data a table of emission intensities may readily be computed.

Nela Research Laboratory,
Cleveland, Ohio,
May, 1919.

When the manuscript of this paper was sent to the printer, a copy was forwarded to Dr. Coblentz who later informed the authors that the "Crescent *Aero*" burner used in the color-matching tests at the Bureau of Standards, was not the same as the one used in his work on visibility. The burner used in the visibility work had an opening of 8 mm., whereas that used in the color-match had a 10 mm. opening. He also stated as the result of a correction in the blue end of the spectrum, his revised data give a distribution agreeing with that of a black body at 2360°K to within 3% from 0.50μ to 0.74μ.

ON THE SPECTRUM OF X-RAYS FROM AN ALUMINIUM TARGET.[1]

By William Duane and Takeo Shimizu.

Synopsis.

Aluminium Spectrum.—The object of the research has been to investigate the question whether or not the frequencies of the K series are the highest X-rays frequencies characteristic of a chemical element. Several investigators have found experimental evidence, which they interpret as indicating the existence of characteristic emission or absorption of frequency higher than those in the K series. For aluminium the wave-lengths corresponding to these frequencies are stated to be $.37 \times 10^{-8}$ cm. and $.49 \times 10^{-8}$ cm. The authors of this paper have examined the emission spectrum of aluminium between the wave-lengths $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. Four small peaks appear on the curves, indicating characteristic radiation at wave-lengths $.622 \times 10^{-8}$ cm., $.705 \times 10^{-8}$ cm., $.975 \times 10^{-8}$ cm. and 1.18×10^{-8} cm. The first two belong to the K series of molybdenum, and undoubtedly come from the metallic molybdenum in the Coolidge carthode in the X-ray tube. The last two belong to the L series of lead, and undoubtedly come from the lead screens containing the slits through which the X-rays passed before they reached the X-ray spectrometer.

No other peaks appear on the curve, and this leads to the conclusion that, within the range examined, aluminium has no emission lines the intensities of which amount to as much as 2 per cent. of the general radiation in the neighborhood.

SEVERAL investigators[2] have obtained experimental evidence, which they interpreted as indicating the existence of the emission or the absorption of X-radiation characteristic of a chemical element (the J-series), and of higher frequency than those in its K-series. Barkla and white (*l.c.*) measured the coefficient of absorption of X-rays in copper, aluminium, paper, water, and paraffin-wax. On platting these coefficients against the wave-lengths, and also on platting the coefficients for one substance against those for another (copper) certain breaks in the curves appeared, similar to, but very much smaller than the breaks that occur at the critical absorption wave-lengths associated with the K and L series of characteristic X-rays. From these breaks they inferred the existence of X-radiation characteristic of aluminium, oxygen and carbon at the wave-lengths $.37 \times 10^{-8}$ cm., 39×10^{-8} cm. and $.42 \times 10^{-8}$

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[1] A paper read at the New York meeting of the American Physical Society, March 1, 1919.

[2] C. G. Barkla, Roy. Soc. Phil. Trans., 217, pp. 315–360, Aug. 29, 1917. C. G. Barkla and Margaret P. White, Phil. Mag., 34, pp. 270–285, Oct., 1917. C. M. Williams, Roy. Soc. Proc., 94, pp. 567–575, Aug. 1, 1918.

cm. respectively. In these experiments the X-rays were not homogeneous, and the wave-lengths were not measured by means of an X-ray spectrometer. They were calculated from the relation between the wave-length and the mass coefficient of absorption in aluminium. At a meeting of the American Physical Society in December, 1914, one of us proposed the equation

$$\frac{\mu}{\rho} = 14.9 \times \lambda^3 \ (\lambda \text{ in ångströms})$$

for this purpose, and it has been used to measure the wave-lengths of the general radiation.[1] It gives the effective wave-length, which may be defined as the wave-length of the homogeneous radiation that would have the same coefficient of absorption as the general radiation in some standard substance.

Williams (*l.c.*) measured the coefficient of absorption of *homogeneous* X-rays in aluminium and in copper. The homogeneous beam of rays was produced and its wave-length measured by means of an X-ray spectrometer. He took the precaution of so regulating the voltage applied to the X-ray tube that no X-rays in spectra of higher order than the first should be reflected by the crystal at the grazing angles used.

He found a small break in the coefficient of absorption wave-length graph for aluminium at $\lambda = .49 \times 10^{-8}$ cm., which is 30 per cent. longer than the wave-length determined by Barkla and White.

Williams also found a break in the corresponding graph for copper, but the break is in a direction opposite to that occurring at the characteristic absorption wave-lengths in the K and L series.

On account of the great importance in the theories of radiation and the structure of matter of knowing exactly what the highest X-ray frequency characteristic of a chemical element really is we decided to investigate the spectrum of the X-rays *emitted* by an aluminium target.

The Wave-lengths of the X-rays we have examined lie between $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. This is the portion of the spectrum usually obtained from an ordinary X-ray tube excited by a difference of potential of 71,200 volts, and includes with ample margins the region in which the above mentioned scientists found the breaks in the absorption curves, and in which, therefore, the J characteristic radiation is supposed to lie.

As the shortest wave-length in the K series of aluminium is 7.98×10^{-8} cm. the range of frequencies in our experiments extends from 6.34 to 43.8 times the highest frequency in this K series.

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[1] Duane & Hunt, PHYS. REV., Vol. 6, No. 2, p. 169, Aug., 1915.

To produce X-rays of higher frequency than the upper limit in our experiments requires a difference of potential greater than 71,200 volts. At this difference of potential the X-ray tube began to show signs of weakening, and we did not push the research further for fear of destroying it.

The thickness of the glass walls of the X-ray tube fixes the lower limit of frequency. X-rays longer than about 1.26×10^{-8} cm. are almost completely absorbed by the glass.

In making the measurements we used the apparatus described in the Physical Review for December, 1917, p. 624. This consisted of an X-ray spectrometer with a calcite crystal, the beam of X-rays being defined by two slits in lead blocks, which lay between the X-ray tube and the spectrometer. The electric current through the tube came from a high tension transformer connected to a system of kenotrons and condensers, which produced approximately a constant voltage.

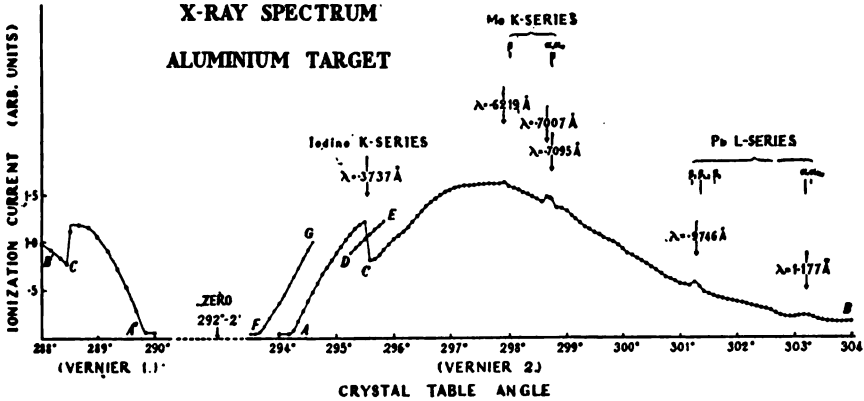

Fig. 1.

The X-ray tube (kindly sent us by Dr. W. D. Coolidge) was similar to the ordinary Coolidge tube, except that an aluminium target replaced the usual tungsten target. The cathode in these tubes consists of a hot tungsten wire coiled in the form of a circular disk and surrounded by a shield of metallic molybdenum.

The figure contains the curves obtained by platting the currents in the spectrometer's ionization chamber against the readings of the verniers attached to crystal table.

We took readings at intervals of 5 minutes of arc, having purposely regulated the breadths of the slits so as to eliminate the possibility that a peak representing characteristic radiation might lie between two such readings.

Curves *AB* and *A'B'* represent an experiment in which the difference of potential applied to the tube amounted to about 53,200 volts. The points *A* and *A'*, where the curves begin to rise above the natural ionization leak of the instrument, correspond to the frequency which, when multiplied by Planck's action constant, *h*, equals the energy acquired by an electron in the X-ray tube as given by the product of the difference of potential into its electric charge.[1]

The position of the zero on the scale was estimated from these two points *A* and *A'*. A correction of 45″ of arc must be subtracted from the double grazing angle of incidence 2θ on account of the scale's eccentricity, when calculating the wave-lengths from the usual formula $\lambda = 6.056 \times \sin\theta \times 10^{-8}$ cm.

At the points marked *C* and *C'* occur sharp breaks in the curves. These are due to the fact that the ionization chamber contained methyl iodide. The critical ionization wave-length of iodine is $.3737 \times 10^{-8}$ cm.,[2] and X-rays of shorter wave-length than this ionize the gas much more strongly than X-rays of longer wave-length do.

As the break in the absorption curve obtained by Barkla and White (*l.c.*), from which they inferred the existence of characteristic *J* radiation occurs at wave-length $.37 \times 10^{-8}$ cm., and as the break in our curve at *C* might completely mask a peak representing characteristic emission lines, we repeated our measurements over wave-lengths in this neighborhood using ethyl bromide in the ionization chamber instead of methyl iodide. The curve *DE* represents this experiment, and no peak appears in it that can be interpreted as indicating characteristic emission lines.

Further no peak appears on the curve *AB* in the neighborhood of wave-length $.49 \times 10^{-8}$ cm. at which Williams (*l.c.*) found a break in the absorption curve.

There are, however, two small peaks at wave-lengths $.6219 \times 10^{-8}$ cm. and $.7051 \times 10^{-8}$ cm. Within the limits of experimental error these are the wave-lengths of the β and α lines in the K series of molybdenum, and it is reasonable to suppose that these peaks are due to the presence of the metallic molybdenum in the Coolidge cathode. It is uncertain whether they are to be ascribed to primary radiation from a thin film of molybdenum deposited on the target during the excessive heating process to which these X-ray tubes are subjected during their exhaustion, or to tertiary, etc., rays excited in the target by secondary rays from the molybdenum in the cathode.

At wave-lengths $.9746 \times 10^{-8}$ cm. and 1.177×10^{-8} cm. appear two more small peaks indicating characteristic radiation. These peaks correspond exactly with the β and α lines in the L series of lead, and, doubtless, are due to secondary rays from the lead blocks containing the slits through which the X-rays passed before they reached the spectrometer.

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[1] Duane & Hunt, PHYS. REV., August, 1915, p. 169.

[2] Duane and Hu, PHYS. REV., June, 1918, p. 491.

In order to extend the research to X-rays of shorter wave-length than those produced by a difference of potential of 53,200 volts, we took a series of readings with a difference of potential of 71,200 volts. The curve *FG* represents these measurements, and it contains no peak.

Molybdenum and lead are much more efficient radiators of X-rays than aluminium is, for their atomic numbers are higher, 47 and 82 respectively, instead of 13 for aluminium. Further, if the voltage applied to the X-ray tube lies considerably above that required to produce the characteristic rays of its target (as is the case with molybdenum and lead in our experiments), the characteristic radiation is many times more intense than the general radiation in its neighborhood. It is not surprising, therefore, that secondary or tertiary characteristic radiation from molybdenum and lead can produce effects amounting to a few per cent. of the general primary radiation from aluminium, upon which it is superposed, as indicated by our curves.

No peaks representing the emission of characteristic X-rays appear on our curves other than those corresponding to the K series of molybdenum and the L series of lead, and we conclude, therefore, that aluminium has no characteristic lines in its emission spectrum, between the wave-lengths $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. that amount to as much as 2 per cent. of the general radiation in the neighborhood and that can be produced by the voltages we employed.

Harvard University.

STUDIES OF THE ADSORPTION OF GASES BY CHARCOAL. II.[1]

BY HARVEY B. LEMON AND KATHRYN BLODGETT.

SYNOPSIS.

Problem.—When an activated and previously evacuated specimen of cocoanut shell charcoal is exposed to an excess of a mixture of oxygen and nitrogen, the two gases are unequally adsorbed. It is desired to determine how the equilibrium pressure at constant volume depends upon the composition of the mixture; the ratio of the mass of gas to the mass of charcoal being the same in all cases.

Results.—(1) There is a nearly linear relation between the logarithm of the equilibrium pressure and the per cent. of oxygen in the mixture. (2) The two gases in the mixture are not adsorbed independently. (3) A given volume either of a pure gas or of a mixture is less completely adsorbed if admitted at one time than if it is admitted in two portions, the second half being admitted after the adsorption of the first has reached equilibrium.

RELATIVE ADSORPTION OF MIXTURES OF OXYGEN AND NITROGEN. EQUILIBRIUM EFFECTS.

THE close connection between the boiling temperature of a gas and the amount which is adsorbed in charcoal at a low temperature has been recognized for many years. In 1907 H. Baerwald wrote:[2] "The more easily a gas is condensed, the more abundantly, then, it is adsorbed, the harder it is to expel it again from the charcoal, and the longer the process of adsorption lasts"; a statement which has been found true, in general, for all gases except hydrogen, whose adsorption is far greater than would be expected from its low boiling point.[3] Oxygen and nitrogen consistently follow this rule. Oxygen which boils at − 182.7° C. is adsorbed and expelled so much more readily than nitrogen which boils at − 195° C., that Dewar, after finding that the gas adsorbed by charcoal from air was 57 per cent. oxygen, recommended "that one of the most rapid means of extracting a high percentage of oxygen from atmospheric air is to adsorb it in charcoal at low temperatures and then to expel it either rapidly or slowly by heating the mass of charcoal to the ordinary temperature."[1]

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[1] This article is published with the approval of Major General William L. Sibert, Director of Chemical Warfare Service, U. S. A.

[2] H. Baerwald, "Ueber die Adsorption von Gasen durch Holzkohle bei tiefen Temperaturen," Ann. d. Physik, 23, 90–91, 1907.

[3] M. Georges Claude, "Sur l'adsorption des gaz par le charbon aux basses temperatures," Comptes Rendus, 158, 863, 1914.

The object of the present experiment was to investigate the adsorption of *mixtures* of oxygen and nitrogen, to learn if the adsorption increased continuously from that for pure nitrogen to that for pure oxygen as the percentage of oxygen present in the mixtures was continuously increased. It was also found profitable to investigate the behavior of the constituents when adsorbed separately, in order to determine whether or not they acted independently when adsorbed together as a mixture.

The apparatus was built in such a way that a few grams of charcoal could be heated to a high temperature under very low pressure; then when the charcoal had been cooled to the temperature of liquid air, 975 c.c. of gas could be admitted suddenly and the pressure measured as it varied during adsorption from 90 cm. to 0.0001 cm. To accomplish this a MacLeod gauge and barometer were built together as shown in the diagram (Fig. 1), and were joined to a glass tube *T*. To this tube were also joined a half-liter glass bulb *B* to contain a large part of the gas before adsorption, and three Pyrex glass bulbs to hold the charcoal. The total volume of *B*, *T*, barometer and gauge was 975.14 c.c. Each charcoal bulb was sealed to a ground glass joint with sealing wax and was separated from *T* by a stopcock. A discharge tube joined on between the stopcock and bulb was useful to indicate any leak in the cock. During "outgassing" an electric oven was placed around the three bulbs, the temperature being indicated by a Leeds & Northrup potential

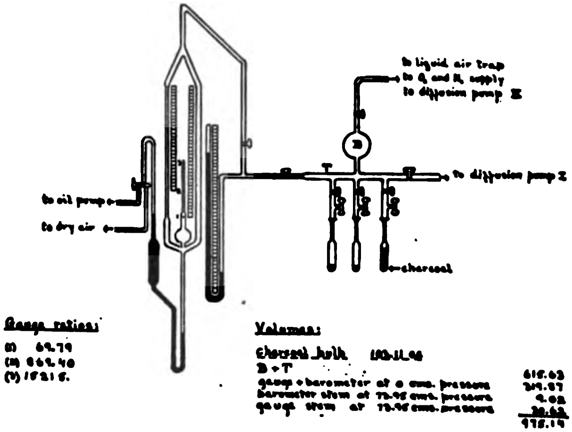

Fig. 1.

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[1] Sir James Dewar, "The Adsorption and Thermal Evolution of Gases occluded in Charcoal at Low Temperatures," Proc. Roy. Soc., 74, 126, 1904.

point thermometer, and they were connected through T to a diffusion pump; during cooling the bulbs were shut off from T and each cooled in turn, just before using, by placing around it a container filled with liquid air. The bulb B was connected through a cock and through a liquid air trap to the oxygen and nitrogen supply and to a second diffusion pump which permitted thoroughly pumping one gas out of the connecting tube and trap before the other was admitted.

The charcoal used in the investigation was some that had been used previously in a long series of experiments described in a previous paper and was therefore completely activated and constant. Preliminary experimenting showed that 6.5 grams of charcoal was the amount needed

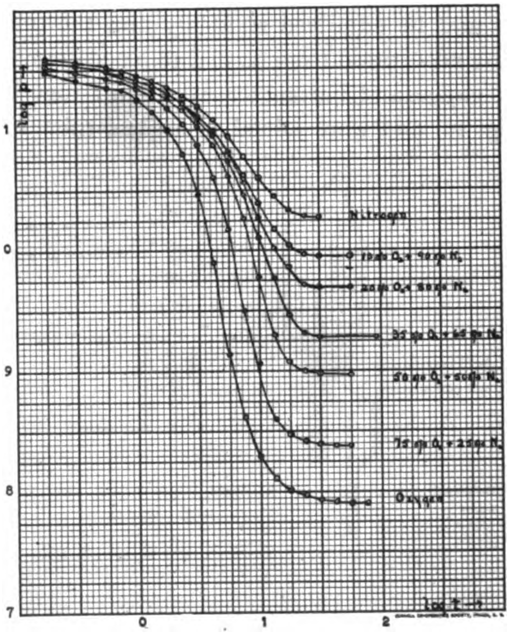

Fig. 2.

Adsorption of nitrogen, oxygen, and mixtures of the two gases.

to adsorb 975 c.c. of gas satisfactorily, for a larger amount gave such complete adsorption of both oxygen and nitrogen that the first requisite of the experiment, *i. e.*, a difference in the adsorption of the two gases, was lost.

Considerable precautions were taken to insure the purity of the gases used. Both oxygen and nitrogen were periodically examined spectroscopically and were used only when they were found to be of a high degree of purity. The oxygen was generated from pure peroxide of hydrogen, sulphuric acid and potassium bichromate, then passed through caustic solution and a phosphorus pentoxide drying tube, and finally liquefied in the trap before being admitted to bulb *B*. The nitrogen was generated by heating a mixture of ammonium sulphate and sodium nitrate solutions. The gas was passed through both potassium bichromate and potassium hydroxide solutions, a P_2O drying tube, and the liquid air trap which completely removed all oxides of nitrogen.

In order to prepare the charcoal for adsorption by outgassing it, it was heated for four and a quarter hours at 582° C. That time has been found sufficient to expel by far the greater part of the gas in charcoal; and the temperature was as high as the walls of the charcoal tubes could be heated for a long period without collapsing. The pressure was read on the gauge at the completion of outgassing, the bulbs were shut off from the pump when they had cooled to 400, and the pressure was again read at 300 and at room temperature, to safeguard against possible leaks. At room temperature all traces of gas had disappeared, as far

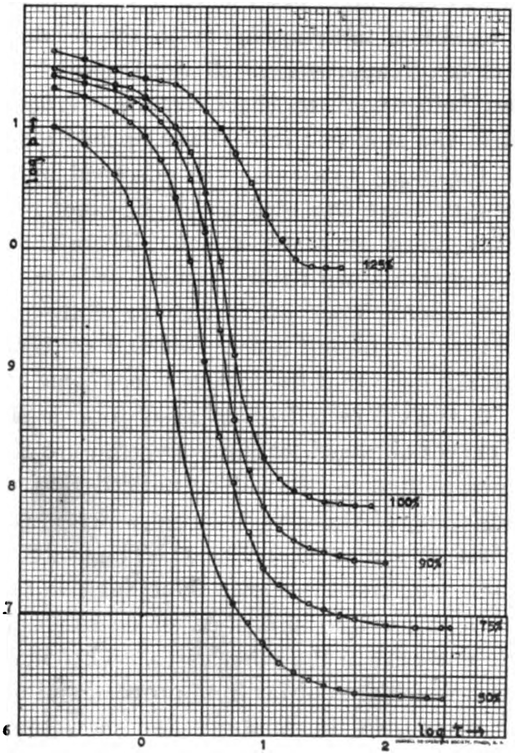

Fig. 3.

Adsorption of various quantities of oxygen alone.

as could be read from the gauge, and the three tubes were shut off from *T*.

The gas before being admitted to the charcoal was contained in *B*, *T*, the barometer and gauge, and its quantity measured by the pressure on the barometer. The same quantity of gas, 975.14 c.c. at 23° C. and 73.95 cm. (*i.e.*, 871.23 c.c. at 0° C. and 76.0 cm.) was used in all the experiments. Care was taken to insure complete mixture of the contents of *B* and *T* before testing.

Before admitting the gas to the charcoal, the charcoal bulb was tested with the discharge tube to make certain that no gas had leaked in from *T* since outgassing, and the bulb was cooled for at least twenty minutes in fresh liquid air. Precautions were taken to insure the freshness of the liquid air which was used in all cases when it was between two and four hours old, and to obtain full consistency in the results the same tube with the same charcoal was used in all the recorded experiments, the other two tubes being used to check results. When the tube was thoroughly cooled, the stopcock into *T* was opened quickly and as the gas rushed into the empty space and was adsorbed by the charcoal the rapidly decreasing pressure was read on the barometer and then on the gauge at intervals timed by a stop-watch. These intervals beginning with 11 seconds and ending with 1 hr. 40 mins. or later, according to the rate of adsorption, where chosen with a view to plotting logs. of pressure against logs. of times to obtain a compact and convenient curve, and were therefore so arranged that the logs. of times came at regular successive intervals. A typical record of an adsorption is shown in Fig. 3.

The series of curves obtained for pure oxygen, pure nitrogen, and five mixtures is shown in Fig. 2. It will be noticed that there is a slight hump at the beginning of the oxygen curve which appears again more strongly in the first oxygen curve in Fig. 3. This is due to the fact that the charcoal is at a temperature below the boiling point of oxygen which introduces conditions for the adsorption of considerable quantities of oxygen which do not exist in the other cases. The curves are regularly spaced at intervals which are not far from proportional to the percentage of one gas, say oxygen, in the mixture. To show the relation of the intervals to the percentages, in Fig. 5 (1) the logs of the final pressures in Fig. 2 have been plotted against percentages of oxygen. The result is a curve very slightly concave. It has been found, then, that the rates of adsorption of mixtures of oxygen and nitrogen vary continuously from the rate for nitrogen to that for oxygen, but that this variation takes place not according to a linear relation, but according to a relation that is very nearly exponential.

Numerical values are tabulated below:

Table I.

Mixtures of Oxygen and Nitrogen.

Oxygen, Per Cent.	Nitrogen, Per Cent.	Final Pressure.	Log p.
100		0.00797	7.902
75	25	.0240	8.380
50	50	.0924	8.966
35	65	.214	9.332
20	80	.489	9.690
10	90	.886	9.948
	100	1.86	0.270

To determine if the two gases acted independently when adsorbed as a mixture, the quantities of each gas present in certain mixtures were experimented upon separately. The system was filled with one gas only, at a pressure which was a given fraction of the usual 73.95 cm., and this gas was adsorbed as before. Fig. 3 shows the series of curves obtained

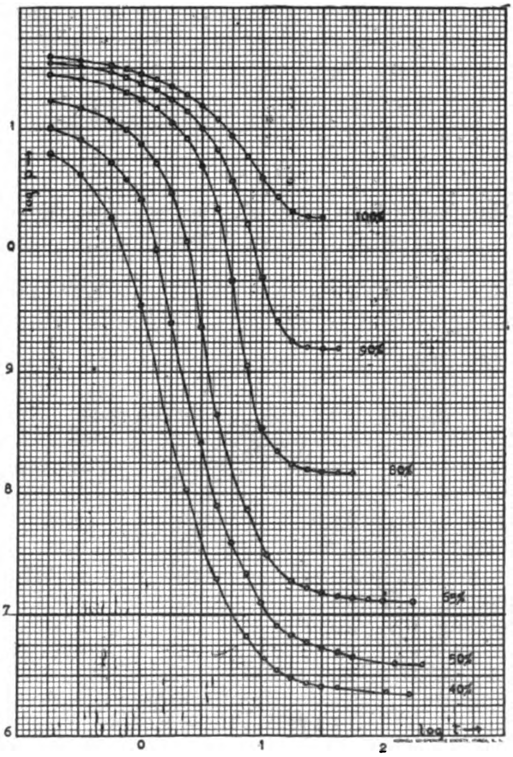

Fig. 4.

Adsorption of various quantities of nitrogen alone.

in this way for oxygen, including one taken with 125 per cent. of the usual quantity of gas. Fig. 4 shows a similar series for nitrogen. In Fig. 5, (2) and (3), the logs of these final pressures have been plotted against percentage of oxygen, decreasing nitrogen percentage being plotted as increasing oxygen percentage. Table II. contains the numerical results in these cases.

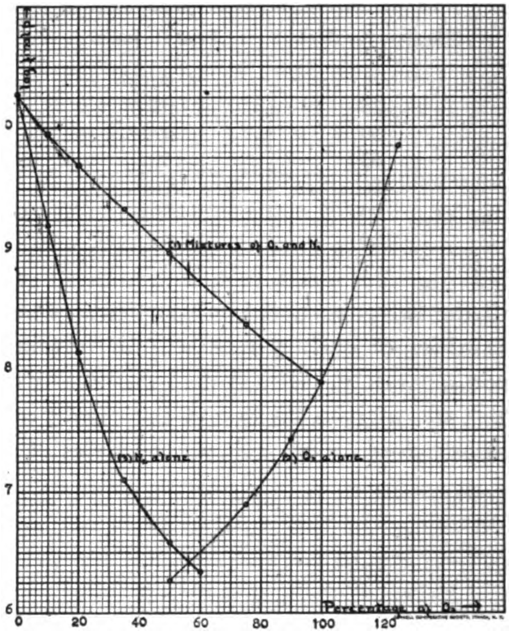

Fig. 5.

Variation of final pressure with gas adsorbed. Oxygen and nitrogen adsorbed separately 2) and (3); together in mixture (1).

TABLE II.

Oxygen and Nitrogen adsorbed separately.

Oxygen, Per Cent.	Final Pressure.	Log *p*.	Nitrogen, Per Cent.	Final Pressure.	Log *p*.
125	0.715	9.854	100	1.86	0.270
100	0.00797	7.902	90	0.157	9.196
90	0.00269	7.430	80	0.0144	8.158
75	0.000785	6.895	65	0.00127	7.102
50	0.000181	6.259	50	0.000386	6.587
			40	0.000220	6.343

(*c*) RELATED PHENOMENA.

In the course of the work it became of interest to know if the final adsorption would be the same if a given amount of a pure gas were adsorbed all at once as if half were adsorbed at one time and the second half admitted later. It was found, as the data will show, that adsorption is more complete in the second case.

	Final Pressure.	Log p.
(*a*) Oxygen, 100 per cent.	0.00797	7.902
(*b*) Oxygen, 50 per cent. 50 per cent.	0.00604	7.781
(*a*) Nitrogen, 100 per cent.	1.86	0.270
(*b*) Nitrogen, 50 per cent. 50 per cent.	0.649	9.812

In this connection it was suspected that charcoal exposed first to 50 per cent. oxygen and then to 50 per cent. nitrogen might yield different results from charcoal exposed in a manner vice versa. The difference was found to be too slight to have any significance, being within the range of experimental error; but as one would expect from the phenomena noticed above, the adsorption was more complete than in the case of a 50 per cent.–50 per cent. mixture of the two gases adsorbed all at one time.

	Final Pressure.	Log p.
1. (*a*) Oxygen 50 per cent., (*b*) nitrogen 50 per cent.	0.0451	8.654
2. (*a*) Nitrogen 50 per cent., (*b*) oxygen 50 per cent.	0.0481	8.682
3. Oxygen 50 per cent., nitrogen 50 per cent.	0.0924	8.966

(*d*) Conclusions Regarding Independence of Gases.

It is readily seen from a consideration of the curves in Fig. 5 that the two gases do not act independently in a mixture. For the logarithm of the sum of the final pressures of each component gives nowhere a point approaching curves (1). This is not an unnatural result, for if the interior surfaces of the charcoal are largely covered with one gas it is plausible to suppose that the forces producing adsorption are different from those that the denuded surface would show.

F. Bergter has stated as a result of his work on oxygen and nitrogen that[1] "In the range of pressure which I have investigated (low pressures of 1 mm. and less) the ability of charcoal to adsorb nitrogen must be increased by the presence of oxygen." He found that by using 9.483 gram of charcoal to adsorb aid at 0.523 mm. pressure, he obtained as a lowest estimate 59 units of nitrogen, while the same charcoal exposed to nitrogen alone adsorbed 29 units. The present work done under altogether different conditions of pressure, volume, and quantity of charcoal used, can neither verify nor refute this statement. But since in the present work the adsorption of a mixture was found inferior to the sum of the adsorptions of the components, it is not probable that Bergter's discovery is applicable to this case, different from his own; for if the ability of the charcoal to adsorb the nitrogen were increased by the presence of the oxygen, then the presence of the nitrogen must to a far greater extent have hindered the adsorption of the oxygen. It seems simple and more natural to accept the hypothesis that each gas hindered the adsorption of the other.

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[1] F. Bergter, "Der Zeitliche Verlauf der Adsorption von Gasen durch Holzkohle," Annalen der Physik, 37, 606, 1912.

RYERSON PHYSICAL LABORATORY,
March, 1919.

THE END CORRECTION IN THE DETERMINATION OF GAS VISCOSITY BY THE CAPILLARY TUBE METHOD.[1]

By A. F. Benton.

Synopsis.

The voluminous data obtained for air from experiments on capillary tubes as flow meters have been used to test the formulae involving end-corrections, proposed by Brillouin and by Fisher. A measure of the validity of a given formula is obtained by assuming the value of the coefficient of viscosity and comparing the mass or gas transpired per second, as found experimentally, with the mass computed from the formula in question. The tubes employed in the tests ranged in size from very long lengths, where the end-effect was practically negligible, down to short lengths that they behaved as orifices.

Under this very exacting test, Fisher's equation for the end-correction is shown to be wholly unsatisfactory and to lead to impossible results. Brillouin's formula however reproduces the observed data fairly well. The agreement becomes excellent if the latter formula is slightly modified by the introduction of a constant, similar to the "orifice coefficient", to take account of the form of the ends of the tube. Finally a criterion is given for determining the negligibility of the end-correction within any desired limits of accuracy.

THE ordinary development of the equation for the flow of fluids through a capillary tube involves the use of the pressures inside the tube, at such a distance from the ends that the stream lines are parallel. The pressures measured in the relatively large end-vessels are greater than those demanded by the formula, since here the velocity of the fluid is practically zero, and the maximum pressure in a moving fluid is obtained at a point where the velocity is least. The possibility of error from this source in the capillary tube method of determining viscosity, was recognized very early. The older writers however have generally tried to make the error negligible by using very long, fine tubes, but in so doing have often introduced relatively great errors in the measurement of the radius.

The first attempt to set up an equation for the correction was made by M. Brillouin.[2] He considers that the kinetic energy of the emergent jet is a measure of the pressure lost at the ends of the tube, and, on this basis, derives the following modified form of Poiseuille's law:

$$p_1^2 - p_2^2 = \frac{p}{\rho}\left[\frac{16\eta LM}{\pi R^4\left(1 + \frac{4\xi}{R}\right)} + \frac{2M^2}{\pi^2 R^4}\right],$$

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[1] Published by permission of the Director of the Chemical Warfare Service.

[2] Leçons sur la Viscosité des Liquids et des Gas, Vol. 1, p. 133; Vol. 2, p. 37.

where p_1 and p_2 are the entrance and exit pressures, respectively; p and ρ are the pressure and corresponding density of the fluid, measured under the same conditions; M is the mass transpired per unit of time; η is the coefficient of viscosity, and ξ the coefficient of slip; and L and R are the length and radius of the tube, respectively. More recently, W. J. Fisher[1] has put forward another formula for the correction, which he regards as a closer approximation to the true state of affairs than Brillouin's:

$$R'T ln\frac{p_1}{p_1'} = -\frac{1}{384}\left[\frac{p_1^2 - p_2^2}{\eta p_1 L}\right]^2[R^4 + 12\xi^2R^2 + 6\xi R^3].$$

Here R' is the gas constant, T is the absolute temperature, p_1' is the measured pressure in the vessel at the inlet end, and p_1 is the pressure to be used in Poiseuille's law. A similar equation with subscripts 2 gives the correction to the exit end. In applying this formula to his own data, Fisher finds the correction to be negligible.

Brillouin's correction has apparently been applied only by I. M. Rapp,[2] who has calculated the corrections to his data by both formulæ, and finds them in fairly good agreement. However, the writer is unable to check Rapp's calculations of the Fisher correction, except on the assumption that p_2 is greater than p_2', whereas the formula clearly shows that p_2 must be less than p_2'. It is hard to conceive how the pressure could be greater at a point where the velocity is high (p_2), than at a point where it is practically zero (p_2'). Fisher's correction, properly applied to the data, is negligible. Rapp's use of it has furnished values in fair agreement with Brillouin's formula in this case, only because the corrections are very small. An inspection of Fisher's equation will suffice to show that, on the assumption which Rapp appears to have made, p_2 could quite easily become larger than p_1, in which case the substitution of these values in Poiseuille's law would give a negative coefficient of viscosity. A further examination of the formula will make clear however that, even when properly applied, Fisher's correction can not be universally valid. For it will be observed that in any given trial,

$$ln\frac{p_2}{p_2'} = \frac{p_1^2}{p_2^2} ln\frac{p_1}{p_1'}.$$

If the pressure difference be made very large, it is evident that $ln \cdot p_2/p_2'$ will have a very much larger numerical value than $ln \cdot p_1/p_1'$, and p_2 becomes very small, so that its square is negligible in comparison with p_1^2.

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[1] PHYS. REV., 32, 216, 1911.

[2] PHYS. REV., (2), 2, 363, 1913.

Poiseuille's law, corrected by Fisher's equation, then gives

$$M \propto \frac{p_1^2 - p_2^2}{p_2'} \propto \frac{p_1^2}{p_2'},$$

from which it follows that, since p_1 decreases as the measured pressure difference increases, M should now decrease as the pressure difference increases. A graph of Fisher's correction which illustrates this retroflex effect will be discussed below. This behavior is of course not found experimentally.

In Rapp's data the necessary corrections are so small that the results do not give a satisfactory test of any formula. He has nevertheless deduced an empirical correction from some of his data, concerning which it will only be pointed out here that it is open to the very serious objection that under certain conditions it can give negative values for the viscosity coefficient.

In connection with the use of capillary tubes as flow meters,[1] a considerable quantity of experimental material has been obtained by the author, which throws light on the end correction. The results, though not of the highest accuracy, are, as will appear from what follows, entirely adequate for the present purpose. Tubes of approximately 0.035 cm. and 0.07 cm. radius were employed, and tests were made on eight different lengths of each, from 1 m. down to 1 cm. Only those values have been used which lay below the critical region.[2] In testing the various formulæ against these results, which were all obtained with air, the coefficient of viscosity has been assumed known, and the weight of gas transpired per second is compared with the observed weight. Table I. gives a number of points selected at random. Space does not permit of including all the rather voluminous experimental material, but the magnitude of the correction in each case is indicated by the difference between columns 1 and 4. It is evident that Brillouin's formula is in fair agreement with the observed values, but Fisher's correction does not even approximately represent the data. These facts are strikingly shown in the accompanying figure, in which the radius was approximately 0.034 cm. and the length was only 0.93 cm. The circles indicate observed points. The retroflection of the curve for Fisher's correction, anticipated above, is here very well illustrated. This extreme case has been chosen solely for convenience in graphical representation.

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[1] These are described in a paper on "Gas Flow Meters for Small Rates of Flow," J. Ind. Eng. Chem., 11, 623, 1919.

[2] The location of the critical region was ascertained by plotting all the results in the usual form, suggested by an application of the "principle of dimensional homogeneity." See for example, T. E. Stanton and J. R. Pannell, Trans. Roy. Soc. London, A, 214, 199, 1914.

The same result is obtained with longer tubes, but in that case the vertex of the curve lies at a higher value of the pressure difference.

Although Brillouin's formula represents all the results fairly well, it gives values for the mass transpired which are higher than the observed values by 3 to 5 per cent. This deviation greatly exceeds the experimental error, and is always in the same direction. The discrepancy is undoubtedly to be accounted for by the fact that all the tubes here used had sharp ends, which produce a very considerable contraction in the cross section of the gas stream. This means in effect that the radius in the last term of Brillouin's formula is smaller than the radius in the preceding term. If C be used to denote the ratio of the radii of the tube and of the vena contracta, respectively, Brillouin's equation becomes

$$p_1^2 - p_2^2 = \frac{p}{\rho}\left[\frac{16\eta LM}{\pi R^4\left(1 + \frac{4\xi}{R}\right)} + \frac{2CM^2}{\pi^2 R^4}\right].$$

TABLE I.

Mass of Air Transpired, in Milligrams per Second.

Mass of Air Transpired, in Milligrams per Second.			
Observed.	**Calculated by the Formula of**		
	Brillouin.	**Fisher.**	**Poiseuille.**
2.23	2.35	2.35	2.38
4.66	4.96	6.27	6.32
8.36	8.90	27.61	27.80
17.29	17.72	75.96	92.41
21.00	21.44	73.84	130.2
25.77	26.64	32.03	194.6
26.38	27.20	28.75	29.20
38.44	39.25	125.9	126.5

C will in general depend on the radius and on the pressure drop, but within fairly narrow limits, it may reasonably be assumed constant, as is evidenced by the fact that the equation in this form represents the observed results very accurately, even when the end-correction is large. C will of course also be a function of the form of the ends of the tube, and if the latter are well rounded out, will be very close to unity, so that in this case it may probably be disregarded altogether, especially since the whole end-correction is ordinarily quite small.

An equation which is essentially a simplified form of Brillouin's has been developed during the course of the experiments, on the assumption that the total pressure difference across the capillary tube consists of two parts, the frictional resistance within the tube and the resistance of the orifices at the ends. If the latter be denoted by h_0, the former by h_L, and the total resistance by H, a combination of Poiseuille's law with an approximate expression[1] of the orifice formula gives

$$H = h_L + h_0 = \frac{k_1 \eta L M}{\rho R^4} + \frac{k_2 C M^2}{\rho R^4},$$

or

$$\rho H R^4 = k_1 \eta L M + k_2 C M^2.$$

k_1 and k_2 are constants, and C is the familiar orifice "coefficient." Since this equation is not strictly valid, and since, with sharp ends, C is a function of H and R, the values of k_1 and k_2 were determined experimentally rather than theoretically. Application of the method of least squares to the forty-nine observations which were below the critical region, gave

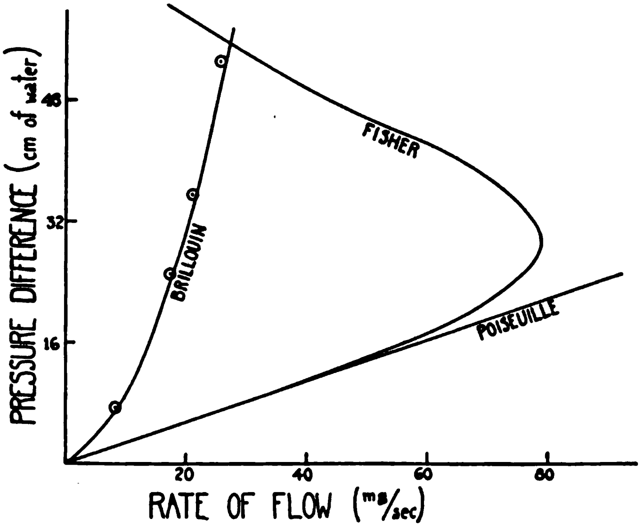

Fig. 1.

$$\rho H R^4 = 2.580 \eta L M + 0.1248 M^2.$$

This equation represents the observed results, even in the most extreme cases, where the experimental error is relatively great, within 2 per cent. Where high pressure differences are used, it will of course give only a rough approximation. In such cases the more exact form given by Brillouin should be employed.

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[1] This simplification is permissible here because relatively small pressure differences were employed. See R. J. Durley, Trans. A. S. M. E., Vol. 27, p. 193, 1906.

From these equations it is a very simple matter to derive a criterion for the negligibility of the end-correction, within any desired accuracy. For example, if the correction is to be less than 0.1 per cent., it is evident that $2.580\eta ML$ must be greater than $1,000 \times 0.1248M^2$. That is, $\eta L/M$ must be greater than 48.4. Since $\eta L/M$ is a dimensionless product, its value is independent of the system of units in which it is expressed, as long as their interrelations remain unchanged. In fact Brillouin's equation and the simplified form of it are each equally valid whether metric or English units are employed.

AMERICAN UNIVERSITY EXPERIMENT STATION,
WASHINGTON, D. C.

THE RADIATING POTENTIALS OF NITROGEN.

By H. D. Smyth.

Synopsis.

1. A formula is derived by which accurate corrections allowing for the distribution of velocities of the impacting electrons may be applied to the observed values of the radiating potentials.

2. Measurements on nitrogen revealed—

(a) a very strong effect at 8.29 ± 0.04 volts.

(b) a very doubtful effect at 7.3 volts.

(c) an effect appearing only at lower pressures but strongly at 6.29 ± .06 volts.

3. These results are explained as follows:

(a) The wave-length λ corresponding to 8.29 ± .04 is 1490.7 ± 10 and this effect is ,therefore, identified with the doublet found by Lyman at 1492.8 and 1494.8.

(b) This gives $\lambda = 1700$ and may be identified with the second doublet 1742.7 and 1745.3 attributed to nitrogen but by some thought due to silicon.

(c) The value $\lambda = 1965 \pm 20$ from 6.29 ± .06 volts is taken to correspond to the beginning of the band spectrum at 1870.9. The discrepancy is attributed to the presence of nitrous oxide.

According to another theory the effect at 6.29 is considered due to lines in the region 2,000–3,000 A.U. coming from neutral atoms. The value 6.29 in this case is taken as the speed necessary before the electrons can split up the molecules.

4. From 3 (a) assuming line spectra to come from atoms, we have as an upper limit to the heat of dissociation of a gram molecule of nitrogen 190,000 calories.

From the second theory in 3 (b) we have as a possoible actual value for this heat of dissociation 145,000 calories.

5. Qualitative evidence was obtained supporting Davis and Goucher's discovery of true ionization in the neighborhood of 18 volts.

I. Introduction.

The substances whose minimum radiating and ionizing potentials have been investigated fall naturally into two classes, first, the monatomic gases and metallic vapors, in which collisions below a certain velocity are elastic, and second, the diatomic gases, in which collisions at all velocities are inelastic or at least partially so, as in the case of hydrogen.

The investigations for substances of the first class have recently been summarized and discussed by McClennan.[1] The conclusions drawn are, briefly, as follows. A vapor, when bombarded by electrons of velocity V emits a radiation $eV = h\nu$. The first radiation produced is the line at the head of the single *t* principal series, and thereafter shorter and shorter wave-length radiations set in as the bombarding electrons attain correspondingly higher velocities. Finally, when the speed of the electrons reaches a value corresponding to the frequency $\nu = (1.5, S)$, the convergence frequency of the singlet principal series of the element in question, ionization occurs.

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[1] J. C. McClennan, "The Origin of Spectra," Proc. London Phys. Soc., XXXI., Part I., pp. 1–29, 1918.

For diatomic gases the mechanism of ionization or the production of radiation is obviously more complicated than where only a single atom is involved. Either it is necessary first to dissociate the molecules and then, by a second impact, to cause radiation or ionization, or only one collision is necessary. Again, if the effect is the result of one impact, it may occur in two ways; an electron may be displaced and the molecule broken up simultaneously, or an electron may be displaced without affecting the bond between the atoms.

Possibly as a result of the above, the spectra of the diatomic gases are very complex and, for the most part, not resolved into series. This makes the prediction of ionizing and radiating potentials difficult. Another source of error especially serious in the case of diatomic gases arises from the uncertainty in applying the necessary correction for velocity distribution, as discussed later in the paper.

In consequence of these complications, although values of the ionizing and radiating potentials have been determined for a number of different diatomic elements and compounds, little has been accomplished toward connecting them with the spectra. In spite of the simple structure of the hydrogen molecule and the extensive data on the spectrum available, the experimental results obtained have not yet been reconciled with those predicted by Bohr's theory or by the quantum relation[1] $eV = h\nu$.

Though the spectrum of nitrogen has not been resolved into series, in the region of the ultra-violet with which we are concerned the line spectrum is very simple. Lyman,[2] in his investigation of the extreme ultra-violet, found for nitrogen no lines except two doublets, one consisting of the lines 1492.8 and 1494.8 and the other[3] of the lines 1742.7 and 1745.3. Applying the quantum relation to these, we have for the corresponding radiating potentials, 8.28 and 8.27, and 7.04 and 7.08 volts. The longest wave-length of the band spectrum which Lyman found in this region is 1870.9 corresponding to 6.6 volts. Now the generally accepted value for the radiating potential of nitrogen is 7.5

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[1] Bergen Davis and F. S. Goucher, PHYS. REV., No. 10, p. 101, 1917.

[2] Lyman, "Spectroscopy of Extreme Ultraviolet," p. 113, 1914.

[3] Possibly due to silicon.

volts while Davis and Goucher[1] found a second more intense radiation at 9 volts.[2]

In view of the discrepancy between these experimental values and those calculated from the spectrum, it was thought worth while to attempt a more exact determination.

II. Apparatus.

The apparatus used had been designed for a somewhat different purpose but was found fairly satisfactory.

As shown in Fig. 1, it consisted of a glass tube 1.5″ in diameter, with a filament *F* sealed in at one end, a gauze *G* in the middle and a disc *P* sealed in from the other end, all of platinum. Electrical connections were so arranged that the electrons coming off from the hot filament *F* were accelerated by a field between *F* and *G* but met a stronger retarding field between *G* and *P*. The difference between accelerating and retarding fields remained constant. The gauze *G* was connected to a Leeds & Northrup high sensitivity galvanometer which measured the electronic current between *F* and *G*. The disc *P* was connected to a Dolazalek electrometer of sensitivity about 3,000 millimeters per volt, making the capacity of the electrometer system of the order of magnitude of 50 cm. The potentials were read on a Robt. W. Paul voltmeter.

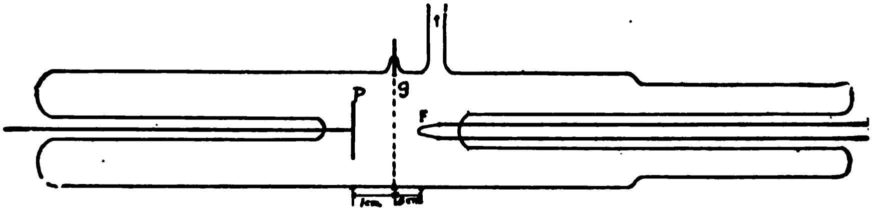

Fig. 1.

By coating the filament with barium oxide increased emission was obtained.

The pressure in the apparatus was regulated by a Gaede pump and measured with a McLeod gauge.

Nitrogen was prepared by heating sodium nitrite and ammonium chloride with distilled water and was introduced through a drying tube. The apparatus was evacuated and washed out several times with nitrogen before measurements were made.

The setting in of radiation was detected in the usual manner by the photoelectric effect on *P* causing an increase in the speed of deflection of the electrometer. It was found impossible to eliminate all zero drift from the electrometer. This made it necessary to start readings far enough below the break point to give a good zero. The accelerating potential was run from 3 or 4 volts up to 11 or 12 and then down again, the intervals between readings near critical points being as small as .2 of a volt. The values going up and coming back were averaged. The electronic current measured by the galvanometer was kept constant in the earlier runs by adjusting the temperature of the filament but, in the later runs, the temperature of the filament was maintained constant and the galvanometer current allowed to vary with the accelerating potential, thus tending to accentuate the sharpness of the break.

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[1] Bergen Davis and F. S. Goucher, Phys. Rev., No. 13, pp. 1–5, 1919.

[2] The fact that these are radiation effects and not ionization seems to have been proved by Davis and Goucher and is taken for granted in this paper.

III. VELOCITY DISTRIBUTION CORRECTION.

The most serious source of error in experiments of this type is due to the fact that the electrons reaching the gauze will not all have exactly the velocity corresponding to the accelerating field.[1] The factors causing this trouble are the potential drop along the filament, initial velocity of emission and, in the case of diatomic gases, inelastic impacts.

Now, in the case of monatomic gases the elastic impacts make possible a very effective method of eliminating this error. In this case, the electrometer current rises rapidly as the accelerating field passes the critical value, reaching a maximum when all the electrons have attained the critical speed, and then falls off again as more and more of the ionizing collisions take place on the filament side of the gauze. When the accelerating potential becomes great enough to allow two ionizing collisions by one electron another maximum occurs, and so on. Thus, by measuring the intervals between successive maxima the true value of the ionizing potential can be found.[2]

With diatomic gases, however, this method is impossible since, with every increase of the accelerating field, some electrons which have lost energy by inelastic impact will attain the critical speed and there will be a continuous increase in the electrometer current. It becomes necessary, therefore, to actually measure the velocity of the electrons coming across and then make a correction.

Franck and Hertz did this but felt so uncertain as to the right method of making the correction that they claimed an accuracy of only one volt[3] for their results.

Goucher[4] eliminated errors due to the potential drop along the filament by introducing an equipotential electron source consisting of a platinum thimble surrounding a tungsten heating element. With this arrangement in mercury vapor he found that over 70 per cent. of the electrons had velocities corresponding to the applied voltage and, further, that the number having a velocity corresponding to .5 volt greater than the applied field was too small to be measured. He therefore made no correction for velocity distribution. In the later work of Davis and Goucher[1] on nitrogen, the same type of electron source was used and the correction considered unnecessary. As before stated the values obtained were 7.5 and 9 volts.

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[1] J. Franck and G. Hertz, Verh. d. D. Phys. Ges., 15, p. 37, 1913.

[2] J. Franck and G. Hertz, Verh. d. D. Phys. Ges., 16, p. 457, 1914.

[3] J. Franck and G. Hertz, Verh. d. D. Phys. Ges., 15, p. 39, 1913.

[4] F. S. Goucher, PHYS. REV., No. 8, p. 561, 1916.

Bishop,[2] working with a filament, took the point of maximum slope of his electron current curve for his velocity distribution correction. That is, he took the most probable velocity of the electrons. He found the value of 7.5 volts for nitrogen and the same for N_2O.

Hughes and Dixon,[3] who obtained the values 7.7 volts for nitrogen and 9.3 for nitric oxide, took as their velocity correction the highest speed detectable on their velocity distribution curve.

Attempts to apply velocity distribution corrections in preliminary tests of the present apparatus proved that the methods mentioned above give quite different results, and that the discrepancy between them varies with the filament temperature and gas pressure. Furthermore, when correcting by the method of Hughes and Dixon, the relative sensitivities of the apparatus for radiation and for velocity distribution measurements and, also, the scale to which the measurements were plotted could be altered so as to vary the value of the corrected "break" point by as much as a volt, without any obvious way of selecting the correct value from among the various possible ones.

This experience led to a closer study of the problem with a view of determining the method of handling data from the two types of measurement which would give the most nearly correct result. The variation of zero drift sets a practical limit to the scale of plotting of either curve. Therefore, it remains to determine an appropriate scale for the more sensitive measurement and this is done by finding the relation between the sensitivities of the apparatus for the two types of measurement. Obviously, the "break" point in the radiation curve is due to the radiation from the smallest number of electrons which can produce a large enough radiation effect to be detected by the apparatus. Since not all electrons capable of producing radiation do so, owing to failure to collide or for other reasons, this "smallest" number of electrons is larger than the least number which can be detected if they are permitted to strike the receiving electrode, as in the velocity distribution measurements. The problem, therefore, is to find how many electrons must pass the gauze with sufficient energy to cause radiation in order that one may produce radiation. In other words, how many times less sensitive is the apparatus for radiation than for velocity distribution measurements and how should the latter be plotted in order that the greatest velocity shown should give the correction appropriate to the "break" point in the former?

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[1] Bergen Davis and F. S. Goucher, Phys. Rev., No. 13, p. 1, 1919.

[2] F. M. Bishop, Phys. Rev., No. 10, p. 244, 1917.

[3] A. Ll. Hughes and A. A. Dixon, Phys. Rev., No. 10, p. 495, 1917.

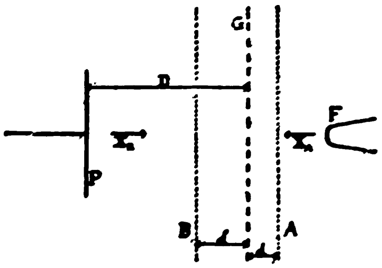

Fig. 2.

The following analysis leads to a very usable expression for the ratio of the sensitivity of the electrometer system for radiation to that for velocity distribution experiments.

Let:

N = number of collisions per centimeter path at 1 mm. pressure;

X_A and X_B = the electric intensities in the accelerating and retarding fields, respectively;

n = number of electrons per unit time reaching the gauze G from the filament side;

$f(V)dV$ = the probability of an electron reaching the gauze with a speed between V and $V + dV$;

V_0 = the minimum radiating velocity;

A and B be two planes parallel to G distant d and d' on either side of it, where

$$d = \frac{V - V_0}{X_A}$$

and

$$d' = \frac{V - V_0}{X_B};$$

$f(V_0)dV$ = the probability of an electron reaching A with velocity between V_0 and $V_0 + dV$.

In all cases, velocities are expressed in terms of equivalent volts.

Evidently

$$nf(V_0)dV(1 - \epsilon^{-pNd}) = n(\epsilon^{pNd} - 1)f(V)dV$$

is the number of electrons which pass A with velocities between V_0 and $V_0 + dV$ and collide before reaching G.

After passing the gauze the electrons will go a distance

$$d' = (V - V_0)/X_B$$

before losing their ability to cause radiation. From these we have

$$nf(V)dV(1-\epsilon^{-pNd'})$$

as the number of radiating impacts after passing the gauze by electrons which reached the gauze with speeds between V and $V+dV$.

We have, then, for the total number of impacts at a speed greater than V_0, by electrons going in the direction away from the filament,

$$M_1 = n\int_{V_0}^{V_m}(\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_B}})f(V)dV, \tag{1}$$

where V_m is the maximum speed of any electron reaching G.

There will also be a small number of electrons which will go through, be stopped, and acquire sufficient acceleration in the opposite direction to produce radiation. For these we have

$$n' = n\epsilon^{-pN\left(\frac{V-V_0}{X_B}\right)}f(V)dV$$

as the number which do not collide before their velocity is retarded to V_0, and

$$n'' = n'\epsilon^{-pN\frac{2V_0}{X_B}}$$

as the number which will regain a velocity V_0 in the opposite direction before collision.

Then

$$n''(1-\epsilon^{-pN(d'+d)})$$

is the number which, having passed G with velocity between V and $V+dV$, escape collision until they have reversed their direction and regained a speed greater than V_0, finally colliding before their speed is again reduced to V_0.

On substitution we have

$$\begin{aligned} dM_2 &= n'\epsilon^{-pN\frac{2V_0}{X_B}}(1-\epsilon^{-pN(d'+d)}) \\ &= n\epsilon^{-pN\frac{2V_0}{X_B}}\cdot\epsilon^{-pN\left(\frac{V-V_0}{X_B}\right)}[1-\epsilon^{-pN\left(\frac{V-V_0}{X_B}+\frac{V-V_0}{X_A}\right)}]f(V)dV \qquad (2) \\ &= n[\epsilon^{-pN\frac{V+V_0}{X_B}} - \epsilon^{-pN\left(\frac{2V}{X_B}+\frac{V-V_0}{X_A}\right)}]f(V)dV. \end{aligned}$$

Therefore

$$M_2 = n\int_{V_0}^{V_m}[\epsilon^{-pN\frac{V+V_0}{X_B}} - \epsilon^{-pN\left(\frac{2V}{X_B}+\frac{V-V_0}{X_A}\right)}]f(V)dV \tag{3}$$

is the number of collisions by electrons coming in the reverse direction and colliding while they have a velocity between V_m and V_0.

We have, for the total number of collisions by electrons having velocities greater than V_0,

$$M = n\int_{V_0}^{V_m}[\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_R}}]f(V)dV$$
$$+ n\int_{V_0}^{V_m}[\epsilon^{-pN\frac{V+V_0}{X_B}} - \epsilon^{-pN\left(\frac{2V}{X_B}+\frac{V-V_0}{X_A}\right)}]f(V)dV. \tag{4}$$

Now if we take the probability of the production of radiation and its detection by photoelectric effect to be $(V - V_0)/kV_0$, where the value of kV_0 may be found from the results of Johnson[1] we have for the effective number of radiating impacts when the maximum electronic speed is V_m,

$$M' = n\int_{V_0}^{V_m}\frac{V - V_0}{kV_0}[\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_B}}]f(V)dV$$
$$+ n\int_{V_0}^{V_m}\frac{V - V_0}{kV_0}[\epsilon^{-pN\frac{V+V_0}{X_B}} - \epsilon^{-pN\left(\frac{2V}{X_B}+\frac{V-V_0}{X_A}\right)}]f(V)dV. \tag{5}$$

We now turn to the velocity distribution measurements. If D is the distance from the gauze G to the plate P and V_R is the retarding potential, the number of electrons getting to the plate will be, for given values of V_m (*i.e.*, V_A, the accelerating potential) and V_R,

$$M_P = n\int_{V_R}^{V_m}\epsilon^{-pND}f(V)dV. \tag{6}$$

Now, the form of the function $f(V)$ will vary with V_m (or V_A) and its value will depend also on the particular value of V substituted.

For a given value of V_m we will have, from (6),

$$\frac{dM_P}{dV_R} = -n\epsilon^{-pND}f(V)_{V=V_R}. \tag{7}$$

Consider the part near the foot of a typical velocity distribution curve, as shown by Fig. 3. The curve is a section of a parabola, while actual experimental measurements are indicated by dots. It is evidently sufficiently accurate, for the present purposes, to consider the lowest part of the distribution curve to be parabolic, so that its slope is taken to be proportional to the horizontal distance from the foot. Use of this property gives the graph shown in Fig. 4.

Let $V_{R'}$ be some particular value of V_R; then

$$(M_P)_{V_R=V_{R'}} = -\tfrac{1}{2}(V_m - V_{R'})\left(\frac{dM_P}{dV_R}\right)_{V_R=V_{R'}}.$$

Also, we have, for any value of V_R less than V_m,

$$\frac{dM_P}{dV_R} = \frac{V_m - V_R}{V_m - V_{R'}}\left(\frac{dM_P}{dV_R}\right)_{V_R=V_{R'}}.$$

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[1] J. B. Johnson, PHYS. REV., X., p. 609, 1917.

Combining these two equations, we have

$$(M_P)_{V_{R'}} = -\frac{1}{2}\,\frac{(V_m - V_{R'})^2}{V_m - V_R}\,\frac{dM_P}{dV_R}.$$

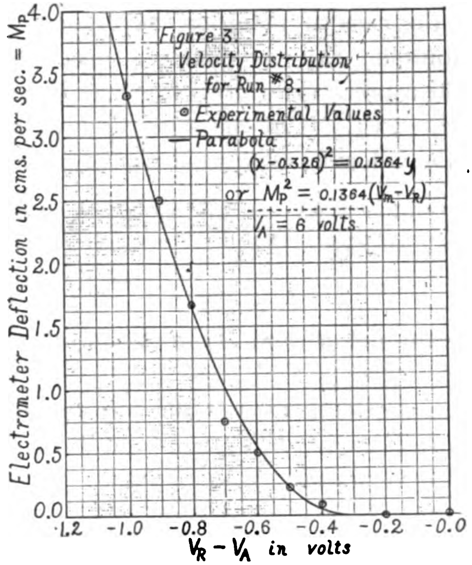

Fig. 3.

Substituting for dM_P/dV_R from (7), and solving for $f(V)_{V=V_R}$, we have

$$f(V)_{V=V_R} = \frac{2(M_P)_{V_{R'}}(V_m - V_R)\epsilon^{pND}}{n(V_m - V_{R'})^2}.$$

We are endeavoring to find the ratio of corresponding values of M'

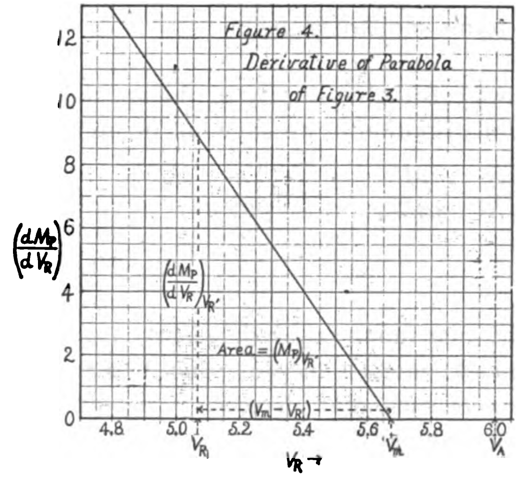

Fig. 4.

and M_P, *i.e.*, of values at equal distances from the break point. For this condition we must have

$$V_m - V_0 = V_m - V_{R'}$$

and $(V - V_0)$ in the case of radiation corresponds to $(V_m - V_R)$ in the velocity distribution. We can therefore substitute the value found above for $f(V)$ in the expression for M' if we write $V_m - V_0$ for $V_m - V_R$ and $V - V_0$ for $V_m - V_R$. If at the same time we divide through by $(M_P)_{V_{R'}}$, we have the following expression for the required ratio:

$$\frac{M'}{M_P} = 2\epsilon^{pND} \int_{V_0}^{V_m} \frac{1}{(V_m - V_0)^2} \frac{(V - V_0)^2}{kV_0} [\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_B}} + \epsilon^{-pN\frac{V+V_0}{X_B}} - \epsilon^{-pN\left(\frac{2V}{X_B}+\frac{V-V_0}{X_A}\right)}]dV$$

for corresponding values of M' and M_P in the neighborhood of the break point.

If, in the above expression, we put the constant factors outside the integral and set $pN/X_A = a$, $pN/X_B = a'$ and $V - V_0 = x$, and if we take, $a = a'$, which is very nearly true we have

$$\frac{M'}{M_P} = \frac{2\epsilon^{pND}}{kV_0(V_m - V_0)^2} \int_0^{x_m} x^2[\epsilon^{ax} - \epsilon^{-ax} + \epsilon^{-2aV_0}(\epsilon^{-ax} - \epsilon^{-3ax})]dx.$$

By integration and the expansion of the powers of ϵ (either before or after integration), this equation takes the following form, which includes all terms as far as those in a^3:

$$\frac{M'}{M_P} = \frac{2a\epsilon^{pND}(V_m - V_0)^2}{kV_0} [1 - aV_0 - \tfrac{4}{5}a(V_m - V_0) + \tfrac{7}{9}a^2(V_m - V_0)^2 + \tfrac{16}{5}a^2V_0(V_m - V_0) + 2a^2V_0^2 + \cdots].$$

Practically, the terms in a^2 are negligible, since a is of the order of magnitude of 0.03.

If, therefore, we call the sensitivity of the apparatus for radiation R and for velocity distribution S, we have, for practical purposes,

$$\frac{R}{S} = \frac{M'}{10M_m} = \frac{\epsilon^{pND}a(V_m - V_0)^2}{5kV_0}(1 - aV_0) = \frac{\epsilon^{pND}a(V_m - V_R)^2}{5kV_0}(1 - aV_0), \qquad (8)$$

where V_m is the maximum speed detectable in velocity distribution measurements and the factor 1/10 is introduced because, in our apparatus, the dimensions were such that P receives only about that proportion of the radiation starting at the gauze.

Equation (8) is evidently not exact, owing to the approximations introduced in order to obtain a solution. It is believed, however, that none of these approximations is seriously in error, so that the results given by (8) should be of the right order of magnitude.

The necessary procedure, therefore, is to plot the velocity distribution on the same scale on which the radiation curve is to be plotted and determine V_m from this. Then multiply the values of the electrometer current for each value of V_R by the corresponding values of R/S and replot. The break point in this new curve determines the correction to be used on the radiation curve. The necessity of plotting a corrected velocity distribution curve is made evident by Table I. which gives the values of S/R calculated by equation (8) for different values of $V_R - V_m$, taking $k = 2$ for nitrogen.[1]

Table I.

$p = 0.015$ mm. $V_0 = 7.5$ volts.

$(V_m - V_R)$.	S/R.	$(V_m - V_R)$.	S/R.
.1	795,000	1.1	6,500
.3	87,500	1.2	5,500
.5	31,000	1.3	4,700
.6	22,000	1.4	4,050
.7	16,000	1.5	3,500
.8	12,500	1.6	3,200
.9	9,500	1.7	2,750
1.0	8,000		

IV. Experimental Results.

It was found impossible to eliminate all zero leak from the electrometer. Consequently readings had to be taken at low accelerating potentials to establish a zero line. Errors due to a variation in this zero leak as well as errors of observation were pretty well eliminated by the method of taking a series of readings with fields increasing and then decreasing and plotting the mean. A typical set of readings is given in Table II., below.

where

V_f = P.D. across filament in volts;
G = current to gauze in amperes $\times 10^{-8}$;
T = time in seconds for electrometer to deflect one cm.;
$\overline{T}$ = mean of T and T' observations as V_A was increased and then decreased respectively;
$1/\overline{T}$ = rate of deflection of electrometer in cms. per second.

The values of V_A and $1/\overline{T}$, that is the accelerating potential and the current to the electrometer, were used in making the curves. The first

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[1] J. B. Johnson, *loc. cit.*

TABLE II.

Observations for Run No. 12.

$p = 0.01$ mm., $V_R - V_A = 2.5$ volts, $V_f = 0.8$ volts.

V_A.	G.	G'.	T.	T'.	$\bar{T}$.	$1/\bar{T}$.
1	6	6	81	81	81	.0124
2	16.4	17	79	86	82.5	.0121
3	21.2	21	76	86	81	.0124
4	23	22	82	80	81	.0124
5	23.7	22.4	73	89	81	.0124
5.5	24.1	22.5	71	74	72.5	.0138
5.75	24.4	22.4	76	71	73.5	.0136
6	24.7	22.3	66	61	63.5	.0158
6.25	25	22.1	63	60	61.5	.0163
6.5	25.2	22.1	60	56	58	.0172
6.75	25.5	21.9	55	55.6	55.3	.0181
7	25.8	21.7	50	53	51.5	.0194
7.25	26.2	21.6	46.4	45.4	45.9	.0218
7.5	26.6	21.5	44	44.6	44.3	.0220
7.75	26.9	21.3	41	41	41	.0244
8	27.2	21	35.2	35	35.1	.0285
8.25	27.4	21	29.8	30.2	30	.0330
8.5	27.7	20.9	23.4	27.5	25.5	.0392
8.75	28	20.8	20.2	22.1	21.1	.0474
9	28.2	20.7	16.8	19.4	18.2	.0550
9.5	28.5	20.8	10.6	12.5	11.5	.0870
10	28.7	20.8	7.5	9.0	8.2	.122
11	29.2	21	1.8	3.0	2.4	.417
12	29.5	21.1	.58	.7	.64	1.563

detectable departure from the zero line was taken as the break point. Thus, in run no. 4, Fig. 5, the break point comes at 9.3 volts. In order to correct this we examined the velocity distribution curve for run no. 4 in Fig. 6. Here we saw that the uncorrected values (curve *a*) gave a break point at $V_R - V_A = 0$ and therefore $V_m - V_A = 0$. Using this value to get $(V_m - V_R)$, and so to calculate the corrected values of the current by applying equation 8, we got the curve *b* which has its first break point at $V_R - V_A = -1.0$. We concluded, therefore, that the first electrons which are effective in producing detectable radiation have a velocity one volt less than the applied field, giving us 8.3 as the true value of the radiating potential from this run.

Similarly for run no. 5 we found the observed break to be at 9.0 and the corrected value to be 8.0 volts.

In run no. 6, Fig. 7, on the other hand, we found the corrected value to come at 6.5 (5.9 observed), but there is a sharp increase in the slope

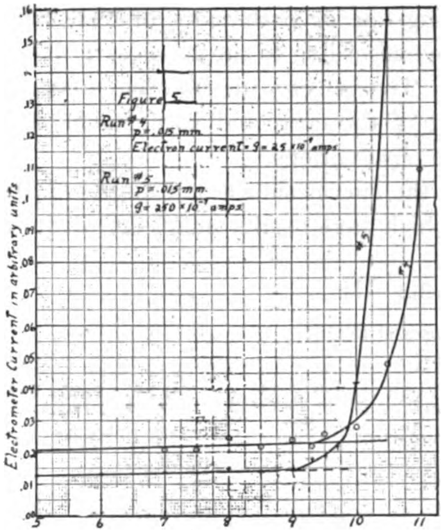

Fig. 5.

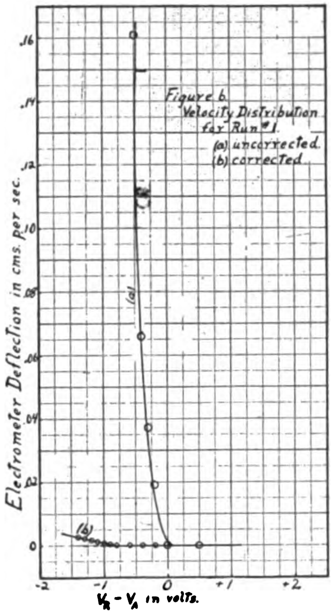

Fig. 6.

of the curve at 8.6 (8.0 observed). This run therefore apparently has two breaks corresponding to two critical speeds.

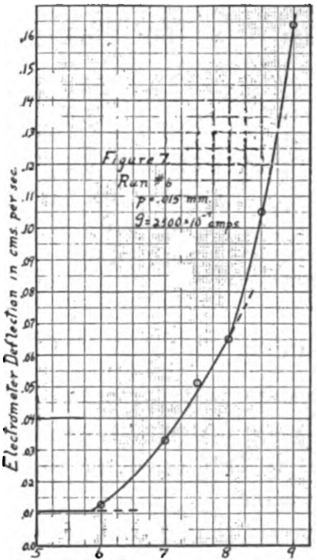

Fig. 7.

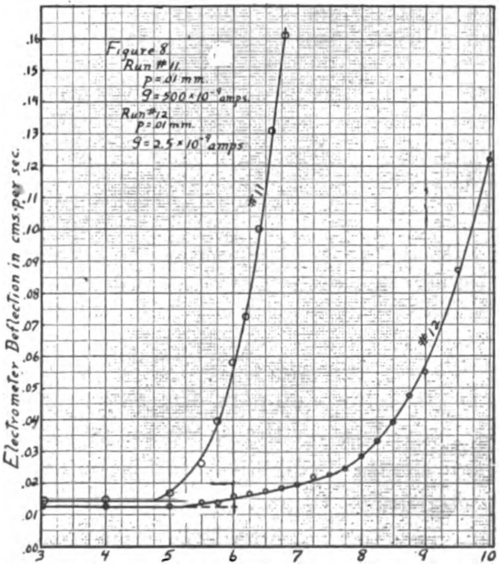

Fig. 8.

In runs nos. 11 and 12 the effect at the lower voltage was so great as to make it impossible to detect any other. For these runs the corrected values are 6.15 and 6.4 volts.

From these five typical curves shown, it is clear there are two distinct critical points, each curve showing one or both more or less sharply. The values observed were weighted according to the sureness with which the break point could be picked. The velocity distribution curves were treated similarly and the weight of the corrected value taken as the product of the weights of the two observations.

In Table III. the results from the curves shown and discussed above are grouped with all other runs which gave results sufficiently definite to have weight.

TABLE III.

Experimental Results.

No.	p (mm.).	G.	Observed Breaks with Weighting.		Corrections with Weighting.	Corrected Breaks with Weighting.	
1	.048	25.	9.4 (4)		−0.4 (3)	8.5 (12)	
2	.038	25.	9.2 (3)		−0.9 (3)	8.3 (9)	
3	.027	25.	9.3 (2)	7.5 (?)	−1.0 (3)	8.3 (6)	
4	,015	25.	9.3 (3)		−1.0 (3)	8.3 (9)	
5	.015	250.	9.0 (4)		−1.0 (3)	8.0 (12)	
6	.015	2500.	8.0 (1)	5.9 (2)	+0.6 (3)	8.6 (3)	6.5 (6)
7	.015	2500.		5.4 (5)	+0.7 (2.5)		6.1 (8)
8	.045	750.	9.3 (2)		−.08 (4)	8.5 (8)	
9	.045	140.	9.2 (1)		−1.0 (4)	8.2 (4)	
10	.026	250.	8.8 (2)	7.4 (1)	−0.7 (5)	8.1 (10)	6.7 (5)
11	.01	500		4.75 (4)	+1.4 (3)		6.15 (12)
12	.01	2.5	7.75 (?)	5.2 (4)	+1.2 (3)		6.4 (12)
13	.01	40	6.5 (?)	4.2 (3)	+1.9 (3)		6.1 (4)
Mean values..........................					....	8.285 ±.045	6.285 ±.061

Note.—Runs 7, 11 and 13 and some not given above show uncertain indications of a third break at about 7.4 volts; *G* is the electronic current in amperes × 10^{-9}.

V. DISCUSSION OF RESULTS.

Let us now compare the experimental results just presented with Lyman's data on the spectrum of nitrogen in the extreme ultra-violet given at the beginning of this paper. This is best done by writing corresponding values opposite each other in a table.

1. *The Break at* 8.29 *Volts.*—In this table we see that the effect which we got at all pressures tried and with various currents, that is the most

TABLE IV.

Spectrum Data.			Radiation Experiment Data.	
Remarks.	**λ (Obs.).**	**V_0 (Calc.).**	**V_0 (Obs.).**	**λ (Calc.).**
Doublet almost certainly due to nitrogen	1492.8 1494.8	8.28 8.27	8.29 ± .04	1490.7 ± 10
Doublet attributed to nitrogen but possibly due to silicon..............	1742.7 1745.3	7.08 7.09	7.3 (?)	1700 (?)
Beginning of band spectrum..........	1870.9	6.6	6.29 ± .06	1965 ± 20

intense effect, corresponds with greater accuracy than could be hoped for, to the most certain doublet of the line spectrum of nitrogen.

2. *Possible Effect at 7.3 Volts.*—The failure of the second value to coincide with that for the other doublet is no greater than the uncertainties of its determination. This radiation was apparently the weakest of the three and, while showing up well on one or two curves was, on the whole, rather doubtful. Attention should again be called to the doubt concerning the origin of this doublet as determined by Lyman.

3. *Explanation of 6.29-Volt Break.*—Two different theories were developed to account for this effect, one depending on the application of the quantum relation to the band spectrum and the other involving the idea of dissociation and the subsequent production of spectral lines. They are discussed at length in what follows.

(*a*) To explain the experimental result of a break occurring only at low pressures and coming at 6.29 volts instead of the 6.6 calculated from the band spectrum, we must consider the effect of a probable impurity. Kreusler[1] found that nitrogen prepared in a manner almost identical with that used in the present experiment had a small quantity of N_2O present as an impurity. He found that this increased absorption at λ = 1,860 from 2.2 per cent. for atmospheric nitrogen to 14.3 per cent. He also made measurements on pure nitrous oxide and found 88.4 per cent. absorption at λ = 2,000, apparently increasing beyond his powers of measurement at 1,930 and 1,860.

It is probable, therefore, that at the higher pressures, the radiation λ = 1,965 corresponding to 6.3 volts loses so much energy by absorption in the 1 cm. space between the gauze and the plate that it is not detectable. This explains the fact that there was no trace of this break at pressures above 0.026 and that it was only detected in one case at pressures exceeding 0.015.

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[1] H. Kreusler, Ann. d. Phys., 6, p. 419, 1901.

The discrepancy between the value 6.3 found and the 6.6 corresponding to the beginning of the band spectrum is not so easily accounted for. It is a well known fact, however, that the presence of small impurities greatly affects the intensity of emission spectra. We have just seen that there is such an effect in the absorption spectrum of nitrogen in this region. Is it not possible then that the presence of N_2O may cause less refrangible bands up to $\lambda = 1,965$ to come out with sufficient strength to be detected? The chances for sufficient experimental error to account for a discrepancy of 0.3 volt do not seem great especially in view of the extremely good agreement in the case of the highest break. It is hoped that this point may be cleared up by further work taking every precaution to get absolutely pure nitrogen.

(*b*) The other explanation of our 6.3 volt break is quite different. There are lines in the spectrum of nitrogen in the region between 2,000 and 3,000 which would produce photoelectric effect and the most refrangible of which is $\lambda = 2,052$.[1] In the previous discussion it has been assumed that these arise from systems that are not present in this experiment, such as charged atoms or molecules, since otherwise we would have a photo-electric effect at lower voltages. If we allow the possibility of the production of some of these lines by a neutral atom, as we must for the doublet previously considered, then we must first have dissociation and then a radiating impact. Now the speed necessary for an electron to produce these radiations is between 4 and 6 volts, but the energy necessary to dissociate a nitrogen molecule is unknown. The velocity necessary for an electron to dissociate a hydrogen molecule is found by calculation from Langmuir's[2] results to be about 3.6 volts and it is known that nitrogen is much harder to dissociate. It is possible therefore that 6.3 is the speed necessary to dissociate the molecules and thus make it possible for the electrons of lower speed to produce radiation. This would correspond to a heat of dissociation of a gram molecule of nitrogen of 145,000 calories, whereas Langmuir[3] found for hydrogen the value 84,000.

This theory necessitates a new explanation of the effect of pressure, since these longer wave-length radiations will not be so strongly absorbed. It must be supposed that the chances for recombination of the atoms at higher pressures than 0.026 are so great as to make the radiation negligibly small.

As to the soundness of the assumption of radiation from neutral atoms, we have the following statement of J. J. Thomson regarding the lines of the hydrogen spectrum: "All theories concur in regarding the atom and not the molecule as the source of these lines, but according to Wien's theory the atom radiates when in the neutral state, while Stark maintains that the radiation is emitted when the atom has a positive charge: according to his view the lines emitted by the neutral atom are far away in the ultra-violet." [1] Stark[2] found no lines attributable to a neutral nitrogen atom but hardly carried his work below 4,000 A.U. From his work on mercury, however, he concluded that the line 2,536.7 was due to a neutral atom.[3] It is evident then that there is no evidence against our assumption but rather indications of its probability.

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[1] Lyman, "Spec. of Extreme Ultraviolot," p. 83, 1914.

[2] I. Langmuir, J. of Am. Chem. Soc., 37, pp. 417–458, 1915.

[3] Ibid., p. 457.

A second assumption implied in this theory is that the neutral molecules will not set up ultra-violet radiation when struck by electrons with speeds below 6.3. On this point, I have found no evidence.

4. *Upper Limit to Heat of Dissociation of Nitrogen.*—A necessary consequence of the principle that the line spectrum is due to atomic nitrogen is the determination of an upper limit for the energy necessary to dissociate a nitrogen molecule. If the effect resulting rom bombardment by electrons with velocities of 8.24 volts is due to atoms, the molecules must be dissociated by the impact of electrons of this or lower speed. The value 8.3 would give as an upper limit for the heat of dissociation of a gram molecule of nitrogen about 190,000 calories. As has been stated, Langmuir[4] found the value for hydrogen at constant volume to be 84,000 calories.

VI. Ionization at 18.5 Volts.

Goucher and Davis found that what had previously been called ionization in nitrogen was really radiation but that true ionization did occur at 18.5 volts. Although the apparatus used in the present investigation was not well suited for testing this point, by greatly reducing its sensitivity a distinct increase in the slope of the electrometer current curve in the neighborhood of 18 volts was observed, thus supporting the more accurate work of Davis and Goucher.

It is realized that the present results were not obtained under experimental conditions ideal for getting sharp break points or reducing corrections to a minimum. The attempt was made, on the other hand, to employ experimental conditions in which the corrections would be as varied as possible, in order to test the soundness of the formula.

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[1] J. J. Thomson, "Positive Rays," pp. 96–97, 1913.

[2] J. Stark, Ann. d. Phys., 55, pp. 29–74, p. 73, 1914.

[3] J. Stark, Ann. d. Phys., 52, pp. 241–302, p. 247, 1913.

[4] I. Langmuir, *loc. cit.*, p. 457, 1915.

Although the corrections in different tests differed by as much as 3 volts in extreme cases, the corrected values of the break point were quite consistent. It appears, therefore, that the above analysis is justified and necessary, and that the final values obtained are trustworthy.

The author wishes to express his indebtedness to Professor K. T. Compton, whose supervision and assistance have made this work possible.

Palmer Physical Laboratory,
Princeton, N. J.
July, 1919.

THE REFLECTION FACTORS OF TUNGSTEN AT INCANDESCENT TEMPERATURES.

By W. Weniger and A. H. Pfund.

Synopsis.

A direct method has been developed for measuring the reflection factors of substances at incandescent temperatures. Curves are given showing the variation in the reflection factor of tungsten for wave-lengths from 0.67 μ to 4.0 μ and for temperatures up to 2067° K.

The reflection factor of tungsten was found to increase with temperature for wave-lengths less than about 1.27 μ and to decrease with temperature for wave-lengths longer than this. The transition from the approximately constant positive temperature coefficient in the visible to the approximately constant negative coefficient in the infra-red takes place in the rather narrow region lying between about 0.7 μ and 2.0 μ. The numerical values of the reflection factor agree fairly well with Worthton's values in the visible and with the theoretical equation first applied to long wave-lengths by Hagen and Rubens, beyond 2 μ in the infra-red.

This study of the reflection factors of tungsten was made as part of the general study of the properties of tungsten undertaken by Nela Research Laboratory. Closely related papers on the emissive powers in the visible and ultra-violet have already been published by Worthing[1] and Hulburt[2] respectively. The present paper deals with the reflection factors at incandescent temperatures in the red end of the visible spectrum and the near infra-red. The most direct method of attack was adopted, involving spectral energy measurements of a beam reflected from a polished tungsten surface both when hot and when cold.

Apparatus and Method.

The success of the method depended entirely on obtaining a tungsten mirror that could be heated without appreciable warping. A few trials were made with massive mirrors, including an attempt to heat such a mirror by means of an electronic discharge impinging upon its back; but it was found simpler and entirely practicable to use smaller mirrors that could be heated by means of an electric current passing through them. Three different types of such mirrors were made: (*a*) squirted filament, circular hole along axis, outer surface hexagonal, width of face 0.75 mm.; (*b*) swaged filament 1.16 mm. in diameter, central portion rolled flat and bent into a form resembling that of the old-fashioned paper fasteners known to the trade as "flat heads," mirror face 25 x 2 mm.; (*c*) drawn wire, 0.9 mm. in diameter, bent so as to form three sides of a long narrow rectangle, ground flat so that its section became a semicircle. All were heated in vacuo to a temperature higher than that used in the experiments, were rough-polished on cloth with sulphur-flour and rouge, and finished on a pitch surface with the finest rouge; and were mounted in evacuated glass bulbs. The part of the polished surface used was always sufficiently removed from the lead-in wires so that its temperature was uniform. The data taken on mirrors *a* and *b* were considered preliminary; those on *c* are recorded below. All mirrors showed the same general behavior. In this connection two experimental difficulties should be mentioned: (1) it was very difficult to make stem seals that would not check, due to the frequent changes in the large heating currents; (2) it was found that the mirror surface developed furrows, which, however, obliterated only a small percentage of the surface, the parts between the furrows retaining their polish.

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[1] Phys. Rev., 10, 1917, p. 377.

[2] Astrophys. Jour., 45, 1917, p. 149.

The spectrobolometer was of the Wadsworth fixed arm type; instrumental dimensions are given in the legend accompanying Fig. 1. The bolometer and galvanometer were provided with auxiliary apparatus so that the galvanometer could be used either as a deflection or as a zero

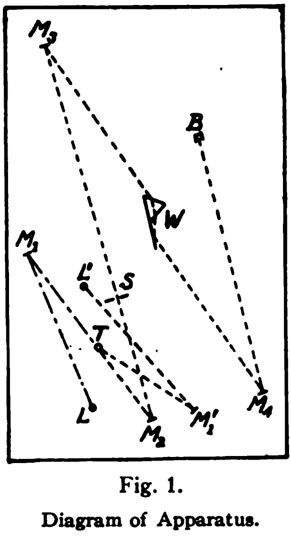

Fig. 1.

Diagram of Apparatus.

L, *L'*, two positions of flat filament (0.2 × 2.5 cm.) tungsten lamp in cylindrical bulb, operated at 15 amp.

M_1, M_1', two positions of concave mirror, $f = 20.0$ cm., diam. $= 9.3$ cm. *L* and *M* were mounted on a single tripod which could be placed in either one of two sets of hole-slot-plane plates. The primed position served for measurements of the beam reflected from *T*, and the unprimed for measurements of the beam incident on *T*.

T, tungsten mirror, mounted as a lamp filament in a selected tubular glass bulb; see text. Angle of incidence of beam on mirror 10°.

M_2, concave mirror, $f = 16.0$ cm., diam. $= 15$ cm.

S, slit, 0.5 mm. × 20 mm. Subtends 0.13 μ in the most condensed part of the spectrum near 1.63 μ.

M_3, M_4, concave mirrors, $f = 76$ cm., diam. $= 15$ cm.

W, fluorite prism in Wadsworth mounting. Prism angle 59° 59.8′.

B, bolometer, platinum strips (0.5 × 25.0 mm.) Smoked above acetylene flame. Resistances when total bolometer current is 0.12 amp., are 5.303 and 5.296 ohms. Balance coils 10 ohms each, part of Wolff five dial bridge. Galvanometers: Four-coil, astatic, 7-sec. period, volt sensitivity, 670 mm. per microvolt at a scale distance of 1 meter. *S-H* d'Arsonval, critically damped, volt sensitivity, 10 mm. per microvolt at a scale distance of 1 meter. Galvanometer scale actually used at distances of 2.5 m. from the four-coil instrument and 5 m. from the d'Arsonval.

instrument. When used as a deflection instrument, its sensibility could be reduced by a group of shunts, but when used as a zero instrument it was always kept at full sensibility. For use in the latter manner, the exposed strip of the bolometer was shunted by a high variable resistance (10,000 Ω in parallel with the strip and 3,000 of this 10,000 in parallel with from 1 to 110,000 Ω) so that at a balance with the exposed strip illuminated, the current in each of the four arms of the bridge was the same as when the shunt was disconnected and the strip shaded. The changes in current due to exposures of the strip, as is customary, were assumed proportional to the incident radiant flux and their values were calculated from the experimentally determined values of the shunt resistance necessary to bring this current back to its normal value.

The path of the beam before reaching the slit of the spectrometer is indicated in Fig. 1 by $L'M_1'TM_2S$. To insure that the tungsten mirror T was in exactly the same position in the image of the source formed by M', when hot as when cold, two telescopes were mounted permanently, one near M_2 giving a view of the front of the mirror, and the other off at right angles, giving a view of the edge of the mirror, the cross hairs of both being set on marks that could be easily identified. A shutter was placed near M_1'. The natural radiation from the tungsten mirror was allowed to enter the slit at all times and the increase in the galvanometer deflection due to opening the shutter was recorded. Readings were usually made in this order: mirror at room temperature, mirror incandescent, mirror at room temperature.

The actual galvanometer deflections or resistance readings varied in magnitude from day to day, due to using different currents in the bolometer and in the source, etc., and are, therefore, not recorded below. Between 0.7 μ and 3.0 μ no deflections less than 2 cm. nor greater than 20 cm., were used; readings were made to 0.1 mm.; each figure recorded is based on the mean of at least ten deflections. Spectrometer settings were checked at the beginning and end of each run by the yellow and red lines of helium and occasionally by the carbon dioxide emission band at 4.4 μ. The temperature-current relations for the tungsten mirrors were determined with a Holborn Kurlbaum optical pyrometer by Dr. Worthing of this laboratory.

Data and Discussion.

The data are recorded in Table I. and plotted in Figs. 2 and 3. In a narrow region bounded approximately by the wave-lengths 0.7 μ and 2.0 μ a change of temperature produces a striking change in the relation between reflection factor and wave-length. Within this interval an

Table I.

Relative Reflection Factors of Tungsten,

i.e., $\dfrac{\text{Reflection factor at high temperature}}{\text{Reflection factor at room temperature}}$.

Wave-length in μ.	1380° K.	1632° K.	1859° K.	2067° K.
0.67	1.06	1.074	1.087	1.098
0.70				1.090
0.76				1.089
0.82				1.080
0.90				1.065
1.05				1.040
1.10				1.026
1.16				1.014
1.20				1.012
1.27	1.000	1.000	1.000	1.000
1.39				0.976
1.50				.954
1.63				.928
1.76				.909
1.90	0.933	0.919	0.902	.890
2.00	.925	.908	.891	.877
2.23				.876
2.44				.876
2.90	.923	.906	.889	.876
2.99				.878
4.00				.880

increase in temperature causes an increase in the reflection factor for wave-lengths less than about 1.27 μ and a decrease for wave-lengths

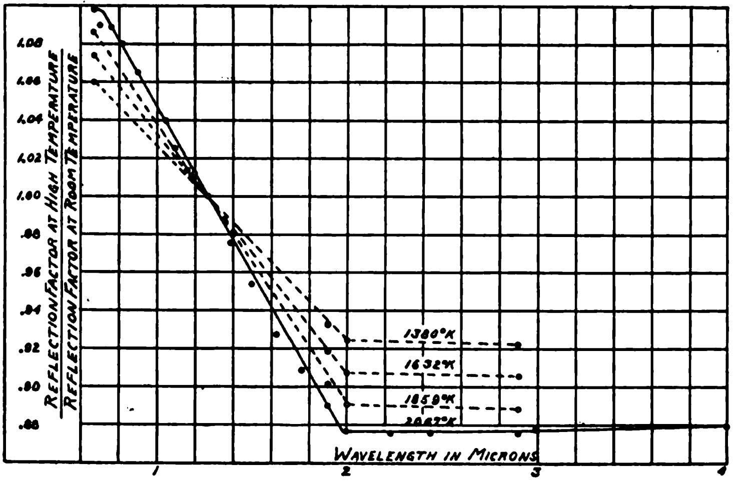

Fig. 2.

Relative Reflection Factor.

greater than this, the change in each case being greater the higher the temperature.

Previous experimental work has shown that in the visible part of the spectrum, the reflection factor of tungsten increases with temperature; thus Worthing[1] for wave-length 0.665 μ using an optical pyrometer method which for this region of the spectrum is much more delicate than that employed by the present authors, gives data yielding a change in the reflection factor of 5.0, 6.2, 7.3, and 8.2 per cent. at temperatures of 1380, 1632, 1859, and 2067° K. respectively; the corresponding values found here are 6.0, 7.5, 8.7, and 9.9 per cent.

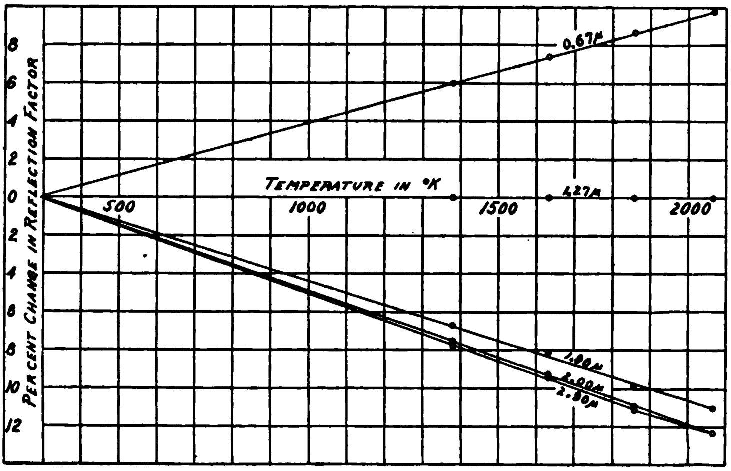

Fig. 3.

Change in Reflection Factor with Temperature.

As is well known, Hagen and Rubens,[2] showed that for long wave-lengths the reflection factors of metals obey the formula derived by Drude,[3] from Maxwell's theory:

$$R = 100 - 3650\sqrt{\frac{\rho}{\lambda}}$$

R is the reflection factor in per cent., ρ is the resistivity in ohm-cm., and λ the wave-length in μ. That this formula fits the present data fairly well beyond 2.0 μ may be seen from Table II.

The resistivities are taken from Langmuir's table.[4] The values of "R Observed" at room temperature (293° K.) are taken from Coblentz'[5]

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[1] Phys. Rev., 10, 389, 1917.

[2] Ann. d. Phys., 4F, 11, 873, 1903; Phys. Zeitschr., 11, 139, 1910.

[3] Lehrbuch der Optik, 2d ed., p. 349.

[4] Phys. Rev., 7, 154, 1916.

[5] B.B.S., 14, 312, 1918.

TABLE II.

Comparison of Observed and Theoretical Values of the Reflection Factor of Tungsten for long Wave-Lengths.

Temperature.	Resistivity Ohm-Cm.	Change in R (Fig. 2).	R Observed.	R Computed.	$\frac{R \text{ Computed}}{R \text{ Observed}}$.
			$\lambda = 2.0\,\mu$		
293° K.	5.51×10^{-6}		90.0%	94.0%	1.045
1380	37.46	− 7.5%	83.3	84.2	1.011
1632	46.01	9.2	81.8	82.5	1.009
1859	54.19	10.9	80.3	81.0	1.009
2067	61.99	−12.3	79.0	79.7	1.009
			$\lambda = 2.5\,\mu$		
293° K.	5.51×10^{-6}		93.8%	94.6%	1.009
1380	37.46	− 7.6%	86.6	85.9	.992
1632	46.01	9.3	85.0	84.3	.992
1859	54.19	11.0	83.5	83.0	.994
2067	61.99	−12.3	82.3	81.8	.994

most recent table; the rest of the numbers recorded in these columns are the result of applying to these values the percentage changes determined by the present investigation. The columns headed "R Computed" are obtained directly from the formula written above. The approximate constancy of the ratio of the computed to the observed values, which holds also at 3 μ, indicates that the formula holds for tungsten at considerably shorter wave-lengths than for other metals previously studied. In fact it appears that the formula holds up to the point at which the sharp break occurs in the curves in Fig. 3, that is up to the region in which the rapid transition takes place from a decrease to an increase in the reflection factor with an increase of temperature.

The general character of the changes found in the reflection factor of tungsten are not peculiar to this metal; McCauley[1] using an indirect method based on Kirchhoff's law, $E/\epsilon = 1 - R$, in which he measured E, the radiant flux emitted by the substance at a certain temperature and wave-length and obtained ϵ, the flux emitted by a black body at the same temperature and wave-length, from Planck's equation, found similar changes in the reflection factors of platinum, palladium and tantalum.

NOTE ON THE REFLECTION FACTOR AT ROOM TEMPERATURE.

The authors measured the reflection factor of tungsten at room temperature by a direct method with the apparatus in front of the slit arranged as indicated in Fig. 1, so that the transition from a measurement of the incident beam to that of the reflected beam could be made readily, with the two optical paths identical except for the reflection from the tungsten mirror.

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[1] Astrophys. Jour., 37, 164, 1913.

A curve showing the relation between reflection factor and the wavelength plotted from data taken after the conclusion of the high temperature experiments differs markedly from a similar curve taken when the mirror was new. Not only does the earlier curve show slightly higher values of the reflection factor, as might be expected on account of the fissures that appeared in the surface during use, but the earlier curve also shows the two depressions at about 0.8 μ and 1.4 μ shown by Coblentz,[1] whereas these seem to be absent in the later curve. As the actual behavior of the metal throughout the infra-red and particularly in this region of the spectrum is of considerable interest, a further study, using this direct method and greater dispersion, is contemplated.

Nela Research Laboratory,
Cleveland, Ohio,
June 28, 1919.

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[1] *Loc. cit.*

REFLECTION OF ELECTRONS FROM METAL SURFACES.

BY H. M. DADOURIAN.

SYNOPSIS.

A beam of electrons from a hot-lime cathode was introduced into the space between two parallel plates so as to be incident against one of them at a large angle. Then the velocity distribution among the resulting secondary electrons was studied by observing the number of electrons which, overcoming different retarding potentials reached a Faraday cylinder. The experimental results obtained tend to show that the number of secondary electrons having velocities between zero and a given velocity v may be expressed as the sum of two numbers one of which is proportional to the number of electrons having the velocity v and the other of which is proportional to the energy of X-rays in the space between the two plates having the wave length which according to the quantum theory corresponds to the velocity v.

THE following investigation was undertaken in order to study the velocity distribution among the secondary electrons produced by the incidence of a beam of cathode rays against a metal surface, and to determine the relation between the velocity of impact against a metal target and the velocity of reflection of an electron.

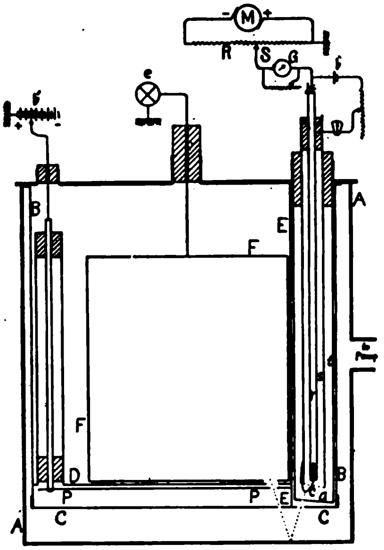

Fig. 1.

A sketch of the apparatus used is shown in Fig. 1, where *AA* is a copper cylinder about 4 inches in diameter; *BB* is a brass cylinder provided with a transverse partition *D*, a longitudinal partition *EE*, and a detachable cap *CC*; *FF* is a Faraday cylinder, carefully insulated and connected to an electrometer of the Doletzalek type; *c* represents a hot-lime cathode which consists of a narrow strip of platinum connected at one end to the rod *r* and at the other end to the tube *s*.

The platinum strip had a small patch of barium deposited upon it by burning away a tiny speck of Bank of England sealing wax. The heating current of the cathode was supplied by a battery, *b*, of low voltage, while the cathode potential was furnished by a motor generator, *M*, the terminals of which were connected to the ends of a graphite rod, *R*, which had a resistance of about 95,000 ohms. At full speed the generator maintained a fairly constant potential of about 1,500 volts between the ends of the graphite rod; consequently any desired potential up to 1,500 volts could be applied to the cathode by moving the slide-contacts along the graphite rod. The constancy of the current of cathode electrons from the hot-lime cathode to the anticathode was observed by means of the galvanometer *G*.

When the apparatus is in operation a beam of cathode rays passes through a hole in the base *a* of the tube *t*, which forms the anticathode, and produces secondary electrons by striking the brass bottom of the cylinder *AA*. Some of these electrons pass through the holes in *CC*, *PP*, *D* and *FF* into the Faraday cylinder and cause the electrometer needle to deflect.

The holes in *a*, *CC* and *PP* were, respectively, 0.5 mm., 1.0 mm. and 3.0 mm. in diameter. The right-hand hole in *CC* and those in *D* and *FF* were large enough not to affect the passage of electrons. The distances from the bottom of the apparatus of the center of the source of electrons on the hot-lime cathode, of the base *a*, of the cap *CC* and of the plate *PP* were, respectively, 19 mm., 13 mm., 10 mm. and 18 mm. The centers of the holes and of the source of electrons were in the same vertical plane. The distances of the centers of the holes in *a*, *CC* and *PP* from the vertical line through the center of the source of electrons were 2 mm., 12 mm. and 14 mm., respectively.

The distribution of velocity among the secondary electrons was studied by applying different retarding potentials to the plate *PP* and observing the corresponding rate of deflection of the electrometer. This rate of deflection was obtained by taking the time of passage of an image reflected from the electrometer mirror over 20 cm. of a scale placed at a distance of one meter from the electrometer. The image was allowed to pass over 5 cm. of the scale before measurement of time was begun.

The results of some of the experiments are indicated by the curves of Figs. 2 to 5, where the abscissas denote the retarding potentials applied to the plate *PP* and the ordinates the rate of deflection of the electrometer. The potential applied to the hot-lime cathode during each experiment is indicated in the figures. The curves of Fig. 2 and Fig. 3 were obtained with the cap *CC* on, and those of Fig. 4 and Fig. 5 with it off.

It will be observed that the greater part of each of the curves has the general appearance of an exponential curve. Let us suppose for the moment that the experimental curves are true exponential curves represented by the equation

$$I = I_0 e^{-\lambda V}, \tag{1}$$

where λ is a constant, I_0 represents the number of electrons which enter the Faraday cylinder when no retarding potential is applied, and I the number when a retarding potential V is applied to the plate PP. The number of electrons which are prevented by the retarding potential from entering the Faraday cylinder is then given by

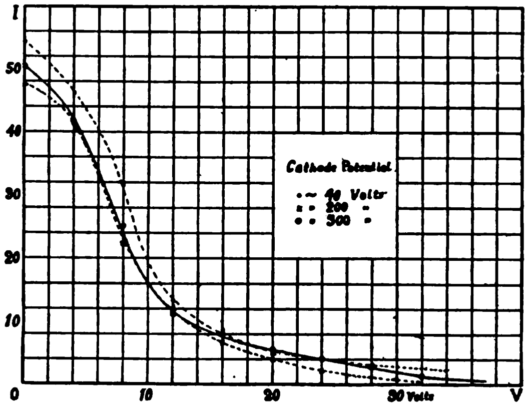

Fig. 2.

$$I_0 - I = I_0(1 - e^{-\lambda V}). \tag{2}$$

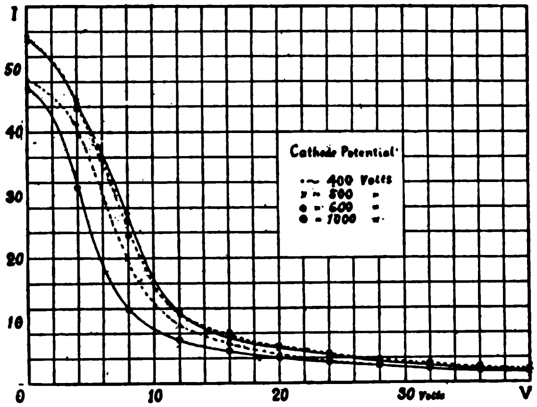

Fig. 3.

Therefore the number, n, of electrons having the energy corresponding to a potential V is obtained by differentiating equation (2)

$$n = -\frac{dI}{dV}$$
$$= \lambda I, \tag{3}$$

or

$$I = \frac{1}{\lambda} n. \tag{4}$$

In other words the total number of electrons which enter the Faraday cylinder having velocities between zero velocity and that corresponding to a potential V is proportional to the number of electrons having the higher velocity.

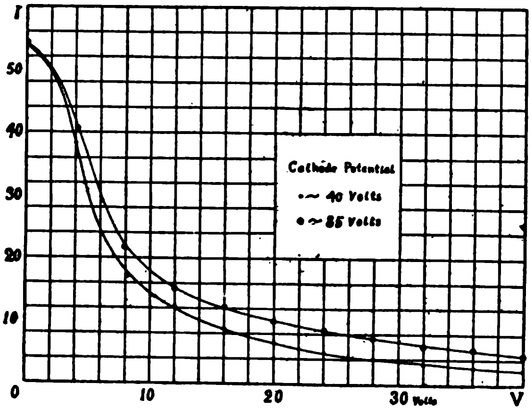

Fig. 4.

The curves differ, however, from true exponential curves in two respects. First, in nearly all cases there is a point of inflection near the I-axis. Secondly, beyond the inflection point the ordinates decrease more rapidly for smaller abscissas than would be expected if the curves were represented by equation (1). The first may be due to errors in

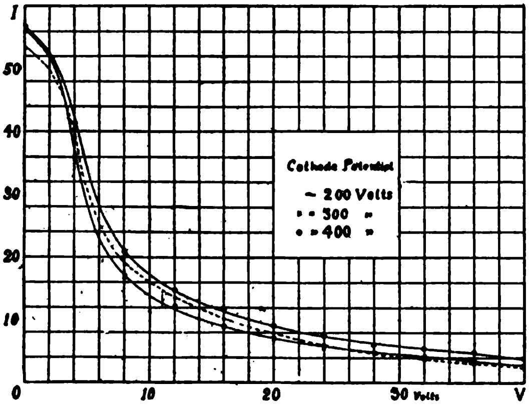

Fig. 5.

measuring the interval of time taken by the image from the electrometer mirror to cover 20 cm. of the scale. For zero retarding potential this interval of time amounted to between two and three seconds; consequently it could not be determined very accurately by the stop watch used for the purpose. The change in curvature may, on the other hand, be real, in which case the velocity distribution curve obtained by differentiating the current-potential curve has a maximum point.

The second difference may be accounted for by supposing that the current-potential curves are the resultants of two exponential curves. In fact a very close approximation to the experimental curves is obtained by putting

$$I = I_1 + I_2, \tag{5}$$

where

$$I_1 = I_0'e^{-\lambda' V} \tag{6}$$

and

$$I_2 = I_0''e^{-\lambda'' V}. \tag{7}$$

On this assumption the total number of electrons, I, entering the Faraday cylinder and having velocities from zero up to that corresponding to the potential V equals the sum of two numbers, each of which is proportional to the amount of a different type of energy in the space between the bottom of the cylinder AA and the cap CC (Fig. 1). The meaning of this statement will become clearer when we consider the transformation of energy which takes place in this space. Energy is admitted into this space in the form of energy of motion of cathode electrons moving with a velocity corresponding to the cathode potential. This energy changes progressively from energy of electrons into energy of soft X-rays and vice versa. In the process of this transformation energy is degraded to the lower forms represented by electrons of slower velocities and softer X-rays. The total number of electrons having velocities between zero and that corresponding to any potential V is then equal to the sum of two numbers one of which is proportional to the number of electrons having velocities given by the expression

$$\tfrac{1}{2}mv^2 = Ve, \tag{8}$$

where m, e and v are respectively the mass, the charge and the velocity of the electrons; while the other number is proportional to the amount of energy of X-rays of a frequency given by the quantum relation

$$h\nu = Ve, \tag{9}$$

where h is Planck's constant and ν is the frequency of the rays.

While working on an investigation on total ionization[1] J. B. Johnson obtained certain results which the present writer interpreted as being due to multiple reflection of electrons between two metal plates, and on this hypothesis he computed the velocity of reflection and found it to be two-thirds of that of the velocity of impact of the electrons. Mr. Johnson made an attempt at the time to repeat his results, but was unable to do so, and being pressed for time turned his attention to the problem of total ionization. It was thought worth while, therefore, to try to obtain results which would agree with those which Johnson could not repeat and which would lead to the determination of the relation between the velocities of impact and of reflection of an electron.

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[1] J. B. Johnson, PHYS. REV., 10, 609, 1917.

If the velocity distribution of electrons in the space between the bottom of the apparatus of Fig. 1 and the cap *CC* were due primarily to multiple reflections from the two surfaces the ordinates of the curves of Figs. 2 to 5 would have decreased in steps as the retarding potential was increased. Then the relation between the velocities of impact and of reflection could be computed from the potentials at which the sudden drops in ordinates occurred. The absence of sudden drops in the ordinates of the experimental curves obtained in these experiments indicates that the velocity distribution is due to several factors, such as suggested in the preceding paragraph, which tend to make the velocity distribution curves continuous.

In conclusion the author wishes to thank Professor George B. Pegram, of Columbia University, for extending to him the facilities of the Phœnix Physical Laboratory where this investigation was carried out, and also Professor Bergen Davis for helping him to secure the apparatus necessary for the experiments and for many other courtesies which made the author feel at home in a strange laboratory.

Trinity College,
Hartford, Conn.,
August, 1919.

REMARKS ON DR. L. SILBERSTEIN'S RESULTS IN HIS PAPER "ON THE DISPERSION OF THE DIAMOND."[1]

BY ALBERT C. CREHORE, PH.D.

SYNOPSIS.

Dr. Silberstein begins with the assumption that dispersion in general is due to the interaction of the electromagnetic field with the electrical charges on the nuclei of atoms. In the particular case of diamond he deduces from this assumption that the charge on the nucleus of the carbon atom is 2.22 times the electronic charge. He admits that this result is not in harmony with the pan-electronists belief in the indivisibility of the electron. This conclusion justifies us in bringing into question his fundamental premises. It is the author's belief that spectrum effects are in general principally due to the revolving electrons rather than the nuclei of atoms. Reasons for this are given in a book by the author now in press. In the case of diamond, if the Silbertsein ratio, 2.22, which is corrected to read 2.246, is multiplied by the characteristic number of the tetrahedron, namely 8/3ds, the result becomes 5.99, which is very close to the whole number, 6, the atomic number of carbon. The 8/3ds would be required if the effect is due to the revolving electrons. It is pointed out that the dimensions of equations derived from electromagnetic theory by most modern writers are not correct without expressing the specific inductive capacity of the medium. In particular this point is illustrated by Dr. Silberstein's equation (9), and the subject is fully treated in the author's book referred to.

IN a paper "On the Dispersion of the Diamond," Dr. Silberstein assumes that the dispersion of the diamond is due to the action of the nuclei of the carbon atoms, and he proceeds to derive the so-called "atomic coefficient" of carbon on this assumption. In the opening paragraphs he says: "Consider a space-lattice of points, indefinitely extended in all directions. Let all these points be occupied by the centers of *equal* atoms, each containing a single electron whose charge and mass are *e*, *m*."

The conclusion[2] arrived at as a result of this premise makes "*the atomic coefficient of carbon*, entering into its

$$\text{`atomic refractivity' } C = \frac{k_0}{u_0 - u},\text{"}$$

$$k_0 = 4.07 \times 10^{10} \text{ cm. gr.}^{-1}.$$

The numerical value of k_0, which is called ϵ, when attributed to a single electron placed at each point of the space-lattice, as given on page 404, is

$$\epsilon = 1.83 \times 10^{10} \text{ cm. gr.}^{-1}.$$

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[1] L. Silberstein, Phil. Mag., Vol. XXXVII., No. 220, April, 1919, p 396.

[2] *Loc. cit.*, p. 405.

The quotient of k_0 above obtained from the diamond, and supposed to be due to the carbon atom, by this ϵ is about 2.22, giving the numerical ratio

$$k_0 \text{ (for diamond)} = 2.22\,\epsilon.$$

Dr. Silberstein remarks on this result: "This exceeds the nearest multiple ($2\,\epsilon$) of the proper electronic value by as much as $0.22\,\epsilon$, a feature certainly not agreeable to the pan-electronists. I do not propose here to attempt to bring it down to an exact multiple by some artificial assumptions."

Believing that the pan-electronists have a very strong case, it seems necessary to question the conclusion of Dr. Silberstein. Assuming that his deductions are all correct on the premises, this is equivalent to doubting the truth of his assumptions.

Briefly, it is my belief that the discrepancy between the ratio 2.22 obtained by Dr. Silberstein and a whole number is due to a cause so fundamental that it negatives the original premises that the dispersion of the diamond is due to the nuclei of the atoms at all. It is my belief that the dispersion is due to the revolving negative electrons rather than the positive nuclei, and further that all spectrum effects including both light and x-ray spectra are due primarily to the motion of the revolving electrons when displaced out of their normal circular orbits. The reasons for this belief are given in a book[1] by the author now in press.

In the present case these views are supported by the result of Dr. Silberstein, for, if we multiply the ratio that he obtains, namely 2.22 above mentioned, by the numeric 8/3ds, a value close to a whole number, 6, is obtained, which might be expected for the diamond, this being the atomic number of carbon. Now, this 8/3ds is a characteristic number relating to the regular tetrahedron, as has been pointed out in several papers.[2]

If we go back and alter the assumption made at the beginning by Dr. Silberstein, and say that in place of the single charge that he assumes to be at each point of the lattice, thus making equal atoms, we now require a group of four revolving electrons, revolving in orbits inclined to each other as the four medial lines of a regular tetrahedron, the result will introduce the required factor 8/3ds. It is not supposed, however, that four such electrons exist in a single atom, but that four atomic centers are required to furnish them, one electron from each atom; for, it has been shown that the directions of the axes of revolution of the electrons in the atoms of the diamond are such that the whole diamond may be divided into four equal groups of atoms,[1] each group having the direction of its axis of revolution parallel to one of the medial lines of the regular tetrahedron.

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[1] A. C. Crehore, "A New Theory of the Atom," D. Van Nostrand Co., New York, 1919.

[2] Crehore, PHYS. REV., Vol. IX., No. 6, Sec. Ser., June, 1917, pp. 458–460; Vol. XII., No. 1, Sec. Ser., July, 1918, pp. 18–21; *loc. cit.*, A New Theory of the Atom, Appendix A.

To compare the k_0, which is called the atomic coefficient, however, with the effect of a single stationary electron, one at each point of the space-lattice, we may retain the same expression for k_0, that is for ϵ, that Dr. Silberstein uses, namely,

$$k_0 = \frac{1.008}{3} \times \frac{e'/c}{m_0} \times \frac{e'/c}{m_H}, \tag{10}$$

where e' denotes the charge on one electron proper. The numerical value of this expression as given by him on page 404 is $\epsilon = 1.83 \times 10^{10}$ cm. gr.$^{-1}$. The numerical values of e', m_0 and m_H used by him to obtain this value are not stated. Let us, first, recalculate this numerical value making use of the theoretical values of e, m_0 and m_H as given in my book[2] referred to, chapter X.

$$e = 4.763 \times 10^{-10},$$
$$m_H = 1.658 \times 10^{-24},$$
$$m_0 = 0.898 \times 10^{-27}.$$

The even number, 3×10^{10}, has been employed throughout for the velocity of light in deriving these theoretical numerical values of e, m_0 and m_H, and will be employed again here because the true value of the velocity of light is very close to and a little less than this even number, and authorities apparently differ as to its more exact value. The Smithsonian Tables give its value due to Weinberg as $c = 2.99870 \times 10^{10}$, and I believe that Millikan has used a value $c = 2.999 \times 10^{10}$ which is the same if the last two digits above are omitted. This differs from the even number 3 by only one unit in the fourth significant figure. With these values we obtain for the Silberstein, ϵ,

$$\epsilon = 1.8107 \times 10^{10}, \text{ instead of } 1.83 \times 10^{10} \text{ above.}$$

Let us next recalculate the Silberstein k_0 for the diamond using his formula

$$k_0 = \frac{1}{3}\frac{M}{d}a,$$

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[1] Crehore, Phil. Mag., Vol. XXIX., June, 1915, pp. 763, 766–767; Vol. XXX., Aug., 1915, p. 257, "Construction of the Diamond with Theoretical Carbon Atoms."

[2] The book referred to has been delayed because of the printer's strike. These numerical values of e, m_H and m_0 are also derived in Section III. of a series of articles entitled "The New Physics," the contents of which was published in The Journal of Electricity, San Francisco, Cal., October, 15, 1919, which begins the series.

and the same numerical values as he gives, namely

$M = 12$, atomic weight of carbon,

$d = 3.515$, mean density of diamond,

$a = 3.5740 \times 10^{10}$, obtained from Marten's observations on the diamond and the Silberstein formula.

We find

$$k_0 = 4.06714 \times 10^{10}.$$

The decimal places are retained here merely on the supposition that the numbers from which this is derived are correct so far as given, the error being unknown.

Introducing, now, the new hypothesis that the dispersion is wholly due to the revolving electrons and not to the nuclei of the atoms, it is necessary to embrace four atoms in one fundamental unit before we have a right to assume that the dispersive centers are all equal. Such groups will be equal, but not the individual atoms because of the orientation of their axes with respect to each other, which must be taken into the account. Each electron on the average in the tetrahedron formation is just 2/3ds as effective[1] as it would be without any orientation. If, therefore, we multiply the Silberstein formula above given by 2/3ds on account of the reduced effectiveness of the electrons due to the tetrahedral formation, and also multiply the value of M by four so as to embrace four atoms in a unit instead of a single one, the formula becomes

$$k_0 = \frac{8}{3}\,\frac{1}{3}\,\frac{M}{d}\,a = \frac{8}{3} \times 4.06714 \times 10^{10} = 10.8457 \times 10^{10}$$

for the diamond. Taking the ratio of this coefficient to the value of ϵ above obtained on the single electron hypothesis, namely 1.8107×10^{10}, the ratio is obtained as follows:

$$\text{Ratio} = \frac{10.8457}{1.8107} = 5.99.$$

The ratio obtained by Dr. Silberstein as above stated, assuming the dispersion to be due to the nuclei, was 2.22, which led him to make the remark above quoted, thus bringing into question a fairly well established fact, namely, the indivisibility of the electron. The new ratio just obtained is very close indeed to the whole number, 6, that is, to the atomic number of carbon, which is supposed to be 6. The difference between 5.99 and 6.00 may easily be accounted for by an error in the assumed mean density of the diamond without assuming that any error at all has entered through the other constants. Dr. Silberstein remarks in a footnote, page 405, that "Martens does not quote the density of his piece of diamond." The required density for the diamond, which will make this ratio exactly 6 is

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[1] *Loc. cit.*

$$d = 3.509,$$

which differs by only six units in the fourth digit from the value adopted by Silberstein.

In submitting these remarks on the Silberstein theory there is no claim to have produced a complete new theory on the new hypothesis that the dispersion is due primarily to the revolving electrons with any such thoroughness as Dr. Silberstein has developed his theory. The fact, however, that the factor 8/3ds reduces his ratio with such exactness to the whole number, 6, which is also the atomic number of carbon, is of considerable interest in itself. The coincidence is too near to be merely the result of chance.

In conclusion I cannot refrain from referring to the dimensions of the equations used by Dr. Silberstein in this paper in common with other writers on electromagnetic theory, because we have another example here of the common practise of omitting to write the specific inductive capacity in all equations derived from electromagnetic theory. Referring to the equation (9) page 403, namely

$$\mu^2 = 1 + \frac{a}{u_0 - u}, \tag{9}$$

the dimensions of each term should be the same, namely the same as μ^2, that is, dimensionless in terms of length and of time. For, index of refraction is essentially a ratio between two sines or two velocities, and is, therefore, dimensionless in terms of any fundamental units. The numeric, 1, in the second member satisfies this requirement; but the last term, $a/u_0 - u$, does not satisfy it. The value of a, as given in equation (8), is

$$a = \frac{NB}{4\pi^2c^2} = \frac{Ne^2}{4\pi^2c^2m}.$$

The meaning of N is the number of carbon atoms per unit volume, and its dimensions may be taken as $N = L^{-3}$. The dimensions of e^2 on the electrostatic system of units are $L^3MT^{-2}k$, the k being the specific inductive capacity of the medium. Dividing e^2 by c^2m gives the dimensions of e^2/c^2m as Lk. Multiplying by the dimensions of N gives for the dimensions of his, a, $L^{-2}k$.

The dimensions of $u = 1/\lambda^2$ are simply L^{-2}, and not $L^{-2}k$, which are the dimensions of a. The quotient $a/u_0 - u$, therefore, has the dimension, k, and is not dimensionless like the other terms in the equation.

Of course, the numerical value of the expression is not affected at all by the omission of the *k* so long as the second and the centimeter are used as our units of time and of space, but, if these units should be changed, then the *k* is no longer unity, and it does make a great difference in the numerical value. This matter is explained at some length in the book above referred to, and it does not seem necessary to enlarge upon this point further here.

This last criticism on the matter of dimensions is not at all peculiar to Dr. Silberstein's paper, but applies to most writers on electromagnetic theory at the present time. And, indeed, no criticism whatever has been made of Dr. Silberstein's work so far as deductions from his premises are concerned. His conclusions, however, seem to lead him to question what many consider to be a well-established fact, and the object of these remarks is to discover a way out of this difficulty.

AN EXPERIMENTAL METHOD FOR THE PRODUCTION OF VIBRATIONS.

By C. V. Raman.

Synopsis.

Forced Vibration of strings; the Melde experiment. Improvements of Fleming's modification are described which enable the amplitude of the forced vibration to be varied, which minimize lateral vibration, and which make it possible to obtain linear, circular or elliptical vibration as desired. This improved apparatus is useful for studying or demonstrating the various types of motion possible for a stretched string. Five photographs illustrate the results which may be obtained.

Forced Vibration of strings. The phenomenon observed by Jones and Phelps, that when two or more different types of vibration are simultaneously obtained, the ratio of the number of ventrals egments may differ from the ratio of the frequencies of maintenance, is briefly discussed.

PROF. J. A. FLEMING[1] has recently devised and described a very simple but ingenious and compact piece of apparatus to illustrate the close analogy existing between the transmission of mechanical vibrations along a loaded string and the propagation of alternating electric currents along a telephone cable having inductance coils inserted in it at equidistant intervals. For this purpose, the well-known method of producing forced vibrations on strings due to F. Melde is not suitable; for, when a long heavily loaded string is used, very large forks would be required, and the reaction of the string when under tension would generally result in stopping the fork even if the latter be electrically driven. Further, it is also desirable to have a means of altering the frequency of vibration which is not possible with Melde's method except by changing the fork. Prof. Fleming therefore replaced the fork in the Melde experiment by a small continuous-current electric motor, on the shaft of which a disk carrying an excentrically placed pin is fixed. The pin actuates a light crank shaft, which is connected at the other end to a rocking lever. The string is maintained in vibration by having its extremity attached to a small hook fixed to the crank-shaft. With an arrangement of this kind, it is possible to impose either a linear or circular vibratory movement on the extremity of the string as desired.

In experimenting with an apparatus of the type described above, the present author found that its usefulness is considerably enhanced by certain improvements which he has made in it. In Fleming's apparatus, as illustrated in his paper, the excentric pin is permanently fixed to the disk carried on the shaft of the motor. The amplitude of the obligatory circular or linear vibration imposed on the extremity of the string is consequently invariable. This is a disadvantage, as in the practical use of the apparatus, it is often useful to be able to alter the amplitude of the obligatory motion quickly to any value, either large or small, as may be desired. Further, the author has found that with an apparatus of the Fleming type, when the shaft of the motor is in rapid rotation, the rocking lever and crank-shaft tend to develop vibrations in a direction transverse to their plane, and this introduces undesirable complications in the forced vibration of the extremity of the string. To remedy these defects and make the apparatus a real precision instrument for the study of vibrations, the design of the working parts has been modified as shown in Fig. 1. The disk fixed to the shaft of the motor has a slotted plate screwed to it a little above the general surface, and the crank-pin is carried by a sliding piece which can be put at any desired point and fixed by a single half-turn of the screw-head shown in the figure. The rocking lever moves between well-oiled guides which prevent any lateral vibration, the whole being firmly bolted to the body of the motor as shown in the photograph of the apparatus (Fig. 2). Linear movements may be obtained by attaching the string to one or other of a number of holes in the rocking-lever. A sliding piece on the crank-shaft (not shown in the figure) which carries a hook and can be fixed at any point enables either circular or elliptical vibrations to be imposed on the extremity of the string if desired.

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[1] Physical Society, London, Proc., December, 1913, p. 61.

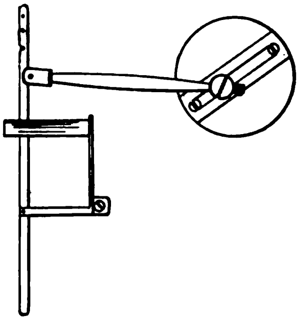

Fig. 1.

The arrangement (modified as described above) has proved to be a most convenient and satisfactory piece of apparatus. It is especially useful in the experimental study or demonstration of the interesting types of vibration maintained by a variable tension, described by the author in previous communications in this Review.[1] These types are obtained when the string is attached to the rocking-lever in such a manner that its oscillation is longitudinal to the string. Under these conditions, as has been shown in the papers quoted, the string may be set in vibration when its frequency in any of its natural modes is sufficiently nearly equal to any multiple of half the frequency of the variation of tension.

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[1] Phys. Rev., December, 1912, and July, 1914.

Each of the possible modes has its own characteristic features, and as moreover, the frequency of vibration of the string depends on the number of ventral segments into which it divides up and may thus have simultaneously more than one value, a very large variety of different modes of vibration may be obtained at will by adjusting the tension or length of the string, or by altering the position of the crank-pin of the motor so as to increase or decrease the magnitude of the imposed variation of tension.

The motion of individual points on the string may be very conveniently studied by using a black cord with white dots woven in it as suggested by A. T. Jones and M. E. Phelps in a recent paper in this REVIEW.[1] The form of the figures thus observed reveals at a glance not only the relation of frequency between the longitudinal and transverse movements of any point on the string, but also their phase-relation which appreciably varies with the experimental conditions. Much longer and heavier strings can be used with this apparatus than with an electrically-maintained fork, and by lighting up with the beam from an electric arc, very effective demonstrations are possible. To illustrate the character of the phenomena observed, photographs of the motion of individual points on the string in the first five types of vibration have been secured and are reproduced in Figs. 3 to 12. Figs. 3 and 4 show the first type of vibration in which the ratio of frequencies is 1 : 2, the phase-relation being slightly different in the two cases. In Figs. 5 and 6 the frequency relation is 2 : 2, the curve in Fig. 6 being a deformed ellipse. In Figs. 7, 8 and 9 we have the third type in which the frequency ratio is 3 : 2, the difference in phase being specially marked in Fig. 9. The fourth type in which the frequency relation is 4 : 2 is shown in Figs. 10 and 11. The fifth type is shown in Fig. 12, and exhibits the frequency relation 5 : 2.

It may be useful here to consider the explanation of the interesting phenomenon noticed by Jones and Phelps in their paper,[2] that when two or more of the different types are simultaneously maintained, the ratio of the number of ventral segments into which the string respectively divides up often differs from the ratio of the frequencies of maintenance. For instance, it may be found in one case that the string divides up simultaneously into two and five segments either in the same plane or in perpendicular planes, the frequencies being respectively one half and three halves of the frequency of variation of tension. Apparent anomalies of this kind are due to the fact that we are here dealing with *forced* oscillations, and though the string practically vibrates in its normal

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[1] PHYS. REV., November, 1917.

[2] *Loc. cit.*

Physical Review, Vol. XIV., Second Series.
November, 1919.

Plate I.
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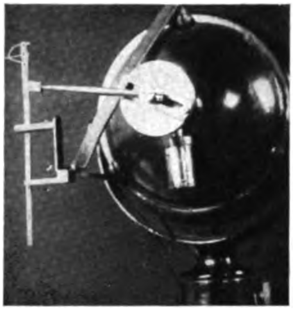

Fig. 2.

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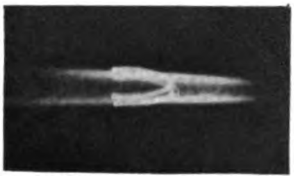

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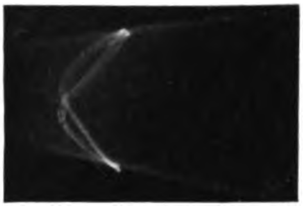

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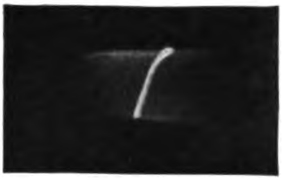

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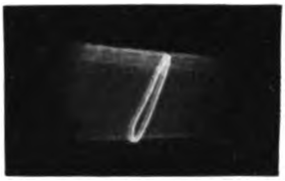

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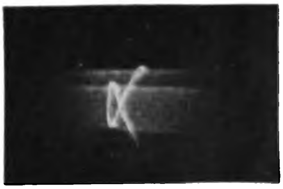

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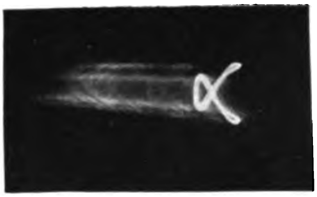

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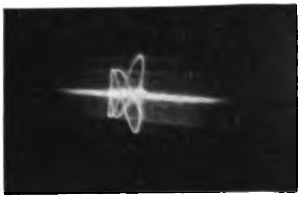

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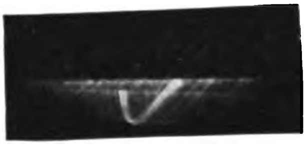

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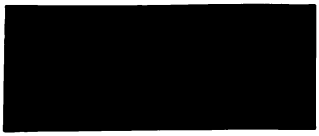

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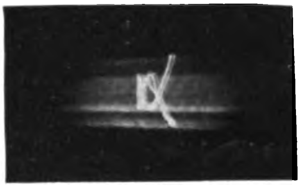

Figs. 3–12.

C. V. RAMAN.

modes, the *frequencies* of vibration stand in fixed relations to the impressed forces and may, especially in the case of the first two or three types, differ very appreciably from the natural frequencies of oscillation. Maintenance is, in fact possible when the natural frequency lies anywhere within a certain range which includes the frequency of maintenance, this range being considerable for the first type, and becoming smaller and smaller as we proceed up the series of types with higher frequencies of maintenance. The magnitudes of the ranges depend on the magnitude of the imposed variations of tension, as may be readily shown with the apparatus described in the present paper, by adjusting the position of the crank-pin, and thus increasing or decreasing the longitudinal motion of the end of the string. With very small variations of tension, the modes with the higher frequencies of maintenance are hardly obtainable, the resonances are much sharper, and the ranges even of the first few types become greatly restricted. With larger variations of tension, the ranges increase, and for the first few types may become very considerable. It will be found that large amplitudes may be obtained even without any exact adjustment for resonance.

The photographs reproduced with the paper were secured for the author by Mr. Rajendra Nath Ghosh, for whose assistance his thanks are due. Mr. Ghosh is now engaged in developing a detailed theory showing the manner in which the amplitudes and phases of the various types of vibration depend on the experimental conditions. The results will, it is hoped, be published in due course.

210, Bowbazar Street,
Calcutta, India,
May 20, 1919.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A BAROSCOPE FOR MEASURING THE PRESSURE IN EXPLOSION WAVES.

By Dayton C. Miller.

THE baroscope is designed to measure the maximum pressure in an explosive wave, such as that occurring in the air near the muzzle of a large caliber gun, when a shot is fired. The pressure may vary from a few ounces to a thousand pounds per square inch. The conditions of the experiment require that the instrument be rugged and portable.

A diaphragm of elastic material is very tightly clamped between heavy bronze rings, Fig. 1, leaving the central portion free to vibrate. The diaphragm is open to the air on one side, while the other side is entirely enclosed by a rigid air-tight housing.

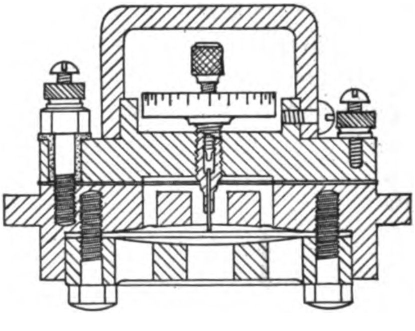

Fig. 1.

The housing carries an insulated micrometer screw; the end of the screw is split and holds by very light friction a sliding contact pin; this pin consists of a fine sewing needle, the gold-plated end of which is placed in contact with the center of the diaphragm. A protecting cover is provided for the graduated head of the screw.

When the diaphragm is caused to vibrate, as by the pressure wave from the explosion, the contact pin is pushed inward by an amount equal to the amplie

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

tude of the maximum displacement. The amount of this displacement is then determined by connecting the baroscope in an electric circuit with an indicating galvanometer, such as a sensitive voltmeter, and turning the micrometer screw forward till contact with the diaphragm is established.

For resetting the instrument, the micrometer screw is withdrawn to its zero position and the pin is again brought into contact with the diaphragm by means of a small push-screw working in the axis of the micrometer screw. The push-screw is finally withdrawn a turn or two to give opportunity for the pin to move inward at the next pressure measurement.

The baroscope is calibrated in terms of static pressure by means of a hydraulic press provided with a standardized gauge. The baroscope is clamped in a fitting on the upper end of the hydraulic cylinder, there being an air cushion between the water and the diaphragm. Each instrument is then calibrated when it is in the same condition as in the field tests.

For the rapid plotting of a field of pressure, it is desirable to have a number of baroscopes to determine the pressure at various points simultaneously. One instrument can be used in a fixed position for successive experiments, thus, tying together the various sets of readings. Two or more baroscopes may be placed in one position to give an average value as far as instrumental variations are concerned.

For pressures from a few ounces to a thousand pounds per square inch, the diaphragm may be of hard-rolled, spring brass, 75 mm. in outside diameter, having an effective diameter inside the clamping ring of 50 mm. Four different thicknesses have been employed, from 0.38 mm. to 1.66 mm. thick. These have free periods varying from about 1,000 to 5,000. A static pressure of 1 kilogram per square centimeter produces a displacement of the center of the thinnest diaphragm of 0.57 mm., and of the thickest one, 0.016 mm.

The baroscopes described may be used within a few feet of the muzzles of the largest guns, where a less rugged apparatus would be destroyed. For locations where it is safe for a person to remain, a baroscope on the same principle, but using a glass diaphragm, has been used. In this case the movements of the diaphragm are recorded on a moving photographic film by means of the author's phonodeik, and a complete record of the wave form, together with a time scale from a tuning fork interrupter, is obtained. This apparatus is also calibrated in terms of static pressure by means of an air pump and a mercury gauge. With a glass diaphragm of 30 mm. free diameter and 0.085 mm. thick, a change of pressure of 0.16 gram per square centimeter gives a displacement of the light spot on the photographic film of 1 centimeter.

The results of investigations made with the baroscope will be given in full in another paper.

Case School of Applied Science,
Cleveland, Ohio,
April 25, 1919.

HIGH FREQUENCY CURRENTS ON WIRES.[1]

BY J. O. MAUBORGNE.

A PAPER describing original experiments in multiplex telegraphy and telephony was published in 1911 by Major (now Major General) George O. Squier. Previous to this, and even for some time afterwards, the opinion prevailed that the use of high frequency currents as "carriers" for transmission of energy over wire and cable circuits of appreciable length was impossible because of the excessive attenuation which would occur. In fact, even up to the present date consensus of opinion in the scientific world has been that it would be impracticable to use currents of frequencies greater than thirty thousand cycles. The experiments of 1910–1911 made by General Squier disproved the idea that it was not possible to use high-frequency currents under these circumstances and that simultaneous operation of the "wire" and high-frequency systems could be carried on without causing interference to each other. It was further demonstrated that several high-frequency units, tuned to different frequenices, could operate either simultaneously or independently on the same wire line. The frequencies utilized by General Squier at that time did not exceed one hundred thousand cycles, due only to the form of generator used.

Recently this subject has assumed an important aspect from a military standpoint and it was decided to conduct further experiments with the object of determining whether or not it was possible to adapt certain existing types of radiotelephone and telegraph apparatus to multiplex operation. In Signal Corps radio apparatus of the continuous wave type, the three electrode vacuum tube is employed as a generator; the powers dealt with are small (in the neighborhood of six watts) and the frequencies are of the order of 500,000 cycles per second, which is much higher than have hitherto been used for multiplex transmission; such frequencies, up to the present time, have been considered entirely impossible for use on account of the computed excessive attenuation mentioned above. The particular type of apparatus employed in these tests is known as the SCR-67, which is the ground set of the standard ground-airplane radiotelephone equipment.

To provide a practical means for carrying out of the tests, a wire running from Washington to Baltimore, Md., was furnished by the Postal Telegraph Company. One multiplex station was established at the Signal Corps Radio Laboratory, Bureau of Standards, Washington, and a corresponding station at Dixon's Park, Curtis Bay, Md., approximately three miles from the Postal office in Baltimore. The total wire distance between the two stations was sixty miles.

Satisfactory two-way communication was obtained. Speech was received at both stations, loud and with exceptional clearness, the distortion, common to long-distance wire telephony, being completely absent. The tuning at the receiver was definite, comparable in every respect to that when receiving signals from a "sharply" tuned radio station. This fact will permit of the operation of a number of multiplex units, each tuned to a different frequency on the same line without the use of filter circuits. The carrier frequency employed in these tests was 600,000 cycles per second (wave-length 500 meters); the effective line current, measured at each of the transmitting stations, averaged 60 milamperes.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

The range of the SCR-67 when operating as a ground "radio" set in communication with a corresponding SCR-67 is, under ordinary conditions, 10 miles. Thus by confining and directing the flow of energy along a wire the range of communication is materially increased.

Office of the Chief Engineer,
Engineering and Research Division, Signal Corps,
Washington, D. C., April 25, 1919.

The Production of Vacuum Tubes for Military Purposes.[1]

By N. H. Slaughter.

At the outset of this country's participation in the European Was, it became evident that there would be required for the signalling devices needed by the Army, enormous quantities of vacuum tubes. (I am assuming that the use of the term "vacuum tube" to designate the three-electrode thermionic device originally called the "audion" is sufficiently understood to obviate the necessity of any further explanation.) The problem was not essentially a "research" problem, as the term is usually understood, but was rather a problem of design and production of vacuum tubes already partly developed in research laboratories.

Previous State of the Art.—In April, 1917, the state of the vacuum tube art was more or less as follows:

In the United States the development of vacuum tubes had progressed to the point where these tubes were in considerable use as detectors and amplifiers of radio signals for amateurs and to a limited extent for government radio stations and for use as amplifiers or "telephone repeaters" by the American Telephone and Telegraph Company. In addition, transmitting tubes had been developed sufficiently to use in experimental radio telephone sets. These vacuum tubes, however, were not a standardized product, were difficult to manufacture, and varied widely in their individual characteristics.

In France the stimulus of war had accelerated the development of vacuum tubes so that there was in production and use by the French signalling units, tubes of a comparatively satisfactory type. The French had adopted the practice of employing only a single type of tube for all purposes. However, the necessity for adopting a tube which was a compromise in so far as the various operating characteristics are concerned and a further compromise from the standpoint of ease and cheapness of manufacture, had so handicapped the design of the tube that the resultant product was far from ideal in its performance of any of the particular duties imposed upon it. In Great Britain the vacuum tube art had progressed along much the same lines as in France, but the military forces had not adopted this device to the same extent as the French.

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[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

Basic Specifications.—In order to arrive at the basic specifications for vacuum tubes for the United States military forces, careful study was made of the characteristics of the French tubes, with the result that the specifications for the American vacuum tubes show marked resemblance in some respects to the French tubes. The fundamental basis of the specifications for any military device comprise ruggedness, reliability, minimum of adjustment and for portable equipment the irreducible minimum of power consumption. In addition to these requirements, it was necessary to standardize the filament voltage and current input, the plate voltage and current input and numerous other detailed points involved in the design of the vacuum tubes. Because of the great difference in the operating requirements for transmitting and receiving tubes, it was decided to adopt two types, one for transmitting purposes and the other for receiving purposes. For the receiving tube the filament voltage was chosen at a value which would conform with the French, namely, the voltage obtained from a two-cell lead storage battery, while the filament current was specified as the minimum which could be obtained and allow the tube to meet other requirements. The plate voltage of the receiving tube was chosen as the voltage given by a 15-cell dry battery, this value being arrived at as the minimum which would insure proper operation of the receiving tubes and yet not require an excessively heavy plate battery. The value of plate current was correspondingly limited to less than 1 mil in order to make it possible to utilize a small size of dry cell for supplying this current. Fortunately the above specifications were substantially met by a design of tube already partially standardized by a commercial company. The most noteworthy improvements that have been effected in the receiving tube during the process of development and manufacture of this tube for the Signal Corps, have been a new type of structure for supporting the electrodes, which new type is enormously more rugged than the type existing in April, 1917; and a reduction of filament current to less than 20 per cent. of the original value. The development of the tube in other respects has been largely in the nature of minor improvements to secure better uniformity and a tube better adapted to quantity production methods. The use of receiving tubes requires satisfactory performance as detectors of radio frequency signals and amplifiers of radio frequency or radio frequency signals, together with the ability to operate as an oscillating detector for heterodyne reception.

The basis for the electrical specifications for transmitting tubes was an output of 5 watts when operating as a high-frequency oscillator. Other specifications called for 300 volts plate potential and values of filament voltage and current of 8 volts and 1.35 amperes. The latter values were arrived at after a considerable period of experimentation to determine the best possible values which could be assigned for these quantities.

The operation of the transmitting tube required performance as oscillators for the generation of high-frequency current for transmitting and as modulators for the control of this high-frequency current for radio telephony.

Production.—Prior to July, 1917, the production of vacuum tubes in the United States had never exceeded more than a few hundred tubes per week. This production had been accomplished on what was practically a laboratory hand-made basis. The problem of producing vacuum tubes at rates of several thousands per day was truly a stupendous one when viewed from the standpoint of the production situation at that time. The design of the two types of tubes required by the Signal Corps was put into the hands of the engineers of the three leading vacuum tube manufacturers of the country and sufficient initial production was authorized to enable the process of manufacture of the tubes to be worked out as the designs were perfected. The final specifications represent a compromise between the characteristics desired from an operational standpoint and the characteristics adapted to quantity production methods. In November, 1918, deliveries of Standard Signal Corps vacuum tubes were being made at a rate of more than one hundred thousand tubes per month.

The difficulties encountered by the manufacturers in undertaking the production of vacuum tubes were largely labor difficulties, on account of the fact there existed no personnel trained in the processes required for the quantity production of these tubes. The equipment required for machine production of the various parts of the tube, the pumps required for evacuation, and the extensive testing facilities required for insuring that the product came up to the standards prescribed, combined to make the production of vacuum tubes a problem of extreme difficulty. The greatest credit is due to the engineers of the commercial companies engaged in the production of these tubes for the ingenuity and enthusiasm with which they solved the problems incident to this work.

A glance at the specifications for acceptance of vacuum tubes would indicate a large number of tests to which a given vacuum tube is subjected before being considered an acceptable tube, but the development of suitable testing equipment and methods has resulted in the simplification of this problem so that the government inspector is enabled to inspect several thousand tubes per day without difficulty.

Performance.—The performance of the standard Signal Corps vacuum tubes in thousands of airplane and land radio sets is conclusive proof of the satisfactory character of these tubes for military purposes. As an example, it may be stated that data collected from many thousands of flights made by airplanes using the radio telephone set for voice-commanded squadron drills, indicates that there is much less trouble from the complete radio equipment than is experienced from the extremely efficient airplane engines. When we consider that the complete radio sets contain many other sources of trouble in addition to vacuum tubes, it will be realized that this is a remarkable tribute to the designers and manufacturers of the vacuum tube.

While predictions as to the future military policy of the United States are not in order at the present time, it can safely be said that the vacuum tube has taken its place as an essential part of military equipment and that the progress of its future development will be a matter of the greatest military importance to the country. Even more important, however, is the influence which the war-time development of this device will exert on the scientific and commercial progress of electrical engineering in the immediate future.

NEW BOOKS.

A Handbook of Physics Measurements. By E. S. Ferry in Collaboration with O. W. Silvey, G. W. Sherman, Jr., and D. C. Duncan. Two volumes. New York, John Wiley and Sons, Inc., 1918. Pp. ix + 251 and x + 233.

These two volumes contain the description of over one hundred experiments, affording ample selection for a laboratory course in general physics such as is given to engineering students. The authors have aimed "to furnish the student . . . with a self-contained manual of the theory and manipulation of those measurements in physics which bear most directly upon his subsequent work in other departments of study and upon his future professional career." For example, experiments in harmonic oscillations, both damped and undamped, suitable for electrical engineers, others such as combustion calorimeters and study of efficiency of pipe coverings, suitable for mechanical engineers, are given.

The experiments of each chapter are preceeded by rather full theoretical treatment which the student is expected to digest before he begins his experimental work. In their own laboratory, the authors expect the student to bring to the laboratory for the inspection of the instructor, a written analysis of the experiment with a discussion of the theory involved, before he starts the experimental work.

The first volume contains experiments in mechanics, the properties of matter, and optics; the second has experiments in vibratory motion, sound, heat, electricity, and magnetism.

In the second volume, calculus methods are used whenever in the opinion of the authors, its use would result in economy of time and effort, but even then only in a relatively small number of experiments.

The general plan of the books as well as the detailed working out of each topic shows that the authors have put a great deal of thought and effort into these manuals. O. M. S.

Sound, Light, Electricity and Magnetism. By W. B. Anderson. New York, McGraw-Hill Book Co., Inc., 1919. Pp. xiii + 794.

This volume is of similar scope and treatment to that on "Mechanics and Heat" by the same author, and is intended as part of a first course in college physics. The book is written primarily for engineering and agricultural students; the practical side of the subject is emphasized and there is included a well selected list of problems. Both author and publisher have done a most excellent piece of work. The text is very clear, paragraphs are numbered and headed and there are some 373 well-made illustrations.

The subject matter is perhaps somewhat more voluminous than is usually offered in a first college course, although there is provision for a shorter course by omitting judiciously chosen sections in finer print. The calculus is not used and the question might perhaps be raised whether it is not better to defer treatment of some of the more complex subjects and their applications until the student has acquired at least an elementary knowledge of this branch of mathematics.

G. K. B.

Second Series. *December, 1919.* *Vol. XIV., No. 6*

THE

PHYSICAL REVIEW.

THE OPTICAL CONSTANTS OF LIQUID ALLOYS.

By C. V. Kent.

Synopsis.

This paper describes a determination of the two optical constants of molten metals, the index of refraction, ν, and the index of absorption, κ, from the reflection of polarized light at molten-metal surfaces, placed in an inactive atmosphere.

The *purpose* of this experiment is to throw some light upon the behaviour of free electrons in metals.

Method.—The constants, ν and κ, are computed from the phase change Δ, and the azimuth change ψ, of polarized light reflected from the surface of the liquid metal at an angle of incidence ϕ. A Jamin circle is employed, the azimuth of the analyzer being determined by a bi-field of new design.

A furnace for melting the metals is made up by winding a few yards of nichrome wire on a hollow iron cone, thus maintaining a temperature of about 400° C. The whole furnace is operated in a water-cooled container through which a stream of hydrogen is made to flow continuously.

The metals employed are bismuth, cadmium, tin, lead and binary alloys of these. As a source of light, three strong rays from a quartz mercury arc are used.

Results.—A good optical surface is obtained, free from the scratches and impurities of polishing materials.

Values of $\nu^2 - \kappa^2$ and $2\nu\kappa$, are plotted as functions of the composition (atomic concentration) of each alloy. The parameters of the free electrons are also computed and plotted as functions of the composition of the alloy. These parameters are: (1) the product Ne^2/m, where the symbols have the meanings usually given in the electron theory, and (2) the frequency of impact of a free electron with the molecules of the metal.

From these optically determined free electron parameters, the electrical resistivities of the pure liquid metals and alloys are computed. In every case the agreement with the known electrical resistivity is within the limits of experimental error. A possible explanation of the disagreement between the values of resistivities for solid metals as computed from optical data and from direct electrical measurement, is suggested.

The results seem to indicate that the simple free electron theory of Drude and others is applicable to molten metals.

I. INTRODUCTION.

THE optical constants of metals (*i.e.*, the indices of refraction and absorption) are of some importance in the free electron theory. They afford a fairly direct method of obtaining a knowledge of the approximate magnitudes of some of the free electron parameters. However, the values of these optical constants as obtained experimentally for metals in the solid state, are unexpectedly unsatisfactory and inconsistent.

This is due to several causes. It is obvious that the necessary experimental data are most readily obtained by determining the changes produced in light when it is reflected from the metal to be examined; these changes being of intensity and phase. The accuracy of results is thus limited by the photometrical sensibility of the eye.

Furthermore, it is almost impossible to obtain plane surfaces which are *optically* pure, by ordinary grinding and polishing. Scratches and traces of abrasive agents, imperceptible to the naked eye, produce surfaces quite different from those of the pure metal. Minor,[1] Erochin,[2] Duncan[3] and others have minimized this source of error by using mirrors deposited on glass and examining the glass-metal inter-face. Their data show, however, that the optical constants vary with the method of producing the surfaces, whether by chemical processes, cathode deposition or casting. For these reasons, results obtained by different observers with the same metal, or even by the same observer with successive surfaces vary from 5 to 20 per cent.

The theoretical interpretation and correlation of data for different metals in the solid state is also open to objection. For example, the parameters controlling electron motion within the space lattice of a metallic crystal will be functions of direction of motion. It seems reasonable, therefore, to expect different values of the optical constants for different orientations. Thus, the optical constants of an extended solid metal surface are perhaps merely averages for the various crystals exposed.

In the course of an optical investigation of alloys, I decided to eliminate the above-named difficulties as far as possible, by restricting the work to liquid surfaces in an inactive atmosphere. By so doing, I could obtain surfaces which were perfectly smooth and practically free from contamination. Any small impurities would be either dissolved in the metal and be negligible in their effect, or, floating as solid aggregates, could be removed by mechanical means.

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[1] R. S. Minor, Ann. d. Phys., 10, 581, 1893.

[2] P. Erochin, Ann. d. Phys., 39, 213, 1912.

[3] R. W. and R. C. Duncan, PHYS. REV., 1, 294, 1913.

The investigation was further restricted to pure metals and bi-metallic alloys in which there were no compounds. In this simple case, the alloy may be considered from a theoretical point of view to consist of three mutually dissolved components: (1) The molecules of the first component metal and (2) of the second, each preserving the characteristics peculiar to the molecules of the pure metals, and (3) that portion to which metallic conduction is due. This portion is the free electrons common to all metals, and contributed in the case of the alloys here tested, by the two component metals.

II. Previous Work.

A few determinations of the optical constants of alloys have been made, most of them for the various kinds of speculum metal. Bernouilli[1] has worked with the binary mixtures Ag-Te, Ag-Sn, Cd-Hg, Cu-Sn, Cu-Zn and Cu-Ni; in each case, only a small portion of the latter component was present. Littleton[2] investigated the effect of the carbon content in various steels, and in another paper has taken up the systematic study of iron-nickel, nickel-silicon, iron-manganese, aluminum-copper, copper-nickel and iron-copper alloys. His efforts were chiefly directed to tracing the effects of the presence of compounds of the components present. Recently L. K. Oppittz[3] has studied the binary alloys of silver with copper and platinum.

Practically no results have been obtained for liquid metals with the exception of mercury. Drude mentions his apparently unsuccessful attempts to work with melted tin; he also gives some values of the optical indices for mercury alloyed with 2 per cent. tin, and for liquid Wood's metal. No other data for liquid metals has been found.

III. Method.

This is the familiar method employed in the investigation of opaque substances. Light incident upon the reflecting surface to be examined, is plane polarized in a plane at an angle of 45 degrees to the plane of incidence. The difference of phase between the two components of the reflected light is determined by a phase compensator which again plane polarizes the light passing through it. The azimuth of the plane of polarization of this light is determined by an analyzer.

Let the angle of incidence be ϕ, the phase difference introduced by reflection be Δ, and the azimuth of the plane of polarization of the reflected light with respect to the plane of incidence be ψ. In conformity with the terminology of many investigators in this subject, I have adopted the optical constants ν and κ, which are equal to Drude's n and nk respectively. The following convenient reduction formulæ may then be used to calculate ν and κ.

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[1] A. Bernouilli, Zeitschr. f. Elektrochemie, 15, 647, 1909.

[2] J. T. Littleton, Phys. Rev., 28, 306, 1912; 33, 453, 1912.

[3] L. K. Oppittz, Phys. Rev., 10, 156, 1917.

$$\cos 2P = \sin 2\psi \cos \Delta, \qquad \tan Q = \tan 2\psi \sin \Delta,$$

$$S = \sin \phi \tan \phi \tan P,$$

$$\nu^2 - \kappa^2 = S^2 \cos 2Q + \sin^2 \phi = A,$$

$$2\nu\kappa = S^2 \sin 2Q = B.$$

Hence

$$2\nu^2 = \sqrt{A^2 + B^2} + A,$$

$$2\kappa^2 = \sqrt{A^2 + B^2} - A,$$

ν, κ, and the reflecting power at perpendicular incidence may then be found.

IV. DESCRIPTION OF APPARATUS.

Monochromatic Illuminator.—A Cooper-Hewitt quartz tube mercury arc was used as a source of illumination. Its light was separated into its component colors by a Wadsworth prism-mirror train. The three intense lines, 404 $\mu\mu$, 546 $\mu\mu$ and 578 $\mu\mu$ (violet, green and yellow) were used. The colored slit image of the slit in the Wadsworth train covered the collimator slit, the collimator lens being completely filled at all times with the light employed.

Spectroscope and Polarization Train.—The spectroscope was a "Cercle de Jamin" from the Société Genevoise and is evidently intended for this sort of work. As liquid surfaces were used the circle was turned into a vertical plane. The verniers of the telescope and collimator gave their position on the spectroscope circle to 30″.

The polarizer and analyzer were mounted on the objective lens end of the collimator and telescope respectively. The nicols were of the Glan-Thompson type with about 1 cm. aperture. They were perfect in action. Their divided circles could be ready by verniers to 6′.

The compensator was mounted between the polarizer and the reflecting surface. It was of the Soleil-Babinet type; the micrometer head of the screw moving the quartz wedge was divided into 100 parts, each corresponding roughly to about 1/2 degree. Thus by estimating to tenths of a division, an accuracy of about 3′ could be obtained.

The writer's experience with this compensator is identical with that of R. W. and R. G. Duncan, in that it is impossible to obtain uniform darkness over the whole field of view. With the compensator and analyzer adjusted for minimum illumination, if the telescope eyepiece was removed, the field of view presented a blotched or striated appearance. With the eyepiece in place, the collimator slit image was traversed by a dark and fairly narrow diagonal band, which shifted position very rapidly with slight changes in the compensator setting. It is probable that this defect in the action of the compensator is due to some irregularity of crystal structure in the quartz plates.

In order to use the compensator for its purpose the practice was adopted of taking all compensator readings when the center of the black band lay under the intersection of the telescope crosshairs. This was done both in the preliminary calibration of the compensator, *i.e.*, in the determination of the phase shift per division on the compensator micrometer head for each color of light used, and in the regular experimental work. In this latter work, what was measured was the phase shift necessary to bring the diagonal band back under the cross hairs when it was shifted by reflection. From the results of a number of preliminary experiments the writer is convinced that no errors were introduced by this method of using the compensator.

Bi-field.—Total extinction of the light after it was reflected could never be produced in this experimental work by any adjustments of compensator and analyzer. This was due to the following causes: the imperfection of the compensator described above; films of moisture or metal on the observation windows of the housing of the heater in which the metal was melted; sources of scattered light such as dust particles, etc., in the optical train between the nicol prisms. An azimuth bi-field was therefore absolutely necessary for determining the azimuth position of the analyzer.

An entirely new form of bi-field was invented by the writer for this purpose in 1913. It was subsequently described by Dr. F. E. Wright,[1] of the Geophysical Laboratory. As the mounting and use by the writer is quite different from the adaptation of Dr. Wright, a brief description will be given.

The bi-field (see Fig. 1) consists of two slips of glass mounted on, and rotating with the analyzer. They are situated, of course, in the beam of light from the polarizer to the analyzer. Satisfactory slips are easily cut from a good strain-free microscope slide. The slips abut along a common edge and are inclined to each other at a somewhat obtuse angle, perhaps 120 degrees. Their common edge is in (or perpendicular to) the plane of polarization of the analyzer and their faces make equal angles with the direction of the beam of light passing through them and the analyzer. The common edge is also inclined at an angle with the direction of the beam of light; the inclination is variable in order to change the sensibility of the bi-field.

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[1] F. E. Wright, Wash. J. Sci., 4, 309, 1914.

When the bi-field is in use, a short focus telescope is substituted for the eyepiece of the spectroscope telescope and focused upon the common edge of the glass slips. The field of view is thus divided in halves by this edge. The "crossed" or extinction position of the analyzer is denoted by equality of brightness of the halves of the field. The arrangement acts exactly as the usual quartz bi-field of variable sensitivity.

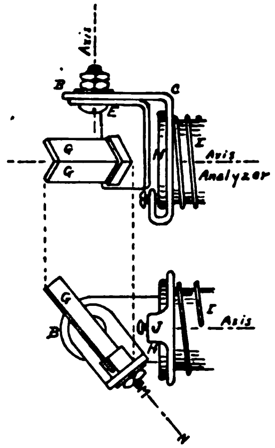

Fig. 1.

The dividing line between the halves of the field is practically eliminated (1) by mounting the glass slips with the exterior obtuse angle toward the incoming beam of light, and (2) by grinding the adjacent edges of the slips or plates to a rough fit. The interior edge of one plate is finished very sharply and overlaps the other. Thus this finely finished edge serves as a common dividing line and is practically invisible.

Multiply reflected light is present in the field of view except in a narrow band on each side of the dividing line. As the sensitivity is much reduced by this multiply reflected light, only the narrow bands are observed when obtaining the position of equal brightness.

In order to properly adjust the bi-field to the position described above it is mounted as shown in the accompanying figure. (Fig. 1).

The theory of its action is simple. Consider plane-polarized light incident upon one of the glass plates and resolve it into its two components in, and perpendicular to, the plane of incidence. By transmission these are unequally diminished with no phase difference introduced, according to the ordinary Fresnel laws. After transmission these again combine to form light of slightly diminished intensity, and on account of the inequality of diminution, plane-polarized in a slightly different plane from that of the incident light. In other words the light has been slightly diminished in intensity and *rotated through a small angle.* The light passing through the other plate will be likewise decreased in intensity and its plane of polarization also rotated, but in the opposite direction. If, now, the analyzer is in the "crossed" or total extinction position, the two glass plates are symmetrically situated, both with respect to the analyzer and the plane of polarization of the incident light.

In this case the lights passing through the two plates are equal in brightness and their planes of polarization equally rotated, but in opposite directions, with respect to the plane of polarization of the analyzing nicol. Therefore when they are observed through the analyzer the two halves of the field will be of equal brightness. When the analyzer (and glass plates) are in any other position, equality of brightness will not obtain.

The action of this bi-field is the same when it is mounted on the polarizer instead of the analyzer, and in the same position with respect to the plane of polarization of this nicol. The common edge may also be in a plane perpendicular to the plane of polarization of the nicol on which it is fastened. Owing to the mounting used, all adjustments of the bi-field for its proper action can be made optically by simple methods which need not be described here. The angle between the planes of polarization of the beams of light passing through the two plates, can be varied from 0 to about 12 degrees.

The arrangement described seems to have all the advantages of the quartz bi-field of variable sensibility. In addition its cost is insignificant and it is very easily constructed.

V. Furnace and Container.

These were mounted on a heavy laboratory tripod beside the spectroscope circle. They could be easily swung out from between the collimator and telescope at any time for adjustment, etc.

The necessary details of the heater and container for the hydrogen atmosphere above the liquid metal may be seen from the accompanying diagram (Fig. 2).

The heater, *Q*, consisted of about 8 m. of no. 28 nichrome wire imbedded in an asbestos water-glass cement. The liquid metal was held in the iron cone *Q*, whose surface was rubbed with steatite to prevent adherence. *S* is a protecting asbestos rim; an overhanging roof protects the binding posts *R*, from spilled metal. Wires from the heating coil are led outside the container through the tube *D*, in the base. The asbestos bottom of the heater is cut away over the exit tube *E* to allow for egress of detritus and samples of the metal used.

The container was made in two parts, both water-cooled. When in use these were cemented together by soft wax. The base of the container consisted of a sheet metal cylinder 12 cm. in diameter and 2 cm. high. The tubes *A* and *B* in this, are for the ingress and egress of water, *D* for the heater wires, and *E* for the dumping of detritus and samples into a beaker of water below, into which it projected. Hydrogen, which flowed slowly through the apparatus during observations, bubbled out through this tube.

The upper portion of the container was made of two concentric sheet iron cylinders soldered together at the bottom edge. The space between these was utilized for cooling water, the outer cylinder being open at the top. *F* and *G* are two short tubes to which the observation windows (two strain-free microscope slides) were fastened with soft wax. Light to and from the reflecting liquid metal surface passed through these plates practically perpendicularly. Deviations of a degree or so from this perpendicularity were found to produce no change in the analyzer settings.

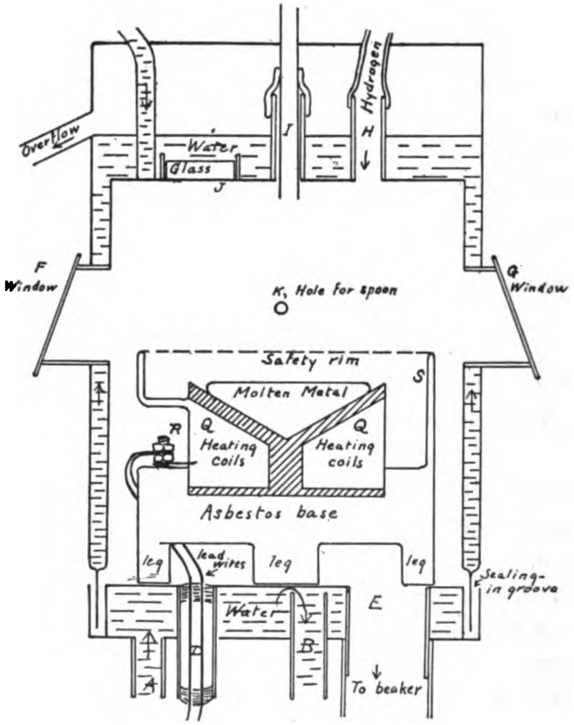

Fig. 2.

Tube *H* at the top of the container, was connected with a Kipp hydrogen generator. A glass tube slipped through tube *I*, the sliding joint being rendered gas tight by a gum rubber collar. This tube was used for the introduction of new metal when changing the composition of the alloy under observation. For this purpose it was pushed down until the lower end was immersed beneath the surface of the liquid metal. The component metal to be added was poured down through it in granular form, the tube corked and lifted up to its former position. Thus the interior of the container was at no time open to the air. *J* is an additional glass window, for the general illumination of the interior of the container, if it should be needed.

At *K*, the sides of the inner and outer container walls were pressed together and soldered, and a small hole bored through both. A short brass collar was soldered over the hole on the outside. Over this was fastened a rubber finger stall. The handle of the "skimming spoon" (to be described later) was passed through this hole into the finger stall and could be manipulated from the outside.

It was impossible to obtain a clear liquid surface when the metal was first introduced into the heater and melted. The surface was therefore freed from oxide and other coarse impurities originally present by skimming with a specially formed spoon, whose handle projected into the finger stall on the outside, as previously described. The bowl of the spoon was a sheet iron cone of practically the same shape and dimensions as the cone in which the metal was melted. A hole 1 mm. in diameter was bored at the bottom point of this cone. The stiff wire handle which passed through *K* was covered with a short piece of clay pipe stem within the finger stall. The spoon was freely movable from outside, over, and in the molten metal. It was used much like the old fashioned cream skimmer; when it was dipped into the metal and raised, the clean metal ran back through the hole, leaving the gross impurities in the spoon; these were then dumped down the exit tube *E*.

VI. Preliminary Adjustment of the Optical System.

The ordinary adjustments of the spectroscope need no description. It should be stated, however, that as the horizontal plane of the liquid reflecting surface was fixed, the optic axis of the spectroscope circle had to be brought into coincidence with it, by means of the leveling screws on the base of the instrument.

Orientation of the Plane of the Polarizer.—The ordinary means of finding this by means of reflection from a plane glass surface was found to be too inaccurate. A method was therefore devised which was quite satisfactory and is believed to be new. The compensator was removed from the instrument and the light from the polarizer, reflected from a clean pool of mercury (or other good metal surface), was examined by the analyzer. Both nicols were rotated alternately until total extinction was secured. In this case, the light coming from the metal mirror to the analyzer must have been plane polarized. Metal surfaces, however, always produce a difference of phase between the components of the reflected light in, and perpendicular to, the plane of polarization. It is evident therefore that one of these must have been absent in the reflected and also the incident light. It is therefore also evident that the planes of polarization of the two nicols must likewise be in and perpendicular respectively to the plane of incidence of the light. In which of the two planes the light from the polarizer is polarized, can be easily determined if necessary by various simple methods which need not be discussed here.

Successive positions of the polarizer as obtained by this method always agreed within the least count of the polarizer circle, *i.e.*, 6′.

The zero phase difference readings of the compensator were found to be very slightly different for the three colors used. This instrument was also found to possess a small temperature coefficient which was negligible, however, in comparison with other sources of error.

Adjustment of Bi-Field.—This is entirely optical. With the compensator removed, the analyzer and polarizer were "crossed." The common edge of the bi-plate was set approximately perpendicular to the axis of the beam of light and the bi-plane turned until its faces made *unequal* angles with the beam. The whole bi-plate mounting was then rotated on the cap of the nicol holder until total extinction of the beam was produced *through* the plates. It should be stated, that for this work the short focus telescope previously mentioned, was used as eyepiece. The common edge now lay in the plane of polarization of the analyzer, and the mounting was therefore clamped to the cap. Next, the common edge being inclined 10° to 20° to the direction of the beam of light, the bi-plate was rotated around this edge until the intensity of the halves of the field was the same. The plates now made equal angles with the beam of light, and the bi-plate bolt was clamped to the arm on which it was supported.

VII. Experimental Procedure and Calculations.

Preparing the Surface.—After the metal had been placed in the heater and the container sealed over it, hydrogen was run through the apparatus gently for about half an hour to wash out the air. The circulation of the cooling water was then started and the furnace current turned on. As no change of the optical constants of the metals with temperature could be detected, the temperature was merely kept well above the melting point, at about 400° C.

After the first fusion, the metal was cleared of dross with the skimming spoon. Traces of oxygen from the surrounding atmosphere or surfaces in the container, seemed to act on the newly skimmed surfaces until completely exhausted. When the surface was quite clean it became extremely mobile with minute pieces of oxide floating here and there. At this stage the container was swung into place beside the spectroscope circle and raised to the proper height, and the compensator and analyzer were roughly adjusted for minimum intensity of illumination. The surface was then examined without the telescope eyepiece. Surface impurities showed prominently as bright flecks or long streaks and were removed as far as possible by skimming. Those ultimately remaining usually travelled to the convex edge of the liquid as previously noted by Bidwell with similar surfaces. At the higher temperatures, actual reduction of the final oxide particles seemed to take place.

Readings.—These usually agreed within the limits of experimental error when made upon freshly cleaned surfaces. The surfaces of many alloys showed a progressive decrease of phase when left standing, although the analyzer readings were practically unaffected. It was therefore desirable to take the readings as rapidly as possible and the following order of taking observations was accordingly adopted:

1. The polarizer was set with its plane of polarization at 45° with the plane of incidence. Using the short focus telescope and bi-field, the analyzer was then accurately "crossed" with the reflected beam of light. Using the regular telescope eyepiece, the compensator was adjusted until the dark band in the slit image lay under the telescope cross hairs. Ten such compensator readings were taken for each of the three colors used.
2. The polarizer was turned through 90° and the compensator readings of 1 above repeated.
3. Using the bi-field and the short focus telescope as an eyepiece, five readings of the analyzer for a photometrically balanced field, for each of the three colors used, were taken.
4. The polarizer was turned back through 90° and 3 repeated.
5. 1 was repeated with yellow light to test possible change in the surface.

Owing to the perfection of the nicols and their divided circles it was found to be unnecessary to turn either polarizer or analyzer to positions approximately 180° from those already used. For positions of the polarizer 90° apart, the mean compensator readings were found to be consistently different by an amount varying from a few tenths to about 1° of phase difference. The cause of this consistent difference is unknown, but undoubtedly lies in the imperfections of the compensator.

Calculation of Optical Constants.—Mean values were taken in all calculations. The determinations of the angle of incidence and the phase difference indicated by the compensator are sufficiently obvious. The difference between the two mean analyzer positions for the two polarizer azimuths of 3 and 4 above was taken as 2ψ. It may have been the complement of 2ψ but as the substitution of this complement in the reduction formula used to calculate the optical constants merely produced a change of sign in the values of these constants, this uncertainty gave no trouble. For metals, $\nu^2 - \kappa^2$ is always negative, and $2\nu\kappa$ positive. The reflecting power at perpendicular incidence was calculated from the formula[1]

$$R = \frac{\nu^2 + \kappa^2 + 1 - 2\nu}{\nu^2 + \kappa^2 + 1 + 2\nu}.$$

VIII. Results.

The alloys used were bismuth-lead, cadmium-lead, tin-lead and bismuth-mercury. The metals were obtained from the chemical laboratory and were pure.

1. *Bismuth-lead.*—These were very satisfactory from an optical standpoint. After skimming well, the alloy surfaces remained clean indefinitely, as the hydrogen atmosphere seemed to reduce any oxide flecks remaining.

2. *Cadmium-lead.*—The cadmium evaporated quite freely from these alloys and formed bright mirrors on the observation windows in the course of an hour or so. The depolarizing effect of the films made the compensator band faint and hazy and gave the field of the bi-plate a mottled appearance, but very good readings could be taken when the mirrors were so opaque that no details of the alloy surfaces could be seen.

3. *Tin-lead.*—Work with these alloys was extremely difficult. A few seconds after the formation of a new and clean surface, fine lines of what seemed to be minute bubbles appeared and spread until the whole surface was covered with a viscous scum, probably oxide. This formation was never entirely prevented, even when the atmosphere was practically pure hydrogen. It is probably due to oxygen originally in solution in the tin. On this account, the surface of the pure tin (or of alloys rich in this component) was renewed after every two or three readings. These readings were reproducible and the phase difference indicated by the compensator was a maximum.

4. *Bismuth-mercury.*—The accuracy of results with these alloys is relatively poor. This is due to the condensation of mercury on the observation windows, and the low temperatures necessarily employed, which prevented any reducing effect by the hydrogen.

Composition of the Alloys.—As an accuracy of more than one per cent. was not necessary, simple gravimetric determinations of the composition were made. The samples obtained from each alloy were dissolved in nitric acid. Lead was estimated as sulphate in each case, bismuth as tetroxide, cadmium as nitrate, and tin as oxide. No chemical analysis of the bismuth-mercury alloy was made. In the following tables the alloys will be defined by the atomic per cent. of one component.

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[1] Houston, A Treatise on Light, p. 417.

Tables.—The experimentally determined values of the angle of incidence ϕ, twice the azimuth angle of the reflected light (2ψ), and the phase difference, Δ, are given for each of the alloys examined. From these are calculated the optical constants $\nu^2 - \kappa^2$, $2\nu\kappa$, ν, κ and R the reflecting power at perpendicular incidence.

Table I.

Bismuth-lead Alloys.

Alloy.		ϕ.	2ψ.	Δ.	$-(\nu^2-\kappa^2)$.	$2\nu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
				579μμ.					
	A	70° 32′	63° 47′	125° 48′	9.64	20.27			
Bismuth (pure)	A	70° 36′	63° 37′	124° 41′	9.36	19.47			
	B	70° 35′	64° 3′	124° 41′	9.77	19.72			
Mean values					9.62	19.72	2.48	3.97	64.4
6 per cent. lead	C	70° 35′	64° 36′	126° 9′	10.52	20.53			
	D	70° 35′	63° 58′	124° 38′	9.69	19.40			
Mean values					10.10	19.97	2.52	4.00	64.5
19 per cent. lead	E	70° 33′	64° 6′	124° 44′	9.79	19.37			
	F	70° 33′	64° 30′	126° 50′	10.51	21.06			
Mean values					10.15	20.21	2.48	4.03	65.0
26.4 per cent. lead	I	70° 33′	65° 11′	126° 23′	11.08	20.52	2.57	4.09	65.2
48 per cent. lead	J	70° 33′	65° 16′	126° 14′	11.64	21.24			
	K	70° 33′	65° 36′	127° 17′	11.67	21.22			
Mean values					11.66	21.23	2.56	4.20	65.7
60 per cent. lead	M	70° 33′	66° 14′	127° 1′	12.24	20.85			
	N	70° 33′	65° 54′	128° 17′	12.19	22.10			
Mean values					12.22	21.47	2.55	4.26	66.8
75 per cent. lead	O	70° 33′	67° 28′	128° 48′	14.03	22.17			
	P	70° 33′	67° 12′	128° 14′	13.58	21.70			
Mean values					13.81	21.94	2.62	4.37	67.4
86 per cent. lead	Q	70° 33′	67° 23′	128° 53′	13.96	22.26	2.50	4.48	69.0
Lead (pure)	R	70° 37′	67° 46′	129° 17′	14.63	22.74			
	S	70° 36′	68° 10′	129° 17′	15.04	22.48			
Mean values					14.84	22.61	2.56	4.53	69.2

Alloy.		ϕ.	2ψ.	Δ.	$-(\nu^2-\kappa^2)$.	$2\nu\kappa$.	ν.	κ.	Refl. Power Per Cent.
546μμ.									
Bismuth (pure)	A	70° 32′	63° 53′	123° 36′	9.39	18.51			
	A	70° 36′	63° 47′	123° 14′	9.25	18.26			
	B	70° 35′	63° 43′	121° 56′	8.96	17.34			
Mean values.......	....	....	....	....	9.20	18.04	2.35	3.84	63.8
6 per cent. lead.....	C	70° 35′	64° 3′	123° 41′	9.54	18.53			
	D	70° 35′	64° 4′	122° 46′	9.39	17.87			
Mean values.......	....	....	....	....	9.47	18.20	2.35	3.87	64.1
19 per cent. lead....	E	70° 33′	64° 2′	123° 8′	9.42	18.13	2.35	3.86	64.1
26.4 per cent. lead..	H	70° 33′	65°	125° 58′	10.80	20.17			
	I	70° 33′	65° 9′	125° 46′	10.90	19.97			
Mean values.......	....	....	....	....	10.85	20.07	2.44	4.10	65.9
48 per cent. lead....	J	70° 33′	65° 26′	124° 4′	10.78	18.54			
	K	70° 33′	65° 42′	122° 2′	10.51	16.94			
Mean values.......	....	....	....	....	10.65	17.74	2.24	3.96	65.2
61 per cent. lead....	L	70° 33′	66° 28′	124° 14′	11.69	18.36			
	M	70° 33′	66° 25′	126° 6′	12.13	19.97			
	N	70° 33′	66° 12′	126° 24′	12.05	20.25			
Mean values.......	....	....	....	....	11.96	19.53	2.34	4.18	67.3
75 per cent. lead ...	O	70° 33′	67° 29′	126° 24′	13.30	19.94			
	P	70° 33′	66° 56′	126° 0′	12.66	19.73			
Mean values.......	....	....	....	....	12.98	19.84	2.32	4.28	68.4
86 per cent. lead....	Q	70° 33′	67° 40′	127° 0′	13.65	20.33	2.33	4.37	66.1
Lead (pure)........	R	70° 36′	68° 18′	127° 29′	14.57	20.71			
	S	70° 37′	68° 18′	127° 23′	14.56	20.65			
Mean values.......	....	....	....	....	14.56	20.68	2.42	4.46	69.5
404μμ.									
Bi...............	A	70° 32′	63° 43′	111° 48′	7.06	11.80	1.80	3.23	60.6
Pure.............									
6 per cent. lead.....	D	70° 35′	62° 50′	110° 54′	6.62	11.66	1.85	3.15	69.0
18 per cent. lead....	E	70° 33′	63° 49′	111° 54′	7.09	11.77	1.82	3.23	60.3
26.4 per cent. lead..	I	70° 33′	65° 24′	111° 54′	7.85	11.40	1.73	3.29	62.1
48 per cent. lead....	J	70° 33′	66° 1′	112° 24′	8.31	11.52			
	K	70° 33′	66° 13′	113° 54′	8.79	12.15			
Mean values.......	....	....	....	....	8.55	11.84	1.74	3.40	63.5

Alloy.		ϕ.	2ψ.	Δ.	$-(\nu^2-\kappa^2)$	$2\nu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
				579$\mu\mu$.					
61 per cent. lead....	L	70° 33′	66° 56′	112° 36′	8.83	11.33			
	M	70° 33′	66° 41′	114° 6′	9.08	12.11			
	N	70° 33′	65° 32′	114° 30′	8.57	12.61			
Mean values.......	....	....	....	....	8.83	12.02	1.74	3.45	64.0
75 per cent. lead....	O	70° 33′	65° 32′	115° 18′	8.77	13.02			
	P	70° 33′	67° 40′	114° 48′	9.78	12.15			
Mean values.......	....	....	....	....	9.28	12.59	1.78	3.53	64.6
86 per cent. lead....	Q	70^b 33′	68° 30′	115° 6′	10.30	12.01	1.66	3.61	67.0
Pure lead..........	R	70° 36′	69° 6′	116° 18′	11.11	12.48			
	S	70° 37′	69° 54′	115° 34′	11.23	11.74			
Mean values.......	....	....	....	....	11.17	12.11	1.63	3.72	68.5

Table II.

Cadmium-lead Alloys.

Alloy.		ϕ.	2ψ.	Δ.	$-(\nu^2-\kappa^2)$	$2\nu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
				579 $\mu\mu$.					
Cadmium (pure) ...	A	70° 37′	82° 38′	126° 36′	25.66	8.78			
	B	70° 37′	83° 56′	127° 0′	26.74	7.49			
	C	70° 37′	82° 8′	127° 6′	25.99	9.61			
	D	70° 37′	82° 56′	126° 0′	25.19	8.19			
Mean values.......	....	....	....	....	25.72	8.15	0.82	5.14	89.8
12 per cent. lead....	F	70° 37′	81° 22′	126° 42′	25.14	10.23	1.00	5.11	86.7
22 per cent. lead....	H	70° 36′	77° 23′	127° 42′	23.30	14.88			
	I	70° 36′	77° 54′	127° 0′	23.06	13.86			
Mean values.......	....	....	....	....	23.19	14.37	1.43	5.02	81.6
33.5 per cent. lead..	J	70° 36′	75° 43′	128° 24′	22.46	16.96	1.69	5.03	79.3
52.5 per cent. lead..	K	70° 36′	73° 53′	128° 12′	20.56	18.24	1.86	4.90	76.2
64 per cent. lead....	L	70° 36′	72° 26′	127° 30′	18.98	19.03	1.99	4.79	75.0
76 per cent. lead....	M	70° 36′	69° 58′	127° 54′	16.44	20.36	2.21	4.62	72.0
79.5 per cent. lead..	N	70° 36′	69° 40′	128° 15′	15.59	20.00	2.21	4.53	71.2
Pure lead..........	....	....	....	....	14.84	22.61	2.56	4.53	69.2

Alloy.		ϕ.	2ψ.	Δ.	$-(\lambda-\pi)$	$2\nu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
				546 $\mu\mu$.					
Cadmium (pure)....	A	70° 37′	82° 18′	123° 24′	22.32	7.32			
	B	70° 37′	83° 56′	124° 0′	23.52	6.33			
	C	70° 37′	82° 11′	124° 10′	22.81	8.07			
	D	70° 37′	82° 40′	124° 12′	23.20	7.68			
Mean values.......	....	....	....	... ...	23.16	7.45	0.76	4.87	88.6
12 per cent. lead....	F	70° 37′	81° 24′	123° 36′	22.09	8.64	0.90	4.79	86.4
22 per cent. lead....	H	70° 36′	77° 18′	125° 16′	21.14	13.16			
	I	70° 36′	77° 54′	125° 40′	21.02	12.57			
Mean values.......	....	....	....	....	21.08	12.87	1.14	4.78	83.4
33.5 per cent. lead..	J	70° 36′	75° 48′	126° 36′	21.07	15.38	1.58	4.86	79.0
52.5 per cent. lead..	K	70° 36′	74° 4′	126° 4′	19.25	16.25	1.72	4.71	76.7
64 per cent. lead....	L	70° 36′	72° 13′	125° 17′	16.17	17.11	1.92	4.46	72.2
76 per cent. lead....	M	70° 36′	70° 10′	125° 37′	15.52	18.14	2.04	4.44	71.1
79.5 per cent.......	N	70° 36′	70° 4′	125° 29′	15.38	18.10	2.05	4.42	71.6
Pure lead..........	....	....	....	....	14.56	20.68	2.42	4.46	69.5
				404 $\mu\mu$.					
Cadmium (pure)....	A	70° 37′	82° 12′	109° 6′	12.62	4.00			
	B	70° 37′	85° 23′	107° 18′	12.16	2.22			
	C	70° 37′	82° 11′	109° 27′	12.79	4.07			
Mean values.......	....	....	....	....	12.52	2.76	0.39	3.56	93.1
12 per cent. lead....	E	70° 37′	82° 25′	108° 6′	12.17	3.73			
	F	70° 37′	82° 12′	109° 54′	13.01	4.14			
Mean values.......	....	....	....	....	12.59	3.94	0.55	3.59	85.6
22 per cent. lead....	H	70° 36′	78° 28′	111° 48′	13.11	6.37			
	I	70° 36′	80° 0′	112° 6′	13.63	5.73			
Mean values.......	....	....	....	....	13.37	6.05	0.81	3.74	81.4
33.5 per cent. lead..	J	70° 36′	77° 0′	113° 42′	13.57	7.78	1.02	3.82	78.2
52.5 per cent. lead..	K	70° 36′	75° 5′	113° 15′	12.65	8.58	1.15	3.74	75.2
64 per cent. lead....	L	70° 36′	73° 10′	112° 51′	11.74	9.30	1.27	3.66	72.4
76 per cent. lead....	M	70° 36′	70° 40′	113° 6′	9.56	10.47	1.52	3.45	66.6
79.5 per cent. lead..	N	70° 36′	70° 2′	113° 48′	10.71	10.94	1.52	3.61	68.6
Pure lead..........	....	....	....	....	11.17	12.11	1.63	3.72	68.5

Table III.

Tin-lead Alloys.

Alloy.		**ϕ.**	**2ψ.**	**Δ.**	**$-(\nu^2-\kappa^2)$**	**$2\nu\kappa$.**	**ν.**	**κ.**	**Refl. Power, Per Cent.**
				579$\mu\mu$.					
Tin, pure..........	A	70° 33′	76° 38′	131° 40′	26.24	19.21			
	H	70° 41′	76° 55′	131° 54′	27.24	19.47			
Mean values.......	....	....	....	....	26.74	19.34	1.77	5.47	81.1
7 per cent. lead.....	I	70° 41′	77° 22′	131° 3′	24.68	20.12	1.89	5.32	79.3
21 per cent. lead....	B	70° 33′	75° 40′	131° 48′	25.28	20.32	1.89	5.37	79.7
37.5 per cent. lead..	C	70° 33′	73° 38′	130° 30′	21.83	20.62	2.03	5.09	76.9
62 per cent. lead....	D	70° 33′	71° 18′	129° 37′	18.67	21.28	2.20	4.85	73.9
85.5 per cent. lead..	E	70° 36′	70° 47′	129° 30′	17.86	21.46	2.24	4.78	73.1
Pure lead..........	....	.....—	....	....	14.84	22.61	2.56	4.53	69.2
				546$\mu\mu$.					
Tin, pure..........	A	70° 33′	76° 4′	131° 27′	25.38	19.53			
	H	70° 41′	77° 13′	130° 7′	25.68	17.32			
Mean values.......	....	....	....	....	25.53	18.43	1.73	5.34	80.8
7 per cent. lead.....	I	70° 41′	76° 22′	129° 11′	24.00	17.27	1.70	5.18	80.3
21 per cent. lead....	B	70° 33′	74° 47′	127° 53′	22.15	18.48	1.83	5.05	78.1
37.5 per cent. lead..	C	70° 33′	73° 10′	127° 53′	19.54	18.36	1.91	4.81	75.9
62 per cent. lead....	D	70° 33′	71° 18′	127° 20′	17.17	18.78	2.04	4.62	73.3
85.5 per cent. lead..	E	70° 36′	71° 24′	126° 36′	17.20	18.40	2.00	4.60	73.5
Pure lead..........	....	....	....	....	14.56	20.68	2.42	4.46	69.5
				404$\mu\mu$.					
Tin, pure..........	A	70° 33′	76° 41′	116° 53′	15.03	9.09			
	H	70° 41′	75° 29′	117° 13′	14.93	10.07			
Mean values.......	....	....	....	....	10.98	9.58	1.18	4.05	77.6
7 per cent. lead.....	I	70° 41′	74° 58′	116° 13′	14.19	9.92	1.25	3.97	76.0
21 per cent. lead....	B	70° 33′	76° 32′	117° 42′	15.42	9.52	1.16	4.10	78.3
37.5 per cent. lead..	C	70° 33′	74° 44′	116° 42′	13.22	9.51	1.24	3.84	74.9
85.5 per cent. lead..	F	70° 36′	72° 7′	115° 30′	12.37	11.11	1.46	3.81	71.5
Pure lead..........	....	....	....	....	11.17	12.11	1.63	3.73	68.5

TABLE IV.

Bismuth-mercury Alloys.

Alloy.	ϕ.	2ψ.	Δ.	$-(\nu^2-\kappa^2)$	$2\nu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
				579μμ.				
Bismuth, pure..........	....	....	....	9.70	19.72	2.48	3.97	64.4
Bi-Hg (40 per cent. Hg?)	70° 36′	64° 36′	124° 6′	10.32	19.11	2.38	4.00	67.7
Mercury, pure (Meier's results)...........	....	....	....	16.7	14.2	1.62	4.39	75.3
				546μμ.				
Bismuth, pure..........	....	....	....	9.20	18.04	2.35	3.84	63.8
Bi-Hg (40 per cent. Hg?)	70° 36′	65° 40′	122° 54′	10.92	17.91	2.24	3.99	66.1
Mercury, pure (Meier's results)...........	....	....	....	15.2	12.3	1.48	4.17	74.9
				404μμ.				
Bismuth, pure..........	....	....	....	7.06	11.80	1.80	3.23	60.6
Bi-Hg (40 per cent. Hg?)	70° 36′	67° 2′	110° 3′	8.33	10.44	1.59	3.29	63.6
Mercury, pure (Meier's results)...........	....	....	....	9.2	5.8	0.92	3.17	73.3

TABLE V.

Ne^2/m and *W*.

Bismuth-lead Alloys.

	Per Cent. Lead.	0.	6.	19.	26.4.	48.	61.	75.	86.	100.
$Ne^2/m\times$ const.......	579 μμ	131	132	134	133	137	137	137	139	143
	546 μμ	133	133	132	146?	124?	137	137	137	142
	404 μμ	148	147	149	141	146	144	152	147	149
$W\times$ const.	579 μμ	281	274	276	261	259	252	232	234	223
	546 μμ	283	278	279	276	247	247	235	233	223
	404 μμ	308	323	309	275	268	265	267	234	220

Cadmium-lead Alloys.

	Per Cent. Lead.	0.	12.	22.	33.5.	52.5.	64.	76.	79.5.	100.
$Ne^2/m\times$ const.......	579 μμ	92	94	101	105	112	114	122	120	140
	546 μμ	93	93	103	115	114	113	122	121	141
	404 μμ	95	99	115	122	123	125	129	137	149
$W\times$ const..........	579 μμ	52	64	97	105	123	152	186	121	223
	546 μμ	51	65	100	123	137	156	184	186	223
	404 μμ	46	64	92	110	140	162	214	204	229

Tin-lead Alloys.

	Per Cent. Lead.	0.	7.	21.	37.5.	62.	85.5.	100.		
Ne^2/m × const.	579 $\mu\mu$	125	123	127	125	127	125	143		
	546 $\mu\mu$	134	127	129	125	126	124	141		
	404 $\mu\mu$	136	150	132	150	——	140	150		
W × const.	579 $\mu\mu$	114	123	126	147	173	179	223		
	546 $\mu\mu$	120	119	138	151	175	171	223		
	404 $\mu\mu$	136	150	132	150	——	182	220		

Bismuth-mercury Alloys.

	$\frac{Ne^2}{m}$ × const.			W × const.		
	Bismuth.	*Bi—Hg.*	Mercury.	Bismuth.	*Bi—Hg.*	Mercury.
579 $\mu\mu$......	131	127	92	281	257	135
546 $\mu\mu$......	133	125	88	283	245	127
404 $\mu\mu$......	148	129	89	308	242	123

The observational errors inherent in katoptric methods give possible variations of 3 or 4 per cent. in the values of ν and κ, and correspondingly larger variations in $\nu^2 - \kappa^2$ and $2\nu\kappa$. These limits suppose perfect metal surfaces. From an inspection of the values obtained by the writer for different surfaces of the same liquid surface it will be seen that this limit is practically attained. As observations upon different surfaces of solid metals of the same composition differ among themselves by 10 to 30 per cent. in some cases, even for the same observer, the experimental advantages of using liquid surfaces are manifest. There are no scratches or surface impurities due to abrasive material, no differential polishing of the components of an alloy, and no question of variation of surface due to heat treatment. The surfaces are always plane and can be renewed at pleasure.

The determinations made with violet light (404 $\mu\mu$) were exceedingly difficult to make on account of eye-fatigue. The values of the optical constants for this light are therefore regarded merely as a check on the other determinations.

IX. Discussion of Results.

Interpretation of the above results is most conveniently made on the simple free electron theory of Drude. His conception of the free electrons ricochetting from molecule to molecule under the influence of an outside applied electromagnetic field is inadequate, especially in the light of the more recent developments of the electron theory. Nevertheless, his classic treatment is capable of explaining and coordinating metallic phenomena qualitatively with surprising success.

Drude's[1] equations for the optical constants of the metals are as follows:

$$n^2(1-k^2) = 1 + \Sigma\frac{\theta_h'}{1-\left(\frac{\tau_h}{\tau}\right)^2} - 4\pi\,\Sigma\frac{m'N}{r^2+\left(\frac{m'}{\tau}\right)^2}, \qquad (1)$$

$$2n^2k = 4\pi\tau\Sigma\frac{Nr}{r^2+\left(\frac{m'}{\tau}\right)^2}. \qquad (2)$$

The first summation in equation (1) is a term wholly due to the molecules of the metal. The other two summations in both equations express the optical effect of the free electrons, being stated wholly in terms of their parameters and τ, the period of vibration of the light used, divided by 2π. In these latter summations, m' is equal to m/e^2, m and e being the mass and charge of each of any group of free electrons. N is the number per cm^3. of each group. r is equal to N/σ, σ being that portion of the total "conductivity" of the metal due to these particular electrons. σ should be equal to the electrical conductivity as usually defined for steady currents.

In this work, the following simplifying assumptions are usually made:

(*a*) In equation (1), the terms $1 + \Sigma\cdot\theta_h'/[1-(\tau_h/\tau)^2]$ have been assumed to possess a value independent of the free electrons present, *i.e.*, these terms have a numerical value equal approximately to that of $n^2(1-k^2)$ for a heavy insulator. As selective reflection is very small for the metals used in this investigation, n may be taken as about 1.6 and k as 0. Thus $n^2(1-k^2)$ for the molecules of the metal, or its equivalent $1+\Sigma\cdot\theta_h'/[1-(\tau_n/\tau)^2]$ will be numerically equal to about 2.5. Since $n^2(1-k^2)$ is a much larger negative quantity for metals, rather wide variations for this assumed value of 2.5 will not seriously affect the final results.

(*b*) The free electrons are assumed to be of one group, *i.e.*, they are negative electrons possessing identical mass, charge, average velocity, etc. The positive electrons are essentially molecules possessing positive charges. Being relatively large and heavy, they contribute practically nothing to the electrical and optical characteristics peculiar to metals. Their optical effect is thus entirely contained in the first summation of equation (1) given in (*a*).

If we divide numerators and denominators of the fractions in (1) and

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[1] P. Drude, Theory of Optics (Mann's translation), p. 398

(2) by m'^2 and substitute m/e^2 for m', the above equations become

$$\nu^2 - \kappa^2 = 2.5 - \frac{4\pi \frac{Ne^2}{m}}{\frac{r^2e^4}{m^2} + \left(\frac{1}{\tau}\right)^2}, \tag{3}$$

$$2\nu\kappa = 4\pi\tau \frac{\frac{Ne^2}{m}}{\frac{r^2e^4}{m^2} + \left(\frac{1}{\tau}\right)^2}, \tag{4}$$

where ν and κ have been used for Drude's n and $n\kappa$ respectively. r has been given as equal to N/σ. σ, the electrical conductivity, is expressible in the free electron theory of Drude by $Ne^2/m \cdot T/2$, in which T is the average value of the time between successive impacts of a free electron with successive molecules. This period is equal to the reciprocal of the number of impacts of the electron with molecules per second, and will be the same for all electrons. If we substitute $1/W$ for $T/2$, σ is equal to Ne^2/mW. W is thus a sort of quasi-frequency of impact of the free electrons. r is now given by mW/e^2. If this is substituted for r in equations (3) and (4), and ω for $1/\tau$ (*i.e.*, ω for 2π times the frequency of the light used), these equations become

$$\nu^2 - \kappa^2 = 2.5 - \frac{4\pi \frac{Ne^2}{m}}{\omega^2 + W^2}, \tag{5}$$

$$2\nu\kappa = 4\pi \frac{W}{\omega} \frac{\frac{Ne^2}{m}}{\omega^2 + W^2}. \tag{6}$$

W and Ne^2/m may be computed from these equations and the values of the optical constants. The reasons for the choice of W rather than T as a free electron parameter will perhaps be evident in the discussion following.

Optical Constants in the Solid and Liquid States.—Figures 3, 4, 5 and 6 show the relations between the optical constants for the solid and liquid states of the pure metals. The data for the solid metals has been obtained from various papers. It is seen that with the exception of cadmium, these constants are roughly doubled by fusion. Figs. 7, 8 and 9 show the variation of the constants $\nu\kappa$ and $\nu^2 - \kappa^2$ for liquids, with the atomic concentration of one component. In general, these are smooth, nonlinear curves. Littleton has assumed that simple mixture in solid alloys will be shown by the straight line variation of these con-

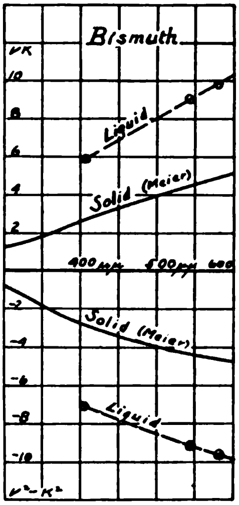

Fig. 3.

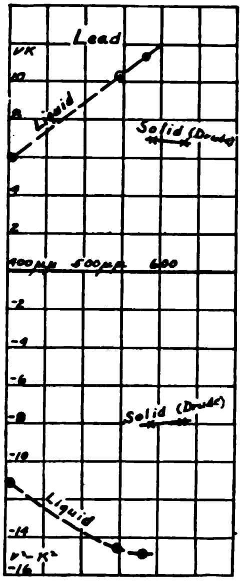

Fig. 4.

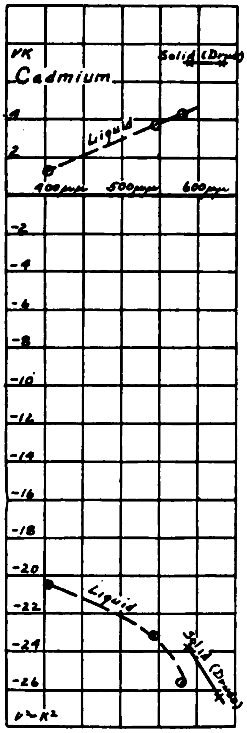

Fig. 5.

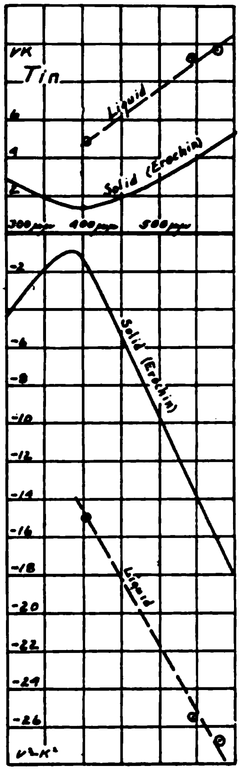

Fig. 6.

stants with the composition. This, of course, will be so where the alloy consists of two types of crystals mechanically mixed.

Theoretical Interpretation of Results.—In Table V. the calculated values of Ne^2/m and W are given for all the pure metals and alloys investigated. The units used are arbitrary. In Figs. 10, 11 and 12 they have been plotted against the atomic concentration of one component, for each series of alloys respectively.

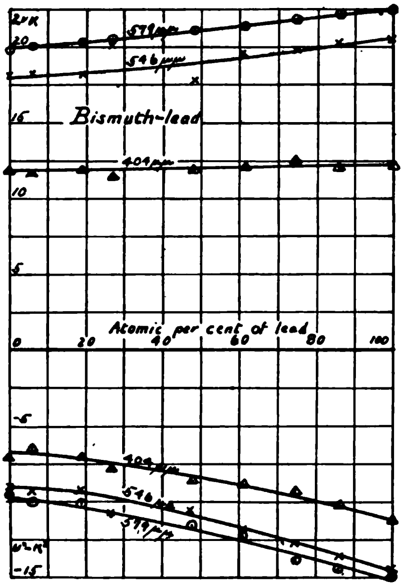

Fig. 7.

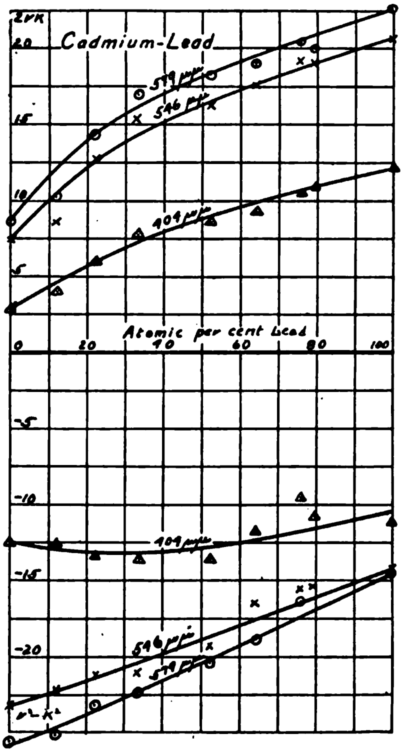

Fig. 8.

Within the limits of error, the values of Ne^2/m calculated from the optical constants for yellow and green light (579 μμ and 546 μμ) seem to lie on the same straight line for each series, except for the pure lead points in Figs. 11 and 12. While the results for violet light (404 μμ) are altogether too uncertain to be used in determining any functional relationship of Ne^2/m, there seems to be no doubt that in general they lie above the other points. Ne^2/m, however, should be the same, whatever the wave-length of the light used in determining it. The discrepancy is probably due to the following causes: (*a*) the effect of atomic resonance may not be even approximately represented by 2.5; (*b*) selective reflec-

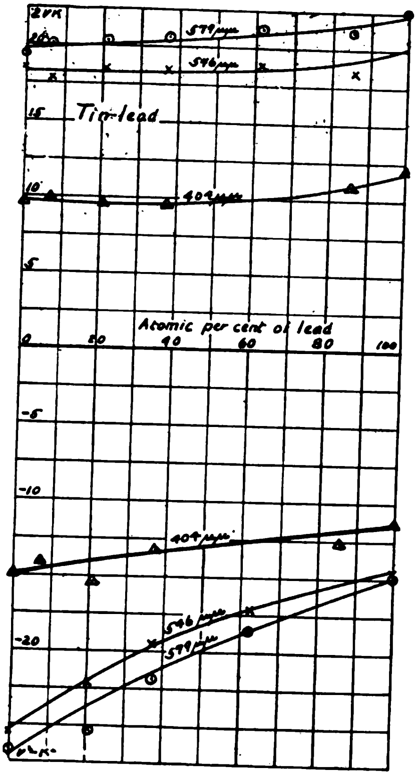

Fig. 9.

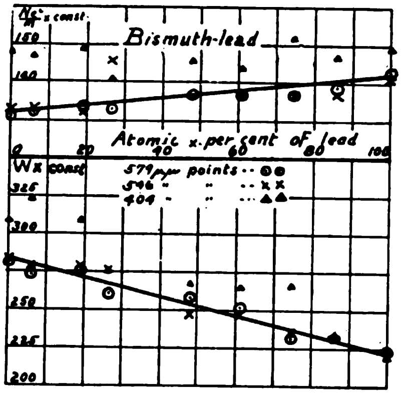

Fig. 10.

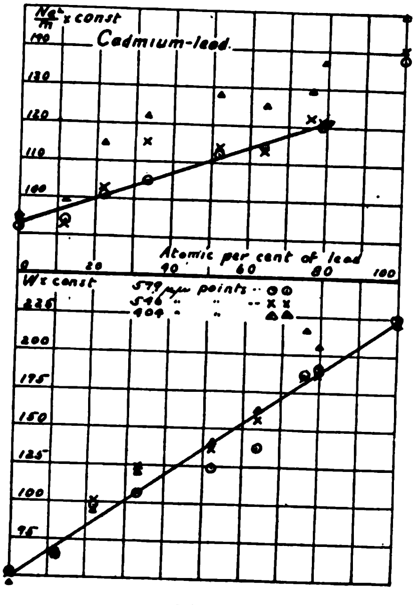

Fig. 11.

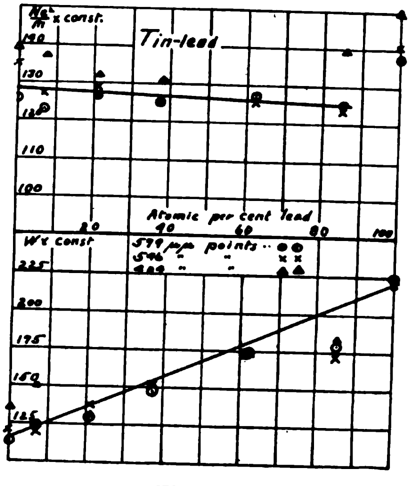

Fig. 12.

tion may be present in the ultra violet, not far beyond the visible. This is the case for most solid insulators. As selective reflection is due to the resonance of bound electrons within the atom, its presence in the atoms of most metals may be inferred. The above effect may be indicated for bismuth by the slight irregularities at the violet end of the solid curves for this metal. In the case of tin, Erochin has placed the center of an absorption band at 340 $\mu\mu$. Selective reflection in the ultra-violet usually increases the value of the term in the expression of $\nu^2 - \kappa^2$ for violet light, which represents the effect of the bound electrons of the atoms, and will introduce a somewhat similar term into the expression of $2\nu\kappa$.

The probable linearity of the variation of Ne^2/m with atomic concentration is to be expected. The molecules of a liquid metal are distinct entities, practically independent of each other. The fields of force of their component electric charges are thus almost entirely internal. The conducting electrons may therefore be regarded as truly "free," and as a definite portion of the metal. Thus each component of the alloy carries to the alloy a certain number of free electrons, identical in characteristics, and the number of these in the mixture is the sum of the numbers present in each component before the alloy is formed.

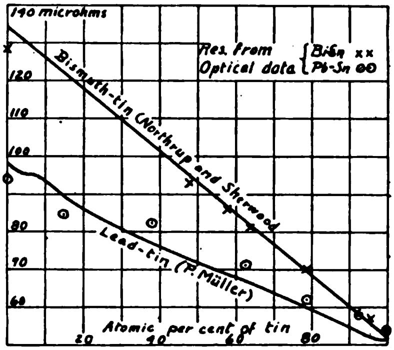

Fig. 13.

The pure lead points in the Cd-Pb and Sn-Pb figures are not collinear with the other points, and the difference can hardly be ascribed to experimental error. The cause of this variation, therefore, probably lies in some peculiarity of constitution of these alloys. Such a peculiarity is indicated by the resistances of this series of alloys, as given by P. Müller.[1] In the tin-lead series, the resistance rises uniformly with the lead content up to about 90 per cent. lead. Beyond this the resistance curve is lifted up in a small but distinct maximum (see Fig. 13). Müller gives the resistances of the cadmium-lead series to 6 per cent. cadmium only, but the same rise is evident. The fall of resistance from the maximum to pure lead is probably due to the anomalous increase in the number of free electrons indicated by the optical results.

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[1] P. Muller, Metallurgie, 7, 755, 1910.

In the solid state, these alloys are solid solutions at the lead side of the phase diagrams. The temperature of 400° at which the observations were made, is not far above the temperature of complete fusion, and it is quite possible that the tendency to atomic grouping shown by the solutions is carried over into the molten state. This tendency is analogous to the formation of an easily dissociated compound, and the process of grouping will result in a similar manner in the absorption of free electrons into the groups. Thus the alloy from 0 per cent. to 90 per cent. lead may be considered as a simple mixture of cadmium (or tin) and this quasi-compound, and the atomic concentration—Ne^2/m curve will be a straight line within this range. The alloy series from 90 per cent. to 100 per cent. lead should act as a mixture of the so-called compound and pure lead, in a similar manner.

Turning to the curves which show the variation of W with the atomic concentration, the agreement of the values of W given by the three lights used is very good, except for alloys rich in bismuth. The differences for this metal series may be due to either or both of the causes given for the Ne^2/m variations. It should be possible to bring the values of both Ne^2/m and W into good agreement for the three lights used, by assuming suitable values of the molecular or bound electron terms in the optical equations. However, in view of the rather large errors inherent in optical measurements on metals, it has seemed hardly worth while to do this.

The linearity of the W curves is striking, and is also a natural consequence of the liquid condition of the alloys. The frequency of impact of a free electron with the comparatively inert molecules ($2W$) is proportional to the "total surface of impact" of the molecules. If we introduce into the assemblage of like molecules of the one metal molecules of another metal which are of a different size, the "total surface of impact," and therefore the frequency of impact of a free electron, are both obviously linear functions of the atomic concentration of either component.

It should be noticed that there is no indication in these curves of any peculiarity of cadmium-lead and tin-lead alloys rich in lead, such as that shown by the variation of Ne^2/m. The tendency towards atomic grouping which would account for this variation cannot be very great. This is shown in P. Müller's work. As the temperature is increased the irregularities of the concentration-resistance curves of these alloys disappear. Owing to this looseness of grouping there is no marked shielding of one atom by another. This will be especially true if the targets of impact are the atomic nuclei. Under these conditions the frequency of impact of any free electron will be unaffected by such tendencies, and the variation of W with atomic concentration will be linear. If true compounds were formed, however, the close association of atoms might be expected to influence this free electron parameter.

Thus the variation of the free electron functions, Ne^2/m and W in these binary alloys and the coherence of results, seem to confirm very strongly the admissibility of the application of the simple form of the electron theory to liquid metals. However, any satisfactory optical theory should be able to show a quantitative relation between the electrical and optical characteristics of metals. Any of these characteristics or any of the parameters of the conducting electrons, should be determinable from a sufficient number of the remainder. For instance, Drude,[1] Schuster,[2] W. Meier[3] and others have calculated the number of free electrons per atom in various solid metals from the optical constants and electrical conductivity. Rubens and Hagen[4] tested the relationship of the electrical conductivity and reflecting power of metals given in the equation $1 - R = 2/\sqrt{\sigma T}$, in which R is the reflecting power, σ the electrical conductivity and T the frequency of the light used. This equation is not valid for visible light but holds very well for the infra-red ($25.5\ \mu$) with one or two exceptions. These exceptions have been held to be due to the neglect of "resonance of the molecules" in the electromagnetic theory of Maxwell, from which the equation is derived. Now Drude's equations, forms of which are used in this paper, are recognized as a better representation of the optical behavior of metals, than Maxwell's original equations. $1 - R = 2/\sqrt{\sigma T}$ may be derived from Drude's equations if we assume W very large in comparison with ω, *i.e.*, that the time between successive impacts of an electron may be neglected in comparison with the period of the light used in the optical determinations. Thus the lack of agreement of Maxwell's equation with facts in the case of bismuth and one or two other metals is probably due to the omission of a term which has nothing to do with "resonance of the molecules."

In general it may be said that when solid metals are used, the data obtained from electrical measurements agree rather poorly with those from optical measurements in the visible spectrum. This may be illustrated very well for the solid state of the metals which were investigated in this work. From equations (5) and (6) we may obtain

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[1] P. Drude, Ann. d. Phys., 14, 936, 1904.

[2] A. Schuster, Phil. Mag., 7, 151, 1904.

[3] W. Meier, Ann. d. Phys., 31, 1016, 1910.

[4] Rubens and Hagen, Ann. d. Phys., 11, 873, 1903.

$$\frac{Ne^2}{m} = -\frac{\omega^2}{4\pi}\frac{(\nu^2 - \kappa^2 - 2.5)^2 - (2\nu\kappa)^2}{(\nu^2 - \kappa^2 - 2.5)}$$

and

$$W = -\frac{\omega 2\nu\kappa}{(\nu^2 - \kappa^2 - 2.5)}.$$

The electrical restivity (in electrostatic units) is Wm/Ne^2 and substituting from the equations above,

$$\text{Res. (e.s.u.)} = \frac{4\pi}{\omega}\frac{2\nu\kappa}{(\nu^2 - \kappa^2 - 2.5)^2 - (2\nu\kappa)^2}.$$

I have used the data of the various writers from which the solid metal curves of Figs. 1, 2, 3 and 4 were drawn to calculate the restivities. These are expressed in microhms and compared with the electrical resistivities of the same solid metals as given by the Smithsonian Tables, as follows:

	Bismuth.	**Lead.**	**Cadmium.**	**Tin.**
Light used	600 $\mu\mu$	600 $\mu\mu$	560 $\mu\mu$	550 $\mu\mu$
Res. (microhms) by opt. methods	238	159	58	76
Res. (microhms) by Smiths. Tables	119	22	7.5	14

The lack of agreement indicated above has been explained on the supposition that the resistivity at frequencies approaching that of light is markedly different from that at ordinary frequencies. Livens[1] and others have developed this functional relation. Owing to the simple conditions existing in molten metals, it should be possible to use their optical constants to gain considerable information as to this possible variation of resistivity, and as to the identity of the electrical and optical parameters given in the elementary equations. On this account I have calculated the resistivities of the pure molten metals from the optical data I have obtained. These and various electron parameters are given in the following table. Only the optical results for yellow and green light were used, those for violet light having altogether too large experimental errors. Meier's[2] results were used for mercury. The resistivities so obtained, are compared with those found by Northrup and Suydam[3] for bismuth, lead, cadmium and tin at 400°. While no particular effort was made to keep the temperature of the liquid metals constant during the optical investigation, it was never very far from this value. The resistivity of mercury at ordinary temperatures was used.

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[1] G. H. Livens, Phil. Mag., 30, 112, 1915.

[2] W. Meier, *loc. cit.*

[3] E. F. Northrup and V. A. Suydam, Journ. Franklin Inst., 175, 153, 1913.

TABLE VI.

	Bismuth.	**Lead.**	**Cadmium.**	**Tin.**	**Mercury (Meier).**
$Ne^2/m \times 10^{-28}/9$	132	142.5	92.5	129.5	90
N (No. free electrons per cm.^{3}) $\times 10^{-22}$	14.7	15.9	10.3	14.5	10.0
No. atoms per cm.^{3} $\times 10^{-22}$	2.9	3.1	4.3	3.5	4.1
No. free electrons per atom	5.1	5.1	2.4	4.1	2.5
$W \times 10^{-12}/6\pi$	282	223	51.2	117	131
Impacts of free electron per sec. $(2W) \times 10^{-14}$	106	84	19.5	44	50
Impacts per period of green light (550 $\mu\mu$)	19.3	15.3	3.5	8.9	9.0
Wm/Ne^2 (resistivity, e.s.u.) $\times 6.60 \times 10^{-7}$	2.14	1.57	0.56	0.90	1.46
Resistivity in microhms from above data	128	94	33.4	54	87.3
Resistivity in microhms from electrical data	134	98	34	52	94
Ratio "optical" to electrical resistivity	.95	.96	.98	1.04	.93

An auxiliary figure (Fig. 13) is also given which compares the resistivities of tin-lead and tin-bismuth alloys, electrically and optically determined. The electrical data for the two series of alloys are taken from papers by P. Müller,[1] and by E. F. Northrup and R. G. Sherwood[2] respectively. The "optical" resistances of the tin-lead alloys were taken directly from the optical results of this series. As the optical characteristics of tin-bismuth alloys were not investigated, the "optical" resistances of this series were calculated from the electron parameters Ne^2/m and W of the pure metal. Both of these are linear functions of the atomic concentration, provided that no compounds are present in the liquid state, and may be easily determined for an alloy of the two metals of any given atomic composition. The quotient of these two will give the resistivity.

From an inspection of the tables and auxiliary figures, it may be seen that, except for mercury, the electrically and optically determined resistivities agree, in every case, within the limits set by unavoidable experimental error. The optical constants of mercury were not determined by the writer and their accuracy is not known, although it is probably good. There is no evidence of any change in the electrical resistivity, even up to the frequencies of light. Certainly any such variation is not greater than 5 per cent. The agreement of the results, both electrical and optical is a strong confirmation of the existence of essentially "free" electrons in these liquid metals. In view of the probable validity of this conception, I have calculated the values of N, the number of free electrons per cm.^{3}, the number per atom, the frequency of impact of the free electrons ($2W$) and the ratio of this frequency to that of green light ($550\ \mu\mu$). The number of free electrons per atom necessitated a knowledge of the number of atoms per cm.^{3} For each metal this number is equal to Avogadro's number multiplied by the density and divided by the atomic weight. Avogadro's number was taken as 60×10^{22}. The densities of the molten metals are not accurate, as the temperatures at which they were taken were not given in the tables consulted. To calculate the number of electrons per cm.^{3}, e was taken as equal to 4.77×10^{-10} e.s.u. and m to 9×10^{-28} g. It is evident that the uncertainties of the various constants used in these calculations render the values of the number of free electrons per atom only approximations at best. In addition it is probable that the parameters Ne^2/m and W given here, in reality include constant multipliers. These would be introduced in a more exact development of the free electron theory.

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[1] P. Müller, *loc. cit.*

[2] E. F. Northrup and R. G. Sherwood, Journ. Franklin Inst., 182, 477, 1916.

From certain points of view it would seem that the number of free electrons per atom, or at least per molecule, should be an integer for each metal. The accuracy of the present work is not sufficient to settle this point. On the other hand, the uncertainties of the constants used to obtain these relations, affect all alike, and it is possible to obtain these ratios as integers, within the limits of experimental error, by making small and legitimate changes in the above doubtful constants. The results are certainly not inconsistent with the possibility of these integral relations.

From the agreement of the electrical and optical data in Table VI., it seems that changes of resistivity at light frequencies, and molecular resonance (except in infrequent cases) are negligible factors in determining the phenomena of these liquid metals. I can see no reason why they should be of any greater importance in solid metals, and the discrepancies encountered in the explanation of the optical and electrical phenomena of the solid phase must be due to some different cause. The distinguishing characteristic of the solid metal is the space lattice structure of its crystals. This structure demands that the atoms should have strong external fields of force, binding them into a rigid form. The conducting electrons can hardly wander freely in inter-atomic space, between so-called impacts. From various points of view it is hard to avoid the conclusion that conduction in crystalline solids is due to the "handing along" of these electrons from atom to atom. Thus, in the fundamental equations of motion of a conducting electron in the metallic crystal, a term must be present which is a function of the position of the electron within the space lattice. It is probable that the neglect of such a term is the source of apparently discordant electrical and optical results for solid metals.

The small range of temperature and the limited accuracy of the experimental work made impossible the detection of measurement of any temperature effect on the optical constants. Such measurement would be of value from a theoretical standpoint.

Physical Laboratory,
University of Michigan.

ON THE MAINTENANCE OF VIBRATIONS OF WIRES BY ELECTRICAL HEATING.

BY N. C. KRISHNAIYAR.

SYNOPSIS.

The amplitude and phase of the vibrations set up in a stretched wire when a single phase alternating current of 50 cycles per second was sent through the wire have been investigated experimentally, and the results have been compared with the theory given by C. V. Raman.

Amplitude.—(*a*) Sonometer nearly vertical, length of vibrating segment constant, and tension varied by steps. When the tension appreciably exceeded the value corresponding to the free-period of the wire no maintained vibrations were established when the current was turned on. As the tension was decreased, persistent vibrations of small amplitude were set up when the resonance tension was approached. The amplitude progressively increased as the tension was decreased through, and some interval below, the resonance tension. Eventually a critical tension was reached at which there was a sudden complete cessation of maintenance. On reversing the process, by gradually increasing the tension, persistent vibrations were not set up unless the wire was strongly plucked at its middle. (*b*) Sonometer horizontal, tension constant, and length of wire decreased progressively. The phenomena observed were essentially the same as in the preceding case.

Phase.—By adapting the principle of the vibration microscope it was observed that the phase of the maintained vibration relatively to the phase of the current was not the same over the whole range of persistent vibrations. The phase changed rapidly by about 90° near the end of the range corresponding to very small amplitudes. For larger amplitudes and over the greater part of the range the phase remained practically constant.

Theory.—When the differential equation of the motion is solved to the first approximation it is found that the square of the amplitude of the vibrations is a linear func- of the difference of the squares of the forced and free frequencies. Also the phase should be constant. The experimental curves agree with the theoretical parabolas over all of the range except the portion pertaining to small amplitudes. A like statement applies to the phase. Further work is in progress to throw light on the observed points of inflection, and to account for the discrepancies between theory and experiment.

I. INTRODUCTION.

WHEN an intermittent electric current traverses a fine wire, the passage of the current may result in setting the wire in vibration. This phenomenon was described some years ago by Argyropoulos.[1] Recently, the analogous phenomenon obtained by sending an alternating electric current through a freely hanging platinum wire has been studied by Strientz,[1] who observed specially striking effects when the wire was heated to incandescence by the current. The present author has carried out a study of the vibrations produced by sending a single-phase alternating current of fifty cycles per second through fine wires of adjustable length or tension stretched on a sonometer, the special object of the investigation being to ascertain the manner in which the amplitude and phase of the maintained vibration vary with the difference between the natural frequency of vibrations of the wire and the frequency of the alternating current. This inquiry appeared to be called for in view of the peculiar character of the maintenance of vibration in this case. The passage of an alternating current through a wire under tension does not *directly* tend (in the absence of any external magnetic field[2]) to displace it from its position of equilibrium. The production of vibration is therefore an indirect effect due to the fact that the passage of the alternating current heats the wire and thus causes a periodic variation of tension of double the frequency of the current, and this in turn sets up and maintains a vibration having the *same* frequency as the current. In a paper[3] on "Some Remarkable cases of Resonance," published recently in this REVIEW, C. V. Raman has discussed the theory of the maintenance of vibrations by forces of double frequency and has shown that the variation of the amplitude and phase of the maintained vibration within the range of resonance in such cases presents peculiar features. It appeared of interest to investigate whether such special features could be observed also in the case of the vibrations of a wire produced by the passage of an alternating current through it.

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[1] Comptes Rendus, 111 (1890), 525.

II. AMPLITUDE OF THE MAINTAINED VIBRATIONS.

To measure the amplitude of the maintained vibrations, a tiny particle of very thin silvered glass was attached to the wire at its middle point, and illuminated. A scale engraved on a mirror was placed behind the wire at right angles to its length, and the drawn-out reflection in the scale was read through a telescope. Two sets of observations were made of the amplitudes of the forced vibrations; in the first set, the length of the wire was kept constant and the tension was varied; and in the second set, the tension was kept constant and the length was varied.

Constant Length and Varying Tension.—A german silver wire of S.W.G. No. 40 was mounted on a sonometer held nearly vertically. A load of 30.9 grammes (which includes twenty half-gramme pieces strung along the length of a silk fiber) was suspended from the free end of the wire. The wire passed over two bridges 50 cm. apart put on the sonometer. The theoretical tension required to make 50 cm. of the wire vibrate with a frequency of 50 per second was equivalent to a load of 25.9 grammes. Starting with a load of 30.9 grammes on the wire, an alternating current of 0.5 amperes, when put on, did not cause the wire to vibrate, but only heated it, the wire slipping over the bridges in consequence of its expansion. This expansion amounted to a few millimeters and persisted as long as the current was on. It did not of course alter the effective length of the wire between the bridges or its tension. The linear density of the wire would, no doubt, have been altered but not to any great extent. The half-gramme masses were then made to drop off one by one, by touching the silk fiber with a glowing match stick. When the tension was thus reduced to 27.4 grammes-weight, the wire started vibrating with an amplitude of 2 mm. With subsequent diminutions of tension, the amplitude increased and attained the value 3.7 mm. when the tension was equal to the calculated value for resonance (25.9 grammes-wt.), and continued steadily to increase as the tension was further reduced until a maximum value of 10 mm. was obtained with a tension of 21.4 grammes-wt. Further reduction of the tension resulted in a sudden and complete cessation of the maintenance. The tension was then increased by the addition of half-gramme loads, but the vibration would not start by itself. The wire was given a small amplitude by plucking, but the vibration died off. But when it was given a sufficiently large amplitude initially, the vibration successfully reëstablished itself, and was maintained with the originally observed amplitude. Similar results were obtained with thicker german silver wires and also with fine copper or nickel wires, and with other current strengths. The maintained amplitudes increased when larger currents were put on, or when thinner wire or wires of greater specific resistance were used.

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[1] Phys. Zeit., 16, pp 137–138, April, 1915.

[2] The effect of the earth's magnetic field, if any, is small, since the experiment succeeds equally well in all positions of the wire.

[3] PHYS. REV., December, 1912, pp. 453.

Constant Tension and Varying Lengths.—Generally similar results were obtained in this case also. The sonometer was placed horizontally and the free end of the wire passed over a pulley, and had a scale pan attached to it. The pan and its contents weighed 25.9 grammes, the load necessary to make the natural period of no. 40 german silver wire of length 50 cm. equal to the period of the alternating current. Two glass prisms were used as the bridges over which the wire passed, the illuminated point being midway between them. With the bridges 47 cm. apart, no appreciable vibration could be noticed when the current was turned on. On increasing the distance between the bridges to 48 cm. a small maintained vibration was observable. On further increasing the length, even beyond 50 cm., the maintained amplitudes continually increased till a stage was reached at which the amplitude was relatively very large, and any further increase in the distance between the bridges resulted in a sudden and complete collapse of the maintenance. After this stage was passed, shortening the length of the wire a little by bringing the bridges nearer would not restart the vibrations. Small amplitudes when given by plucking died away, but if a sufficiently large initial amplitude were given, the vibration successfully reëstablished itself. Maintenance would start by itself only when the natural period of the wire for small vibrations was less than, equal to, or a very little greater than the period of the current.

When the results of the observations are exhibited graphically we get a figure which is quite unlike the usual type of resonance curve for a vibrator under the action of a periodic force.

As will be seen from Fig. 1, in which the amplitudes for two different current strengths have been plotted against the differences of the squares of the forced and natural frequencies, we have no "peak" or maximum of resonance, but only a continual increase in the amplitude from one end of the range to the other. This phenomenon finds an explanation in the theory of the maintenance of vibrations by forces of double frequency as developed in the paper[1] by Raman referred to above. The equation for normal vibrations under variable tension is

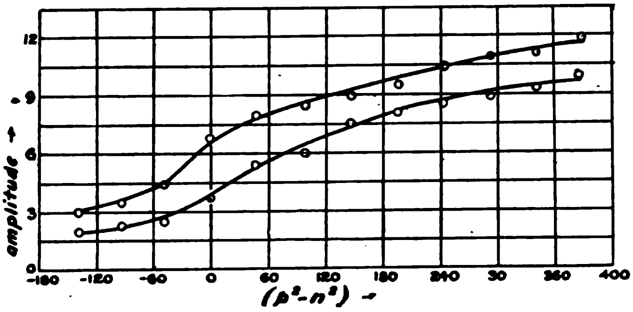

Fig. 1.

$$w + k \cdot\cdot + (n^2 - 2\alpha \sin 2\,pt + \beta w^2)w = 0$$

The solution is

$$w = A_1 \sin pt + B_1 \cos pt + A_3 \sin 3\,pt + B_3 \cos 3\,pt +, \text{etc.}$$

where A_1, B_1, etc., have to be found by substituting for $\ddot{w}$, $\dot{w}$, and w in the differential equation and equating to zero, the coefficients of sin pt, cos pt, etc. To a first approximation we have

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[1] *Note by Editor.*—The reductions suggested require the use of such additional relations as $\sin^3 pt = \frac{3}{4} \sin pt - \frac{1}{4} \sin 3pt$, $2 \cos pt \sin 2pt = \sin pt + \sin 3pt$, etc."

$$\tan E = \frac{B_1}{A_1} = \frac{\alpha - kp}{n^2 - p^2 + F}$$

$$= \left(\frac{\alpha - kp}{\alpha + kp}\right)^{\frac{1}{2}} \text{ where } (n^2 - p^2 + F)^2 = \alpha^2 - k^2p^2$$

and

$$F = \frac{3\beta}{4}(A_1^2 + B_1^2).$$

This may be conveniently written in the form

$$\frac{3\beta}{4}(A_1^2 + B_1^2) = (\alpha^2 - k^2p^2)^{\frac{1}{2}} + (p^2 - n^2).$$

Accordingly, when $(A_1^2 + B_1^2)^{\frac{1}{2}}$, which is the amplitude of vibration, is shown graphically against the values of $(p^2 - n^2)$, we should get a parabolic arc, as shown in Fig. 2 for two different values of α.

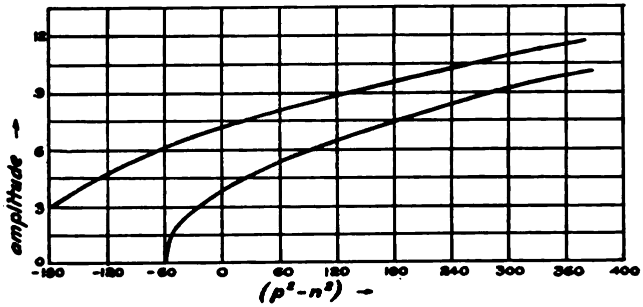

Fig. 2.

It will be seen that the theoretical and the experimental types of resonance curve shown in Fig. 2 and Fig. 1, generally resemble each other except near the extreme end of the range at which very small amplitudes of vibration are obtained. The curves in Fig. 1 show distinct inflections which do not appear in the theoretical curve. The explanation of this small discrepancy will be considered below separately.

III. The Phase of the Maintained Vibrations.

Under the action of periodic forces of the ordinary type, the phase of a maintained vibration alters continually through a series of values of which the extreme difference is 180°, when the natural frequency of the vibration is decreased from a value above to a value below that of the

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[1] Phys. Rev., December, 1912, p. 451.

impressed force.[1] The case is entirely different in regard to vibrations maintained by variable spring or tension. According to the formula noted above, the impressed variation of tension being denoted by $-2\alpha \sin 2pt$, the forced vibration would, to a first approximation, be given by

$$w = A_1 \sin pt + B_1 \cos pt$$

where

$$\frac{B_1}{A_1} = \left(\frac{\alpha - kp}{\alpha + kp}\right)^{1/2}.$$

Thus the phase of the maintained vibration would depend only on α and kp, and if these quantities were absolute constants, the phase would remain unaffected when the natural frequency of the wire is altered. In the cases now under consideration, the variation of tension is due to the heating action of the alternating current, and its magnitude and phase for any given current-strength would depend on the experimental conditions. The temperature variations in stationary wires carrying alternating currents have been discussed by Irwin,[2] Drysdale,[3] Ebeling[4] and others, and have been shown to depend on the capacity for heat and the emissivity of the wire. When, as in the present experiments, the wire is in vibration, it would lose heat to the surrounding atmosphere more quickly than usual, and the magnitude and phase of the temperature variation would, no doubt, to some extent depend on the amplitude of vibration. This complicates matters somewhat, and the theoretical conclusion stated above—that the phase of the maintained vibration should be independent of the natural frequency of the wire—may require modification in practice.

In order to study the phase relation of the maintained vibration to the current passing through the wire, an arrangement similar in principle to the vibration microscope was used. The illuminated point of the vibrating wire was observed from a distance through a telescope, the object glass of which was caused to vibrate to and fro in its own plane with the same frequency as the alternating current. This was arranged by fixing the object glass to the middle points of two vertical wires under tension, placed between the poles of a magnet. The alternating current through the sonometer also passed through the two supporting wires in the same direction and caused the vibration of the object glass. The natural frequency of this vibrating system being much smaller than the frequency of the current, the forced vibration of the object glass may be assumed to have been almost exactly in opposition of phase to the current. The Lissajous figure seen when the illuminated point on the wire is observed through the telescope enables us to infer the phase of its maintained vibration. The observations seem to show that for the current-strengths generally used (0.5 to 0.8 ampere), the phase of the maintained vibration relatively to that of the current is not the same over the whole of the range of resonance, but changes rapidly by about 90° near one end of the range of resonance where the amplitude is very small. Subsequently, over the remaining and much the greater part of the range, the phase remains roughly constant. The general result of the investigation is thus to confirm the indication of the theory of maintenance by forces of double frequency except in the part of the range where the maintained amplitudes are very small.

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[1] Rayleigh's Sound, Vol. 1, p. 48.

[2] Inst. Elect. Engin. Jour., 39, pp. 617–642, Sept., 1907.

[3] Inst. Elect. Engin. Journ., pp. 643–647, Sept., 1907.

[4] Ann.d. Physik., 27, 2, pp. 391–435, Oct., 1908.

As regards the origin of the discrepancies between theory and experiment over part of the range of resonance referred to above, it is difficult to pronounce definitely at present. Possibly they are due to a small forced vibration of the ordinary kind produced by the action of the earth's magnetic field on the wire traversed by the alternating current, and superposed on the much larger vibration due to the heating action of the current. Experiments are now in progress to clear up this point and also to study the cases in which the wire is maintained in vibration under the joint action of the electric heating and of external magnetic fields.

GOVERNMENT COLLEGE,
RANGOON, BURMA,
May 8, 1919.

ON THE DIFFRACTION THEORY OF MICROSCOPIC VISION.

By Phanindra Nath Ghosh.

Synopsis.

The changes that take place in the resultant diffraction pattern when a black crossed-grating on a white ground is viewed through a very long rectilinear slit, the axis of which is inclined at an angle of 45° to the lines of the grating and when the slit width is gradually decreased, are described, and reproductions of photographs of typical stages in the image distortion are given. It is shown that when the slit is sufficiently narrow the diffraction field consists of a series of equidistant bright and dark lines at right angles to the chief axis of the slit.

The elemental case of a single black 90° cross on a white ground, when viewed through a slit which is parallel to the bisector of a pair of vertical angles of the cross, is subjected to approximate mathematical analysis, and the contour lines are given both numerically and graphically. It is explained how a synthesis of the elemental pattern accounts, qualitatively at least, for the more general case.

I. Introduction.

According to Abbe's well-known theory, the condition for the optical image of an illuminated structure being an accurate representation of it is that the aperture of the system should be sufficient to permit the entry into it of the entire diffracted streams of light issuing from the structure. The image differs from the object if only a part of the diffracted streams enter the optical system, and the structure would be altogether unrepresented in the image if the aperture be insufficient to permit the entry of the diffracted streams. The mathematical development of the theory for the case of an ordinary grating has been worked out by Lord Rayleigh,[1] Porter,[2] and Wolfke.[3] There are important differences observed between the cases in which the structure is regarded as self-luminous and those in which it is illuminated by an external source, and these have been considered in detail by Mandelstam[4] and also by Lummer and Reiche.[5] The subject has also been extensively studied by Gordon.[6] A case that is of special interest and has often been used in the illustration of Abbe's theory[1] is that in which the structure is a crossed grating and is viewed through a narrow rectilinear slit. In certain cases, that is, when the slit is held inclined to both sets of lines of the grating, the actual structure of the grating disappears, and is replaced by a spurious set of lines running transverse to the direction of the observing slit. It is proposed in the present paper to consider more fully than has been done by previous writers, the theory of the case of the crossed grating and the manner in which the spurious representation arises, especially the successive stages of the phenomena noticed as the obliquely inclined slit is gradually narrowed down.

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[1] Rayleigh, Scientific Papers, Vol. IV., p. 243.

[2] Porter, Phil. Mag., January, 1906.

[3] Wolfke, Annalen, 34, 1911; 39, 1912; 40, 1912.

[4] Mandelstam, Annalen 35, 1911.

[5] Lummer and Reiche, Annalen, 35, 1911.

[6] Gordon, Photographic Journal, Dec., 1914.

It facilitates an understanding of the case of the crossed grating if we consider the phenomena observed when a single dark cross on an uniformly bright ground is viewed through a rectilinear slit. When a sufficiently wide slit is used, the image of the cross is a faithful representation of the object. But as the slit is gradually narrowed down, keeping it parallel to one of the arms of the cross, the line to which the slit is parallel gets fainter and fainter, its apparent width increases, till when the slit has been sufficiently narrowed down, the image disappears, leaving only the other arm of the cross visible in the field. When the lines of the cross are inclined to the slit, both the arms in the image turn grey, their apparent width increases and the junction of the cross undergoes a curious modification. A clean cut bright line which at first is very narrow appears at the junction of the cross in a direction perpendicular to the observing slit, while above and below the junction two dark lines appear. This is shown in Fig. 1. The phenomenon is most clearly seen when the arms make an angle of 45° with the slit.

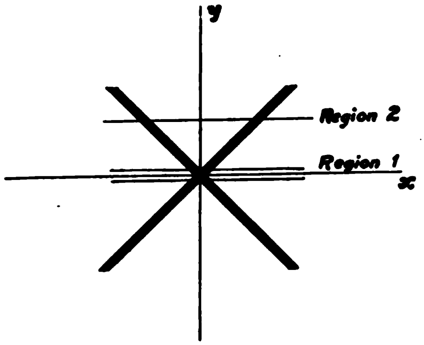

Fig. 7.

When a crossed grating made up with alternate dark and bright bars is used as the object, and the slit placed in front of the lens is inclined at

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[1] See for instance a paper by Winkelmann, Annalen, 19, 1906.

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PLATE I.
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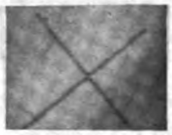

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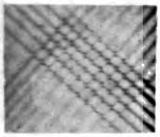

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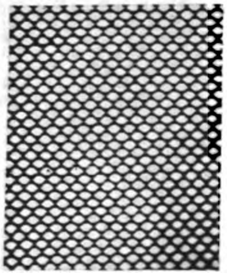

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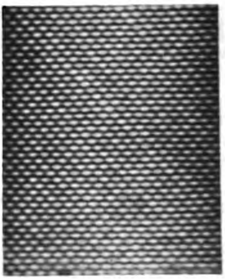

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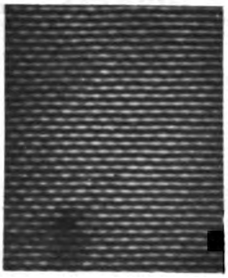

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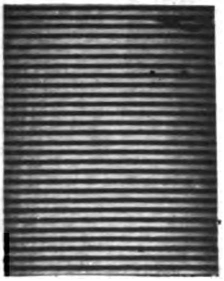

PHANINDRA NATH GHOSH.

an angle of 45° to the lines of the cross and gradually narrowed down, we have at each point of the crossing of the lines the phenomena described above for a single cross, Fig. 2. In the initial stages, the image of the crossed grating appears as a series of zigzag or wavy lines running transverse to the direction of the observing slit. These stages are clearly shown in Figs. 3 and 4. With the widening of the gaps at the junction of the crosses, the wavy lines straighten up (Fig. 5), and finally we have an entirely spurious set of lines (Fig. 6), running transverse to the observing slit, that is, at an angle of 45° to the two sets of lines of the original grating.

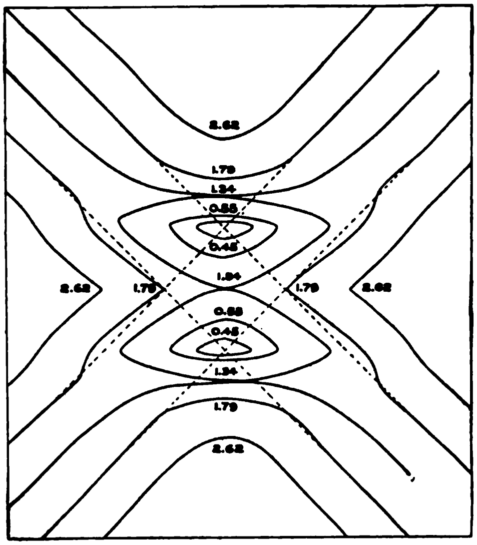

Fig. 8.

2. Mathematical Theory.

In a supplementary paper on the theory of microscopic vision, Lord Rayleigh[1] has given the mathematical theory of the case of a single dark bar on a bright field viewed through a rectilinear slit. We shall here extend the treatment given by Lord Rayleigh to find the contour lines of equal illumination in the image of a dark cross on a bright field formed by a slit held at an angle of 45° to the arms of the cross. In what follows, it will be assumed that the different parts of the luminous ground have no phase relation with each other. To each point on the ground there corresponds a diffraction pattern in the image plane, formed on the line transverse to the direction of the slit containing the geometrical image of the point. The rectilinear slit is assumed to be of great length, so that we can consider independently the diffraction pattern due to each of the strips transverse to the slit into which the luminous ground may be divided.

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[1] Rayleigh, Scientific Papers, V., pp. 118–125.

The diffraction pattern of a radiant point due to a rectilinear slit of width a is a series of points lying on a line transverse to the direction of the slit. The expression for intensity is of the usual form,

$$I = \alpha^2 \frac{\sin^2 \frac{\pi x \alpha}{\lambda f}}{\left(\frac{\pi x \alpha}{\lambda f}\right)^2}$$

or, omitting constants, $\dfrac{\sin^2 u}{u^2}$.

where f is the focal length of the lens used, and λ the wave-length of the light employed. $x = 0$ is the origin of coördinates.

We shall consider separately the region in which the arms of the cross are superposed and that in which they are separate (shown as regions I and II in Fig. 7).

Region I.—Considering this region, if the width of the geometrical image of the cross along the line $y = 0$ be 2α, we find that the width gradually increases to 4α at the line $y = \alpha$. Considering the illumination along any strip in this region, and denoting by 2β the width of the geometrical image of the cross along this strip, we get for regions within the geometrical image,

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du - \int_0^{\beta - u} \frac{\sin^2 u}{u^2} du - \int_0^{\beta + u} \frac{\sin^2 u}{u^2} du. \qquad (1)$$

For any point beyond the geometrical image—

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du + \int_0^{u - \beta} \frac{\sin^2 u}{u^2} du - \int_0^{u + \beta} \frac{\sin^2 u}{u^2} du. \qquad (2)$$

Region II.—Here we have to consider the effect of two dark lines each of width 2α situated along the strip at a distance v from the center of symmetry.

Within the geometrical image of the arms of the cross, the expression for the intensity is,

$$\begin{aligned} I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du - \int_0^{\alpha - u + v} \frac{\sin^2 u}{u^2} du - \int_0^{\alpha + u - v} \frac{\sin^2 u}{u^2} du \\ + \int_0^{u + v - \alpha} \frac{\sin^2 u}{u^2} du + \int_0^{u + v + \alpha} \frac{\sin^2 u}{u^2} du. \end{aligned} \qquad (3)$$

In the space outside the arms of the cross,

$$\begin{aligned} I(u) = \int_{-\infty}^{+\infty}\frac{\sin^2 u}{u^2}du + \int_0^{u-v-\alpha}\frac{\sin^2 u}{u^2}du - \int_0^{u-v+\alpha}\frac{\sin^2 u}{u^2}du \\ + \int_0^{u+v-\alpha}\frac{\sin^2 u}{u^2}du - \int_0^{u+v+\alpha}\frac{\sin^2 u}{u^2}du. \end{aligned} \tag{4}$$

In the space between the arms of the cross,

$$\begin{aligned} I(u) = \int_{-\infty}^{+\infty}\frac{\sin^2 u}{u^2}du + \int_0^{v-u-\alpha}\frac{\sin^2 u}{u^2}du - \int_0^{v-u+\alpha}\frac{\sin^2 u}{u^2}du \\ + \int_0^{u+v-\alpha}\frac{\sin^2 u}{u^2}du - \int_0^{u+v+\alpha}\frac{\sin^2 u}{u^2}du. \end{aligned} \tag{5}$$

The integrals can be made to depend upon the sine integral tabulated by Glaisher in the Phil. Trans. for 1870, knowing that

$$\int_{\pm\infty}^{+\infty}\frac{\sin^2 u}{u^2}du = \pi,$$

and that

$$\int_0^{x}\frac{\sin^2 u}{u^2}du = \int_0^{2x}\frac{\sin u}{u}du - \frac{\sin^2 x}{x} = Si2x - \frac{\sin^2 x}{x}.$$

We have

$$(1) = \pi - Si2(\beta - u) + \frac{\sin^2(\beta - u)}{\beta - u} - Si2(\beta + u) + \frac{\sin^2(\beta + u)}{\beta + u},$$

$$(2) = \pi + Si2(u - \beta) - \frac{\sin^2(u - \beta)}{u - \beta} - Si2(\beta + u) + \frac{\sin^2(\beta + u)}{\beta + u},$$

$$\begin{aligned} (3) = \pi &- Si2(\alpha - u + v) - Si2(\alpha + u - v) + Si2(u + v - \alpha) \\ &- Si2(u + v + \alpha) + \frac{\sin^2(\alpha - u + v)}{\alpha - u + v} + \frac{\sin^2(\alpha + u - v)}{\alpha + u - v} \\ &- \frac{\sin^2(u + v - \alpha)}{u + v - \alpha} + \frac{\sin^2(u + v + \alpha)}{u + v + \alpha}, \end{aligned}$$

$$\begin{aligned} (4) = \pi &+ Si2(u - v - \alpha) - Si2(u - v + \alpha) + Si2(u + v - \alpha) \\ &- Si2(u + v + \alpha) - \frac{\sin^2(u - v - \alpha)}{u - v - \alpha} + \frac{\sin^2(u - v + \alpha)}{u - v + \alpha} \\ &- \frac{\sin^2(u + v - \alpha)}{u + v - \alpha} + \frac{\sin^2(u + v + \alpha)}{u + v + \alpha}, \end{aligned}$$

$$\begin{aligned} (5) = \pi &+ Si2(v - u - \alpha) - Si2(v - u + \alpha) + Si2(u + v - \alpha) \\ &- Si2(u + v + \alpha) - \frac{\sin^2(v - u - \alpha)}{v - u - \alpha} + \frac{\sin^2(v - u + \alpha)}{v - u + \alpha} \\ &- \frac{\sin^2(u + v - \alpha)}{u + v - \alpha} + \frac{\sin^2(u + v + \alpha)}{u + v + \alpha}. \end{aligned}$$

Using these formulæ, the intensities at various points in the field have been computed and are shown below.

Table of Intensities for Slit Width $a = \lambda f/\pi$.

y.	$x=0$.	$x=.5$.	$x=1$.	$x=1.5$.	$x=2$.	$x=2.5$.	$x=3$.
0	1.342	1.53	1.797	2.25	2.62	2.89	3.06
.2	1.087	1.27	1.564	2.01	2.46	2.77	3.006
.5	.77	.95	1.25	1.71	2.21	2.58	2.88
.8	.55	.72	.96	1.41	1.808	2.37	2.72
1.0	.45	.57	.83	1.19	1.71	2.25	2.56
1.2	.878	.87	.96	1.22	1.61	2.08	2.49
1.5	1.358	1.297	1.25	1.47	1.65	1.79	2.25
2.0	2.102	1.88	1.71	1.52	1.34	1.59	1.79
2.5	2.608	2.53	2.24	1.794	1.49	1.34	1.49
3.0	2.968	2.88	2.61	2.25	1.794	1.49	1.34

The contour lines of equal illumination have been drawn from the above data (Fig. 8); they clearly show the oval shaped contours of the dark areas above and below the junction of the cross,and also the increased illumination transverse to the rectilinear slit at the junction of the cross. The crossed grating can be considered as an assemblage of a large number of dark crosses with their junctions lying along a system of lines parallel to the x axis. The form of the lines of equal illumination in then neighbourhood of each cross would be influenced to some extent by neighbouring crosses. At first, the oval-shaped darkspots at the junctions touching each other would form a zigzag boundary running transverse to the slit, and when the slit-width has been sufficiently narrowed down, these zigzag boundaries would be drawn out into continuous lines with bright spaces between them. Thus, finally, in the image, instead of the original crosses, we get the spurious representation of a set of lines transverse to the slit.

In conclusion, I beg to record my best thanks to Professor C. V. Raman for his help and encouragement during the course of the investigation.

University College of Science,
Calcutta, India,
May 7, 1919.

RESONANCE AND IONIZATION POTENTIALS FOR ELECTRONS IN THE MONATOMIC GASES ARGON, NEON AND HELIUM.

By H. C. Rentschler.

Synopsis.

The monatomic gases argon, neon and helium were tested for resonance potential. The method used was that first described by Tate in his work with mercury vapor. As a further check of the results a method similar to Lenard's method was used.

Experiments are described which show the necessity of properly freeing the metal electrodes from adsorbed and occluded gases.

The curves obtained indicate that argon has a resonance potential and that neon and helium do not.

Introduction.

TWO types of inelastic impacts between electrons and atoms of a number of metallic vapors[1] have been found to exist. The first type occurs when the colliding electron displaces an electron of the atom without removing it from the atom. The potential through which the colliding electron must fall to acquire the necessary energy to produce this displacement is known as the resonance potential for the vapor in question. The second type of inelastic impact occurs when the colliding electron removes an electron from the atom and produces ionization. The present investigation was undertaken to test the gases argon, neon and helium for these two types of inelastic impact.

Apparatus and Method.

The arrangement of the apparatus is shown in Fig. 1. A tungsten filament *f* was heated to incandescence by the battery *B*. Surrounding this filament and concentric with it was a nickel net *n* of about 100-mesh to the inch made from 4-mil wire. The diameter of the net was about 1.5 cm. and surrounding this in turn was a nickel cylinder *C* about 2 cm. in diameter.

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[1] Mercury: Tate, Phys. Rev., 7, p. 686, 1916. Goucher, Phys. Rev., 8, p. 561, 1916. Davis and Goucher, Phys. Rev., 10, p. 101, 1917.

Sodium: Tate and Foote, J. Wash. Acad. Sci., 7, p. 517, 1917.

Cadmium: Tate and Foote, Bur. Standards Sci. Paper No. 317.

Sodium, Cadmium, Potassium and Zinc: Tate and Foote, Phil. Mag., 36, p. 64, 1918.

Magnesium and Thallium: Foote and Mohler, Phil. Mag., 37, p. 33, 1919.

Arsenic, Rubidium, Cæsium: Foote, Rognley and Mohler, Phys. Rev., 13, p. 59, 1919.

Before assembling the tube as shown in Fig. 1, the net and cylinder were freed from occluded and adsorbed gases. This was done by sealing them in separate tubes with tungsten filaments along their axes. These tubes were exhausted with a diffusion pump and the net and cylinder bombarded by thermions from their respective filaments until no more gases could be liberated from them even when the bombardment was so violent that they were heated to as near their melting point as was safe. After allowing them to cool in the vacuum, the net and cylinder were taken out of the respective bulbs and mounted in the test bulb as shown in Fig. 1. The test bulb was exhausted by a diffusion pump and heated to about 400° C. to remove gases from the walls of the glass. The net and cylinder were again bombarded by thermions from the filament to remove gases adsorbed while transferring them to the experimental bulb. It is desirable to have a short filament in the experimental tube so as to have as small a potential drop along the filament as possible. With a short filament it would have been difficult to get sufficient bombardment of the cylinder through the net to heat it hot enough to drive out the occluded gases and hence the pre-treatment given the net and cylinder.

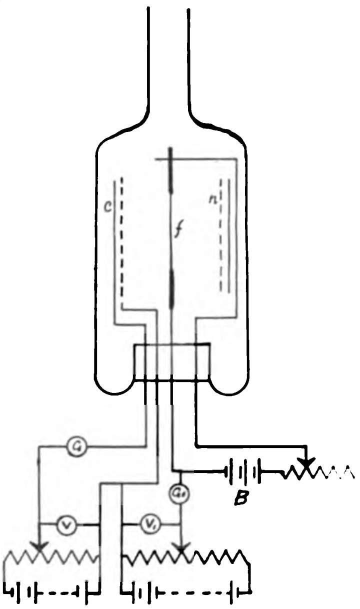

Fig. 1.

In treating nickel to remove the occluded gases a rather interesting phenomenon was noticed. From the length and diameter of the filament the electron current from the filament to the cylinder was calculated. At first the current that could be obtained was only about one seventh of the calculated current. After the bombardment had proceeded for about ten minutes the current gradually increased and after the nickel was entirely free from gas the calculated thermionic current was easily obtained. This was probably due to a poorly conducting coating on the surface of the nickel.

The tube was now filled to the desired pressure with the gas to be tested. A trap immersed in liquid air, between the ionization chamber and the pump prevented mercury vapor from the pump and gauge from reaching the ionization chamber. A constant retarding potential was applied between the net and the outer cylinder and both the current from the hot filament and that part of the current which reached the outer cylinder against the retarding field were measured by the galvanometers G_1 and G, as functions of the accelerating potential between the filament and the net. This was done for different retarding potentials and for different pressures. A decrease in the electron current to the outer cylinder with increase in accelerating potential generally indicates inelastic impact of the electrons while a sudden increase in the total current from the hot filament indicates ionization. A decrease in the cylinder current without a simultaneous increase in the total current from the filament would indicate that the inelastic impact was due to resonance and not ionization.

Second Method.

As a further check on the results, the outside cylinder was maintained at a sufficient negative potential with reference to the filament so that the electrons liberated from the filament were not able to reach the cylinder. This was done either by using a retarding potential always greater than the accelerating potential or by maintaining a constant difference of potential between the filament and outer cylinder (cylinder negative). A current through the galvanometer in the cylinder circuit may be due to positive ions reaching the cylinder or photo-electrons liberated from the cylinder or to a combination of these two effects. Such a current without ionization, indicated by a break in the total current from the filament, must be due to photoelectric action. With the metallic vapors resonance is accompanied by single line radiation. If this radiation is of sufficiently short wave-length to produce photoelectric action this method can be used as a check for either resonance or ionization in as much as ionization is always accompanied by recombination and therefore radiation of the complete spectrum.

Results.

Argon.—The argon used in these tests was purified by circulating the gas through an arc between calcium electrodes until no traces of other gases could be noticed in the spectrum of the argon. Tests were made at various pressures ranging from a fraction of a millimeter of mercury to several millimeters. A few of the curves obtained are shown in Fig. 2.

Curve I shows the variation of the current to the outside cylinder for a retarding potential of 2 volts between the net and cylinder. Curves *2*, *3* and *4* are similar curves for retarding potentials of 3, 4 and 6 volts respectively. The first decrease in these curves occurs at about 3 volts higher than the retarding potentials for the corresponding curve. Thus curve *1* starts to decrease at about 5 volts which is 3 volts higher than the retarding potential for this curve. This decrease is probably produced by a reflection or reëmission of electrons from the adsorbed argon on the surface of the cylinder. Curve *6* is a similar curve obtained when the apparatus was thoroughly evacuated after taking curves *1* to *4*, but before the adsorbed argon was removed from the cylinder. The retarding potential for this curve was 3 volts. After bombarding the cylinder in the high vacuum again the break in curve *6* at 6 volts was no longer obtained, indicating that reflection or reëmission took place from the adsorbed gas on the cylinder.

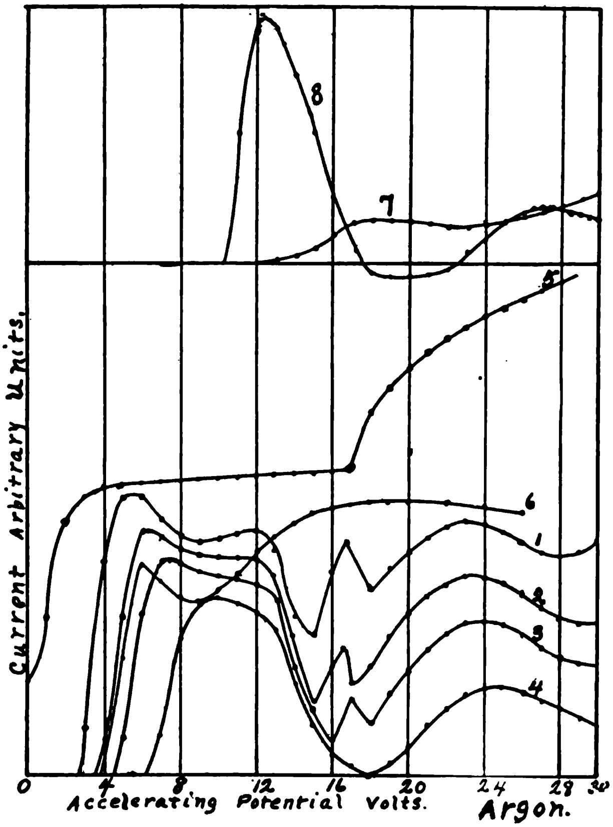

Fig. 2.

All the curves *1* to *4* show a sharp decrease in the cylinder current beginning between 12 and 13 volts accelerating potential. The total current curve *5* indicates ionization at about 17 volts. At this same potential there is a sharp break in the curves *1* to *3* as there should be since the electrons producing ionization collide inelastically. For curve *4* the retarding potential was too high for the current to begin increasing before this decrease at 17 volts occurs. The decrease between 12 and 13 volts indicates resonance. To determine the resonance potential more accurately the effect of reflection of the electrons from the outer cylinder must be eliminated. For curve *8* a retarding potential of 11 volts was therefore used. The inelastic impact due to resonance should show again at twice the resonance potential. Curves *1* to *4* show the effect but the disturbance due to reflected electrons from the cylinder influences the value at which the break in the curve occurs. That is, an electron may lose its energy in producing ionization at 17 volts and then fall through such a potential that upon striking the cylinder it is reflected at a net potential slightly lower than twice the resonance potential. For curve *1* this should occur for an electron velocity corresponding to about 3 volts when striking the cylinder, that is, for an accelerating potential of 17 plus 3 plus 2 (the retarding potential) or about 22 volts. For curves *2*, *3* and *4* the reflection effects should enter at about 23, 24 and 26 volts respectively. The second resonance potential for each of these curves should occur at twice the resonance potential. These two effects overlap and make it impossible to determine the second resonance points from curves *1* to *4*. For curve *8* this disturbance due to reflection or reëmission does not complicate the curve but the change in curvature is so gradual that it is difficult to estimate the second resonance point accurately. A number of curves similar to *5* and *8* were taken for retarding potentials as given below.

Regarding Potential.	First Resonance Point.	Ionizing Potential.
11	12.2	17.
10.6	12	17.2
11.5	12.5	17
10	12	17
10.5	12.5	16.8
11	12.6	17

The average value for the resonance potential is thus 12.3 volts and the ionizing potential 17 volts. The second peak for the six curves similar to 8 at about 26.5 volts all occur at a potential varying not more than half a volt from the average.

To check the results thus obtained method 2 described above was used. The outer cylinder was maintained negative by a few volts with reference to the negative end of the filament while the potential of the net was varied. The current through the galvanometer connected to the outer cylinder as a function of the net potential is represented by curve *7*.

A positive ordinate for curve *7* represents an electron stream away from the cylinder or positive ions to the cylinder. For the other curves the opposite direction is, taken. A curve showing the total current from the filament in terms of the accelerating potential was taken simultaneously with curve *7*. This curve is exactly similar to curve *5* showing ionization at 17 volts. The current of curve *7* starting between 12 and 13 volts is therefore clearly due to photoelectric action on the outer cylinder.

Discussion of Results for Argon.—The only previous determination of the ionizing potential of argon was made by Franck and Hertz.[1] The value given by them is 12 volts not correcting for the initial velocity of emission of the electrons. The present investigation indicates that they were probably dealing with the resonance potential and not the ionizing potential. The method used by them (Lenard's method) does not distinguish between the two. Recently Lyman[2] has identified lines as short as 833 A.U. with argon. From the quantum theory $h\nu = eV$ this would require an ionizing potential of about 15 volts to produce this line as the shortest line of the spectrum. In his experiments Lyman had to use long paths for his light to travel and no doubt absorption produced a decided disturbance and it is probable that there are lines in the argon spectrum shorter than he was able to detect with his apparatus. The shortest helium line he could detect has a wave-length of about 600 A.U. while Richardson[3] and Bazzoni by a photoelectric method find this limit to be between 420 and 470 A.U. The value of the ionizing potential determined by the present experiment indicates the shortest argon line to have a wave-length of about 750 A.U. calculated from the quantum relation $h\nu = eV$.

Neon.—The neon used for these tests was originally prepared by Claude. Last traces of helium were removed by fractionation from cocoanut charcoal cooled in liquid air. The purification was continued until no trace of any other gases could be detected in the spectrum. Different retarding potentials varying from 1 to 18 volts between the net and cylinder were used for pressures varying from a tenth of a millimeter of mercury to 5.9 millimeters. The best results were obtained for the higher pressures. In Fig. 3 a few of the curves obtained for neon are shown. The pressure in this case was 5.9 mm. Curve *1* represents the total electron current from the filament as a function of the accelerating potential between the filament and net. This curve indicates ionization at about 19.5 volts. Curve *2* represents the electron current which reaches the outer cylinder. This current begins for a net potential of about 15.5 volts and increases as the net potential increases until ionization sets in when the current reverses direction. This is due partly to positive ions reaching the cylinder and partly to photoelectric action on the cylinder. This reverse current diminishes continuously as the net potential is raised to about 33 volts. This may be brought about by the combination of several causes—first with increase of net potential the ionization takes place nearer the filament diminishing chances of positive ions reaching the cylinder. In the second place recombination and therefore radiation thus produced takes place further from the cylinder accompanied by greater absorption of the radiation before it reaches the cylinder and also by increased shielding of the cylinder by the net and hence less photoelectric action on the cylinder. Third, as the net potential is raised a value is reached when an electron once having suffered inelastic collision in producing ionization will again have acquired sufficient energy to

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[1] Franck and Hertz, Verh. d. Deutsch, Phys. Ges., 15, p. 34, 1913.

[2] Lyman, Ast. Phys. Jour., 43, p. 89, 1916.

[3] Richardson and Bozzoni, Phil. Mag., 6–34, p. 285, 1917.

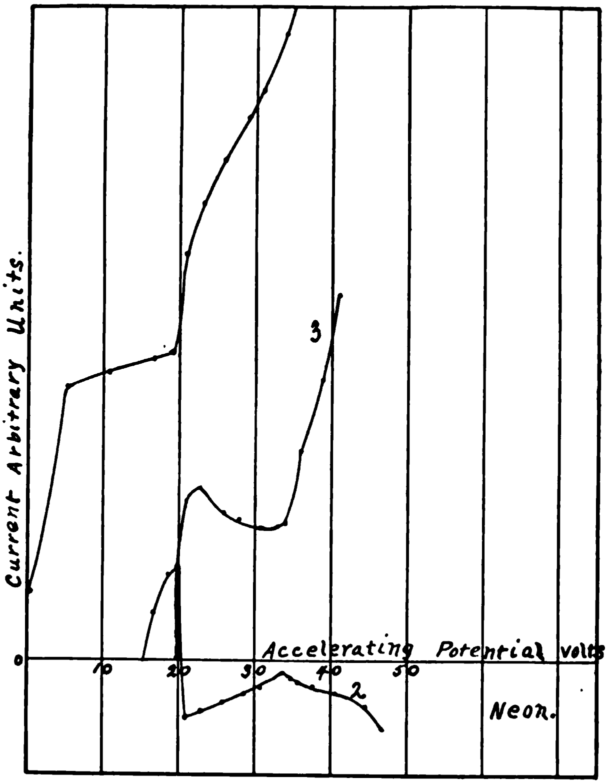

Fig. 3.

reach the cylinder thus diminishing the apparent reverse current. Curve *3* was obtained with the cylinder negative with reference to the filament (method *2*). At the potential of the net at which curve *1* indicates ionization curve *3* shows positive ions reaching the cylinder or photoelectrons leaving it or both. This current reaches a maximum and then diminishes to about 33-volt net potential. The cause for this decrease is probably the same as the first and second explanations given for curve *2*. At about 33 volts there is a break in curves *2* and *3*. This break is noticeable in curve *1* but not as definitely marked in *2* and *3*. These breaks indicate second ionization. Normally this second ionization should occur at twice the voltage at which the first occurs whereas in this case it is about 6 volts less than twice the value. Attempts were made to get a third and fourth break but when the net potential was raised to about 40 volts the cylinder current never reached a steady value for a definite net potential but continued increasing. While this drift of the galvanometer was taking place the filament appeared to be getting noticeably hotter probably due to bombardment of the filament by positive ions thus approaching something similar to arc conditions. This general effect is probably present to some extent as soon as the ionization potential is exceeded and may be a partial explanation for the second ionization point being less than twice the potential of the first.

The results obtained with neon show no indication of a resonance potential. The indications are that the ionizing potential is about 19.5 volts, not allowing for the initial speed with which the electrons leave the filament. The usual way of determining this speed is to take the difference between the potential at which first ionization occurs and that at which the same electron produces ionization twice as true ionizing potential and subtracting from this the potential for single ionization. As pointed out before, this is not practical in the present case.

The only other determination of the ionizing potential of neon is that made by Franck and Hertz using Lenard's method. The value given by them is 16 volts.

Helium.—The helium used was obtained from monazite sand and Thorianite. The gas was purified by passing it through a tube filled with cocoanut charcoal immersed in liquid air. The curves of Fig. 4 show the variation of the electron current from a filament to the concentric cylinder without a net as a function of the accelerating potential from the filament to the cylinder.

For curve *1* the cylinder was nickel. The ionization chamber was baked out but the cylinder was not specially treated by bombardment to remove occluded gases and impurities. After taking a number of curves similar to curve *1* the helium was pumped out and the cylinder heated by making the cylinder positive with reference to the hot filament and thus bombarding it by thermions until no more gas could be removed. After this treatment the curve obtained for the same pressure is represented by *2*. Curve *3* shows similar result obtained with a treated copper cylinder.

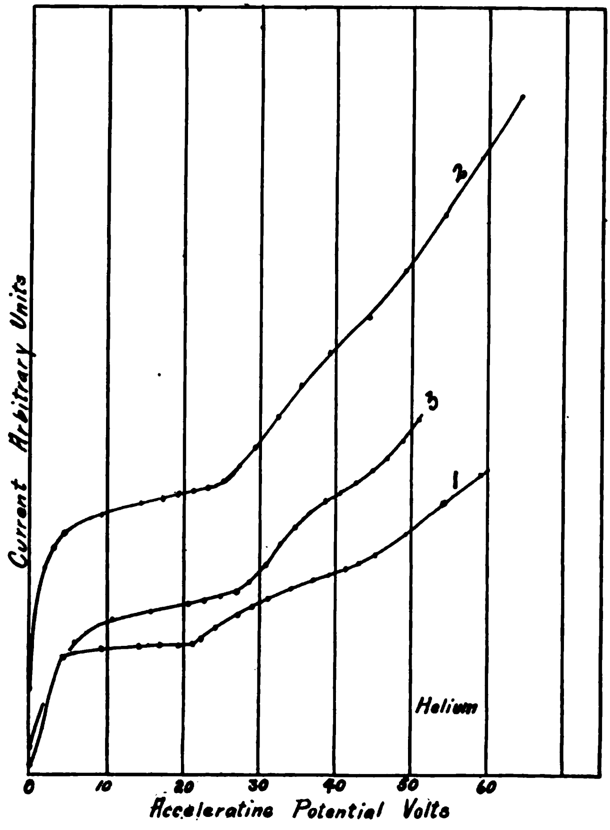

Fig. 4.

To determine whether helium has a resonance potential, the same methods were used as for argon and neon. Tests were made for pressures varying from a fraction of a millimeter to 1.6 centimeters of mercury and for retarding potentials between the net and cylinder varying from zero volts to 32 volts. In some cases an accelerating potential was even used. A few of the curves obtained are shown in Fig. 5.

Curve *1* shows the electron current to the outer cylinder for 1.5 volts retarding potential, curve *2* for zero volts retarding potential and curve *3* for an accelerating potential of 3 volts. The pressure in this case was

3 millimeters of mercury. The net and cylinder were both nickel. Every curve taken with a nickel net and a nickel cylinder shows a sharp increase in the electron current to the outer cylinder starting between 20 and 25 volts, rising to a maximum value and suddenly dropping the instant ionization as indicated by corresponding curve *4* starts. This increase is repeated again between 40 and 50 volts net potential. Curve *5* is a similar curve using a copper net and a copper cylinder for 2-volt retarding potential and a pressure of about 4 millimeters of mercury. The copper net had a mesh of about 60 to the inch as against 100 for the case of nickel. Otherwise the tests using copper elements were exactly similar to those with the nickel net and cylinder. With a copper net and a nickel cylinder curves similar to curve *5* were obtained. It is difficult to see how the difference in the mesh of the net could produce as much difference in the character of the curves as there is between curves *1* and *5*. Again the sharp increases in the current of curves *1*, *2*,

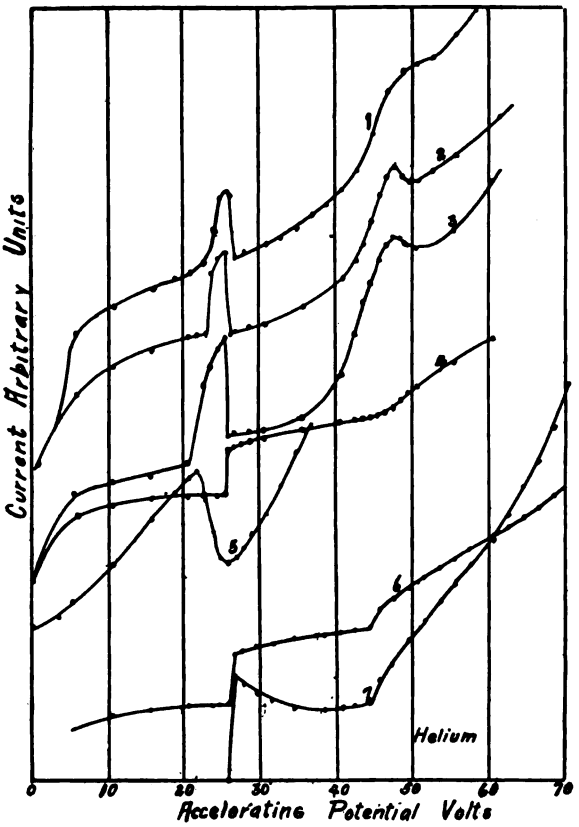

Fig. 5.

etc., starting around 21 volts are equally difficult to explain. Clearly the increase cannot be due to ionization in the space between the net and the outer cylinder starting before the ionization starts inside of the net. This might explain curve *3* but cannot explain *1* and *2* and a number of similar curves taken for retarding potentials up to 10 volts or more—for if ionization took place in this space positive ions would be driven to the outer cylinder and thus produce a smaller effective electron current to the cylinder instead of an increased current. Again it is hardly possible that the increase could be produced by reflection or reëmission of electrons from the net after a certain electron velocity is reached. This might explain curve *1* and similar curves taken with a retarding potential between the net and cylinder, but does not explain curves *2* and *3* because reflection should take place from the cylinder as well as from the net for the cases of *2* and *3*.

Dr. F. Horton[1] and Ann C. Davis in a recent note in Nature state that they find a resonance potential of 20.4 volts and an ionizing potential of 25.6 volts for helium. In order to explain these current increases on the basis of resonance accompanied by photoelectric action on the net and a consequent increase in the number of electrons to the outer cylinder, several assumptions are necessary. First the radiation due to resonance must be strongly absorbed before it reaches the outer cylinder, otherwise this photoelectric action on the cylinder would tend to neutralize the effect of the action on the net. In the second place the electron producing resonance will not have sufficient velocity left to reach the cylinder against a retarding field. The photoelectric action on the net must therefore liberate several electrons for each electron producing resonance in order to explain the increased current effect. That is to say an electron having a velocity corresponding to 20.4 volts must collide inelastically and produce resonance and consequently radiation. The original electron loses its energy and will not reach the cylinder against the retarding field. This radiation falling on the net must liberate several electrons of sufficient velocity to reach the cylinder against the retarding field in order to account for this sharp increase in current to outer cylinder. Considering that not nearly all the radiant energy can fall on the net and that the sharp increase in current is present even when using as much as 10 volts retarding potential this explanation seems out of the question.

Curve 4 represents the total current from the filament to the net and cylinder as a function of the accelerating potential, taken simultaneously with curve *1*. Curve *6* is another total current curve taken simultaneously with curve 7. Curve 7 represents an electron current away from the outer cylinder or a positive current to the cylinder when the cylinder was maintained negative by 10 volts with reference to the negative end of the filament. (Method 2.) This current is zero until ionization (shown by the sudden break in curve 6) starts. This ionization is accompanied by recombination and hence radiation so that some positive ions would be expected to reach the outer cylinder and photo-electrons leave it. For exactly the same reason as in the case of neon, this curve should reach a maximum, then decrease and show a sudden break again when the same electron produces ionization for the second time. This second break as in the case of neon occurs at less than twice the potential at which the first ionization occurs, and the same explanation applies to this as was given for neon. Attempts were made to get a third break in the different curves but as in the case of neon there was a gradual increase in current for a constant accelerating potential at the higher net voltages, which made results meaningless.

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[1] Horton and Ann C. Davis, Nature, Feb. 13, 1919.

The results with helium do not indicate a resonance potential. The sharp increases in the electron current to the outer cylinder are probably due to some unexplained action of the net.

In order to get the exact value of the ionizing potential for helium from these curves several corrections are necessary. First the initial velocity with which the electrons leave the filament must be known. As was pointed out before, this initial velocity cannot be determined in the usual manner and therefore cannot be corrected for. In the second place K. T. Compton[1] and Benade point out that the loss of energy of an electron when it strikes a molecule of helium is not entirely negligible. With the gas under the pressures used in this experiment the average number of collisions of an electron with different molecules before it reaches the net is large and the correction factor due to these collisions may be quite appreciable. This correction could be calculated using Compton and Benede's results if the effect of space charge, etc., upon the electric field between the filament and net are taken into consideration. No attempt was made to do this since other correction factors could not be definitely determined.

Taking the breaks in the various curves to determine the ionizing potentials the values for pressures varying from about 1 millimeter to 1.6 centimeters do not differ by more than 1.5 volts, from the average value of 26 volts obtained from some sixty curves.

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[1] K. T. Compton and Benade, PHYS. REV., 11, p. 184, 1918.

Conclusions.

The results of these tests indicate that argon has a resonance potential of about 12 volts and an ionizing potential of about 17 volts. Neon and helium do not show resonance. The ionizing potentials for neon and helium are higher than those obtained by other observers. The main purpose of these experiments was rather to test for resonance and not to determine the ionizing potentials with great accuracy.

Nutting[1] and Tugman point out that helium persistently gives the same spectrum and that no lines appear or disappear as the excitation is varied. The same appears to be true for neon while the character of the argon spectrum and of the metallic vapors is very materially changed by changing the method of excitation. While this is no direct proof for a resonance potential for argon and not for neon and helium it may perhaps be considered a further verification of the results of this experiment.

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[1] Nutting and Tugman, Bulletin Bureau of Standards, Vol. 7, No. 1, 1910.

ON THE X-RAY ABSORPTION FREQUENCIES CHARACTERISTIC OF THE CHEMICAL ELEMENTS.

By William Duane and Kang-Fuh-Hu.

Synopsis.

Object.—The object of this paper is to record in some detail the data presented to the American Physical Society at its meeting on April 27, 1918.[1]

Blake and Duane[2] had determined the critical absorption wave-lengths associated with the K series of x-rays for most of the chemical elements from bromine (atomic number 35) to cerium (atomic number 58) by measuring the currents in an ionization chamber attached to an x-ray spectrometer. The research reported in this paper extends the measurements to chemical elements of lower atomic number, as far as manganese (atomic number 25). The same apparatus was used as in the previous research, except that the x-ray tube employed had a long glass tube attached to it carrying a thin glass window at its end. This window lay close to the spectrometer slit, and the device materially reduced the absorption of the long x-rays by the air and glass.

Importance in Theory.—These critical absorption wave-lengths have considerable importance in connection with the theories of the structure of matter and the mechanism of radiation, for they represent frequencies of vibration that are the highest x-ray frequencies definitely known to be characteristic of the chemical elements.

Results.—From the experiments it appears that the square root of the critical absorpion frequency is not quite a linear function of the atomic number.

If we calculate the velocity v of an electron in the x-ray tube required to produce the radiation from the equation

$$\frac{1}{2}\frac{m_0v^2}{\sqrt{1-\frac{v^2}{c^2}}} = h\nu \qquad (1)$$

using for ν the critical absorption frequency, we find that v is a linear function of N, namely

$$v = v_0\,(N - 3/2),\ v_0 = .00678xc \qquad (2)$$

for all the chemical elements from manganese to cerium.

Further equation (2) gives the critical velocity for the chemical elements as far as magnesium, if we use data obtained from emission spectra.

It will be noticed that equation (1) contains the expression for the *transverse* mass of the electron.

In Moseley's classical experiments[3] on x-rays he showed that the square roots of the frequencies, ν, of corresponding lines in the characteristic emission spectra of the chemical elements are very nearly linear functions of the atomic number, N, of those elements. Recent researches have confirmed and extended these results. The wave-lengths of about fifteen lines in each of the emission spectra of a large number of chemical elements have been measured and tabulated. The graphs[1] representing $\sqrt{\nu}$ for corresponding lines in these spectra as functions of N are not quite straight. They bend slightly upward, the curvature becoming more marked as the frequency increases.

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[1] Phys. Rev., June, 1918, p. 488.

[2] Phys. Rev., Dec., 1917, 697.

[3] Phil. Mag., April, 1914, p. 703.

Attempts have been made to deduce empirical formulas for ν in terms of N, and some of these,[2] with four arbitrary constants, seem to fit the data very well.

The equations[3] in Bohr's theory of radiation contain the mass m of the electron. Since m varies with the electron's velocity the square roots of these expressions for the frequencies of rotation and vibration are not quite linear functions of N. The deviation from the straight line law is much the same as that which appears in the graphs representing experimental data.

It might be expected that the critical absorption frequencies associated with the emission series of x-rays would bear the most fundamental and, perhaps, the simplest relations to the atomic numbers; for they appear to be the most important frequencies characteristic of the chemical elements. Their importance rests upon the following facts. (*a*) A critical absorption frequency equals the critical ionization frequency associated with the same x-ray emission series.[4] The latter probably means the critical frequency for the characteristic emission of electrons with definite energy from the atom. (*b*) The difference between two critical absorption frequencies equals the frequency of one of the emission lines characteristic of the chemical element.[5] (*c*) A critical absorption frequency, substituted in the quantum equation, $Ve = h\nu$, gives the voltage V required to produce the emission series associated with it.[6] (*d*) A K critical absorption frequency is the highest x-ray frequency known to be characteristic of its chemical element. It lies very close to, but slightly above, the highest frequency in the corresponding emission series.[7]

De Broglie has made a series of interesting and important experiments, in which he measured the critical absorption wave-lengths characteristic of a large number of chemical elements. He used an x-ray spectrometer with a rock salt crystal, and determined the position of the beam of rays reflected from the crystal by means of a photographic plate.

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[1] Friman, Phil. Mag., Nov., 1916, p. 497.

[2] Uhler, Phys. Rev., April, 1917, p. 325.

[3] Phil. Mag., September, 1913, p. 476.

[4] Duane and Hu, Phys. Rev., June, 1918, p. 489; Dec., 1919.

[5] Duane and Shimizu, Phys. Rev., April, 1919, p. 306; July, 1919.

[6] Webster, Phys. Rev., June, 1916, p. 599; and Duane and Hu, *l.c.*

[7] Duane and Hu, *l.c.*

Owing to the difficulty of estimating by this method the effect due to the penetration of the x-ray into the reflecting crystal, and that due to the widths of the spectrometer's slits, Prof. F. C. Blake and one of us[1] decided to undertake a series of measurements using the ionization method of detecting the reflected beam. The data obtained in these experiments include the critical absorption wave-lengths associated with the K series of x-rays for all but two of the known chemical elements from bromine ($N = 35$) to cerium ($N = 58$).

The equation $\nu = \nu_0 (N - 3.5)^2$, in which ν_0 is the Rydberg fundamental frequency, namely 109,675 multiplied by the velocity of light, approximately represents the critical absorption frequencies ν corresponding to these wave-lengths. There appears to be, however, a small systematic variation from the law represented by the equation.

The object of the research reported in this paper has been to extend the measurements to chemical elements of lower atomic number than that of bromine, and we have succeeded in obtaining the critical absorption wave-lengths associated with the K series for all the chemical elements from bromine ($N = 35$) to manganese ($N = 25$).

The magnitudes of the frequencies belonging to chemical elements having small values of N give us the best determinations of the quantity k in formulas of the general form

$$\nu = \nu_0(N - k).$$

The x-rays in the K series of chemical elements of low atomic numbers are comparatively long; and, since the coefficient of absorption of x-rays varies approximately as the cube of the wave-length, the absorption of the rays of long wave-length by the glass walls of the x-ray tube itself becomes of great importance. In order to allow as much radiation of long wave-length to emerge from the tube as possible we designed, and had constructed an x-ray bulb with a long glass side tube attached to it. This tube extended out toward the x-ray spectrometer, and carried at its end a thin glass window, which lay close to the spectrometer's slit. The device markedly reduced the absorption of the x-rays by the glass and air.

Except for this side tube attached to the x-ray bulb the apparatus used in the experiments on chemical elements from manganese to bromine was exactly the same as that employed in the experiment on chemical elements from bromine to cerium. A detailed description of this apparatus may be found in the PHYSICAL REVIEW for December, 1917, on page 624.

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[1] PHYS. REV., December, 1917, p. 697.

The electric current exciting the x-ray tube came from a high-tension storage battery containing 20,000 cells, and the voltage across the tubes terminals was kept constant by slowly changing a water resistance in series with it. An electrostatic voltmeter, calibrated by means of a measured current from the storage battery flowing through a known metallic resistance of 890,000 ohms, served to measure the voltage. The current through the tube, amounting to several milliamperes was measured by a milliameter. Two slits in lead blocks placed between the x-ray tube and the reflecting crystal defined the beam of rays and the third slit, in front of the ionization chamber, was wide enough to allow all the reflected rays to pass. This arrangement eliminates a correction for the penetration of the rays into the crystal. This correction, however, is very small for x-rays of long wave-lengths.

The absorbing screen consisted of a thin layer of the chemical element, or of one of its salts, and lay between the x-ray tube and the first slit.

We made measurements on both sides of the zero line of the spectrometer, and obtained curves for each chemical element representing the ionization currents as functions of the readings of the verniers attached to the crystal table, similar to the curves shown on page 701 of the Physical Review for December, 1917. The sharp drops in these curves correspond to the critical absorption wave-lengths, the angular breadths of the drops representing the effects due to the widths of the slits.

To determine the grazing angles of incidence θ to substitute in the formula for the wave-length

$$\lambda = 2a \sin \theta = 6.056 \times \sin \theta \times 10^{-8} \text{ cm}.$$

we measured from the mid-points in the drops. The breadths of the drops amount to a few minutes of arc, and the grazing angles of incidence to several degrees. The positions of the mid-points can be estimated to within 1/10 per cent. of the values of the grazing angles.

The slopes of the curves above and below the drops depend upon the setting of the instrument, the thickness of the absorbing layer, the relation between the voltage applied to the x-ray tube and that required to produce x-rays as short as the critical rays, etc.

The following Table I. contains the critical absorption data for the K series of all the known chemical elements from manganese ($N = 25$) to cerium ($N = 58$), except the two gases xenon and krypton. The critical absorption wave-lengths appear in the third column, and the square roots of the corresponding vibration frequencies, in the fourth. The $\sqrt{\nu}$ cannot quite be represented by a linear function of N. The values of $\sqrt{\nu}$ differ from such a function by several per cent.

TABLE I.[1]

Chemical Element.	Automic Number.	$\lambda \times 10^8$.	$\sqrt{\nu} \times 10^{-9}$.	Ratio $\beta = v/c$.	$\beta_0 \times 1,000$.
Cerium	58	.3068	3.127	.3828	6.775
Lanthanum	57	.3188	3.068	.3761	6.778
Barium	56	.3307	3.012	.3698	6.785
Cæsium	55	.3444	2.952	.3629	6.783
Iodine	53	.3737	2.833	.3494	6.784
Tellurium	52	.3896	2.775	.3426	6.783
Antimony	51	.4065	2.717	.3358	6.784
Tin	50	.4242	2.659	.3291	6.786
Indium	49	.4434	2.601	.3223	6.786
Codnium	48	.4632	2.545	.3157	6.790
Silver	47	.4850	2.488	.3089	6.789
Palladium	46	.5075	2.431	.3023	6.794
Rhodium	45	.5330	2.373	.2953	6.790
Ruthenium	44	.5584	2.319	.2889	6.797
Molybdenum	42	.6180	2.203	.2751	6.794
Niobium	41	.6503	2.149	.2685	6.797
Zirconium	40	.6872	2.089	.2614	6.791
Yttrium	39	.7255	2.034	.2547	6.792
Strontium	38	.7696	1.974	.2475	6.783
Rubidium	37	.8143	1.919	.2408	6.784
Bromine	35	.9179	1.808	.2272	6.783
Selenium	34	.9790	1.751	.2202	6.775
Arsenic	33	1.0435	1.696	.2135	6.776
Germanium	32	1.1146	1.639	.2067	6.780
Gallium	31	1.1902	1.590	.2002	6.785
Zinc	30	1.2963	1.521	.1922	6.759
Copper	29	1.3785	1.475	.1863	6.774
Nickel	28	1.4890	1.420	.1793	6.766
Cobalt	27	1.6018	1.369	.1730	6.784
Iron	26	1.7396	1.313	.1661	6.780
Manganese	25	1.8892	1.260	.1595	6.787

It is interesting to inquire whether some other quantity connected with x-radiation may not be a linear function of N. If we calculate the velocity of the electron in the x-ray tube required to produce the K emission series from the quantum equation.

$$\tfrac{1}{2}mv^2 = h\nu, \tag{1}$$

assuming that m varies with the velocity according to the law

$$m = \frac{m_0}{\sqrt{1 - \beta^2}}, \qquad \beta = \frac{v}{c}, \tag{2}$$

where c is the velocity of light, we find that the values of v do not differ from those given by the linear equation

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[1] We wish to express our sincere thanks to Professor C. James and to Professor J. L. Howe for sending us some of the rare elements.

$$v = v_0(N - 3/2), \qquad v_0 = .006783c \tag{3}$$

by as much as 1/5 per cent.

The fifth column in the table contains the values of β, and the sixth, of $\beta_0 = v_0/c$. None of the values of β_0 differ from .006783 by as much as 1/5 per cent., except that for zinc.

We have shown by experiment (*l.c.*) that the critical absorption frequency of rhodium does not differ from the highest frequency in its K emission series by more than about 1/4 per cent. If we calculate the critical velocity v from equations (1) and (2) using for ν the highest frequencies given in Siegbahn's tables of emission lines in the K series, we find that equation (3) represents the velocities with considerable precision for all the chemical elements as far down as and including magnesium ($N = 12$).

The following table contains the data for these chemical elements:

Chemical Element.	Atomic Number.	$\lambda \times 10^8$.	$\sqrt{\nu} \times 10^{-9}$.	Ratio $\beta = v/c$.	$\beta_0 \times 1{,}000$.
Chromium	24	2.067	1.205	.1526	6.782
Titanium	22	2.490	1.097	.1391	6.785
Calcium	20	3.072	.9880	.1253	6.773
Potassium	19	3.446	.9437	.1185	6.771
Chlorine	17	4.391	.8265	.1050	6.774
Sulphur	16	5.014	.7734	.09829	6.779
Phosphorus	15	5.804	.7188	.09137	6.768
Silicon	14	6.755	.6663	.08475	6.778
Aluminum	13	7.982	.6131	.07797	6.778
Magnesium	12	9.471	.5627	.07158	6.817

In conclusion we wish to call attention to the fact that $\frac{1}{2}mv^2$ is not the *relativity* expression for the kinetic energy of the electron. The fact that equation (2) gives the *transverse* mass of the electron suggests that possibly v may represent the velocity of an electron travelling in an orbit. These points will be discussed in another paper.

ON THE X-RAY ABSORPTION FREQUENCIES CHARACTERISTIC OF THE CHEMICAL ELEMENTS.[1]

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SYNOPSIS.

Object.—The critical absorption wave-lengths associated with the K series of x-rays have been measured[2] for all but two of the known chemical elements from manganese ($N = 25$) to cerium ($N = 58$). The object of the experiments described in this paper was to extend the measurements to chemical elements of higher atomic numbers, using the same x-ray spectrometer.

Source of Current.—In the earlier experiments the electric current through the x-ray tube came from a storage battery of 20,000 cells, giving an electromotive force of about 43,000 volts when fully charged. A difference of potential of 43,000 can produce x-rays only a few per cent. shorter than the critical absorption wave-length associated with the K series of cerium, and to make the measurements for chemical elements of larger atomic numbers, having shorter critical absorption wave-lengths, requires a higher voltage applied to the x-ray tube. The authors, therefore, employed a transformer, with a set of condensers and kenotrons attached to it for producing approximately a constant difference of potential. Except for this generating plant the apparatus did not differ from that used in the earlier researches.

Results.—The K-critical absorption wave-lengths were measured for eight of the chemical elements from neodymium ($N = 60$) to lead ($N = 82$) both inclusive. The square roots of the vibration frequencies corresponding to their wave-lengths, when platted against the atomic numbers, indicate a still greater departure from the straight line law than do the values previously obtained for elements of lower atomic numbers.

As has been shown (*l.c.*), the equation

$$v = 0.00678\,(N - \tfrac{3}{2})c \qquad (1)$$

represents to within 1/5 per cent. the velocity of the electrons in the x-ray tube required to produce the K emission series of a chemical element of atomic number lower than $N = 58$, if the velocity is calculated by the quantum relation, using the expression for the transverse mass. The values of v calculated by the same formulas for elements of atomic numbers higher than 58 fall slightly below those given by equation (1). The greatest variations from the equation amount to a little over 1 per cent.

SEVERAL papers have been published[2] describing measurements of the critical absorption wave-lengths associated with the K series of x-rays characteristic of all but two of the known chemical elements from manganese (atomic number $N = 25$) to cerium ($N = 58$) both inclusive.

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[1] A paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

[2] Blake and Duane, PHYS. REV., Dec., 1917, p. 697; and Duane and Hu, PHYS. REV., June, 1918, p. 488.

In these experiments the electric current flowing through the x-ray tube came from a storage battery of 20,000 cells, giving, when fully charged, a difference of potential of about 43,000 volts. This difference of potential can not produce x-rays much shorter than the K critical absorption wave-length of cerium, and, as the critical wave-length decreases quite rapidly with increasing atomic numbers, it was found impossible to make good measurements for elements of higher atomic numbers than 58.

The authors therefore set up a generating plant similar to that designed by Dr. A. W. Hull. This consisted of a high tension transformer attached to a system of condensers and kenotrons for maintaining the voltage approximately constant. The plant gave us voltages up to about 115,000 volts and we were able to measure the critical absorption wave-lengths associated with the K series of eight of the chemical elements from neodymium ($N = 60$) to lead ($N = 82$).

Except for the generating plant the apparatus did not differ from that used in the earlier researches (*l. c.*), and the same precautions were taken to eliminate errors due to the penetration of the x-rays into the reflecting crystal (calcite), and to the width of the spectrometer's slits.

Curves were drawn for each chemical element similar to those published on page 701 of the Physical Review for December, 1917, and the angular distances between the mid-points in the sharp drops gave us the grazing angle of incidence, θ, from which to calculate the wave-lengths by the formula

$$\lambda = 6.056 \times \sin\theta \times 10^{-8} \text{ cm.}$$

The following table contains the data. The critical absorption wave-lengths appear in the third column, and the square roots of the corresponding vibration frequencies in the fourth.

The values of the critical absorption wave-lengths given by de Broglie[1] are several per cent. smaller than ours.

Chemical Element.	Atomic Numbers.	$\lambda \times 10^8$.	$\sqrt{\nu} \times 10^{-9}$.	Ratio $\beta = v/c$.	$\beta_0 \times 1{,}000$.
Neodymium......	60	.2861	3.237	.3952	6.756
Terbium.........	65	.2398	3.536	.4282	6.743
Dysprosium.......	66	.2308	3.604	.4356	6.753
Tungsten.........	74	.1786	4.097	.4877	6.726
Osmium..........	76	.1683	4.221	.5003	6.716
Gold.............	79	.1541	4.411	.5194	6.702
Mercury.........	80	.1501	4.469	.5252	6.690
Lead.............	82	.1424	4.588	.5369	6.669

c = the velocity of light.

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[1] Journal de Physique, May-June, 1916, p. 161.

On platting the square roots of the vibration frequencies against the atomic numbers it can be seen that they depart from the straight line law still further than do the values previously obtained for chemical elements of lower atomic numbers.

It has been shown (*l.c.*) that the velocity of the electrons in the x-ray tube required to produce the K emission series obeys the law represented by the equation

$$\beta = \frac{v}{c} = \beta_0(N - 3/2) = 0.006783(N - 3/2), \qquad (1)$$

for chemical elements up to cerium ($N = 58$), provided that we calculate v from the quantum equation.

$$\tfrac{1}{2}mv^2 = h\nu. \qquad (2)$$

In this equation (2) ν is the critical absorption frequency, and m, the transverse mass of the electron, is given by the equation

$$m = \frac{m_0}{\sqrt{(1 - \beta^2)}}. \qquad (3)$$

The fifth column in the table contains the values of $\beta = v/c$ calculated by equations (2) and (3), putting $h = 6.555 \times 10^{-27}$, $m_0 = 8.989 \times 10^{-28}$ and $c = 2.9986 \times 10^{10}$.

In the sixth column appear the values of β_0 calculated from the expression $\beta/(N - 3/2)$. The differences between these values of β_0 and that found previously for chemical elements of lower atomic numbers, namely, 0.006783, increase progressively with increasing atomic number, N, the greatest difference being a little more than 1 per cent.

As suggested in a former paper the fact that the *transverse* mass of the electron enters into the equations may mean that v is really the velocity of the electrons in an orbit. If so, the above variation in the value of β_0 may be due to the magnetic effect of the electrons in a ring on each other, and not to a lack of constancy in their angular momenta.

HARVARD UNIVERSITY.

THE RELATION BETWEEN THE INTENSITY OF GENERAL X-RADIATION AND THE ATOMIC NUMBER OF THE ANTICATHODE.[1]

By William Duane and Takeo Shimizu.

Synopsis.

Object.—The intensity of general x-radiation is known to be approximately proportional to the atomic weight of the element constituting the anticathode, provided the other conditions are the same. But since the atomic weights are, roughly speaking, proportional to the atomic numbers, it may be that the intensity is proportional to the latter instead of the former, considering the important part which the atomic number plays in the phenomena of characteristic x-radiation. The object of the present experiment was to decide this point.

Experiment.—We used the four successive elements iron, cobalt, nickel and copper because of the advantage that the atomic weights of cobalt and nickel are in the reversed order of their atomic numbers. An x-ray tube of special type, which enabled us to bring any one of the four elements in the focus of a Coolidge cathode without changing the geometrical relations between the focused point and the measuring apparatus, was excited by a large storage battery of available voltage up to 42,000. The intensity of the rays coming out of the tube, which were almost free from characteristic radiations, was measured by means of an ionization chamber and electroscope.

Result.—The intensity was found to be proportional to the atomic number and not to the atomic weight for the above four elements. This was particularly clear because the observed intensities for cobalt and nickel were in the reversed order of their atomic weights.

The total intensity of general x-radiation regarded as a function of the velocity of the exciting electrons and also as that of the chemical nature of the anticathode was first treated theoretically by J. J. Thomson[2] in 1907. His conclusion, based upon certain assumptions, was that the total intensity should be (I.) proportional to the fourth power of the velocity of the exciting electrons and (II.) independent of the chemical constitution of the anticathode. The first part of the conclusion was a mere anticipation at that time, but it was proved to be true by later experiments of other investigators, especially those of Beatty,[3] who showed that the above law was valid for the general x-radiation. The second part was, however, contradictory to the known fact that the heavy metals are more efficient radiators of x-rays than the lighter ones. Thomson attributed the cause of this discrepancy between his theory and the fact to the effect of secondary cathode rays.

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[1] A paper presented at a meeting of the American Physical Society, April, 27, 1918.

[2] J. J. Thomson, Electrical Origin of Radiation from Hot Bodies, Phil. Mag., 14, pp. 226–231, Aug., 1907.

[3] R. T. Beatty, Energy of Roentgen Rays, Roy. Soc., Proc., Ser. A. 89, pp. 314–327, Nov. 1, 1913.

The first systematic research on the relation between the intensity of general x-radiation and the substance of the anticathode was made by Kaye[1] in 1908. He showed that the intensity was nearly proportional to the atomic weight of the element for about twenty metals he used. This relation was also confirmed later by the experiment of Beatty (*l. c.*).

The work of Moseley on the x-ray spectra of chemical elements showed the great importance of the atomic number in the domain of nuclear phenomena, and the question arises naturally whether the intensity is proportional to the atomic weights of the elements, as shown by Kaye and others, or to their atomic numbers. Since the atomic numbers are nearly proportional to the atomic weights, the answer could not be given without making experiments specially designed for it. The object of the present investigation is to settle this question.

There are three sets of successive elements in the atomic table, the atomic weights and the atomic numbers of which are in the reversed order. For these elements the parallelism between the two quantities is broken, so that they may advantageously be employed in our present inquiry. One of those pairs is that of nickel and cobalt, the atomic weights of which are 58.68 and 58.97 and the atomic numbers 28 and 27 respectively. It is to be noticed that there existed a controversy about the atomic weights of these elements before the important rôle of atomic number was pointed out by Moseley. Thus Barkla and Sadler[2] proposed in 1907 that the atomic weight of nickel might be higher than that of cobalt, despite the fact that the reverse is considered to be established in chemistry. They based their argument upon the penetrating power of secondary x-rays radiated from different elements. At about the same time, J. J. Thomson,[3] observing the intensity of secondary rays from various elements exposed to x-rays, obtained the results which harmonized with the above view. On the other hand, Hacket[4] found that the intensities of secondary rays from those metals exposed to the β-rays of radium were in the same order as their atomic weights, and was opposed to Barkla and Sadler's proposition.

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[1] G. W. C. Kaye, Roentgen Rays: Emission and Transmission, Roy. Soc., Phil. Trans., Ser. A, 209, pp. 123–151, Nov. 19, 1908.

[2] C. G. Barkla and C. A. Sadler, Atomic Weight of Nickel, Phil. Mag., 14, pp. 408–422, Sept., 1907.

[3] J. J. Thomson, Secondary Roentgen Radiation, Cambridge Phil. Soc. Proc., 14, pp. 109–114, March 6, 1907.

[4] F. E. Hacket, Atomic Weight of Nickel, Nature, 75, p. 535, April 4, 1907.

In the present experiment we have measured the intensity of general x-radiation from the four successive elements iron, cobalt, nickel, and copper. In order that the x-rays from all four metals should pass through exactly the same thickness of glass and follow the same path in space, the x-ray bulb indicated in the figure has been used.

Four similar plates of quadrant shape, one of each of the four metals, were soldered to a circular block of copper *A*, which served as anticathode. The surface consisting of the different metals was carefully turned in a lathe, so that the entire surface made a good plane perpendicular to the axis of a circular copper rod *R*, which was rigidly attached to *A*. This rod *R* could rotate in two copper bearings *BB* fixed to the glass wall of the tube, but could not slip in the direction of its length. A small piece of soft iron *I*, fastened to *R* made it possible to rotate the anticathode from outside of the tube with an electromagnet.

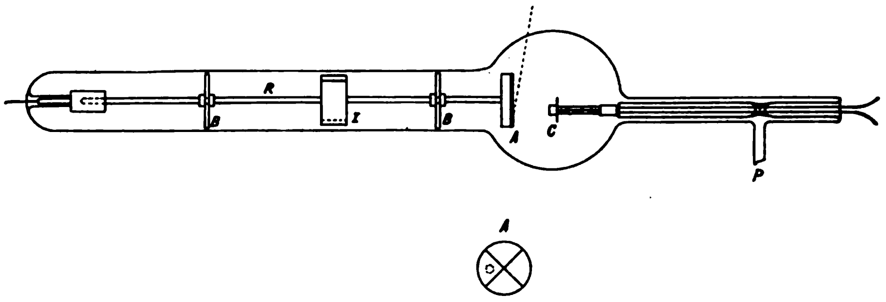

Fig. 1.

A Coolidge cathode *C* was fastened to the tube a little excentric with the axis of *R*, so that when *A* was turned around its axis to a suitable position, the cathode rays from it might strike *A* at the middle of any desired one of the four quadrants, as indicated in the lower figure by a dotted circle.

The side tube *P* was connected to a Langmuir diffusion pump and a pressure gauge.

As the source of high potential, we employed the 42,000-volt storage battery belonging to the laboratory. The potential across the tube was taken from a suitable number of its sections, and carefully kept constant during observation by adjusting a water rheostat connected in series with the tube. A sensitive static voltmeter made it easy to detect a variation of a few ten thousandths of the applied voltage. The current through the bulb was also kept constant by adjusting the heating current of the cathode filament.

The x-rays coming out of the bulb sustaining a small angle with the surface, as indicated by a dotted line in the figure were measured by means of an ionization chamber containing saturated vapour of methyl iodide and a Wilson tilted electroscope. Two lead slits placed between the bulb and the ionization chamber defined the solid angle of the measured x-rays. To secure a good constancy of the geometrical relations, the bulb was set up at a distance of about one meter from the nearer slit.

Our object being to compare the radiation from the four metals with one another, observations were taken for them in immediate succession at a constant voltage. But a repetition of one particular observation after an interval of about a month gave almost the same result as was obtained before, indicating that the geometrical relations between the different parts remained unchanged during that time.

A simple calculation showed that the characteristic rays from the metals were almost completely absorbed by the walls of the bulb, so that the observed intensity was due to the general radiation only for all of the metals. The table contains the observed result.

Volts.	Ionization Current (Volts per Sec.).[1]			
	Cu (29).	Ni (28).	Co (27).	Fe (26).
19,140	.0248	.0240	.0227	.0220
21,430	.0386	.0375	.0344	.0336
24,120	.0544	.0532	.0502	.0492
27,300	.0812	.0790	.0792	.0732
30,110	.1088	.1061	.1021	.0981
32,400	.1327	.1295	.1251	.1210
40,870	.238	.231	.223	.214

Volts.	Relative Intensity, Fe = 1.00 (Calculated from the Above).			
	Cu.	Ni.	Co.	Fe.
19,140	1.127	1.091	1.032	1.00
21,430	1.149	1.116	1.024	1.00
24,120	1.106	1.081	1.020	1.00
27,300	1.109	1.079	1.041	1.00
30,110	1.109	1.082	1.041	1.00
32,400	1.097	1.070	1.034	1.00
40,870	1.110	1.079	1.042	1.00
Mean.............	1.115	1.085	1.033	1.00
Ratio of atomic nos.	1.115	1.077	1.038	1.00
Ratio of atomic wts.	1.138	1.051	1.056	1.00

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[1] The current across the x-ray tube was kept at one milliampere throughout the experiment.

Thus we see that the ratios of the intensities of the general x-radiation produced at the same potential by the different metals are independent of the potential within the limits of errors, at least for the voltages used, and that the intensities are very nearly proportional to the atomic numbers of the elements. This is particularly evident in this case since the order of the atomic weights is not the same as that of the atomic numbers. The intensity follows the order of the atomic numbers and not that of atomic weights.

Harvard University.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE PHILADELPHIA MEETING, OCTOBER 10 AND 11, 1919.

THE ninety-ninth meeting of the American Physical Society was held in Philadelphia on October 10 and 11, 1919. On Friday, October 10, there were three sessions, arranged by the Committee on Technical Physics. These were joint meetings with the American Institute of Electrical Engineers and were held at the Hotel Bellevue-Stratford.

In the session of Friday morning, at which President Ames of the American Physical Society presided, the following program was presented:

A Discussion on the Present Status of Theories of Atomic Structure:

I. Introductory Remarks on Present Theories. SAUL DUSHMAN.

II. Special Applications of the Octet Theory. IRVING LANGMUIR.

III. Spectroscopic Evidence Regarding Atomic Structure. F. A. SAUNDERS.

IV. Bearings of High Pressure Phenomena on the Problem of the Constitution of Matter. P. W. BRIDGEMAN.

On Friday afternoon, under the auspices of the American Institute of Electrical Engineers, the following program was presented:

The Arrangement of Atoms in Metals. A. W. HULL. An X-ray study of crystal structures, a demonstration with models of crystals.

The Oscillating Vacuum Tube as a Generator of Electrical Power. J. H. MORECROFT AND H. T. FRIIS.

Electromagnetic Induction. S. J. BARNETT.

Piezo Electric Effect. A. M. NICHOLSON. (Demonstration.)

On Friday evening there was an informal dinner in the Clover Room of the hotel, followed by a joint meeting of the Institute and the Physical Society, at which two papers were presented as follows:

The Indispensability to each other of Pure and Applied Science. H. A. BUMSTEAD.

Pure Science and Industrial Research. J. J. CARTY.

On Saturday, October 11, 1919, there were two general sessions of the Physical Society held, by invitation, in the auditorium of the Leeds and Northrup Company, Vice-President Lyman presiding. In the intermission the members availed themselves of the invitation to inspect the laboratories of the Leeds and Northrup Company. A complimentary luncheon was provided by our hosts.

The program began with two papers presented by invitation of the Committee, as follows:

Research Development and Technical Control in a Small Industry. M. E. Leeds.

Some Research and Development Problems of the Leeds & Northrup Co. R. B. Smith.

The regular program consisted of eleven papers contributed by members in general, two of these being read by title, as follows:

A Precision Potentiometer. Arthur W. Gray.

On Establishing of the Absolute Temperature Scale. Frederick G. Keyes.

The Crystal Structure of Ferro-Magnetic Metals. Albert W. Hull.

Hydrogen Overvoltage. D. A. MacInnes, Leon Adler and A. W. Contieri.

Note on a Method of Measuring the Decrease of Electron in Thin Films by Means of Characteristic X-Rays. (Read by title.) Bargen Davis.

The Application of Motion Picture Photography to Ballistics and Airplane Performances. F. C. Brown.

Unipolar Induction. W. F. G. Swann.

Magnetic Resistance Effects in Films of Bismuth. F. K. Richtmyer and L. F. Curtiss.

The Spectral Photoelectric Sensitivity of Molybdenite as a Function of the Applied Voltage. (Read by title.) W. W. Coblentz and H. Kahler.

Ionization and Resonance Potentials for Electrons in Vapors of Lead and Cadmium. F. L. Mohler, Paul D. Foote and H. F. Stimson.

The Calculation of Audion Constants. R. W. King.

At the meeting of the Council held on October 11, 1919, the following elections were made: *elected to regular membership*, L. H. Adams, E. Ferrari, M. LaRosa, F. R. Moulton, S. Nakamura, O. Veblen; *elected to associate membership*, A. K. Aster, F. I. Bernhard, E. C. Brady, I. C. Cornog, L. F. Curtiss, R. C. Duncan, A. L. Ellis, J. L. Fearing, Miss M. K. Frehafer, J. C. Karcher, R. H. Kent, V. F. Lenzen, F. H. McBerty, W. C. Pomeroy, Miss J. A. Rodman, T. G. Seidell, A. F. Wagner, W. R. Williams; *transferred from associate to regular membership*, J. H. Morecroft, T. Takamine, E. D. Tillyer.

Dayton C. Miller, *Secretary*.

The Application of Motion-Picture Photography to Ballistics and Airplane Performances.[1]

By F. C. Brown.

This paper shows, with the aid of motion pictures and lantern slides, the results obtained by photographing the reflected image of the Handley-Page airplane simultaneously with the Mark IV bomb. These photographs were taken from the airplane with the motion-picture camera placed directly above the bomb.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

From an enlarged reproduction of the motion picture film, of magnification of six, was calculated the axis of the bomb relative to the trajectory tangent from the time it left the airplane until it struck the water 2,500 feet below. Also, the trail angle of the bomb back of the reflected image of the plane was calculated to an accuracy of about .03 degree. The results show very clearly the free period of the bomb throughout the trajectory, and also a close correlation between the lag and lead of the bomb and the position of the bomb on the trajectory. The so-called skidding effect is measured quite accurately.

The altitude of the airplane is calculated from the size of the reflected image on the film to a probable accuracy of about 15 feet, while the distance that the bomb has fallen is calculated from the diameter of the bomb image on the film to a probable accuracy of 100 feet.

The rolling and pitching of the airplane carrying the bomb at the time of release and when the explosive wave hits the plane is shown to an accuracy of a few hundredths of a degree. The free period of the plane during flight is less than .5 degree on either side of the mean flying position. When the explosive wave hits the airplane, there is a transverse displacement of the wings of nearly 1 degree, but the explosive wave, when the bomb explodes above water, does not affect the pitching of the airplane by as much as .1 degree.

Ordnance Department,
Bureau of Standards.

The Calculation of Audion Constants.[1]

By R. W. King.

GIVEN the characteristic equation of an audion as

$$I_p = \kappa \left(\frac{E_p}{\mu} + E_c \right)^{\eta},$$

it has been found possible to deduce expressions for the constants μ and κ from theoretical considerations which agree closely with experimental results. The calculation has been carried out for four classes of audions.

1. The structure with plane-parallel elements in which the filament is symmetrically placed between grids and plates;
2. The structure with cylindrical anode and grid and a coaxial strand of filament;
3. The structure with cylindrical anode and grid and several parallel strands of filament arranged on an internal coaxial cylinder;
4. The structure similar to Class 3 except that the strands of filament lie on an external coaxial cylinder. These calculations result in the following formulas.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

Class 1.—Given α the distance from plate to grid, β the distance from grid to filament, n the number of grid wires per unit length, r the radius of the grid wires, and A the total plate area,

$$\mu = \frac{2\pi r n}{\log_e \dfrac{1}{2\pi\alpha n}},$$

$$\kappa = 2.33 \times 10^{-6} \frac{A}{\sqrt{\alpha + \beta}} \left(\frac{\mu}{\alpha + (\mu + 1)} \right)^{3/2}.$$

In this and the following values for κ it has been assumed that the exponent η has the value 3/2. This assumption in general gives good numerical agreement.

Class 2.—Given R_p, R_g, R_f the radii of plate, grid and filament respectively, and l the length of the structure,

$$\mu = \frac{\left(2\pi R_g^2 \dfrac{1}{R_g} - \dfrac{1}{R_p} \right) n}{\log_e \dfrac{1}{2\pi r n}},$$

$$\kappa = 14.65 \times 10^{-6} \frac{l R^{1/2}{}_p \, \mu^{3/2}}{[(R_p - R_g) + R_g(\mu + 1)]^{3/2}}.$$

Classes 3 and 4.—The expression for μ is that given under Class 2. The expression for κ is

$$\kappa = 14.65 \times 10^{-6} \frac{R_p l \mu^{3/2}}{\sqrt{\pm(R_p - R_f)} \pm (R_p - R_g) \pm (R_g - R_f)(\mu + 1)^{3/2}}$$

where the signs are taken in such a way as to make all terms positive.

It is evident from the above expressions for μ and κ that these two quantities remain unchanged so long as the *relative* dimensions of any structure remain the same.

The accompanying table shows the numerical agreement obtained with some tubes of Class 1. All dimensions are in centimeters and currents in milliamperes.

TABLE.

α.	β.	n.	r.	A.	μ'.		E_p.	E_g.	I_p.	
					Calc'd.	Obs'd.			Calc'd.	Obs'd.
0.228	0.169	2.52	0.01	12.1	1.97	1.90	120	−10.7	70	64
.466	.169	3.45	.01	12.1	6.6	6.1	300	0	79	69
3.17	.317	6.29	.01	22.6	13.6	13.0	700	0	112	99

Research Laboratories of the American
Telephone and Telegraph Company
and the Western Electric
Company, Inc.,
New York City.

IONIZATION AND RESONANCE POTENTIALS FOR ELECTRONS IN VAPORS OF LEAD AND CALCIUM.[1]

BY F. L. MOHLER, PAUL D. FOOTE AND H. F. STIMSON.

MEASUREMENTS of electron currents in three electrode vacuum tubes of the type previously described have been made in vapors of lead and calcium.

The lead and calcium were boiled in porcelain tubes at temperatures of about 1000° and 900° respectively. Current voltage curves in lead showed a resonance potential of 1.26 volts and an ionization potential of 7.93 volts. Applying the quantum relation $Ve = h\nu$ we find that 1.26 volts corresponds within experimental error to the frequency of a strong infra-red spectrum line at $\lambda = 10{,}291$ Å. giving a theoretical value of the resonance potential 1.198 volts.

In calcium two resonance potentials were found at 1.90 volts and at 2.85 volts of which the first is the most prominent. Ionization occurred at 6.01 volts. The ionization potential corresponds to the limit of the principal series $1.5S\lambda = 2027$ Å. giving as the theoretical value $V = 6.081$ volts. The first resonance is determined by the line $1.5\,S - 2p_2$. $\lambda = 6572.78$ Å. $V = 1.877$ volts. The second resonance corresponds to the line $1.5S - 2P$. $\lambda = 4226.73$ Å. $V = 2.918$ volts.

The spectral relations of the first resonance potential and ionization potential are analogous to the relation found with other metals in this group. Work of other observers shows that both the lines $1.5S - 2P$ and $1.5S - 2p_2$ appear below the ionization potential in most metals of this group.

BUREAU OF STANDARDS
October 1, 1919.

THE SPECTRAL PHOTOELECTRIC SENSITIVITY ON MOLYBDENITE AS A FUNCTION OF THE APPLIED VOLTAGE.[1]

BY W. W. COBLENTZ AND H. KAHLER.

IN a previous investigation it was observed that the spectral photoelectric sensitivity of molybdenite is confined practically within three spectral bands, the maxima of which are separated by equal intervals, when plotted in terms of frequency instead of wave-lengths.

The present investigation was undertaken in order to test the validity of this frequency relation, using for the purpose a quartz prism which gave twice the dispersion formerly obtained with a fluosite prism. The sample of molybdenite was soldered to copper wire terminals, and operated in an evacuated chamber as in previous work. The photoelectric substance, the dry battery and the d'Arsonval gavanometer were joined in series. The deflection caused by the small dark current was overcome by rotating the suspension head of the galvanometer.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

The results obtained with this apparatus appear to verify the previous observations, indicating that the frequency maxima are separated by equal intervals, which decrease with temperature; the arbitrary wave-number being $n = 40$ at 25° C. and $n = 30$ at $-175°$ C.

A New Voltage Phenomenon.—A photo-negative action (resistance increase) has been observed in certain samples of selenium,[1] when exposed to the total radiation from an incandescent lamp. A similar spectral photo-negative response was observed in some samples of stibnite,[2] when exposed to radiations of wave-lengths less than $\lambda = .657\,\mu$. But heretofore no one appears to have observed that, for wave-lengths less than about $0.65\,\mu$, the nature of the photoelectric response depends upon the voltage applied to the substance under test.

We have found in some samples of molybdenite that, for the visible spectrum extending to about $\lambda = 0.647\,\mu$, the electric response is photo-positive or photo-negative, depending upon the voltage applied to the terminals of the receiver. For wave-lengths greater than $\lambda = 0.647\,\mu$, the photoelectric response was observed to be photo-positive whatever the applied voltage.

The region of transition in the spectrum, in which the action changes from photo-negative to photo-positive, is very narrow,—less than 9 Å. U.

The critical voltage is very small as may be inferred from the fact that an increase of 1.3 volts (additional dry battery) changed a positive-negative galvanometer deflective of ± 1 cm. into a purely negative deflection of -24 to -26 cm., which is the photo-negative response under discussion.

There seem to be two contending forces acting. The one which causes the photo-positive response acts quickly and prevails on low voltage. The photo-negative action builds up more slowly and is predominant on high voltages. As a result of these two forces, for certain applied voltages, on exposing the the molybdenite receiver to light of wave-lengths less than $0.647\,\mu$, the galvanometer deflection is first positive, then decreases in value (and may even become negative) when, on shutting off the light stimulus, there is a further deflection in the negative direction, after which the deflection returns to the original zero scale reading.

For example, using the wave-length $\lambda = .5876\,\mu$ as a light stimulus, and applying a potential of 10 volts, the galvanometer deflection was almost entirely positive. On 20 volts the deflection was partly positive and partly negative. On 29 volts the positive response was almost eliminated, and the negative response prevailed. On 33 volts the galvanometer deflection was entirely negative and eight times as large as the observed maximum purely positive deflection, for 10 volts. In another test, at a slightly lower temperature ($-104°$ C.) which raises the critical voltage, changing from 35.6 volts to 37 volts transformed the positive-negative deflection of ± 5 mm. into a negative deflection of -230 mm.

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[1] Ries, Phys. Zeit., 9, p. 569; 1908. Brown, Phys. Rev., 33, p. 1; 1911.

[2] Elliot, Phys. Rev., (2) 5, p. 62; 1915.

These tests were carried out at −100° C. to −178° C. It is of interest to note that the photo-positive action is the same as a resistance decrease caused by a rise in temperature of the material, while the photo-negative action is similar to the building up of a counter-electromotive force (the electrolytic action) previously observed in silver sulphide. An equal energy spectrum was used for the radiation stimulus and if this phenomenon were the result of heating and of electrolytic action, then the photo-positive response should occur in the short wave-lengths where the absorption is greatest, and the photo-negative response should occur in the long wave-lengths where the photoelectric activity is the greatest. This is just the reverse of what has been observed.

No explanation of this phenomenon is attempted at this time . Even if it is "only a gas effect," "electrolytic action" or "surface charge," it is unique in being selective as to the wave-length of the exciting radiation and in being photopositive or photo-negative, depending upon the applied voltage. It therefore requires further investigation.

WASHINGTON, D. C.,
October 2, 1919.

MAGNETO-RESISTANCE EFFECTS IN FILMS OF BISMUTH.[1]

BY F. K. RICHTMYER AND L. F. CURTISS.

THERE is a disagreement among previous investigators concerning the magneto-resistanee effects in films of bismuth obtained by cathodic sputtering. The present work has been undertaken as a preliminary attempt to obtain further information on this subject, which has an important bearing on the electron theory of metallic conduction. Films of various thickness were sputtered from chemically pure bismuth. It has been found that if especial care is taken to avoid heating these films during the sputtering, by allowing the process to go on only intermittently at short intervals, there is little or no change produced in the resistance of such films when placed in a transverse magnetic field of 16,000 gauss. Heating the films will restore this property. If the films are heated to a temperature near the melting point of the bismuth and allowed to cool and this process repeated several times, after each heating the film exhibits a greater change of resistance after each heating than before it, until a certain upper limit is reached, beyond which further heating, if the previous maximum temperature is not exceeded, has little effect. Likewise, heating to successively higher temperatures, beginning at a fairly low maximum and gradually approaching the melting point of the metal in each successive heating, restores the property under investigation to a greater and greater degree. Heating to temperatures under 150° has very little effect towards restoring this property, while heating to temperatures above this value has a correspondingly larger effect. If the film is strongly heated while being sputtered the initial increase of resistance in the magnetic field is comparatively large and further heating produces only slight increases in this initial value. The heating produces no observable change in the negative temperature coefficient of resistance of these films, which was carefully measured before and after each heating.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

UNIPOLAR INDUCTION.[1]

By W. F. G. Swann.

THE paper forms an enquiry as to the extent to which the so called "Moving Line Theory" of electromagnetic induction is the equivalent of the Maxwell-Lorentz Theory. The latter theory gives the force E on a fixed unit of charge in the form:

$$E = -\frac{l\partial U}{c\partial t} - \frac{\partial\psi}{\partial x}, \tag{1}$$

where U is the Maxwellian vector-potential, and ψ is the electro-static potential.

Consider the case of an infinitesimal amperian whirl of negative electricity on which is superposed a distribution of positive electricity which just cancels the electrostatic effect of the negative when the whirl is without translatory motion. As shown in the paper, the theory of relativity requires that, when the whirl is set into translatory motion perpendicular to its axis, there shall arise an electrostatic potential which is the equivalent of that of an electric doublet placed at the center of the whirl. The total electric field is thus composed of the field due to this doublet together with a part depending upon the time rate of change of the vector-potential; and, the inclusion of this electrostatic potential, which comes into existence by a rearrangement of the electric density in the whirl, makes the Maxwell-Lorentz theory agree with the " moving line theory " for this case.

Now in the case of a rotating magnet, the various elementary whirls partake of a translatory motion, an acceleration towards the center of rotation of the the magnet, and the axes of the whirls may also undergo a rotary motion. To the extent that we may assume the constitution of the whirls to be unaltered by the motion except to the extent indicated above, we may calculate the field which would be produced by a symmetrical insulating magnet of unit specific inductive capacity rotating about its axis of magnetization, by applying the " moving-line theory" to each of the elements which constitute the magnet. In doing this, however, only the translatory motion of the elements which constitute the magnet. In doing this, however, only the translatory motion of the elements must be considered. We must exclude any idea of movement of the magnetic lines of force as a result of rotary motion of the elements; for, neither on the basis of physical intuition, nor on the more exact basis of the Lorentzian equations are we justified in expecting any field as a result of mere rotation of the frame of the whirl about its own axis.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

The fundamental difference between the "moving-line theory" as it is ordinarily applied, and the theory in the form in which it would be applied lies in the fact that, in the former, the motion attributed to the magnetic field is the same as if the magnetic lines from the elementary whirls were pinned down at places where they pass through the axis of rotation of the magnet. In other words, the motion of the lines as is calculated as though they partook of the rotational velocity of the *frame* of the whirl about its own axis as well as of the translatory velocity of the whirl. As we have remarked, the former part should be omitted; and, when this is done, we have the "moving-line theory" in its proper form.

In a symmetrical rotating system, the vector-potential does not change with the time so that the second member of the right-hand side of (1) is the only term which contributes to E. The effect of this term is equivalent to a set of electric doublets with their axes radial. We must suppose one doublet for each molecular magnet, and the moments of the doublets are proportional to their distances from the axis of rotation in the case of a uniformly magnetized system.

By Poissen's mathematical theorem on polarized media, the field due to the doublets is equivalent to that of a certain fictitious distribution of electricity throughout the volume, and a distribution of electricity of opposite sign and of equal total amount over the surface.

In a conducting magnet we have two additional facts to consider. The force on a unit of electricity which is in motion with velocity V is given by:

$$F = E + \frac{[TH]}{C} \tag{2}$$

If for the moment we ignore the second member of this force, it will be necessary for the potential to be constant and the field zero throughout the magnet. This can only result by the appearance of real surface and volume distributions which, at each element, are equal and opposite in amount to the fictitious charge corresponding to the element. The result is then a complete cancellation of the fields due to these fictitious charges, not only within the magnet but at all points outside.

If we now introduce the second member of the force F into the consideration, we see that, owing to the motion of the substance of the magnet in its own magnetic field, an electromotive intensity will arise, and this will bring into existence an electrostatic distribution whose field, at all points within the magnet, just cancels the electromotive intensity which caused it. The electrostatic distribution arising in this way is then the only factor which ultimately contributes to the field outside the magnet. Its effect is of course always completely shielded off from any space from which the magnet is shielded by an earthed shield which does not participate in the rotation.

The paper concludes with an application of the above discussion to certain typical experiments, and an extension of the ideas to the case of an elementary whirl rotating as a whole about an axis in itself, other than the primary axis of the whirl.

DEPARTMENT OF PHYSICS, UNIVERSITY OF MINNESOTA.

NOTE ON A METHOD OF MEASURING THE DECREASE OF VELOCITIES IN THIN FILMS BY MEANS OF CHARACTERISTIC X-RAYS.[1]

By Bergen Davis.

It is desired to point out briefly that the phenomenon of characteristic X-rays furnishes a convenient and probably accurate means of measuring the decrease of electron velocities in penetrating metal surfaces.

A method of applying characteristic X-rays for this purpose was described in the Physical Review for June, 1918. The method here described is much simpler than the one suggested.

The method under consideration depends on the fact that there is a definite velocity of electron impact necessary to excite characteristic X-rays of any element.

If a thin film of the metal to be investigated be deposited on the surface of another metal giving characteristic X-radiation within the range of observation, the characteristic radiation will not be excited in the under metal unless the electrons have penetrated the film of known thickness above and reach the metal with the required velocity. This limiting velocity is already known. The velocity with which the electrons enter the film can be calculated from the voltage applied.

The velocity is thus determined on each side of a metallic film of known thickness.

A difficulty that may be encountered is a possible alteration in the thickness of the metal film both by the mechanical action and also by the slow evaporation of the metal as the target becomes heated.

The decrease of electron velocities in penetrating gold and aluminium foils has been measured by the direct magnetic deflection method by Whiddington. The decrease seemed to follow the law:

$$U_x^4 = V_0^4 - ax.$$

The constant a did not appear to depend in any simple way on the properties of the metals. It is very desirable that this constant should be determined for many metals as it enters largely into all theories of X-ray emission.

Department of Physics,
Columbia University.

HYDROGEN OVERVOLTAGE.[1]

By D. A. MacInnes, Leon Adler and A. W. Contieri.

Hydrogen overvoltage is due, primarily, to a layer of supersaturated dissolved hydrogen in the electrolyte surrounding an electrode. If, however, the electrode can hold nuclei of gaseous hydrogen on its surface the supersaturation will not rise to high values, since a portion of the hydrogen will then go into the gaseous phase. Metals with small adsorbtive powers hold small nuclei and have high overvoltages. This explanation of overvoltage is supported by experimental evidence as to the presence of gaseous nuclei, and by observations of fluctuations of overvoltage during the evolution of a single minute bubble. Further, the relation between the radius of the evolved bubbles γ, the surface tension γ, the pressure p and the overvoltage E has been shown experimentally to be, for platinum electrodes,

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

$$2FE = \frac{3RT}{pr\gamma}.$$

The work of Goodwin and Wilson 'unpublished' has shown that the change of overvoltage with pressure follows, nearly quantitatively, the relation given above, for electrodes of nickel, lead and mercury, *i.e.*, the overvoltage rises with a decrease of pressure.

A study has also been made of the effect of the change of gaseous pressure on several chemical and electrochemical processes involving the evolution of hydrogen. The changes in rates of reaction and in reaction efficiencies were found in each case to be in the directions which follow from the change of hydrogen overvoltage with pressure, *i.e.*, a decrease of pressure produces (*a*) a decrease of the rate of solution of metals in electrolytes, (*b*) an increase in the efficiency of reductions by metals, and (*c*) an increased efficiency of metal deposition.

The Crystal Structure of Ferro-Magnetic Metals.[1]

By Albert W. Hull.

It is well known that ferro-magnetism is not a specific property of the atom, that is, the atoms of ferro-magnetic materials are, in general, ferro-magnetic only when in pure metallic condition and between definite limits of temperature. The relative spacing of the atoms is apparently assential. It might have been anticipated, therefore, that ferro-magnetic metals would have the same crystal structure. That this is not the case is evident from the following table, which shows the type of lattice and distance between atoms of these metals as determined from powder photographs. (For method see Physical

Table.

Substance.	Type of Lattice.	Lattice Constant [Side of Elementary Cube or Triangle.]	Distance Between Nearest Atoms.
Chromium...	Centered Cubic	2.91	2.52
Iron........	Centered Cubic	2.86	2.47
Cobalt	Hexagonal "Close-packed"	2.52	2.52
	Face-Centered Cube	3.57	2.52
Nickel.......	Face-Centered Cube	3.54	2.50

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

Review, 10, 661–696, 1917. The full data will be published elsewhere in this journal.)

It will be observed that the ferro-magnetic metal, nickel, has the same crystal structure as copper, while cobalt appears in two forms—one like copper and one like magnesium. Neither is like iron. Chromium, on the other hand, which is not ferro-magnetic has a centered cubic arrangement like iron. Manganese has not yet been accessible in sufficiently pure form to allow of its determination.

It is evident, therefore, that ferro-magnetism does not depend upon any particular arrangement of atoms. It is much more likely that it depends upon the distances between the atoms, which would explain the fact that this property is lost when the temperature is increased beyond a definite value.

Research Laboratory,
General Electric Co.

A Precision Potentiometer.[1]

By Arthur W. Gray.

The potentiometer forming the subject of this communication has been designed especially for the accurate measurement of small electromotive forces, such as those produced by thermocouples; but its range can be extended by employing a volt-box in the usual way.

The instrument possesses the following advantages:

1. The galvanometer circuit is practically free from parasitic thermal electromotive forces.
2. An eliminating switch provides for easily correcting the galvanometer deflections for any parasitic electromotive forces that may happen to exist.
3. There are no sliding contacts in the galvanometer circuit.
4. The resistances of the sliding contacts in the battery circuits are negligible.
5. The operation of balancing the electromotive force to be measured does not change the resistances in series with either the galvanometer or the batteries; consequently, it does not change the galvanometer sensitivity.
6. A three-point switch provides a means of reducing the galvanometer sensitivity to 0.1 or 0.01 of the greatest sensitivity used.
7. The galvanometer is critically damped for each sensitivity when balancing either the standard cell or the electromotive force to be measured, and also when the eliminating switch is closed.
8. The range of the instrument can be multiplied by 10 by shifting switches operable by a single handle.
9. An electromotive force can be balanced as closely as with a slide-wire potentiometer without introducing any of the disadvantages of a slide wire.
10. The potentiometer can be used either as a null-method instrument or as a deflection instrument.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

11. While the manipulation is as convenient as that of a potentiometer in which all the essential parts are contained in one box, the instrument is made up of parts which can readily be used separately for other purposes.

12. Its cost is low in comparison with that of other potentiometers yielding the same accuracy.

For some time past there has been in regular use a potentiometer of this type which measures any E.M.F. up to 10,000 microvolts with a precision of one microvolt. A galvanometer deflection of 5 millimeters indicates 1, 10, or 100 microvolts, depending upon the position of a switch. The operating current is furnished by dry cells.

PHYSICAL RESEARCH LABORATORY,
THE L. D. CAULK COMPANY,
MILFORD, DEL.,
Sept. 19, 1919.

ON ESTABLISHING OF THE ABSOLUTE TEMPERATURE SCALE.[1]

BY FREDERICK G. KEYES.

BY employing the equation of state whose physical basis was set forth by the writer in the Proceedings of the National Academy for 1916, corrections have been made for the constant pressure nitrogen thermometer. The hydrogen and helium equations have in addition been employed to compute the position of the ice point on the absolute scale. The scale corrections are practically the same as those arrived at by Buckingham by the use of the available Joule-Thomson coefficients. The ice point is found, employing the recent Dayand Clement Pt-Ir bulb constant volume nitrogen expansion coefficients, to be about 273.14.

The form of equation for one type molecule systems is a linear function of the temperature at constant volume and accordingly the constant volume thermometer readings would lie on the absolute scale. The recent work of Eumorfopoulos on the boiling point of sulphur employing a quartz bulb with the Callendar constant pressure thermometer is free from the disturbing effect inherent to the porcelain and glass bulb previously employed. The values obtained by Eumorfopoulos, corrected to the absolute scale, lead to a mean value for the S.B.P. of 444.54 whereas the mean obtained with the constant volume nitrogen thermometer is 444.52.

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[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919. Contribution from the Research Laboratory of Physical Chemistry of the Massachusetts Institute of Technology.

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